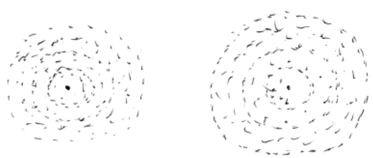
Nuclear energy production
Electron screening

- repulsive Coulomb force is important for estimating ϵ_{aA}.
- free e$^-$ have influence on Coulomb force, i.e. E_{CB}.
- approaching particle will "feel" neutral conglomerate of target nucleus & surrounding e$^-$ - cloud.

Static screening (Salpeter 1954; Austr. J. Phys 7, 373)

- e$^-$ are attracted from nucleus of charge $+Ze$ → e$^-$ have slightly ↑↑n_e near nucleus, and ions are repelled, i.e. n_i ↓↓ ⟶ clustering of e$^-$.



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- particle density n with charge 'q' is modified in the presence of an electrostatic potential ϕ according to

$$n = \underbrace{\bar{n}}_{\#\text{ density for } \phi = 0} \mathrm{e}^{-q\phi/kT} .$$

… derived from Debye-Hückel theory (see e.g. Jackson 1975)

- Typically $|q\phi| \ll kT$ ⟶ approximation:

$$n_i = \bar{n}_i \left(1 - \frac{Z_i e\phi}{kT}\right) , \quad n_e = \bar{n}_e \left(1 + \frac{e\phi}{kT}\right)$$

Which shows directly the decrease of ion density and increase of e$^-$ density.

Nuclear energy production

Electron screening

- total (electrical) charge density σ for all types of ions (n_i)

For $\phi = 0$ (neutral gas) $\bar{\sigma} = 0$ i.e.: $\bar{\sigma} = \sum_i (Z_i e)\bar{n}_i - e\bar{n}_e = 0$,

$\phi \neq 0$: $\sigma = \sum_i (Z_i e) n_i - e n_e$

$$= \sum_i -\frac{(Z_i e)^2 \phi}{kT}\bar{n}_i - \frac{e^2\phi}{kT}\bar{n}_e \ .$$

- Combine last two terms to obtain

$$\boxed{\sigma = -\chi \frac{e^2\phi}{kT} n \ ,}$$

where total particle density n , and average value χ are

$$n = n_e + \sum_i n_i \qquad \chi := \frac{1}{n}\left(\sum_i Z_i^2 \bar{n}_i + \bar{n}_e\right) = \mu \sum_i \frac{Z_i(Z_i+1)}{A_i} X_i$$

Nuclear energy production

Electron screening

$$\nabla^2\phi = -4\pi\sigma \qquad \sigma = -\chi \frac{e^2\phi}{kT} n$$

$$\frac{1}{r^2}\frac{\mathrm{d}}{\mathrm{d}r}\left(r^2 \frac{\mathrm{d}\phi}{\mathrm{d}r}\right) = r_D^{-2}\phi \qquad r_D = \left(\frac{kT}{4\pi\chi e^2 n}\right)^{1/2}$$

The Debye-radius (in some sense the radius of e^- cloud; $r_D = 10^{-8} \ldots 10^{-9}$ cm.)

$$\phi \to \frac{Ze}{r} \quad \text{for} \quad r \to 0$$

$$\phi = \frac{Ze}{r} \exp(-r/r_D)$$

Nuclear energy production
Electron screening

$$\phi = \frac{Ze}{r} e^{-r/r_D}$$

For 'normal stars' $\frac{r}{r_D} \simeq \frac{r_0}{r_D} << 1$; $r_0 = Z_1 Z_2 e^2 / E_0 \simeq 10^{-11}$ cm

classical particle distant at Gamov peak

$$\boxed{\phi \simeq \frac{Ze}{r}\left(1 - \frac{r}{r_D}\right) = \underbrace{\frac{Ze}{r}}_{\text{unshielded potential}} - \frac{Ze}{r_D}}$$

Nuclear energy production
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- Accordingly the Coulomb barrier E_{CB} is reduced by Debye energy E_D

$$E_{CB} = \boxed{\frac{Z_1 Z_2 e^2}{r}} \boxed{- \underbrace{\frac{Z_1 Z_2 e^2}{r_D}}_{\text{Debye energy } E_D}} \quad .$$

