Faraday waves in cold-atom systems with two- and three-body interactions

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Outline

Faraday waves in BEC with engineering 3B interactions

In this work, we study a BEC with 2 and 3-body interactions periodically varying in time. For the time-dependent 3-body interaction, two models are assumed, with quadratic and quartic dependence on the two-body atomic scattering length a_s.

It is shown that parametric instabilities in a Bose-Einstein condensate leads to the generation of Faraday waves (FW), with wavelengths depending on the background scattering length, as well as on the frequency and amplitude of the modulations of a_s. This opens a new possibility to tune the period of Faraday patterns by varying not only the frequency of modulations and background scattering length, but also through the amplitude of the modulations.

The latter effect can be used to estimate the parameters of three-body interactions from the FW experimental results. Theoretical predictions are confirmed by numerical simulations of the corresponding extended Gross-Pitaevskii equation.

Outline

Formalism

Let us consider a 1D BEC with atoms of mass m, with two- and three-body interactions varying in time, which are given by the functions $\Gamma(t)$ and $G(t)$. By also considering a possible time-independent external interaction V_{ext}, with the wave-function $\psi \equiv \psi(x,t)$ normalized to the number of atoms N, we have

$$i\hbar\frac{\partial\psi}{\partial t} = -\frac{\hbar^2}{2m}\frac{\partial^2\psi}{\partial x^2} + V_{ext}(x)\psi - \Gamma(t)|\psi|^2\psi - G(t)|\psi|^4\psi,$$

where $\Gamma(t)$ is related linearly with the two-body $s-$wave atomic scattering length $a_s(t)$, which can be varied in time by considering Feshbach resonance techniques. The possible ways that the three-body strength $G(t)$ can be varied in time will depend on specific atomic characteristics, which are also related to the kind of two-body interaction, as well as induced by some external interactions acting on the condensate.

In dimensionless quantities, with $u \equiv u(x,t) = \sqrt{l_\perp}\psi$ and $V_{ext} = \hbar\omega_\perp \mathcal{V}_{ext}$, we have

$$\mathrm{i}\frac{\partial u}{\partial t} + \frac{\partial^2 u}{\partial x^2} - \mathcal{V}_{ext}(x)u + \gamma(t)|u|^2 u + g(t)|u|^4 u = 0,$$

where the dimensionless time-dependent two- and three-body parameters are, respectively, given by

$$\gamma(t) \equiv \sqrt{\frac{2m}{\hbar^2}} \frac{\Gamma(t)}{\sqrt{\hbar\omega_\perp}} \quad \text{and} \quad g(t) \equiv \frac{2m}{\hbar^2} G(t).$$

By considering no external potential ($\mathcal{V}_{ext} = 0$) the natural scale is the $s-$wave two-body scattering length a_s at $t = 0$, which will define $\omega_\perp$ and the corresponding length $l_\perp = 2a_s$.

Three-body interaction proportional to $[a_s(t)]^2$ (quadratic case)

This case can occur in a model for a BEC with 1D non-polynomial GP equation, confined in a cigar type trap. By a series expansion for small $a_s|\psi|^2$, an effective quintic term can be derived ($G(t) \equiv 2\hbar\omega_\perp a_s^2(t)$). [See Salasnich et al., PRA 65, 043614 (2002) or Khaykovich and Malomed, PRA 74, 02360 (2006)].

A quadratic dependence of $G(t)$ on $a_s(t)$ can also occur for $\Gamma \equiv \Gamma(x,t) \approx \cos(\omega t)\cos(kx)$, corresponding to a time dependent short-scale nonlinear optical lattice. In this case, averaged over short scale modulations in space, the dynamics is described by a GP equation with effective time dependent three body interactions Abdullaev et al, PL A 367, 149 (2007). Another model with quadratic dependence on a_s was also suggested in Mahmud et al PRA 90 041602(R) (2014), considering effective 3-body interactions for atoms loaded in a deep optical lattice.

Three-body interaction proportional to $[a_s(t)]^4$ (quartic case).

