

Collapse and final explosions

Stars with initial mass of less than $\sim 9M_{\odot}$ (this limit depends strongly on mass loss) develop degenerate cores and if shell sources cannot increase M_c to $\sim M_{ch}$ the star becomes a WD.

Other stars undergo core collapse (such as those with neutron stars as remnants) or explosions, thereby ejecting a large part of their mass (supernova).

Evolution of the C-O core

- Further evolution details depend on whether C-O core becomes **degenerate or not** in the ensuing contraction phase.
- Estimating critical core mass M_{crit} , which determines whether contraction will increase T_c or if the core becomes degenerate:
- consider (approximate) EOS **interpolating between both (non-deg. – degen.) regimes**:

$$P \approx P_e = \frac{\mathfrak{R}}{\mu_e} \rho T + K_{\gamma} \left(\frac{\rho}{\mu_e} \right)^{\gamma}$$

$\rho \gg 10^6 \text{gcm}^{-3}$ $\rho \ll 10^6 \text{gcm}^{-3}$
 $(\mu_e \approx 2, \mu_0 \geq 12)$ rel. \rightarrow non-rel. $4/3 \leq \gamma \leq 5/3$
 $10^{15} - 10^{13}$

- rough estimate of central ρ_0 values from hydrostatic equation

$$\bar{\rho} = 3M_c / (4\pi R_c^3) \sim \rho_0$$

$$P_0 \approx \frac{GM_c \bar{\rho}}{R_c} = f GM_c^{2/3} \rho_0^{4/3}$$

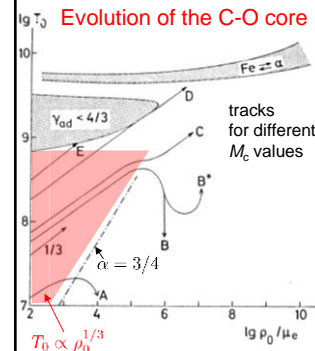
$f(\bar{\rho}/\rho_0) = \text{constant}$

use EOS for P_0

$$\frac{\mathfrak{R}}{\mu_e} T_0 = f GM_c^{2/3} \rho_0^{1/3} - K_{\gamma} \rho_0^{\gamma-1} \mu_e^{-\gamma}$$

dominates for non-degeneracy
both terms are about equal for high-degeneracy

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$$\frac{\mathfrak{R}}{\mu_e} T_0 = f GM_c^{2/3} \rho_0^{1/3} - K_{\gamma} \rho_0^{\gamma-1} \mu_e^{-\gamma}$$

describes $T_0(\rho_0)$ for given M_c .

- non-degenerate region: $T_0 \propto \rho_0^{1/3}$; $T_0 \propto M_c^{2/3}$
- for low ρ_0 and small M_c the temperature T_0 increases up to a maximum value T_{0max} , after which it decreases until $T_0 \rightarrow 0$ (A, B; M_3, M_2 in Ch. 28).

- with ρ_0 **relativistic degeneracy** becomes important; we rewrite EOS as ($\gamma = 4/3 + \chi$ with $\chi \rightarrow 0$ for $\rho/\mu_e > 10^7 \text{g cm}^{-3}$):

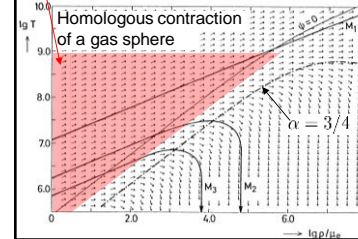
$$\frac{\mathfrak{R}}{\mu_e} T_0 = \rho_0^{1/3} \left(f GM_c^{2/3} - K_{(4/3+\chi)} \mu_e^{-(4/3+\chi)} \rho_0^{\chi} \right)$$

