GTR formelsampling

(by GTR18 students)

Coordinate transformation:

$$dx'^a = \sum_{b=0}^{3} \frac{\partial x'^a}{\partial x^b} dx^b \equiv \frac{\partial x'^a}{\partial x^b} dx^b$$

Vector transformation:

$$A'^a = \frac{\partial x'^a}{\partial x^b} A^b$$

Tensor transformation:

$$F'^{ab} = \frac{\partial x'^a}{\partial x^c} \frac{\partial x'^b}{\partial x^d} F^{cd}$$

Jacobian matrix:

$$J = \frac{\partial x'^a}{\partial x^b}$$

Metric:

$$ds^2 = g_{ab} dx^a dx^b$$

$$dg = g g^{ab} dg_{ab} = -g g_{ab} dg^{ab}$$

Covariant differentiation:

$$DA^a = dA^a + \Gamma^a_{bc} A^b dx^c$$

$$DA_a = dA_a - \Gamma^b_{ac} A_b dx^c$$

$$A^a_{;c} = A^a_{,c} + \Gamma^a_{bc} A^b$$

$$A_{a;c} = A_{a,c} - \Gamma^b_{ac} A_b$$

Christoffel symbols:

$$\Gamma_{abc} = \frac{1}{2}(g_{ab,c} - g_{bc,a} + g_{ac,b})$$

$$\Gamma_{abc} = \Gamma_{acb}$$

$$g_{ab,c} = \Gamma_{bac} + \Gamma_{abc}$$

Geodesic equation:

$$Du^a = 0$$

$$\frac{du^a}{ds} + \Gamma^a_{bc} u^b u^c = 0$$

$$\frac{d^2 x^a}{ds^2} + \Gamma^a_{bc} \frac{dx^b}{ds} \frac{dx^c}{ds} = 0$$

$$\frac{du_c}{ds} = \frac{1}{2} g_{ab,c} u^a u^b$$

Riemann tensor:

$$R^a_{bcd} = \Gamma^a_{bd,c} - \Gamma^a_{bc,d} + \Gamma^a_{ec}\Gamma^e_{bd} - \Gamma^a_{ed}\Gamma^e_{bc}$$

Ricci tensor:

$$R_{ab} = R^d_{adb}$$

Ricci scalar:

$$R = g^{ab} R_{ab}$$

Einstein equation:

$$R_{ab} - \frac{1}{2} R g_{ab} = \kappa T_{ab}$$

Schwartzschild metric:

$$ds^2 = (1 - \frac{r_g}{r}) dt^2 - \frac{dr^2}{1 - \frac{r_g}{r}} - r^2 (d\theta^2 + \sin^2\theta d\phi^2)$$

Equatorial orbits in Schwarzschild metric:

$$u'' + u = \frac{M}{J^2} + 3Mu^2$$

Lemaitre coordinates in Scharwschild metric,

$$d\tau = dt + \sqrt{\frac{r_g}{r}} \frac{dr}{1 - \frac{r_g}{r}}$$

$$d\rho = dt + \sqrt{\frac{r}{r_g}} \frac{dr}{1 - \frac{r_g}{r}}$$

Metric in closed and open Friedman universe:

$$ds^2 = a^2 (d\eta^2 - d\chi^2 - \sin^2\chi (d\theta^2 + \sin^2\theta d\phi^2))$$

$$ds^2 = a^2 (d\eta^2 - d\chi^2 - \sinh^2\chi (d\theta^2 + \sin^2\theta d\phi^2))$$

Friedman equation (+ closed, − open):

$$\frac{3}{a^4}(a'^2 \pm a^2) = \kappa\epsilon$$

Energy conservation:

$$(\epsilon a^3)' + p(a^3)' = 0$$

$$\frac{3da}{a} = -\frac{d\epsilon}{\epsilon + p}$$

Lorentz transformation:

$$\begin{pmatrix} t' \\ x' \end{pmatrix} = \frac{1}{\sqrt{1 - \frac{v^2}{c^2}}} \begin{bmatrix} 1 & -\frac{v}{c^2} \\ -v & 1 \end{bmatrix} \begin{pmatrix} t \\ x \end{pmatrix}$$