Phase transition in the SRG flow of a chiral interaction

Varese S. Timóteo
State University of Campinas - UNICAMP, Limeira, São Paulo, Brazil

Sérgio Szpigel
Presbiterian University Mackenzie, São Paulo, São Paulo, Brazil

Enrique Ruiz Arriola
University of Granada, Granada, Andalucia, Spain

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OUTLINE

- Motivation
- Similarity Renormalization Group (SRG)
- Infrared limit of the SRG evolution (Toy model: 1S0 & 3S1)
- Application: binding energies of light nuclei (Toy & N3LO)
- The on-shell transition (Chiral N3LO: 1S0)
- Final remarks

Chiral N3LO	Toy model
PRELIMINARY	PLB 728 (2014) 596
	PLB 735 (2014) 149
	AoP 353 (2015) 129
	AoP 371 (2016) 398

MOTIVATION

- Infrared limit of the SRG evolution
- Diagonal (on-shell) interaction in momentum space
- Application to few-body and many-body problems
- Phase transition in the SRG flow

MOTIVATION (SRG)

$$\mathcal{H}\ |\psi\rangle \ = \ E\ |\psi\rangle$$

- Pré-diagonalization
- Reduces off-shellness
- Improves convergence in many-body calculations
- Nuclei and Nuclear matter

S. Glazek
K. Wilson

R. Furnstahl
R. Perry
S. Bogner
E. Jurgenson

R. Roth
A. Schwenk
P. Navratil
J. Vary

H. Hammer
K. Hebeler
A. Calci
S. Binder

Renormalization Group Flow

maps the theory at given scale to same theory at different scale

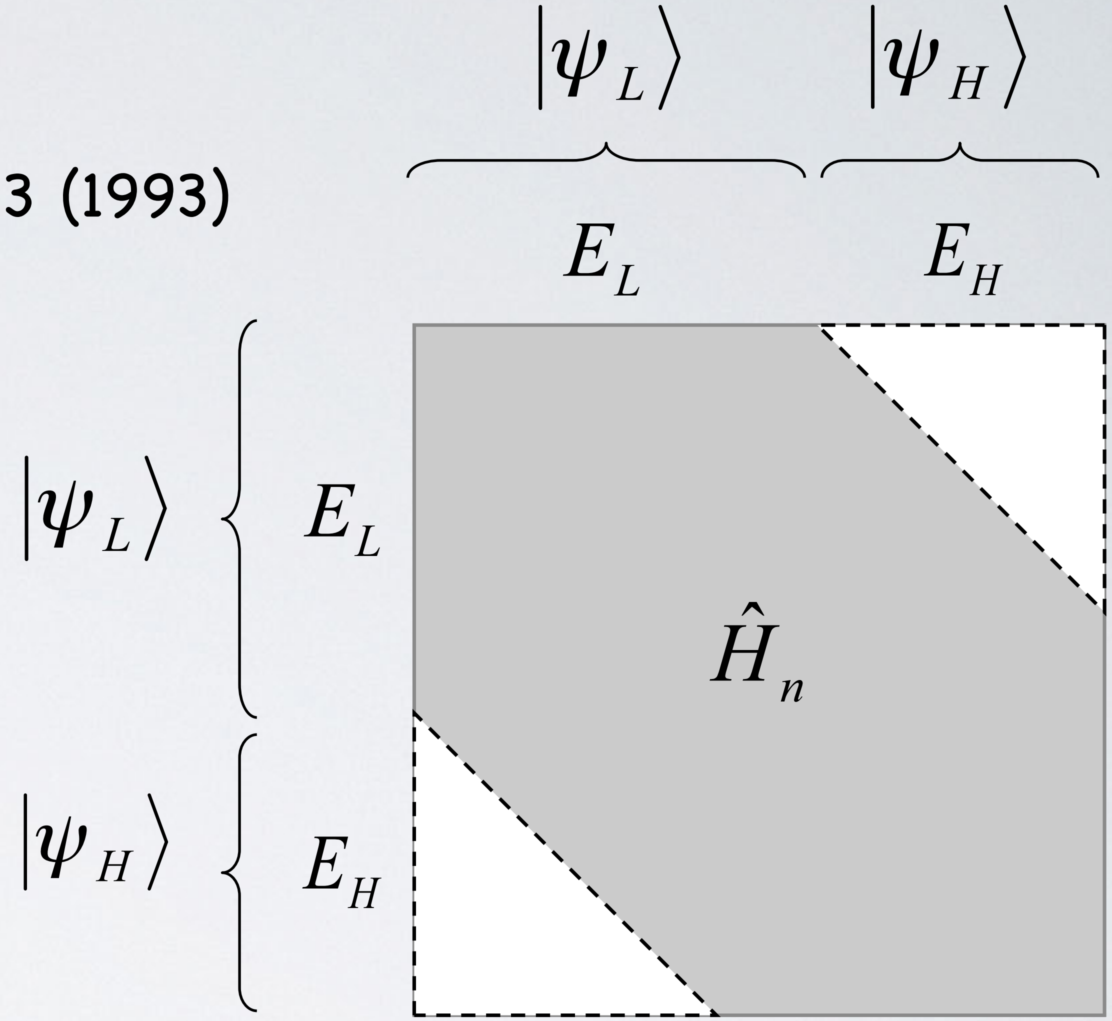
S → S'

resolution scale

Similarity Renormalization Group

Similarity Transformation:

S. D. Glazek and K. G. Wilson, Phys. Rev. D 48, 5863 (1993)



$$H_n[\Lambda_n] = T^{(n)}_{Sim.}\{H_0[\Lambda_0]\}$$

Doesn't remove degrees of freedom

But suppresses states with large energy difference (off-diagonal elements):

$$\langle\psi_L|H|\psi_H\rangle \rightarrow \Lambda_n \leq (E_H - E_L) \leq \Lambda_0$$

↕ **off-shellness**

Unitary transformation: doesn't affect the spectrum → $T^{\dagger}T = 1$

Similarity Renormalization Group

Wegner's formulation:

F. Wegner, Annalen der Physik (Berlin) 3, 77 (1994)

Flow equation:

$$H_s = U(s)\ H\ U^\dagger(s) = T + V_s$$

$$\frac{d}{ds} H_s = [H_s, \eta_s]$$

Flow parameter: $s = \frac{1}{\lambda^4}$ $(0 \leq s \leq \infty)$

similarity cutoff λ: dimension of momentum

Boundary condition: $\lim_{s \to s_0} H_s = H_{s_0}$

Generators for the similarity transformation

Free hamiltonian (kinetic energy): $\eta_s = [H_s, T]$

Diagonal part of the running hamiltonian: $\eta_s = [H_s, H_D]$

SRG - WILSON GENERATOR

(two-nucleon system)

$$\eta_s = [H_s, T] \longrightarrow \frac{d}{ds} H_s = [H_s, [H_s, T]]$$

$$\frac{d}{ds} V_s(p,p') = -(p^2 - p'^2)\; V_s(p,p') + \frac{2}{\pi} \int dq\; q^2\; (p^2 + p'^2 - 2q^2)\; V_s(p,q)\; V_s(q,p')$$