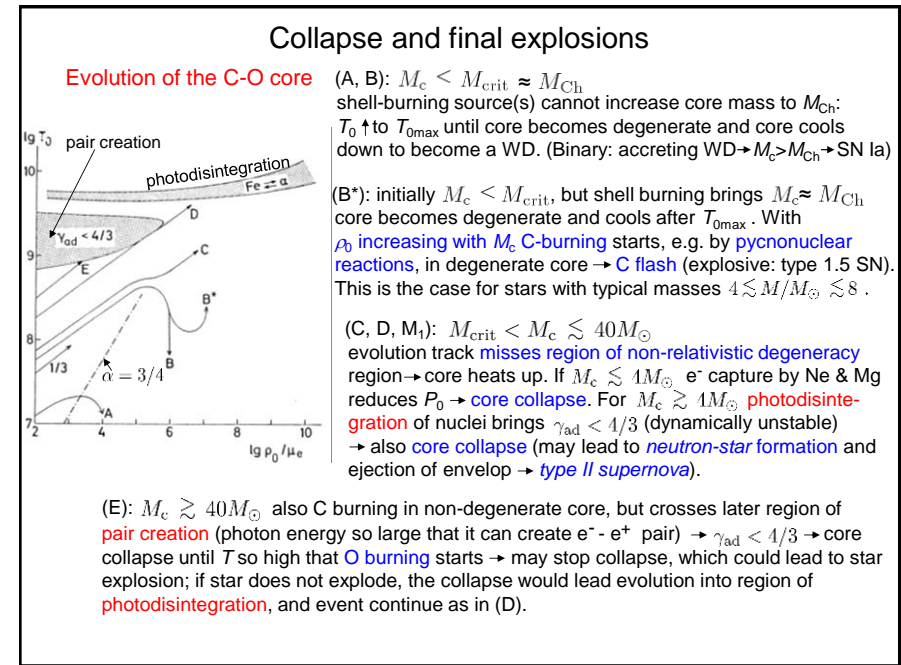
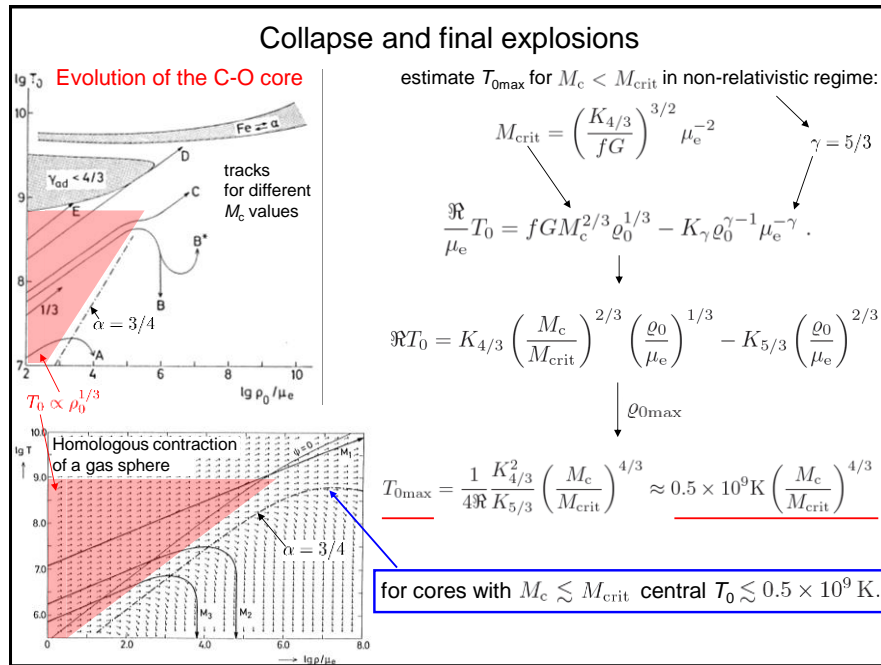
this shows that T_0 increases again with $\rho_0^{1/3}$ for

$$M_c > M_{crit} = \left(\frac{K_{4/3}}{fG} \right)^{3/2} \mu_e^{-2} \approx M_{Ch}$$

if $M_c \lesssim M_{Ch}$: $T_0 \rightarrow T_{0max}$ & decreases afterwards

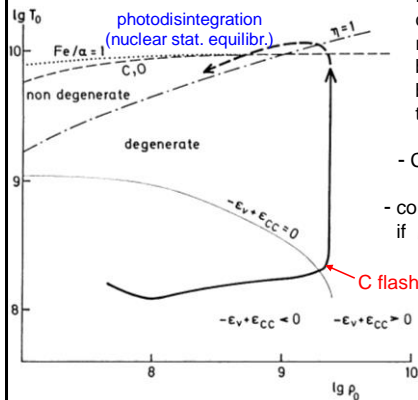
if $M_c \gtrsim M_{Ch}$: T_0 continuously increases with $\rho_0^{1/3}$.





Collapse and final explosions

The carbon flash (B*)



- For $M \geq 8M_{\odot}$ shell-burning source increased core mass nearly to M_{Ch} , thereby also increasing ρ_0 ; released energy of core contraction is transported by e^- conduction towards centre, where $T_0 < T_{max}$ because of neutrino losses, which carry outwards the energy. Once ρ is high enough C burning ignites.

- C ignition either in centre or shell of T_{max}

- core stability crucially depends on neutrino losses ϵ_{ν} ; if $\epsilon_c \gtrsim \epsilon_{\nu} \rightarrow$ core becomes unstable \rightarrow (violent) C flash

C flash [$\rho_0 \sim 2 \times 10^9 \text{ g cm}^{-3}$]

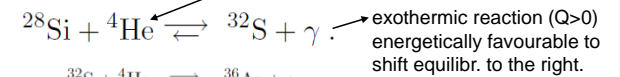
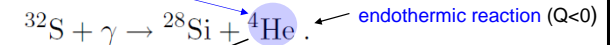
How violent is C flash:

for a mixture of equal C & O, C burning releases $\sim 2.5 \times 10^{17} \text{ erg/g}$ and O twice as much. If all this energy is used to heat the core, it can reach temperatures as indicated by the dashed curve, labelled "C,O"; at this high temperatures, $T \sim 10^{10} \text{ K}$ we have photodisintegration.

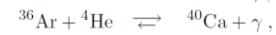
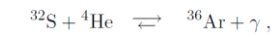
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Photodisintegration and nuclear statistical equilibrium

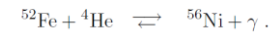
@ T about 10^9 - 10^{10} K photons γ so energetic (MeV) as to cause photodisintegration (photodissociation) in the nuclei in the gas (α decay), such as, for example,



Similar reactions:



\vdots



Processes occur essentially in equilibrium (EQ) with similar numbers of nuclei being dissociated and created; EQ is eventually shifted to heavier nuclei ($> E_b$):

quasi-equilibrium processes (misleadingly called *silicon burning*).