Thereby increasing the probability with which particles can tunnel through the Coulomb barrier, leading to an increase of

$$\langle \sigma v \rangle_{\text{screen}} \propto S(E_0) \int_0^\infty \underbrace{e^{-bE^{-1/2}}}_{\text{Coulomb barrier}} \underbrace{e^{-E/kT}}_{\text{M-B tail of thermal energy}} \underbrace{e^{E_D/kT}}_{\text{screening factor "}f\text{"}} \mathrm{d}E \quad .$$

Increase by "*f*": $\langle \sigma v \rangle_{\text{screen}} = \langle \sigma v \rangle \underbrace{e^{E_D/kT}}_{\text{"}f\text{"}}$

Nuclear energy production

Electron screening

- weak screening:

$$E_{\mathrm{D}} = \frac{Z_1 Z_2 e^2}{r_{\mathrm{D}}} \qquad r_{\mathrm{D}} = \left(\frac{kT}{4\pi\chi e^2 n}\right)^{1/2}$$

$$\frac{E_{\mathrm{D}}}{kT} = \frac{Z_1 Z_2 e^2}{r_{\mathrm{D}} kT} = 5.92 \times 10^{-3} Z_1 Z_2 \left(\frac{\zeta\varrho}{T_7^3}\right)^{1/2} \qquad \zeta = \chi/\mu$$

For solar case, p + p: $\langle\sigma v\rangle_{\mathrm{screen}} \simeq 1.1 \times \langle\sigma v\rangle$

Nuclear energy production

Electron screening

- strong screening:

$$\frac{E_{\mathrm{D}}}{kT} \approx 0.0205[(Z_1 + Z_2)^{5/3} - Z_1^{5/3} - Z_2^{5/3}] \frac{(\varrho/\mu_{\mathrm{e}})^{1/3}}{T_7}$$

Screening factor f important for large ρ & low T

Consider shielded reaction rate

$$f\langle\sigma v\rangle = f_0 \langle\sigma v\rangle_0 \left(\frac{\varrho}{\varrho_0}\right)^{\lambda} \left(\frac{T}{T_0}\right)^{n} \qquad f = \mathrm{e}^{E_{\mathrm{D}}/kT} \qquad \frac{E_{\mathrm{D}}}{kT} \propto \frac{\rho^{1/3}}{T}$$

In the neighbourhood of ρ_0 and T_0: $n = \frac{\eta}{2} - \frac{2}{3} - \frac{E_{\mathrm{D}}}{kT}$; $\lambda = 1 + \frac{1}{3}\frac{E_{\mathrm{D}}}{kT}$

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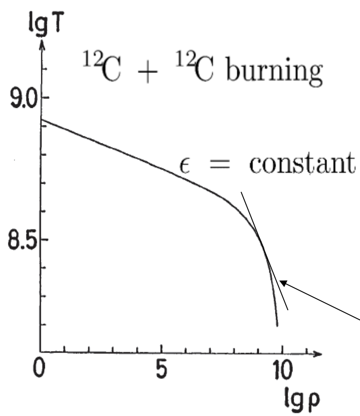
In the neighbourhood of ρ_0 and T_0 : $n = \frac{\eta}{2} - \frac{2}{3} - \frac{E_D}{kT}$; $\lambda = 1 + \frac{1}{3}\frac{E_D}{kT}$



for ρ >> and T << $\longrightarrow \lambda \gg$ & $n \ll$

$\longrightarrow$ "pycnonuclear reactions"

$\longrightarrow \epsilon = \epsilon(\rho)$ and not $\epsilon = \epsilon(T)$

slope = -1, where

$\lambda = n$

Nuclear energy production

Electron screening

Is the static picture correct?