S. Szpigel and R. J. Perry, in "Quantum Field Theory, A 20th Century Profile", ed. A. N. Mitra, Hindustan Publishing, New Delhi (2000)

S.K. Bogner, R.J. Furnstahl, and R.J. Perry, Phys. Rev. C **75**, 061001(R) (2007)

S.K. Bogner, R.J. Furnstahl, R.J. Perry, and A. Schwenk, Phys. Lett. B **649**, 488 (2007)

E.D. Jurgenson, P. Navratil, R.J. Furnstahl, Phys. Rev. Lett. **103** (2009) 082501

$V_{s=0}$ ⟶ regular or regularised

SRG - WEGNER GENERATOR

(two-nucleon system)

$$\lambda = \frac{1}{\sqrt[4]{s}}$$

$$\eta_s = [\ H_s\ ,\ \mathrm{diag}(H_s)\ ]$$

$$\frac{d}{ds} H_s = [\ H_s\ , [\ H_S\ , \mathrm{diag}(H_s)\ ]\ ]$$

$$T\ |p\rangle = p^2\ |p\rangle \qquad [\mathrm{diag}(H_s)]|p\rangle = \epsilon_p |p\rangle$$

$$\frac{d}{ds} V_s(p,p') = \frac{2}{\pi} \int_0^\infty dq\ q^2\ (\epsilon_p + \epsilon_{p'} - 2\epsilon_q)\ H_s(p,q)\ H_s(q,p')$$

TOY MODEL - 1S0 & 3S1

Typical SRG calculation with Av18:

