Strongly interacting trapped systems in one dimension

Artem G. Volosniev

TECHNISCHE UNIVERSITÄT DARMSTADT

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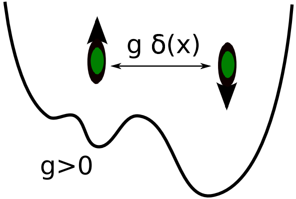
Alexander von Humboldt Stiftung/Foundation

EFB23, Aarhus, August 9, 2016

Example



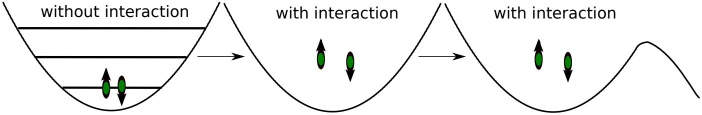
$$H\Psi = E\Psi, \qquad H = \sum_{i=1}^{2}\left(-\frac{\hbar^2}{2m}\frac{\partial^2}{\partial x_i^2} + V_{trap}(x_i)\right) + g\delta(x_1 - x_2).$$

This talk focuses on

- Energy spectrum,
- Properties of eigenstates.

Motivation

Two-body experiments in the group of Selim Jochim (Heidelberg)

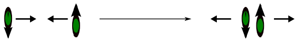


- ^6Li atoms in different hyperfine states,
- $T = 0$,
- approximately one-dimensional harmonic trap,
- tunable short-range interaction ($g\delta(x)$).

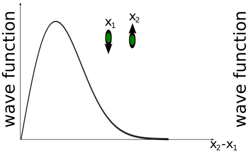
G. Zürn et al. *PRL* 108, 075303 (2012)

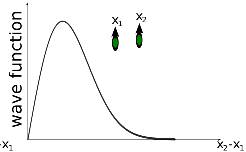
Motivation

At $1/g = 0$: $\Psi(x_\uparrow = x_\downarrow) = 0$



$$H\Psi = E\Psi, \qquad H = \frac{\hbar^2}{2m}\sum_{i=1}^{2}\left(-\frac{\partial^2}{\partial x_i^2} + \frac{x_i^2}{L^4}\right).$$

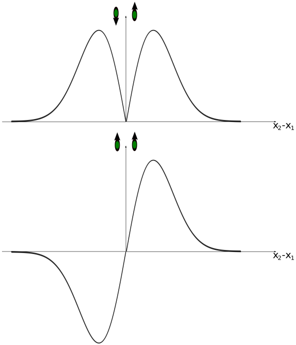




Motivation

M. Girardeau *J. Math. Phys.* 1, 516 (1960)

T. Busch et al. *Foundations of Physics* 28: 549 (1998)



Motivation

These experiments

1. Give opportunity for **simulating few-body systems** that are difficult to solve (**theory** is needed **for comparison**),
2. Give opportunity for **studying** standard **many-body problems** in a **few-body limit** (**theory** is needed **for comparison**),
3. Pave the way for **realizing** such systems (if theory finds some promising **applications**),
4. Push theory.

Outline

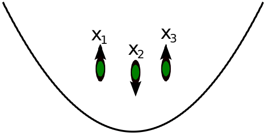
1. Hamiltonian and Main Results.
2. Approach.
3. Properties and Applications.
4. Conclusions.

Hamiltonian

$$H = \sum_{i=1}^{N=N_\downarrow+N_\uparrow}\left(-\frac{\hbar^2}{2m}\frac{\partial^2}{\partial x_i^2}+V_{trap}(x_i)\right)+\sum_{int.pairs} g_{ij}\delta(x_i-x_j).$$

$$H = \frac{\hbar^2}{2m}\sum_{i=1}^{3}\left(-\frac{\partial^2}{\partial x_i^2}+\frac{x_i^2}{L^4}\right)+g\delta(x_1-x_2)+g\delta(x_2-x_3).$$

- two fermions of mass m,
- one impurity of mass m,
- external harmonic potential,
- $V(x)=g\delta(x)$, $g/L \gg \hbar^2/(mL^2)$.