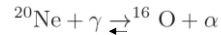
Similar reaction(s): ${}^{20}\text{Ne} + \gamma \rightarrow {}^{16}\text{O} + \alpha$,
(neon burning)



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Nuclear statistical equilibrium (in a plasma of photodisintegration)

@ T about 10^{10}K photons γ so energetic as to break up nuclei by α decay, e.g. :



Processes of **disintegration** (dissociation) and **inverse reaction** similar to ionization and recombination of atoms: **statistical equilibrium** can be describes via **SAHA equation**, ie

$$\frac{n_{\text{O}}n_{\alpha}}{n_{\text{Ne}}} = \frac{1}{h^3} \left(\frac{2\pi m_{\text{O}}m_{\alpha}kT}{m_{\text{Ne}}} \right)^{3/2} \frac{G_{\text{O}}G_{\alpha}}{G_{\text{Ne}}} e^{-Q/kT},$$

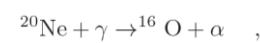
where G_{O}, G_{α} and G_{Ne} are the statistical weights, while Q is the difference of binding energies

$$Q = (m_{\text{O}} + m_{\alpha} - m_{\text{Ne}})c^2.$$

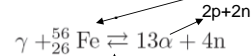
Collapse and final explosions

Nuclear statistical equilibrium (in a plasma of photodisintegration)

@ T about 10^{10}K photons γ so energetic as to break up nuclei by α decay, e.g. :

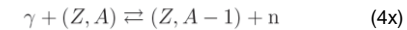
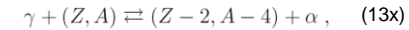


${}^{56}\text{Fe}$ plays important role, because it is the nucleus with highest binding energy.



↑ behave like ionization and recombination

→ Nuclear statistical equilibrium defines ratio n_{Fe}/n_{α}



(13+4) Saha-like equations combined

$$(1) \frac{n_{\alpha}^{13}n_n^4}{n_{\text{Fe}}} = \frac{G_{\alpha}^{13}G_n^4}{G_{\text{Fe}}} \left(\frac{2\pi kT}{h^2} \right)^{24} \left(\frac{m_{\alpha}^{13}m_n^4}{m_{\text{Fe}}} \right)^{3/2} e^{-Q/kT} \quad \text{with} \quad Q = (13m_{\alpha} + 4m_n - m_{\text{Fe}})c^2$$

also

$$(2) \rho = m_{\text{u}} \sum_i A_i n_i = (56n_{\text{Fe}} + 4n_{\alpha} + n_n)m_{\text{u}}$$

Note: for ${}^{56}_{26}\text{Fe}$: $n_{\text{p}}/n_{\text{n}} = 13/15$
 $n_n/n_{\alpha} = 4/13$

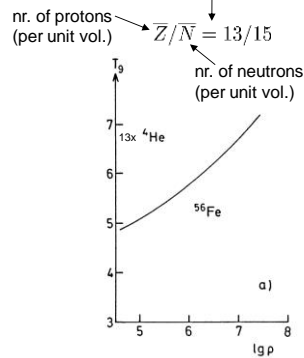
→ for given ratio, e.g. $n_n/n_{\alpha} = 4/13$ (${}^{56}\text{Fe}$), 2 equ. (1)+(2) for 2 unknowns n_{Fe} and n_{α} .

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Nuclear statistical equilibrium (in a plasma of photodisintegration)

@ T about 10^{10} K photons γ so energetic as to create $e^- - e^+$ pairs = photodisintegration, e.g.

for given ratio, e.g. $n_n/n_\alpha = 4/13$ (^{56}Fe), and given ρ & T , **nuclear equilibrium demands:**



In nuclear statistical equilibrium at moderate T one expects nuclei of the iron group, which with increasing T disintegrate into α particles, protons and neutrons.

Collapse and final explosions

Hydrostatic and convective adjustment during C flash

- during He flash star stays very close in hydrostatic equilibrium: convection removes energy.
- for **C flash** the situation is very different: in a single thermal run away, after the C flash (ε_{CC}), $T \uparrow$ so fast that additional reactions (O burning) take place; core regions is then so hot that **statistical equilibrium between Fe & He is reached** \rightarrow degeneracy is removed, pressure increases \rightarrow **central regions expand**.

- **time scale** τ_e determined by change of T & internal energy ($\dot{T}/T \simeq \varepsilon_{CC}/u$): $\tau_e \simeq \frac{c_p T}{\varepsilon_{CC}}$
- other (outer) core regions react on the **hydrostatic time scale**: $\tau_{\text{hyd}} \simeq (G\bar{\rho})^{-1/2}$

- if $\zeta := \tau_e/\tau_{\text{hyd}} \gg 1 \rightarrow$ core follows expansion quasi-hydrostatically
- $\zeta \ll 1 \rightarrow$ outer layers can not expand rapidly enough, **compression wave will move outwards with c_s** (outwards travelling shock wave)

convection in core will (a) transport part of surplus energy to outer layers, (b) bring new fuel into burning layers:

- if $\xi := \tau_e/\tau_{\text{conv}} \gg 1 \rightarrow$ convection carries all energy away from core

$$\tau_{\text{conv}} \simeq \ell_m/v_c$$

in core $\rho > 10^8 \text{ g cm}^{-3}$, $T \simeq 3 \times 10^9 \text{ K}$ one typically finds $\tau_e \simeq 10^{-6} \text{ s}$, $\tau_{\text{hyd}} \simeq 0.1 \text{ s}$ & $\tau_{\text{conv}} \simeq 0.1 \text{ s}$.