Momentum grid: $N = 200 \quad , \quad p_{\max} = 30 \text{ fm}^{-1}$

$\lambda \sim 1 \text{ fm}^{-1}$ computational time: 100 - 1000 hours

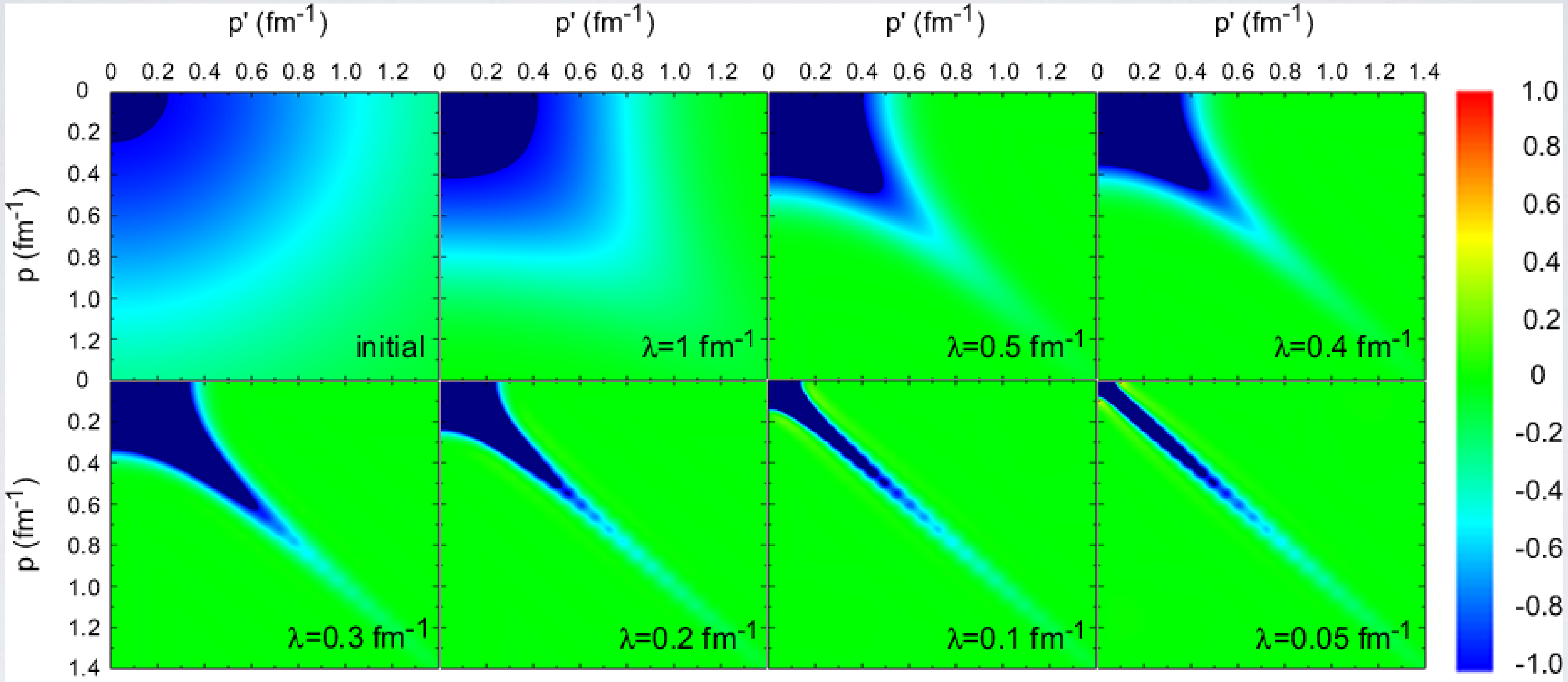
$$V(p,p') = C\ e^{-(p^2+p'^2)/L^2}$$

$p_{\max} = 2 \text{ fm}^{-1}$

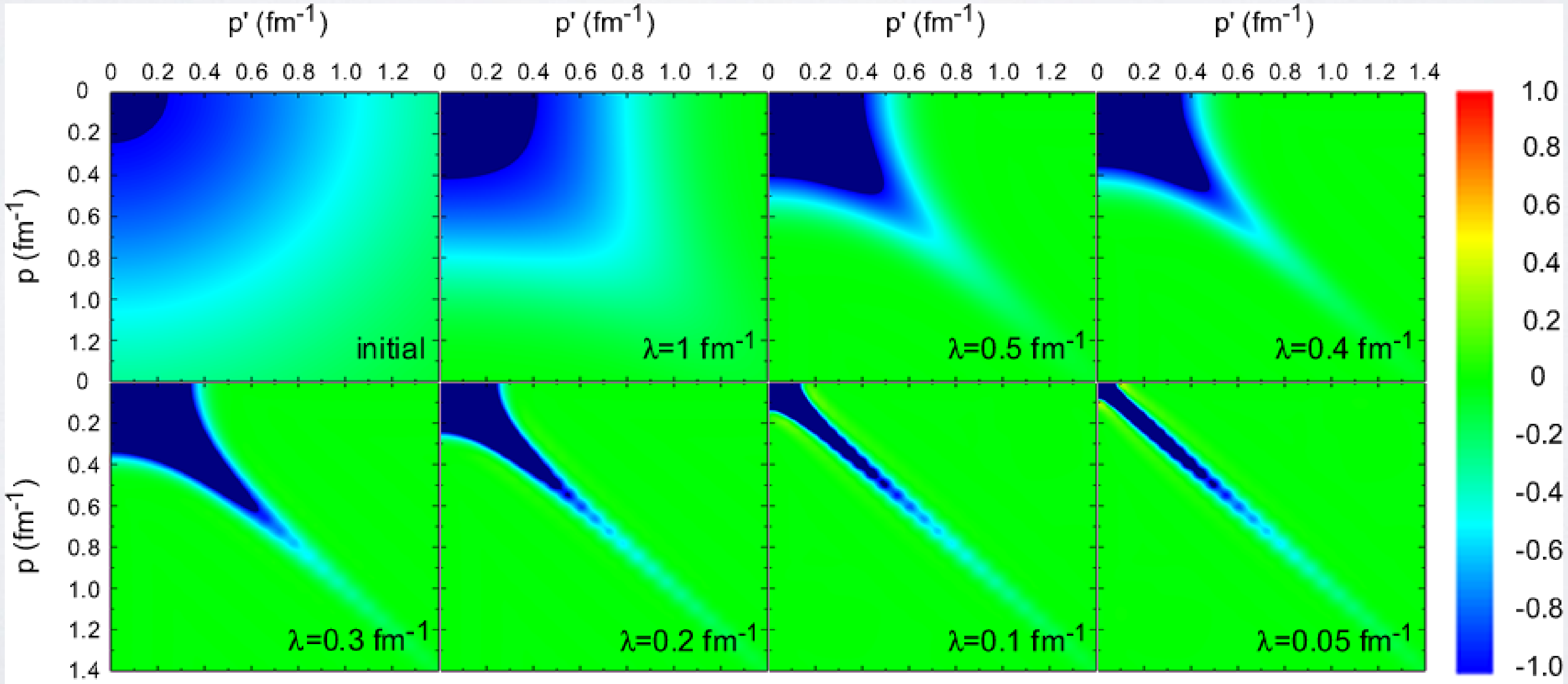
Parameter Units	α_0 (fm)	r_0 (fm)	C (fm)	L (fm^{-1})
$^1S_0$	-23.74	2.77	-1.9158	0.6913
$^3S_1$	5.42	1.75	-2.3006	0.4151