By varying $a_s(t)$ through Feshbach resonances techniques, as the absolute value of this two-body observable becomes very large, one approaches the unitary limit ($|a_s| \to \infty$) where many three-body bound-states and resonances can be found. This behaviour will induce changes in the corresponding quintic parameter of the GP equation, such that we can have $G(t) \sim a_s^4(t)$ [See, for instance, Bulgac, PRL 89 050402 (2002).]

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Modulational instability

For analyzing modulational instability (MI) of the nonlinear solution, we introduce

$$u_0 \equiv u(0,t) \quad = \quad Ae^{\mathrm{i}\theta(t)}, \;\; \text{where } \theta(t) = A^2 \int_0^t \left[\gamma(t') + A^2 g(t')\right] dt'.$$

and look for a solution of the form

$$u(x,t) = [A + \delta u(x,t)]e^{\mathrm{i}\theta(t)}, \;\; \text{with } \;\delta u \ll A.$$

By keeping only linear terms $\delta u \equiv \delta u(x,t)$, we obtain

$$\mathrm{i}\frac{\partial \delta u}{\partial t} + \frac{\partial^2 \delta u}{\partial x^2} + A^2\left[\gamma(t) + 2A^2 g(t)\right](\delta u + \delta u^*) = 0.$$

Next, with $\delta u = v + \mathrm{i}w$, and going to the Fourier components, $(v, w) = \int e^{\mathrm{i}kx}\,(V, W)\,dk$, we have the set of coupled equations:

$$\frac{dV}{dt} = k^2 W, \quad \frac{dW}{dt} = -k^2 V + 2A^2\left[\gamma(t) + 2A^2 g(t)\right] V,$$

leading to

$$\frac{d^2 V}{dt^2} + k^2\left[k^2 - 2A^2\left(\gamma(t) + 2A^2 g(t)\right)\right] V = 0.$$

Influence of the inelastic three-body collisions

Considering inelastic three-body collisions with κ_3, one should add $\mathrm{i}\kappa_3|u|^4u$ and replacing $g(t)$ by $g_c(t) = g(t) + \mathrm{i}\kappa_3$. In this case,

$$u(x,t) = [A(t) + \delta u(x,t)]\, e^{\mathrm{i}\theta(t)}, \quad A(t) = A_0(1 + 4\kappa_3 A_0^4 t)^{-1/4},$$
$$\theta(t) = \int_0^t ds \left[\gamma(s)A^2(s) + g(s)A^4(s)\right].$$

In the above expression for $\theta(t)$, we neglect $\delta u(x,t)$ with the assumption that $A(t) \gg \delta u(x,t)$. Following the same procedure as before, with $\delta u = v + \mathrm{i}w$, for the Fourier component V we obtain

$$\frac{d^2V}{dt^2} + k^2\left[k^2 - 2A(t)^2(\gamma(t) + 2A(t)^2 g(t))\right] V =$$
$$-6\kappa_3 A(t)^4 \frac{dV}{dt} + 15\left[\kappa_3 A(t)^4\right]^2 V,$$

where appears a dissipative term $\sim \kappa_3$, together with a term $\sim \kappa_3^2$.

MI for periodic variations of the scattering length

The periodic modulations of the scattering length in time is given by $\gamma(t) \equiv \gamma_0 + \gamma_1 \cos(\omega t)$, with the three-body interaction term given by $g(t) = c\,[\gamma_0 + \gamma_1 \cos(\omega t)]^{2n}$ ($n = 1$ for quadratic, and $n = 2$ for quartic).

γ_0 refers to the natural two-body scattering length, which can be attractive ($\gamma_0 > 0$) or repulsive ($\gamma_0 < 0$); and γ_1 is the amplitude of the periodic modulation.

We should note that some other works are mainly concerned with 3B repulsion as a way to stabilise the condensate with attractive two-body interaction.

However, in the present case, as we are interested in verify the emergence of FW patterns, we consider the interesting conditions where the time-dependent parameter $g(t)$ is positive ($c > 0$), implying in attractive three-body interaction.