$\rightarrow \zeta \ll 1$, & $\xi \ll 1$ compression wave will start outwards and "convective blocking" (strong damping if motion is $v_c \simeq c_s$) limits spread of energy released in the core.

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Combustion fronts (during C flash)

$\tau_e \simeq 10^{-6} \text{ s}$ at the onset is rather short \rightarrow burning proceeds at such high rates that **fuel** in mass shell is **consumed** essentially **instantly** & layer above has not time to adjust.

layer ahead is heated to ignition either by compression or energy transport, and flash proceeds outwards (burning confined to very thin layer) \rightarrow **outward moving combustion front**.

Two different types of **combustion front**:

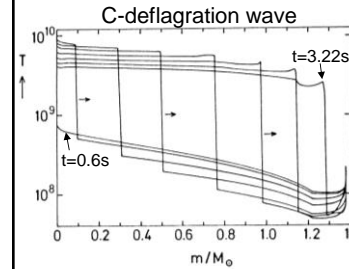
- (a) matter in front enters discontinuity of **compression wave (shock wave)** with supersonic velocity and is compressed & heated; if matter is ignited combustion front coincides with shock front \rightarrow **detonation front**
- (b) if compression in shock wave does not ignite the fuel, then ignition temperature is reached due to energy transport (convection or conduction) \rightarrow slower, subsonic motion of the burning front with a discontinuity in pressure and density drop (inwards!) \rightarrow **deflagration front**
 - \rightarrow speed it controlled by energy transport (convection or conduction).

in both cases deviation from hydrostatic equilibrium mainly confined in thin shell of P, ρ discontinuity, and energy release.

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Combustion fronts (during C flash): numerical results

Numerical results suggest deflagration than detonation front.



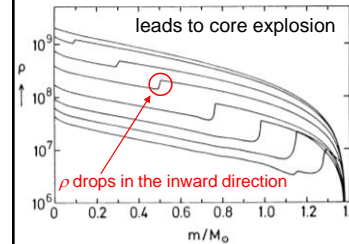
@ $\rho = 2 \times 10^8 \text{ g cm}^{-3}$, relativistic degeneracy gives:

$$P_e \simeq 1.24 \times 10^{27} \text{ dyn cm}^{-2} \rightarrow u_e = 3P_e/\rho \simeq 2 \times 10^{18} \text{ erg s}^{-1}$$

$$\text{C-burning adds only } \sim 27\%, \text{ i.e. } \simeq 5 \times 10^{17} \text{ erg s}^{-1}$$

\rightarrow overpressure rather moderate and shock is not extremely strong.

one problem in modelling is treatment of time-dependent convection.



although deflagration front moves subsonically, it can destroy the (**relativistic** $\gamma_{\text{ad}} = 4/3$) **core**:

$$\text{total Energy } W \simeq 0 \rightarrow |E_g| \simeq \frac{3GM_c^2}{2R_c} \simeq 9 \times 10^{49} \text{ erg.}$$

$$\text{energy release from C-burning (1/2 } M_c): \simeq 3.5 \times 10^{50} \text{ erg.}$$

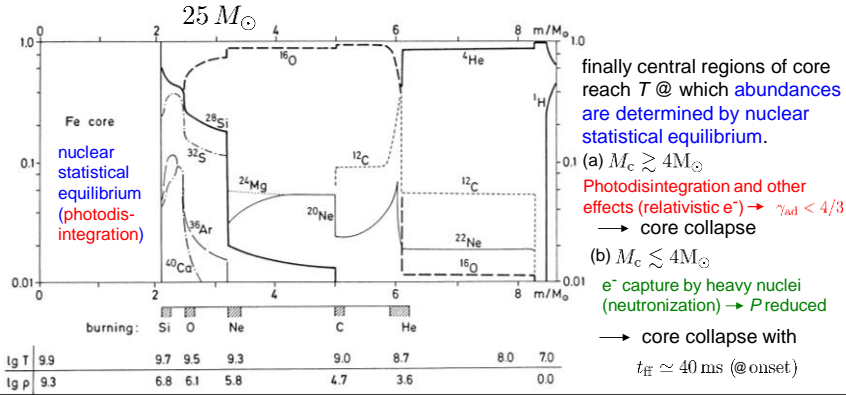
\rightarrow releases enough E to disrupt whole core.