SRG evolution - Toy - 1S0

Wilson

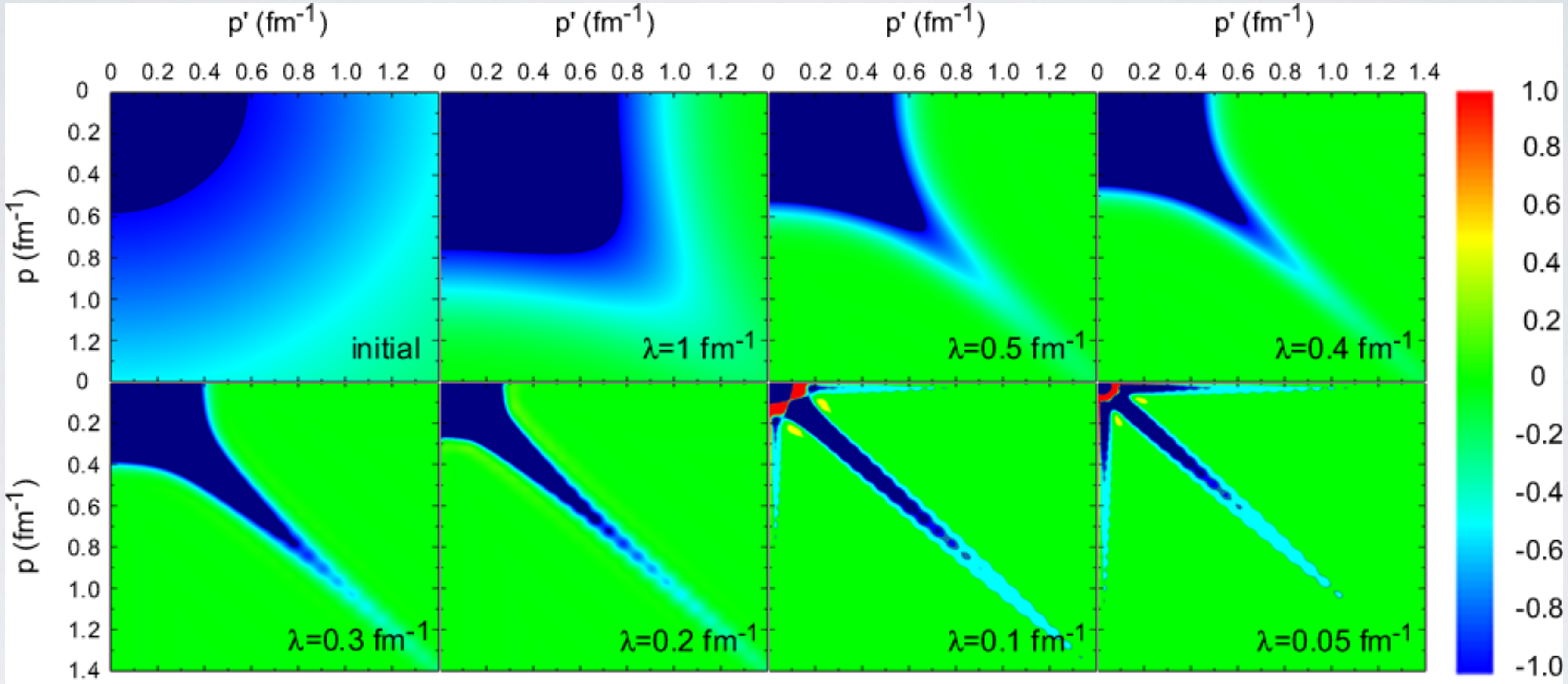


Wegner

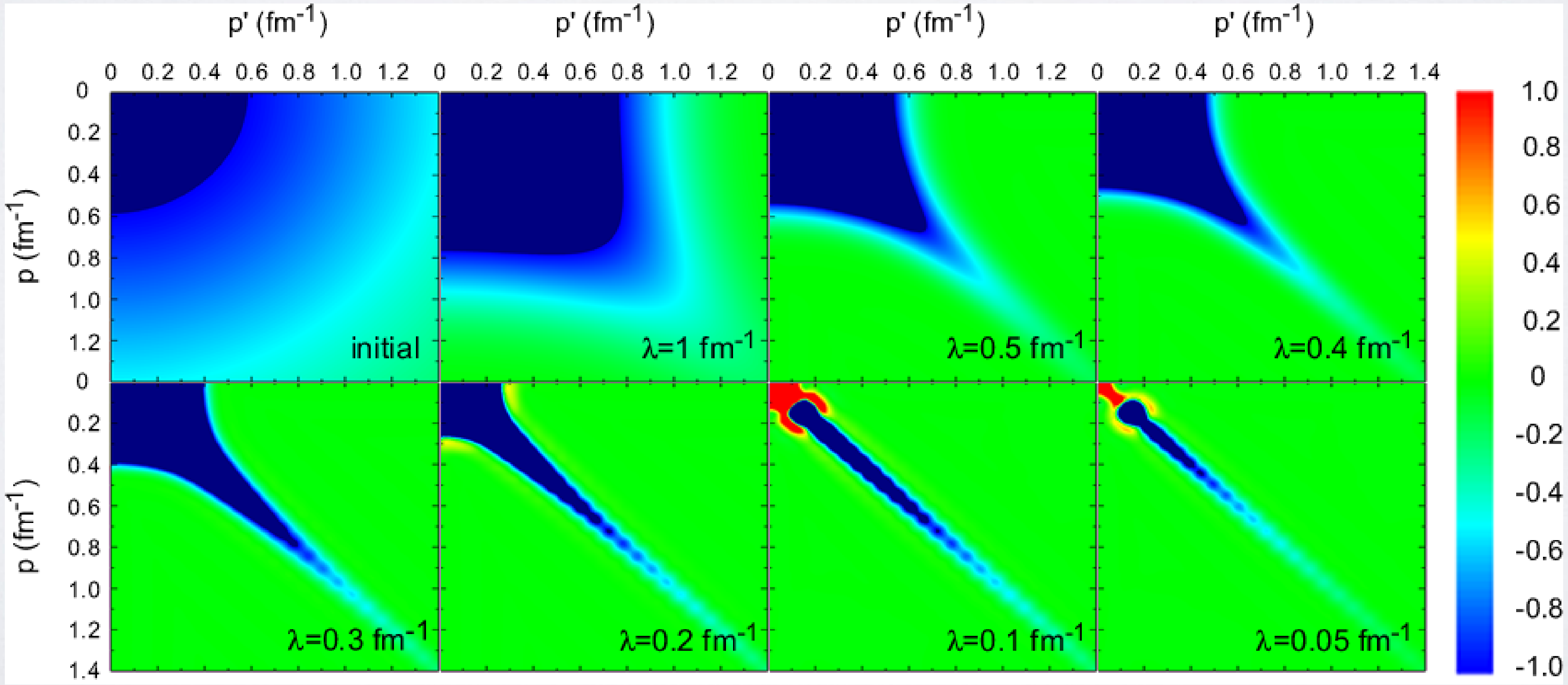


SRG evolution - Toy - 3S1

Wilson

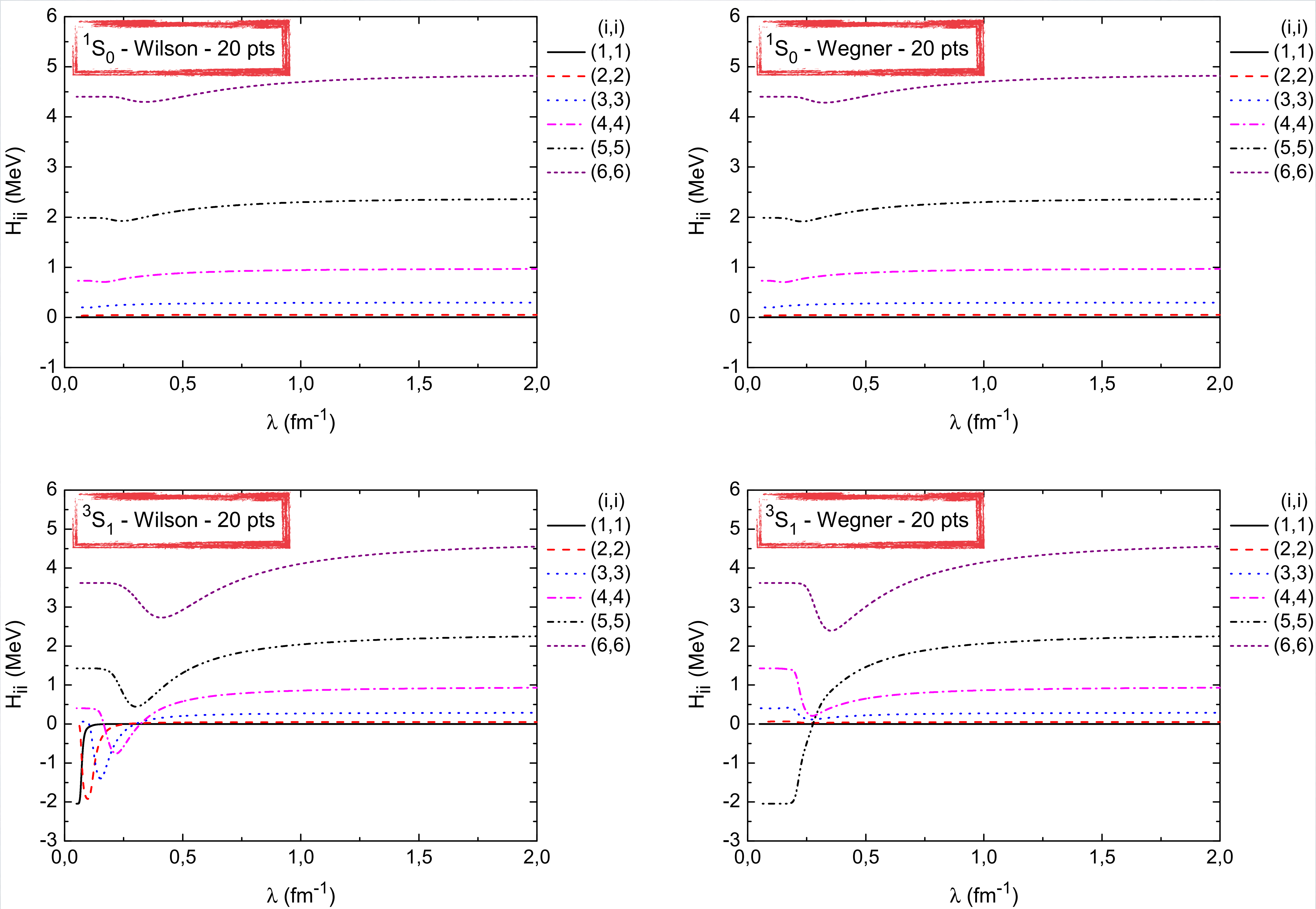


Wegner



H MATRIX ELEMENTS IN THE LIMIT $\lambda \to 0$

E. Ruiz Arriola, S. Szpigel, VST, Physics Letters B **735** (2014) 149



Energy-shift formulas

E. Ruiz Arriola, S. Szpigel, VST, Physics Letters B **735** (2014) 149

$$\mathcal{H}\,|\psi\rangle = E\,|\psi\rangle \longrightarrow \left[p_n^2 + \frac{2}{\pi}\sum_k p_k^2\, w_k\, V_{nk} \right]\psi = P^2\,\psi$$

At $\lambda \to 0$, V is diagonal:

$$P_n^2 = p_n^2 + \frac{2}{\pi}\, p_n^2\, w_n\, V_n^{\lambda=0}$$

$$V_n^{\lambda=0} = \frac{1}{\frac{2}{\pi}p_n^2 w_n}\left(P_n^2 - p_n^2\right)$$

ordering required

$$\delta(p_n) = -\frac{p_n}{\frac{2}{\pi}p_n^2 w_n}\left(P_n^2 - p_n^2\right) = -\,p_n\, V_n^{\lambda=0}$$