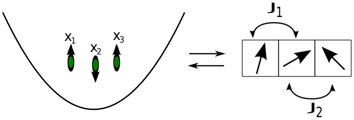


Take home messages

- The system can be studied using the Heisenberg Hamiltonian

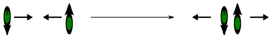
$$H_s = \sum_{j=1}^{N} \frac{J_j}{2} \boldsymbol{\sigma}^j \boldsymbol{\sigma}^{j+1}, \qquad J_i = \frac{f_i[V_{trap}]}{g} \ ,$$

- J_i can be calculated for up to $N \simeq 30$, for $N > 30$ it can be estimated or guessed, allowing one to solve the problem in the leading order in $1/g$,
- The system can be used to simulate non-homogeneous spin chains.



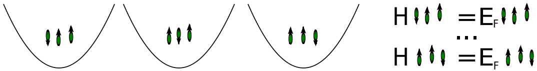
Impenetrable particles (1/g=0)

- At $\frac{1}{g} = 0$: $\Psi(x_\uparrow = x_\downarrow) = 0$ – particles do not exchange positions,

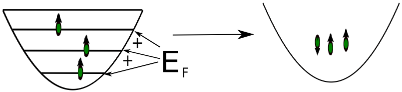


$$H = \frac{\hbar^2}{2m} \sum_{i=1}^{3} \left(-\frac{\partial^2}{\partial x_i^2} + \frac{x_i^2}{L^4} \right), \Psi(x_\uparrow = x_\downarrow) = 0.$$

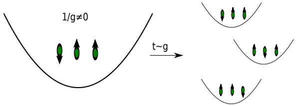
- Degeneracy



- $\Psi_{\downarrow\uparrow\uparrow} \sim \Phi_{\uparrow\uparrow\uparrow}$ (cf. M. Girardeau (1960)),



Eigenstates at $1/g \neq 0$



Degenerate perturbation theory

$$\begin{pmatrix} \delta_1 & J_1 & 0 \\ J_1 & \delta_2 & J_2 \\ 0 & J_2 & \delta_3 \end{pmatrix} \begin{pmatrix} \downarrow\uparrow\uparrow \\ \uparrow\downarrow\uparrow \\ \uparrow\uparrow\downarrow \end{pmatrix} = (E - E_F) \begin{pmatrix} \downarrow\uparrow\uparrow \\ \uparrow\downarrow\uparrow \\ \uparrow\uparrow\downarrow \end{pmatrix}$$

This is a Heisenberg spin chain Hamiltonian.

$\Psi = a_1 \Psi_{\downarrow\uparrow\uparrow} + a_2 \Psi_{\uparrow\downarrow\uparrow} + a_3 \Psi_{\uparrow\uparrow\downarrow}$+corrections($\sim 1/g$).

Eigenstates at $1/g \neq 0$

$$J_1 = J_2 = \frac{\hbar^2}{mg} \frac{\int \mathrm{d}x_1 \mathrm{d}x_2 \left(\frac{\partial \Phi_{\uparrow\uparrow\uparrow}}{\partial x_\downarrow}\right)^2_{x_\downarrow = x_\uparrow}}{\int \mathrm{d}x_1 \mathrm{d}x_2 \mathrm{d}x_3 \Phi_{\uparrow\uparrow\uparrow}{}^2} (= -\delta_1 = -\delta_3 = -\delta_2/2).$$

- J_1 is related to flux at $x_\uparrow = x_\downarrow$,
- Flux is determined by $\int \Psi(0) \frac{\partial \Psi}{\partial x_\uparrow}\Big|_{x_\uparrow \to x_\downarrow} \simeq \int \frac{1}{g} \left(\frac{\partial \Phi_{\uparrow\uparrow\uparrow}}{\partial x_\downarrow}\right)^2_{x_\downarrow = x_\uparrow}$,