Collapse and final explosions

Collapse of cores of massive stars (C,D): $M_{Ch} < M_c \lesssim 40M_\odot$

core 'misses' non-relativistic degeneracy and heats up during contraction until ignition of next heavier element. Core is then either non-degenerate (large M_c) or degenerate, but still in a region "above" $\alpha=4/3$, i.e. gravothermal heat capacity $c^* < 0 \rightarrow$ burning is stable!

after several cycles of nuclear burning and contraction, core will heat up to Si burning; burning in several shell sources produces layers of different chemical elements \rightarrow onion-shell model.



finally central regions of core reach T @ which abundances are determined by nuclear statistical equilibrium.

(a) $M_c \gtrsim 4M_\odot$
 Photodisintegration and other effects (relativistic e^-) $\rightarrow \gamma_{rad} < 4/3$
 \rightarrow core collapse

(b) $M_c \lesssim 4M_\odot$
 e^- capture by heavy nuclei (neutronization) $\rightarrow P$ produced
 \rightarrow core collapse with $t_{ff} \simeq 40$ ms (@onset)

Collapse and final explosions

Collapse of cores of massive stars (C,D): $M_{Ch} < M_c \lesssim 40M_\odot$

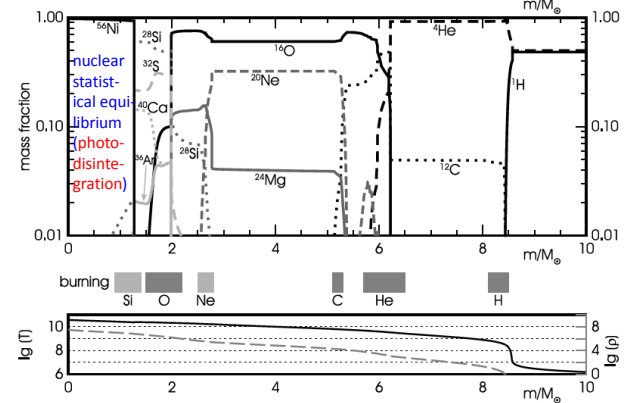
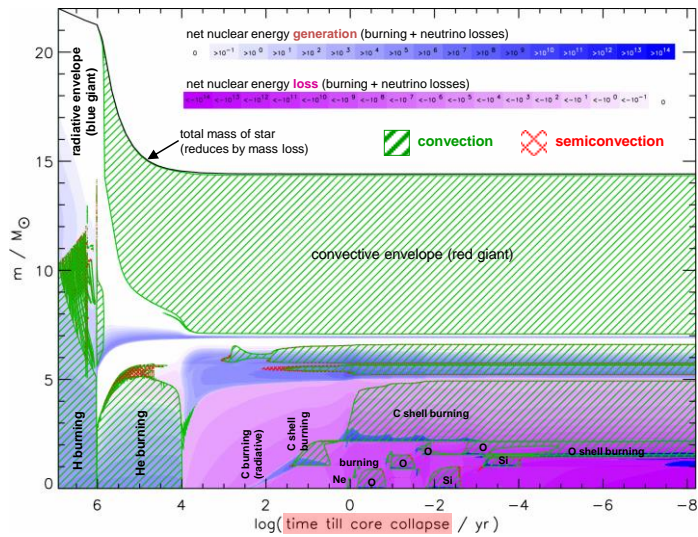


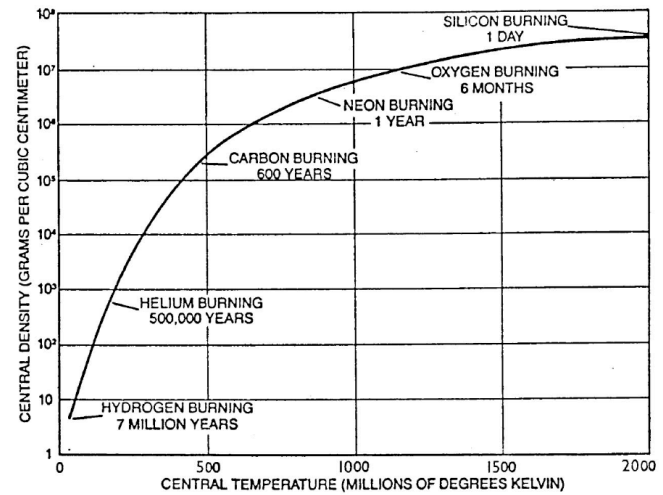
Fig. 36.4. The chemical composition in the interior of a highly evolved model of a population I star with an initial mass of $25M_\odot$, close to the end of hydrostatic nuclear burning. The mass at this time is reduced to $16M_\odot$ due to mass loss. In the upper panel the mass concentrations of important elements are plotted against the mass variable m . Below the abscissa, in the middle of the figure the approximate location of shell sources in different nuclear burning phases is indicated by the grey rectangles. In the lower panel the run of temperature ($\lg T$: scale at left axis) and density ($\lg \rho$: right axis) is given to identify typical burning conditions for these nuclear shells

Collapse and final explosions

Collapse of cores of massive stars (C,D): $M_{Ch} < M_c \lesssim 40M_{\odot}$

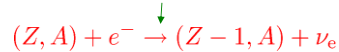


Collapse and final explosions



Collapse and final explosions

Inverse β decay (neutronization, e- capture) @ high densities



High-energy e^- can combine with protons (in nuclei!) to form neutrons. High Fermi energy (sea) inhibits n-decay (no space left in 6-dim. phase space because of Pauli exclusion principle.)