Interaction at $\lambda = 0$ and can be calculated directly from the eigenvalues

Phase-shifts can be computed without solving the scattering equation

Possible orderings

O. Rubtsova, V. Kukulin, V. Pomerantsev, A. Faessler, Physical Review C **81**, 064003 (2010)

$$\delta_n^{\text{Kuk}} = -\pi \frac{P_{n+n_B}^2 - p_n^2}{2 w_n p_n}$$

ascending with shift to discard BS

E. Ruiz Arriola, S. Szpigel, VST, Physics Letters B **735** (2014) 149

$$\delta_n^{\text{Wil}} = -\pi \lim_{\lambda \to 0} \frac{H_{n,n}^{\text{Wil},\lambda} - p_n^2}{2 w_n p_n} = -\pi \frac{P_n^2 - p_n^2}{2 w_n p_n}$$

ascending with no shift

SRG with Wilson generator

$$\delta_n^{\text{Weg}} = -\pi \lim_{\lambda \to 0} \frac{H_{n,n}^{\text{Weg},\lambda} - p_n^2}{2 w_n p_n}$$

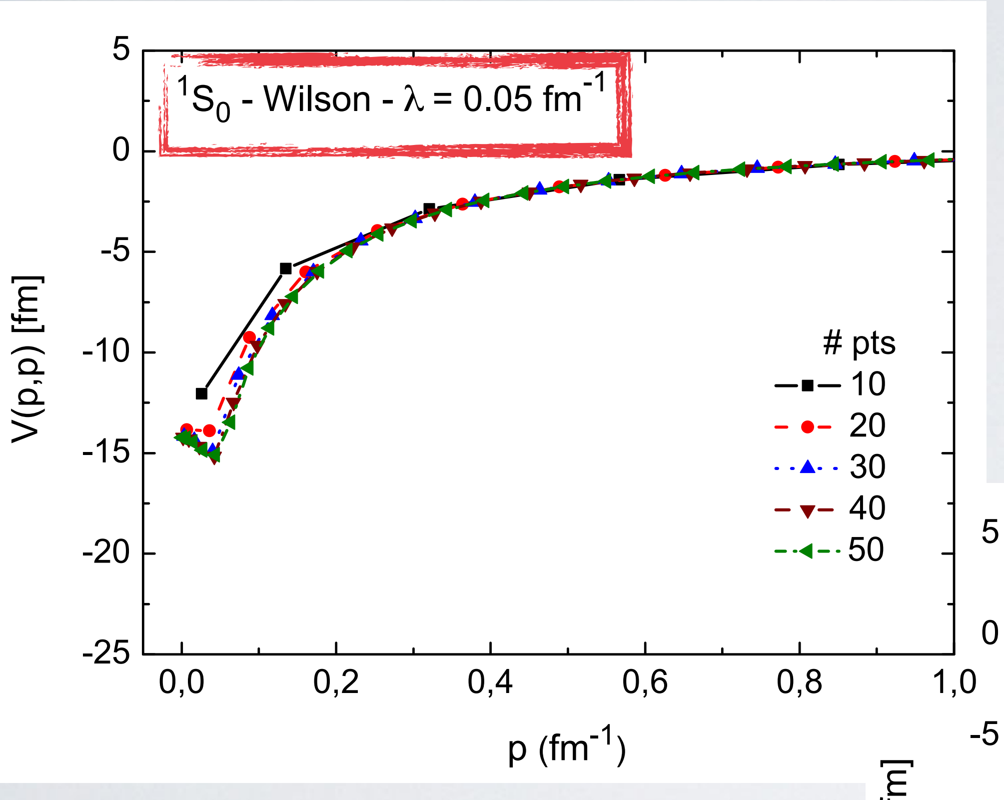
shift below BS,
ascending above BS

SRG with Wegner generator

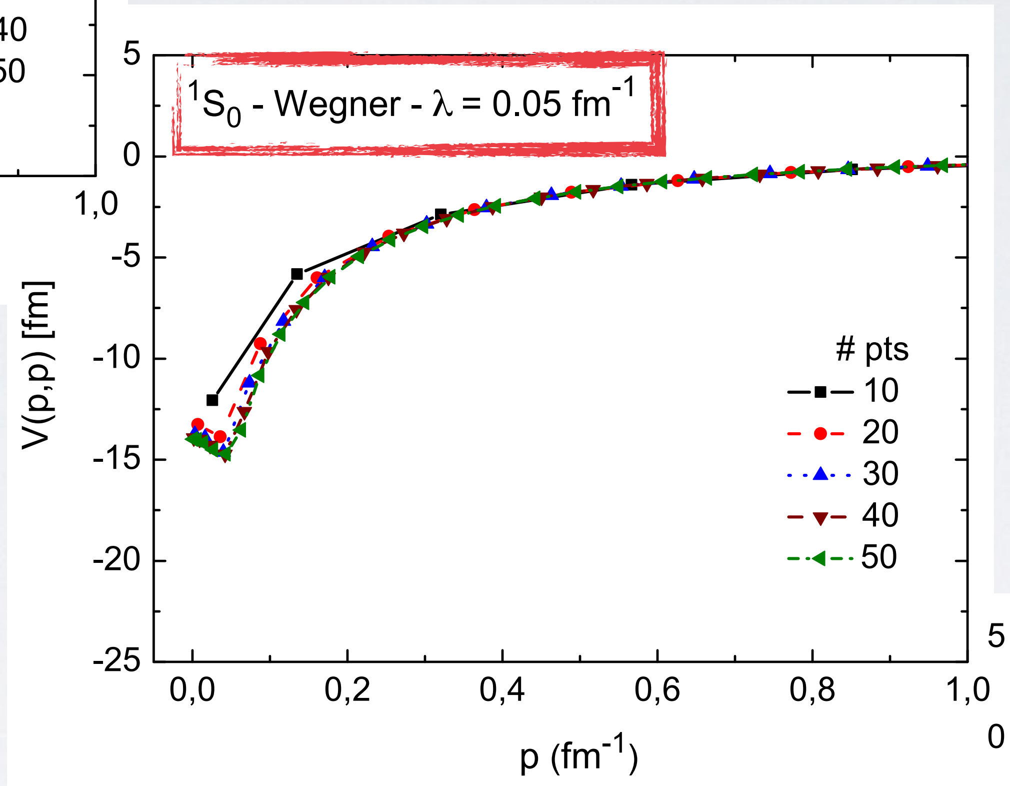
$$\lim_{\lambda \to 0} H_{n,m}^{\text{Weg},\lambda} = \delta_{n,m} \begin{cases} P_{n+1}^2 & \text{if} \quad n < n_c \\ -\gamma^2 & \text{if} \quad n = n_c \\ P_n^2 & \text{if} \quad n > n_c \end{cases}$$