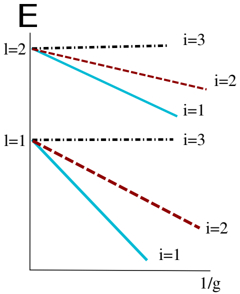
AGV et al. *Nature Commun.* 5, 5300 (2014)
F. Deuretzbacher et al. *PRA* 90, 013611 (2014)
J. Levinsen et al. *Science Advances* 1, e1500197 (2015)
L. Yang et al. *PRA* 91, 043634 (2015)
F. Deuretzbacher et al. arXiv:1602.06816
N. J. S. Loft et al. arXiv:1603.02662

Properties (Energy)

- $1/g = 0$: $E = E_F$,
- $1/g \neq 0$:

  $H \to H_s = \sum_{j=1}^{2} \frac{J_j}{2} \boldsymbol{\sigma}^j \boldsymbol{\sigma}^{j+1}$,

  $E_i \simeq E_F - K_i/g$,
- Same for excited manifolds.

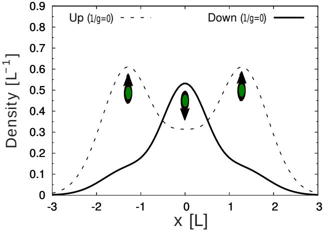


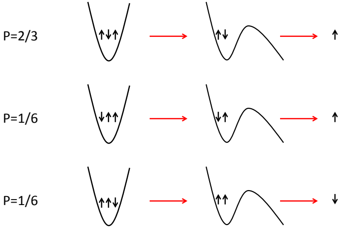
S. E. Gharashi and D. Blume *PRL* 111, 045302 (2013)

E.J. Lindgren et al. *NJP* 16, 063003 (2014)

Properties (Density)

The chain is 'antiferromagnetic' - the impurity in the middle.





Plenary Talk on Thursday by Frank Deuretzbacher.

S. Murmann, F. Deuretzbacher et al. *PRL* 115, 215301 (2015)

Extensions

- More particles,
- More than one impurity,
- Other trapping potentials,
- More components, e.g., $\{\uparrow, \nearrow, \downarrow\}$ – SU(N) spin chains,
- Particles can be strongly interacting bosons, $V_{int}(x_{\uparrow(\downarrow)} - x_{\uparrow(\downarrow)}) = \kappa g \delta(x_{\uparrow(\downarrow)} - x_{\uparrow(\downarrow)})$

$$H_s^b = \sum_{j=1}^{N} [\frac{J_j}{2} \boldsymbol{\sigma}^j \boldsymbol{\sigma}^{j+1} - \frac{J_j}{\kappa} \sigma_z^j \sigma_z^{j+1}], \qquad J_i = \frac{f_i[V_{trap}(x)]}{g},$$

1. $1/\kappa = 0$ – hard-core bosons (fermions),
2. $\kappa = 2$ – Heisenberg XX model,
3. $\kappa \ll 1$ – Ising model,
4. ferro-antiferro transition (P. Massignan et al. *PRL* 115, 247202 (2015)).

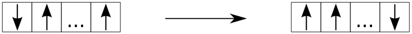
Applications

- Magnetism without lattices (Plenary Talk on Thursday),

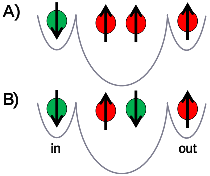
- A reference point for numerical calculations (DMRG, exact diagonalization, etc.), F. F. Bellotti et al. arXiv:1606.09528

- Engineer non-homogeneous chain (J_i are defined by trap).

– Short distance quantum communication (state transfer)



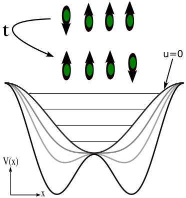
– Conditional state transfer

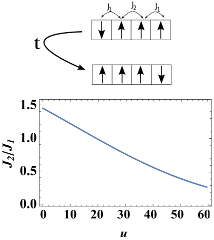


O. Marchukov, AGV, M. Valiente, D. Petrosyan, and N. Zinner. To appear in *Nature Commun.*

State transfer with four particles

$$H_s^b = \sum_{j=1}^{3} [\frac{J_j}{2} \boldsymbol{\sigma}^j \boldsymbol{\sigma}^{j+1} - \frac{J_j}{\kappa} \sigma_z^j \sigma_z^{j+1}].$$