Because process changes a proton into a neutron (although in a nucleus), this process is called **neutronization**.

Electron capture has two important consequences:

- 1) It reduces the number of electrons: μ_e gets larger. Thus, the pressure which is available to stabilize the gravitational contraction of the core is reduced. In other words, the Chandrasekhar mass limit decreases as μ_e increases.
- 2) The neutrinos produced in the reaction can leave the star. These neutrinos carry kinetic energy (usually of a few MeV) which is lost for the core. In fact, these neutrino losses keep the core at relatively low temperatures.

In summary:
the pressure and energy loss due to electron capture accelerates the contraction (collapse).

Collapse and final explosions

Reflection of infall (massive stars: C,D)

Because of infall ρ approaches that of neutron stars ($\rho \approx 10^{14} \text{ gcm}^{-3}$). Matter becomes **incompressible** (EOS is "stiff").

Complete elastic reflection would bring whole collapse only to original state before the collapse; we need extra energy to expel the envelope.

Remnant (neutron star) is somewhat compressed by inertia beyond equilibrium state and afterwards, acting like a "spring", it expands and pushes back the infalling matter.

This creates a pressure wave, steepening if it enters regions of lower density.

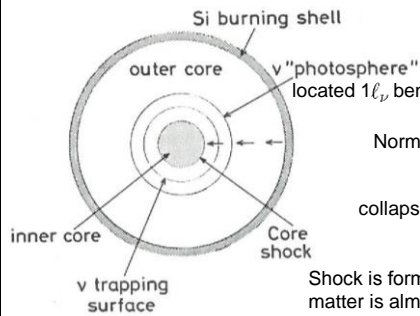
However, a substantial fraction of the energy in this **rebounded pressure wave** will be **used up to disintegrate remaining Fe (in envelope) into free nucleons**, i.e. only a small fraction of the (rebounded) kinetic energy remains in the shock wave for lifting the envelope (also major energy loss due to **neutrinos** \rightarrow only 1% of initial kinetic energy available for lifting envelope).

During (core) collapse, **neutrino production by neutronization** becomes dominant. Because of large density, matter becomes opaque to neutrinos, i.e. free-mean path is reduced and so the neutrino velocity [for $\rho > 3 \times 10^{11} \text{ gcm}^{-3}$ **neutrinos are trapped**, because their velocities are smaller than the infall velocity (free-fall)] \rightarrow **influence further neutronization**, i.e. **neutronization stops @ $\rho \sim 3 \times 10^{12} \text{ gcm}^{-3}$. Collapse stops @ $\rho > 10^{14} \text{ gcm}^{-3}$.**

Collapse and final explosions

Neutrinos in MeV range have mean free path ($\mu=1$)

$$\ell_\nu = \frac{1}{n\sigma_\nu} = \frac{\mu m_u}{\rho\sigma_\nu} \approx \frac{2 \times 10^{20} \text{ cm}}{\rho}$$



infall velocity of matter (small arrows) equals velocity of outward neutrino diffusion

Normal stellar matter : $\rho \simeq 1 \text{ g cm}^{-3}$: $\ell_\nu \simeq 100 \text{ parsec}$.
 $\rho \simeq 10^6 \text{ g cm}^{-3}$: $\ell_\nu \simeq 300 R_\odot$.
 collapsing stellar core: $\rho \simeq 10^{14} \text{ g cm}^{-3}$: $\ell_\nu \simeq 20 \text{ km}$.
 → reabsorbed within star.

Shock is formed at sphere labelled "Core shock", within which matter is almost at rest, after sudden stop of core collapse, once density has reached **nuclear matter density** ($\rho \sim 10^{14} \text{ g cm}^{-3}$). Neutrinos interact with electrons (inelastic scattering) and neutrinos are thermalized, which brings also the weak interaction into equilibrium (already at $\rho \sim 10^{12} \text{ g cm}^{-3}$) - **homologous core**. Sound speed in core is larger than infall velocity. Where both velocity are the same = core shock boundary of homologous core. Thus the **sudden stop of collapse causes shock wave at the surface of the homologous core with $R \sim 30 \text{ km}$** .

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Reflection of infall (massive stars: C,D) : **Supernova explosion**

Shock wave travels outwards through rest of collapsing iron core with energy of about 10^{52} erg . Matter through which shock travels will be dissociated.

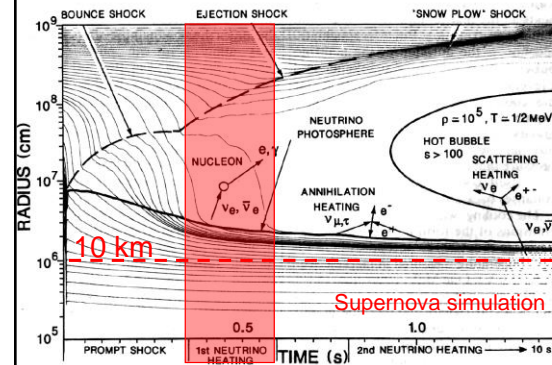
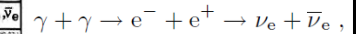


Figure shows radial motion of various layers and locations of travelling shock. Shock stalls @ $r \sim 500 \text{ km}$, but gets revived again after about 0.5s, finally exploding the star.