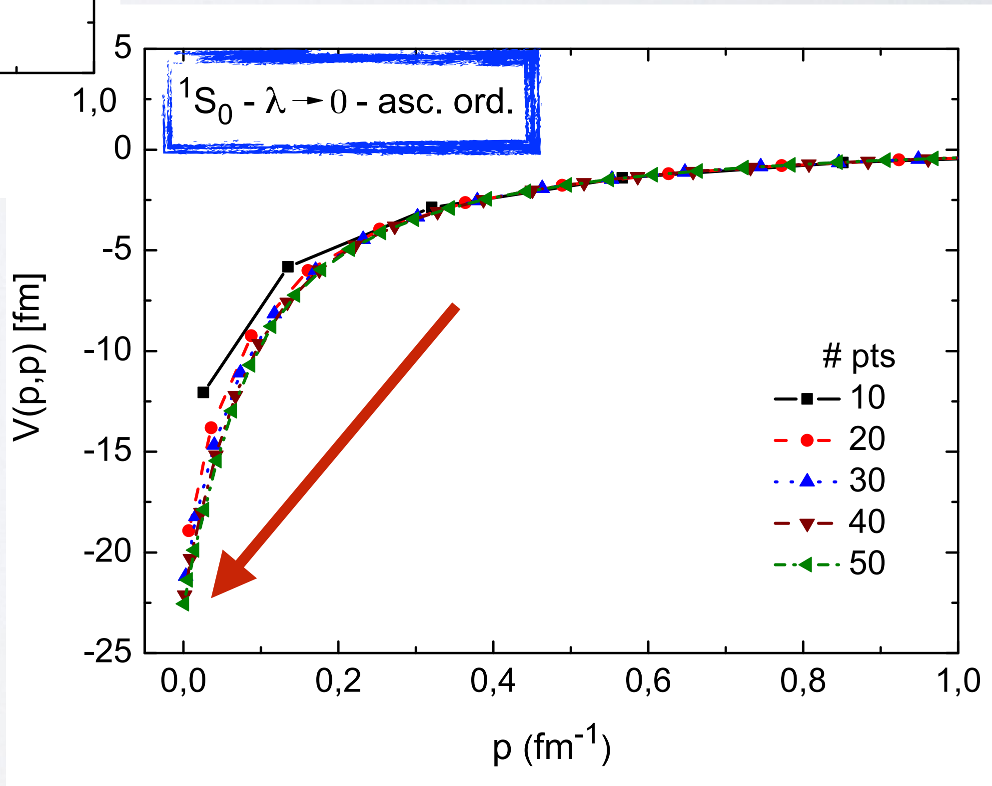
$$B_d = \gamma^2 / M$$

NN toy potential in the infrared limit - 1S0



E. Ruiz Arriola, S. Szpigel, VST, Annals of Physics (2016)



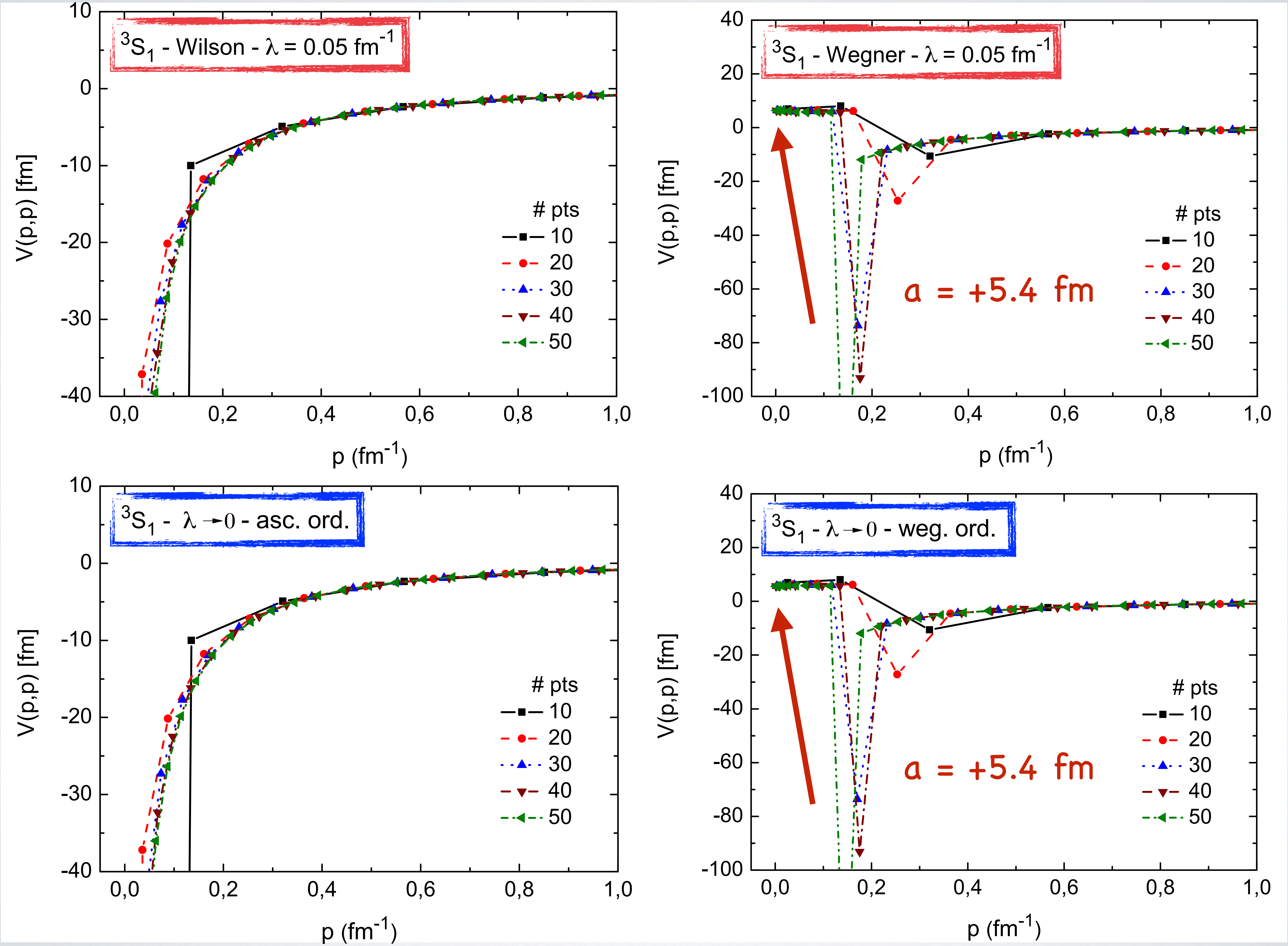


a = -23.7 fm

correct p -> 0 limit: V(0,0) = a

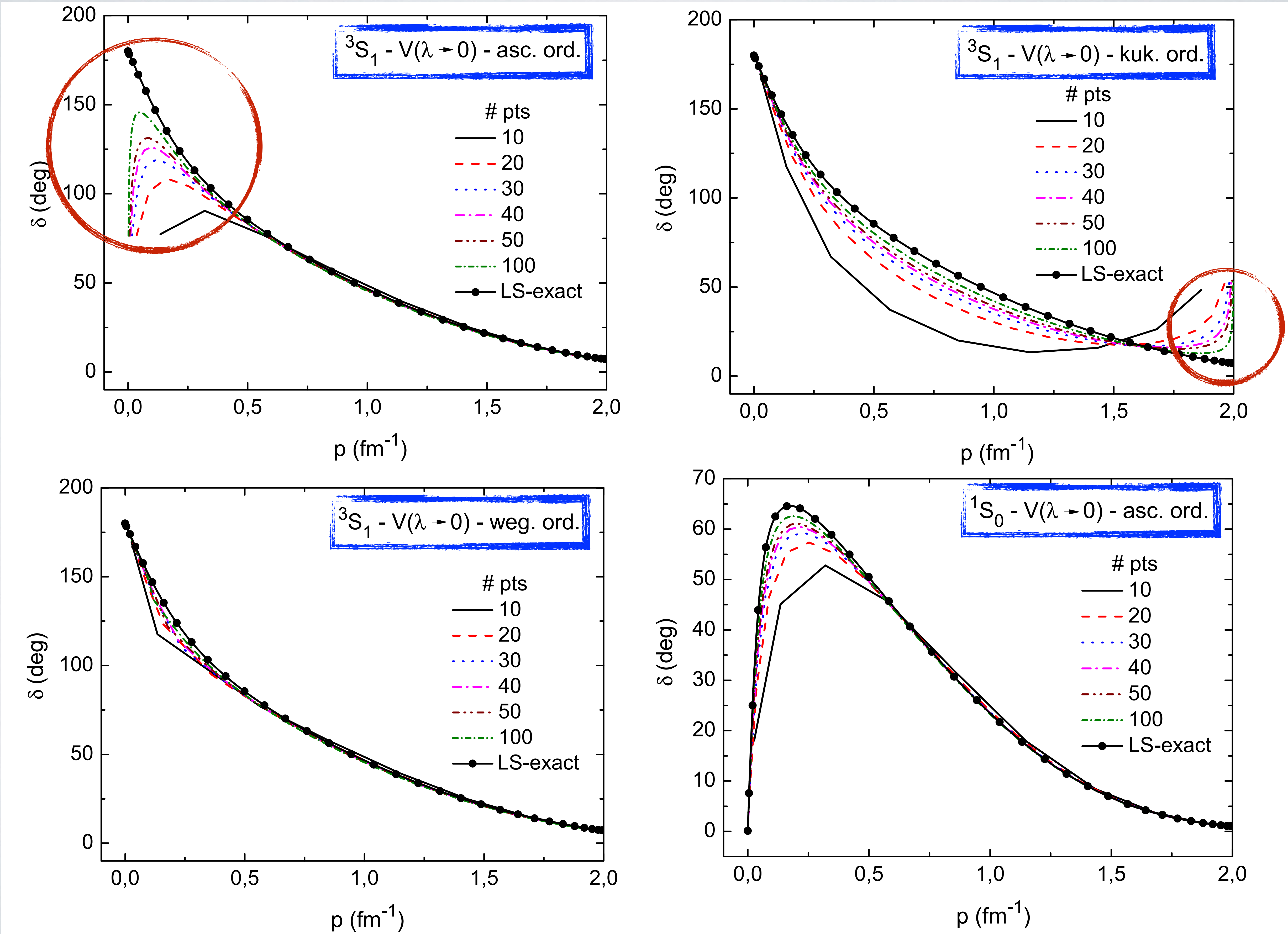