Without dissipation ($\kappa_3 = 0$), we obtain

$$\frac{d^2V}{dt^2} + \Omega^2\left[1 - f_1\cos(\omega t) - f_2\cos(2\omega t)\right]V = 0,$$

where

$$\Omega^2 \equiv k^2\Delta \equiv k^2\left\{k^2 - 2A^2\left[\gamma_0 + A^2c(2\gamma_0^2 + \gamma_1^2)\right]\right\},$$

$$f_1 \equiv \frac{2\gamma_1 A^2(1 + 4cA^2\gamma_0)}{\Delta},\ f_2 \equiv \frac{2c\gamma_1^2 A^4}{\Delta}.$$

Parametric resonances occur for two cases: $\omega = 2\Omega$ ($\eta \equiv 1$) and $\omega = \Omega$ ($\eta \equiv 2$), with the corresponding wavenumbers

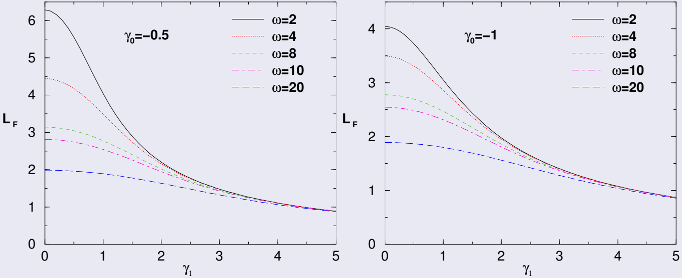
$$k_F^{(\eta)} = \pm\sqrt{\frac{M_\pm}{2} + \frac{1}{2}\sqrt{M_\pm^2 + (\eta\omega)^2}} \equiv L_\eta, \quad \text{with}$$

$$M_\pm \equiv 2A^2\left[\pm|\gamma_0| + A^2c\left(2\gamma_0^2 + \gamma_1^2\right)\right].$$

M_+ is for attractive or zero two-body interactions, $\gamma_0 \geq 0$, and M_- for the repulsive case, $\gamma_0 < 0$. In the present case, as we are analysing the case with $c > 0$, M_- can be set to zero or negative only for repulsive interactions. In the following, we consider only the relevant positive sign for the resonance wavenumber k_F.

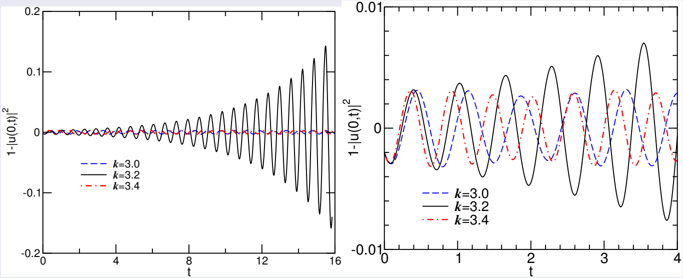
Outline

FW patterns - Period of oscillations $L_F = \frac{2\pi}{k_F}$

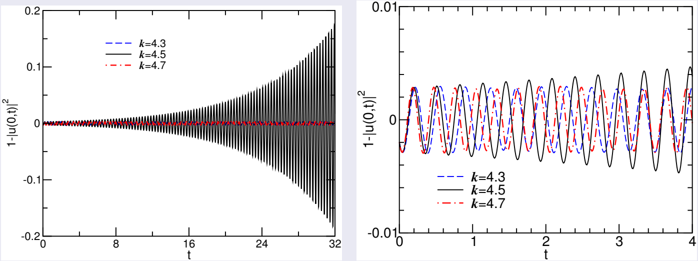


Behaviour of the period of FW oscillations, L_F (= L_1 in case $2\Omega = \omega$, and = L_2, when $\Omega = \omega$), given as functions of γ_1, for a few set of frequencies ω and for two cases of repulsive two-body interactions, when the three-body interaction is proportional to $[a_s(t)]^2$. All quantities are dimensionless with parameters such that $A = c = 1$.