Core (newly-born neutron star) cools rapidly via **neutrino + anti-neutrino pair creation**:



leading to neutrino **absorption by nucleons (p,n)**, thereby transferring energy (heated within $\sim 0.3 \text{ s}$) to the matter which has previously been dissociated by the shock.

Material heated to energy values sufficient to overcome gravitational potential and can therefore be expelled from the star. This mechanism through **neutrino heating** is called "**delayed supernova**" mechanism. **Time-dependent convection important for explosion mechanism!**

Explosion expels matter outside "mass cut" $\sim 1.6 M_\odot$ into ISM. Partially degenerate remnant is new neutron star consisting, after cooling, mainly of fully-degenerate neutrons.

Collapse and final explosions

Reflection of infall (massive stars: C,D) : Supernova explosion

Only for stars with initial mass of $\sim 8 - 30 M_{\odot}$ does explosion from a core collapse (Supernova Type II; there are also Type Ib and Ic; Type Ia is a different mechanism where a WD collapses after accreting matter from a massive binary companion) result in the generation of a neutron stars.

More massive stars also live through a core-collapse neutron star but instead generate most likely a black hole in the centre.

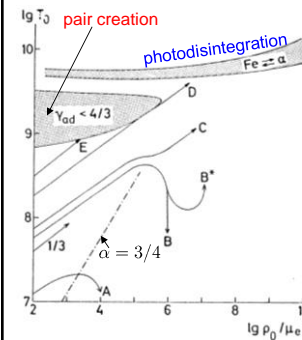
On the other hand stars with $< 8 M_{\odot}$ never reach latest nuclear burning stages and will end up as WDs.

Collapse and final explosions

Very massive stars: (E) $M_c \gtrsim 30 M_{\text{Ch}}$ ($M \gtrsim 80 M_{\odot}$)

evolutionary tracks of massive cores enter region in T - ρ diagram where electron-positron pair production by energetic photons is taking place: $h\nu \geq 2m_e c^2$. This reduces $\gamma_{\text{ad}} < 4/3$ and dynamical instability is reached. Also, for this massive cores, radiation pressure is important.

According to numerical simulations in pair-unstable, collapsing cores, oxygen is ignited explosively, and core runs into unstable regions of photodisintegration.



most numerical simulations result in a disrupted core due to pair-instability.

Some properties of Supernovae (SN)

SN are amongst the brightest objects in the universe and can be brighter than a whole galaxy for weeks.

Energy of visible explosion: $\sim 10^{51}$ erg (= 1 foe [10^{fifty one} erg])

Total energy : $\sim 10^{53}$ erg (most in neutrinos)

Luminosity : $\sim 10^9$ – $10^{10} L_{\odot}$

SN events are rather rare: **some 1 – 10 per century and galaxy**.
(in our Galaxy only a few have been recorded, the last one in the 17th century: Kepler's SN: type Ia).

Classifications of Supernovae (SN)

Observational:

Type I : no H lines (depending on other spectral features: Ia, Ib, Ic,...)

Type II : hydrogen lines

SN progenitor

Type I : 2 possibilities

Ia : **white dwarf accreting matter** from (massive) companion
in binary system

Ib,c: collapse of Fe core in star that blew its H (or He) envelope into
space before the explosion

Type II : collapse of Fe core in normal massive stars (8 – 30 M_{\odot})

electron-capture SN (Crab nebula?)

Classifications of Supernovae (SN)

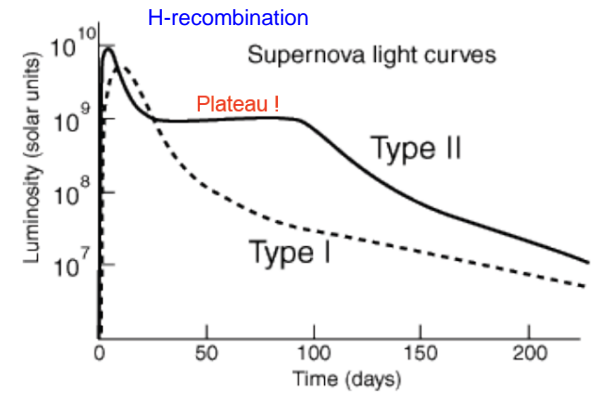
Observational:

Type I No hydrogen	Type Ia Presents a singly ionized silicon (Si II) line at 615.0 nm (nanometers), near peak light		Thermal runaway	
	Type Ib/c Weak or no silicon absorption feature	Type Ib Shows a non-ionized helium (He I) line at 587.6 nm		
		Type Ic Weak or no helium		
Type II Shows hydrogen	Type II-P/L/N Type II spectrum throughout	Type II-P Reaches a "plateau" in its light curve	Core collapse	
		No narrow lines		Type II-L Displays a "linear" decrease in its light curve (linear in magnitude versus time). ^[47]
		Type IIi Some narrow lines		
	Type IIb Spectrum changes to become like Type Ib			