NN toy potential in the infrared limit - 3S1

E. Ruiz Arriola, S. Szpigel, VST, Annals of Physics (2016)



NN phase-shifts in the limit $\lambda \to 0$

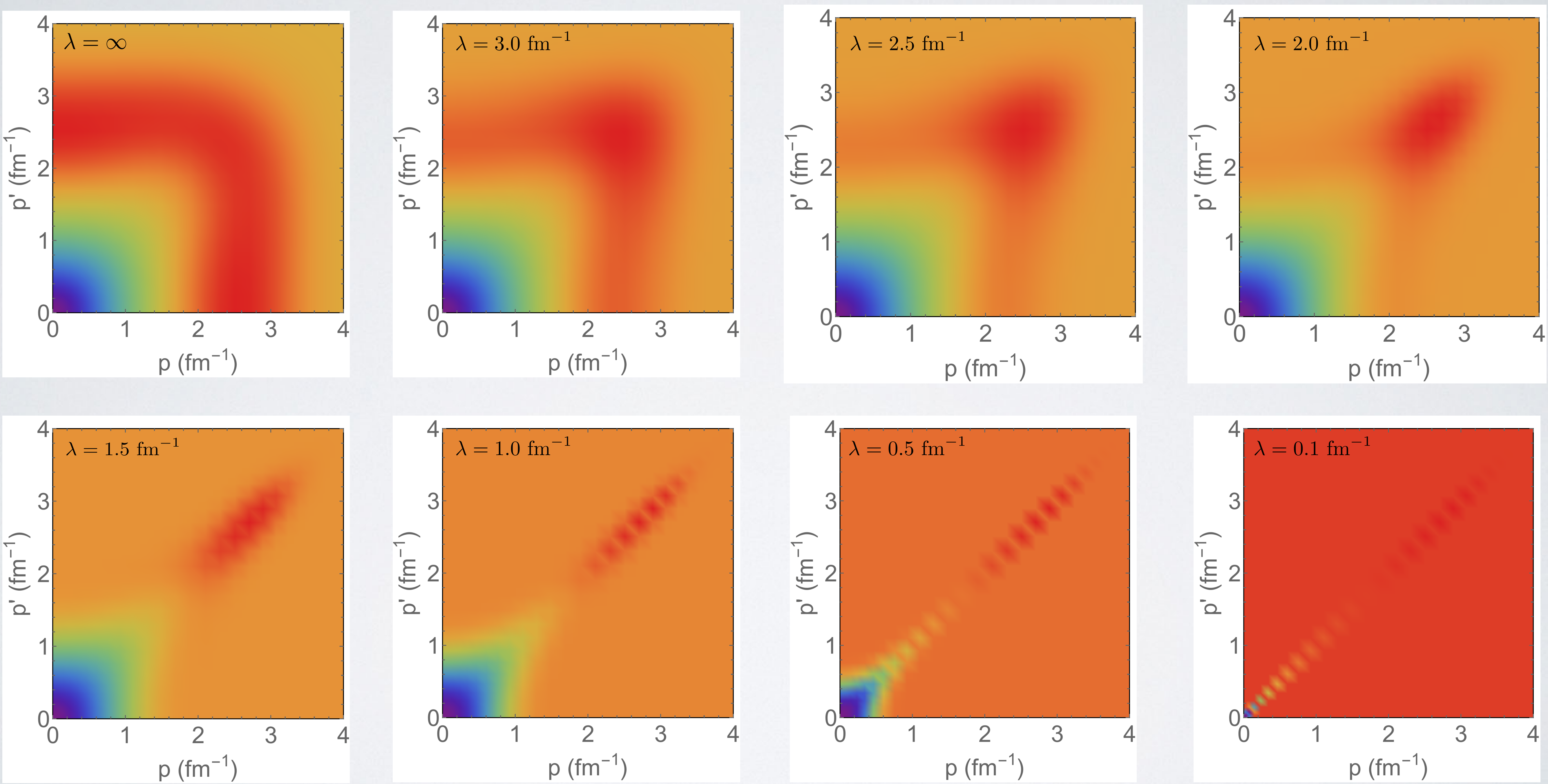
E. Ruiz Arriola, S. Szpigel, VST, Physics Letters B **735** (2014) 149