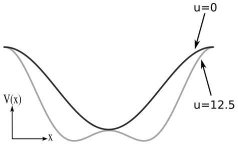
$$V_{trap}(x)\frac{mL^2}{\hbar^2} = -50\sin^2\left[\frac{(x/L-1)\pi}{2}\right] - u\sin^2\left[(x/L-1)\pi\right].$$

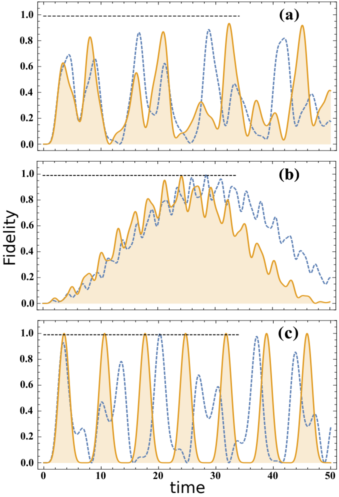
Fidelity of state transfer

Fidelity of transfer, $|\langle\Psi(t)|\uparrow\uparrow\uparrow\downarrow\rangle|^2$

a) $1/\kappa = 0$ ($\uparrow$ - hard-core bosons or fermions)
b) $\kappa = 1/2$ ($\uparrow\uparrow$ interaction is weaker)
c) $\kappa = 2$ ($\uparrow\uparrow$ interaction is stronger)

($u = 0$ (blue, dashed), 12.5 (orange, solid))





Outlook

Directions

- Spin-orbit coupling (Q. Guan and D. Blume *PRA* 92, 023641),
- p-wave interaction (L. Yang et al. *PRA* 93, 051605 (2016)),
- Spin-dependent perturbation (X. Cui and T.-L. Ho *PRA*, 89, 023611 (2014)),
- Time-dependent traps (AGV et al. *PRB* 93, 094414 (2016)).

...

Talks by A.S. Dehkharghani, C. Duncan, N.L Harshman, D. Pecak, T. Sowinski

...

Summary

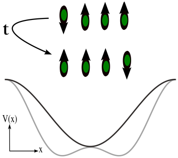
- N-body two-component systems of strongly interacting particles in one dimension can be studied using Heisenberg Hamiltonians

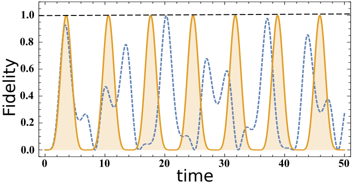
$$H_s^b = \sum_{j=1}^{N} [\frac{J_j}{2} \boldsymbol{\sigma}^j \boldsymbol{\sigma}^{j+1} - \frac{J_j}{\kappa} \sigma_z^j \sigma_z^{j+1}], \qquad J_i = \frac{f_i [V_{trap}(x)]}{g}.$$

- J_i can be calculated (if $N < 30$), estimated (e.g., LDA) or guessed,
- Strongly interacting systems in one-dimension can be used to engineer spin Hamiltonians.

Summary

One can engineer dynamic properties of the system





AGV et al. *PRA* 91, 023620 (2015)

N. J. S. Loft et al. *NJP* 18, 045011 (2016)

Collaboration

D. Fedorov, A. Jensen, N. Zinner, A. Dehkharghani, N. J. S. Loft, L. B. Kristensen, M. E. S. Andersen, Aarhus University (Denmark)

H.-W. Hammer, TU Darmstadt (Germany)

O.V. Marchukov, Seoul National University (South Korea)

D. Petrosyan, IESL-FORTH (Greece)

M. Valiente, Heriot-Watt University (UK)

C. Forssén, J. Rotureau, Chalmers Univ. of Technology (Sweden)

A. E. Thomsen, CP^3-Origins, University of Southern Denmark (Denmark)

J. Lindgren, Université libre de Bruxelles (Belgium)

Tak