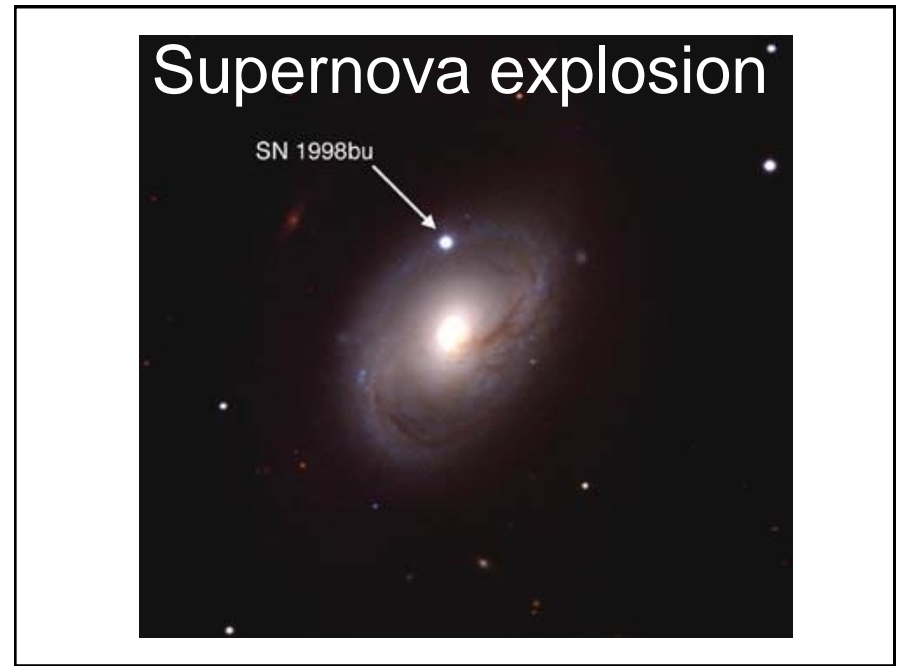
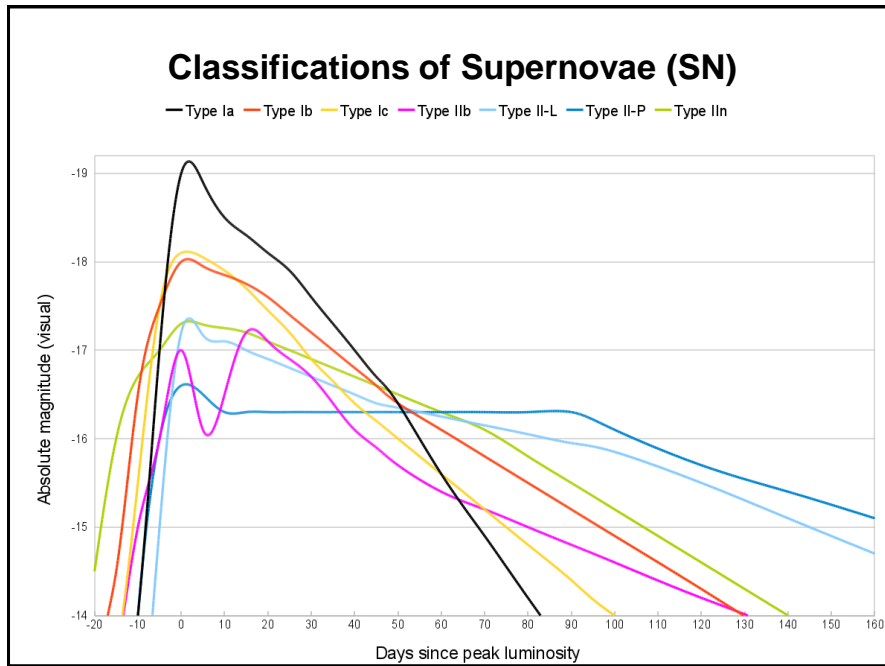
Classifications of Supernovae (SN)

Energy sources: explosion

radiative decay: $^{56}\text{Ni} \rightarrow ^{56}\text{Co} \rightarrow ^{56}\text{Fe}$



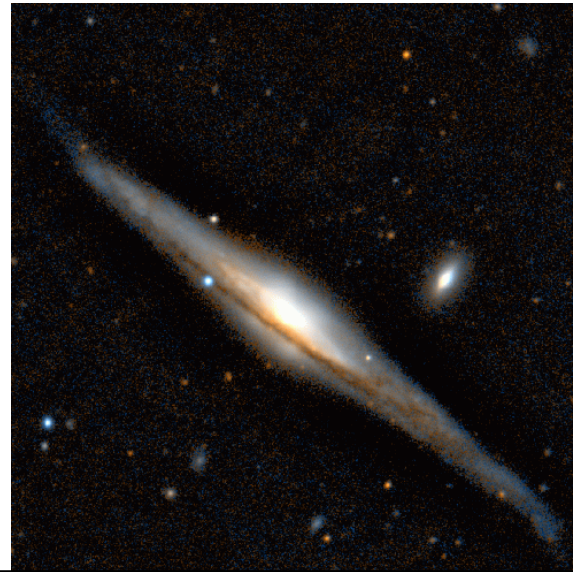
Adapted from Chaisson & McMillan



[Supernova 1994D in NGC 4526](#)



[Supernova 2001cm in NGC 5965](#)



Crab nebula in Taurus
(believed to be electron capture type SN)