Sample FW pattern result - no dissipation



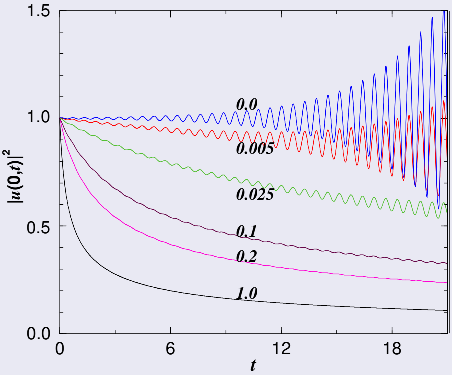
Behavior of the central density $|u(0,t)|^2$, as function of time, showing the emergence of the first parametric resonance (for $\omega = 20$), from full-numerical calculations. In full agreement with analytical predictions for the values of k, it is obtained the resonance for $k = k_F = 3.2$. The other parameters, in this case, are such that $\gamma_0 = 0$, $\gamma_1 = 0.5$, $\epsilon_0 = 0.001$, $A = 1$, and $c = 1$. In the right frame, we show a smaller time interval ($t < 4$) for a clear identification of the plots for the given values of k.



Behavior of central density is displayed as a function of time, showing the emergence of the second parametric resonance ($\omega = 40$), from full-numerical calculations. In this case, again in agreement with analytical prediction, the resonance occurs at $k = 4.5$. In the right frame, for $t < 4$, we also show the plots for the given k, in order to appreciate how the resonance starts to appear. The other parameters are the same as in the previous figure.

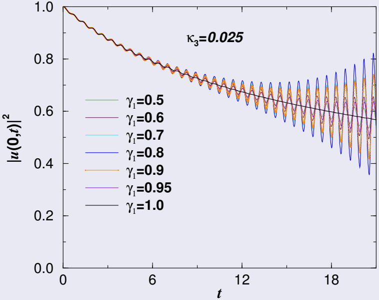
Outline

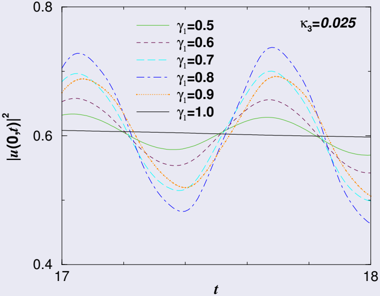
Effect of dissipation



The effect of dissipation in the system we can exemplify by considering $k = 3.17$ at the resonant position. For that, in our full numerical calculation, we add in the quintic parameter g a dissipative imaginary term κ_3, varying it from zero (non-dissipative case shown by the upper results) to $\kappa_3 = 1$ (lower curve), as indicated inside the frame. As expected, the amplitude of the resonance decreases gradually as we increase the dissipation.

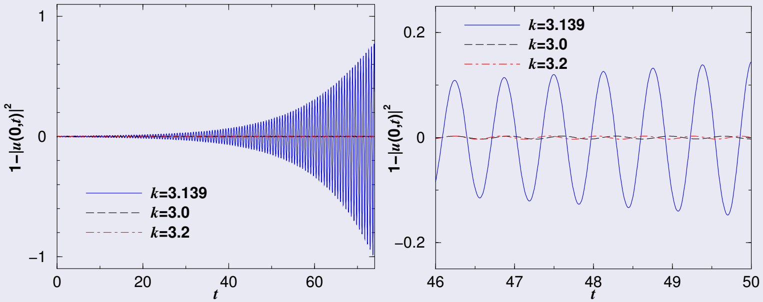
Effect of dissipation - varying the amplitude of modulation γ_1