$$r_D = \left(\frac{kT}{4\pi\chi e^2 n}\right)^{1/2}$$

Number of electrons in Debye sphere

$$N_e = \frac{4}{3}\pi r_D^3 n_e = \frac{4}{3}\pi\left(\frac{kT}{4\pi\zeta e^2\rho/m_u}\right)^{3/2} \rho/(\mu_e m_u)$$

$$N_e = \frac{1}{3\mu_e}\frac{(kT)^{3/2}}{(4\pi\rho/m_u)^{1/2}(\zeta e^2)^{3/2}} = \mu_e^{-1}\zeta^{-3/2}T_7^{3/2}\rho_2^{-1/2}5.6$$

where

$$\zeta = \chi/\mu\,, \quad T_7 = T/(10^7\,\mathrm{K})\,, \quad \rho_2 = \rho/(10^2\,\mathrm{g\,cm^{-3}})$$

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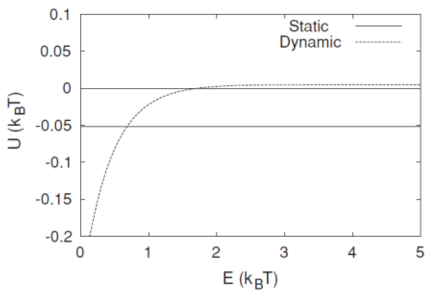
Dynamic screening: Mussack & Däppen 2011; ApJ 729, 96



Screening Energies and the Ratio of Screened to Unscreened Nuclear Reaction Rates for Solar p–p Reactions

Case	Screening Energy U	Reaction-rate Correction
Unscreened	0	1
Statically screened	$U_0 = -\frac{Z_1 Z_2 e^2}{R_D}$	1.042
Dynamically screened	$U_0(E) = k_B T(0.005 - 0.281\ \exp(-2.35\frac{E}{k_B T}))$	0.996

Neutrino energy losses

Have extremely small cross section $\sigma_\nu \simeq (E_\nu / m_e c^2)^2\ 10^{-44}\,\mathrm{cm}^2$.

Neutrinos in MeV range have mean free path (μ=1)

$$\ell_\nu = \frac{1}{n\sigma_\nu} = \frac{\mu m_u}{\varrho \sigma_\nu} \approx \frac{2\times 10^{20}\mathrm{cm}}{\varrho}$$

Normal stellar matter : $\rho \simeq 1\,\mathrm{g\,cm^{-3}}$: $\ell_\nu \simeq 100\,\mathrm{parsec}$.

$\rho \simeq 10^6\,\mathrm{g\,cm^{-3}}$: $\ell_\nu \simeq 300\,\mathrm{R}_\odot$.

collapsing stellar core: $\rho \simeq 10^{14}\,\mathrm{g\,cm^{-3}}$: $\ell_\nu \simeq 20\,\mathrm{km}$. $\longrightarrow$ reabsorbed within star.

Also important for cooling cores of stars.

Neutrino energy losses

Nuclear energy production

Hydrogen burning (energy loss due to escaping neutrinos):

$^1H + {}^1H \rightarrow {}^2H + e^+ + \nu$	$(pp1, 2, 3)$	$Q_\nu = 0.263$ MeV
$^7Be + e^- \rightarrow {}^7Li + \nu$	$(pp2)$	0.80 MeV
$^8B \rightarrow {}^8Be + e^+ + \nu$	$(pp3)$	7.2 MeV
$^{13}N \rightarrow {}^{13}C + e^+ + \nu$	(CNO)	0.71 MeV
$^{15}O \rightarrow {}^{15}N + e^+ + \nu$	(CNO)	1.0 MeV

With ~ 4×10^{-5} erg/cycle Sun produces ~ 2×10^{38} (electron) neutrinos every second!

$\longrightarrow$ Flux of ~ 10^{11} solar neutrinos per cm^2 and second at Earth!

Neutrino losses

Involving nuclear processes

Urca process at extreme densities (no nuclear reactions necessary):

e^- capture:

$$\underbrace{(Z, A)}_{\text{Nucleus with charge } Z \text{ and atomic weight } A} + e^- \rightarrow (Z-1, A) + \nu \,,$$
$$(Z-1, A) \rightarrow (Z, A) + e^- + \bar{\nu} \,.$$

degenerate

Example:

$^{35}\mathrm{Cl}\ (e^-, \nu)^{35}\mathrm{S}$

$^{35}\mathrm{S}\ (e^- \bar{\nu})^{35}\mathrm{Cl}$

Neutrino energy losses