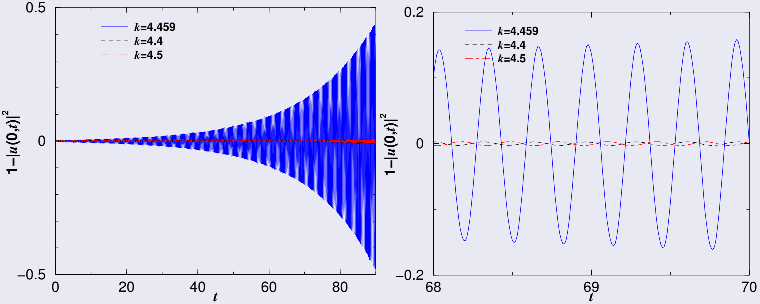


The effect of dissipation can be compensated by varying γ_1. By selecting the case $\kappa_3 = 0.025$ in the previous figure, γ_1 was varied from 0.5 till 1. We noticed that the maximum occurs near $\gamma_1 = 0.8$. The panel in the right, for a small time interval, is for an easy identification of the different curves.

Results for repulsive 2B interactions $\gamma_0 < 0$ and attractive 3B quartic case ($g(t) > 0$)

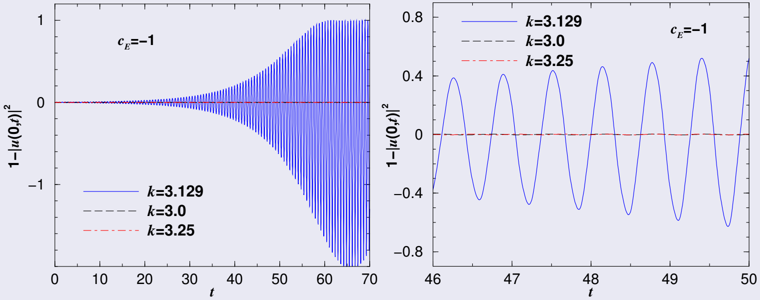


For the repulsive case, with $\gamma_0 = -0.2$ and $\gamma_1 = 0.2$, from full numerical results, we present a case when the three-body interaction $g(t)$ is for the quartic case. The first resonance, for $\omega = 20$, $\epsilon_0 = 0.001$, $A = 1$, and $c_E = +1$ ($g(t) > 0$), is found at $k = 3.139$. In the right panel, for a small interval of time, we show how the amplitude is changing for a small variation of the parameter k.



Second parametric resonance ($\omega = 40$) for the repulsive two-body interaction ($\gamma_0 = -0.2$), with positive three-body parameter ($c_E = +1$) in the quartic case. The resonance appears at $k = 4.45$. This is shown by comparing with results obtained for k smaller and larger than this value, when the oscillation patterns remain almost constant (see right panel). In this case, as compared with the first resonance, the peak of the resonant value is manifested for larger values of t.

1st resonance, for repulsive 3B quartic case ($g(t) < 0$)



First parametric resonance, with $\omega = 20$, for the quartic case, with repulsive two-body interaction ($\gamma_0 = -0.2$), and also negative three-body $g(t) < 0$ ($c_E = -1$). The resonance, as predicted, appears at $k = 3.129$, with the right panel showing that density oscillation remains almost constant for smaller and larger values of k. By comparing with the case $g(t) > 0$, the resonance occurs at smaller values of t.

Outline

Concluding remarks

We have studied the generation of Faraday pattern in a BEC system, by engineering the time dependent three-body interactions. Two models were analysed, according to the mechanism of modulation and behaviour of the three-body interaction with respect to the atomic scattering length a_s (quadratic and quartic power).

Our analysis and numerical simulations show that the time-dependent three-body interaction excites Faraday patterns with the wavenumbers defined not only by a_s and modulation frequency, but also by the amplitude of such oscillation.

In our analysis we have considered both cases of repulsive and attractive two-body interactions. We also present simulations for repulsive three-body interactions in the quartic case, when it is proportional to the fourth power of a_s, considering the case of repulsive two-body interaction, where the behaviour of the resonances can be well identified in agreement with predictions.

For more details, see
Abdullaev, Gammal, Tomio, J. of Phys. B 49, 25302 (2016).

THANK YOU!