SRG evolution - Chiral N3LO - 1S0

D. R. Entem and R. Machleidt, Phys. Rev. C 68 (2003) 041001

E. Epelbaum, W. Glöckle and U.-G. Meissner, Nucl. Phys. A 747 (2005) 362

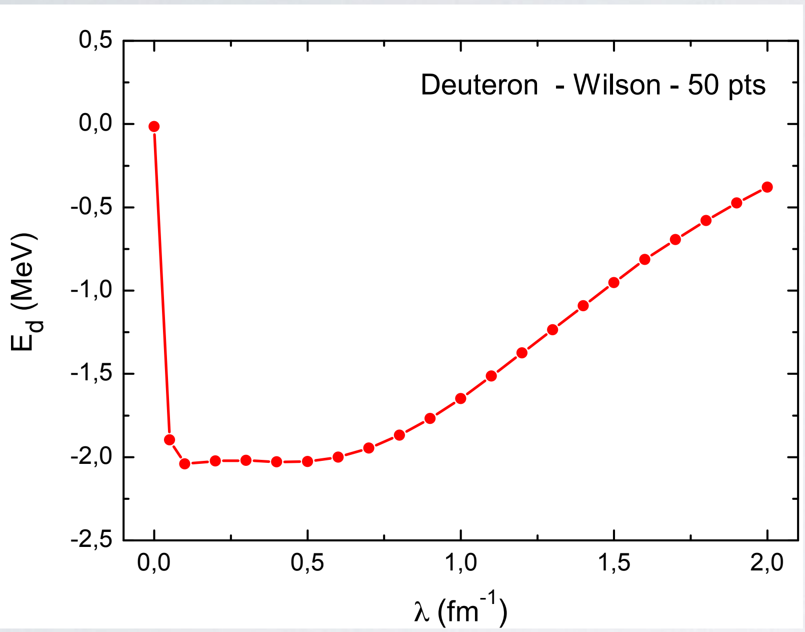


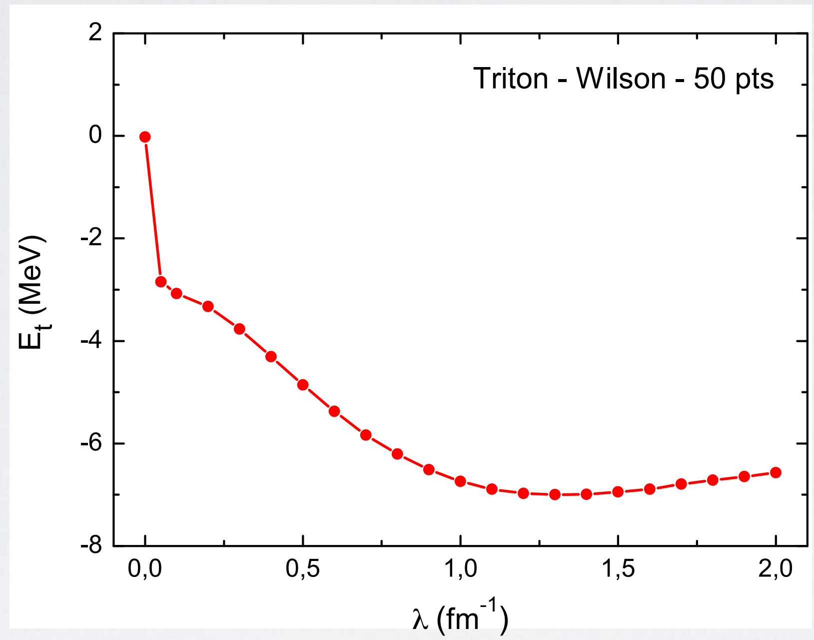
Variational binding energies (Toy)

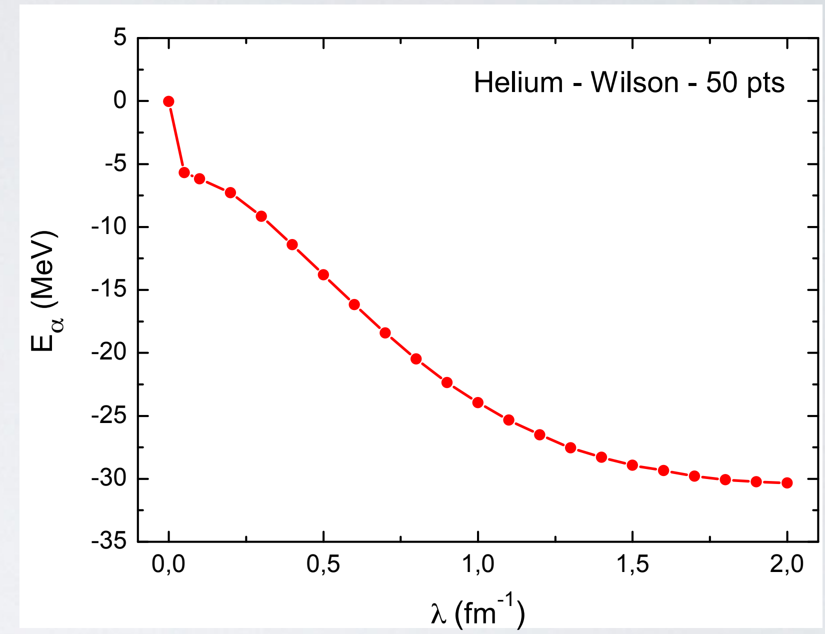
no three-body force

TOY

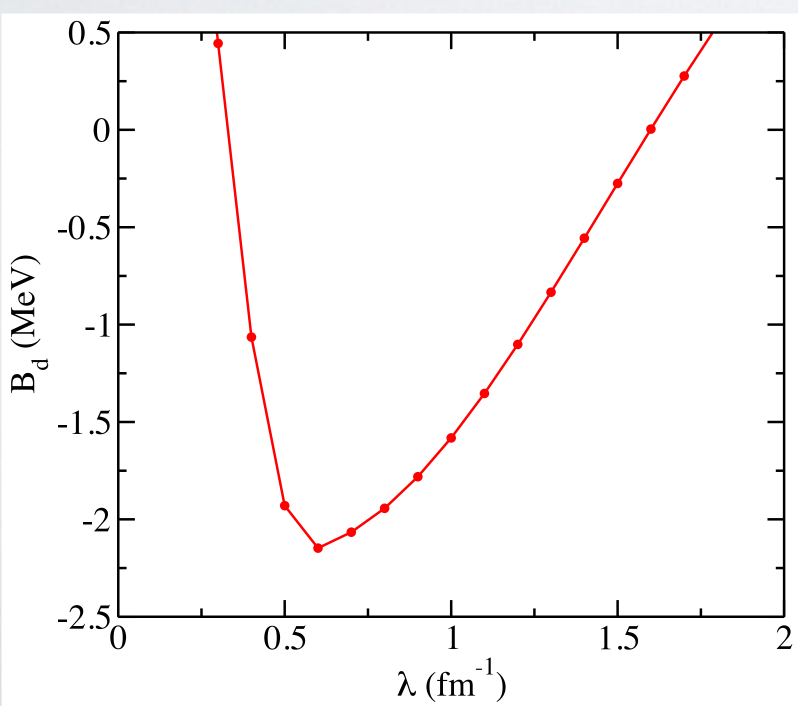
E. Ruiz Arriola, S. Szpigel, VST, Annals of Physics 371 (2016) 398

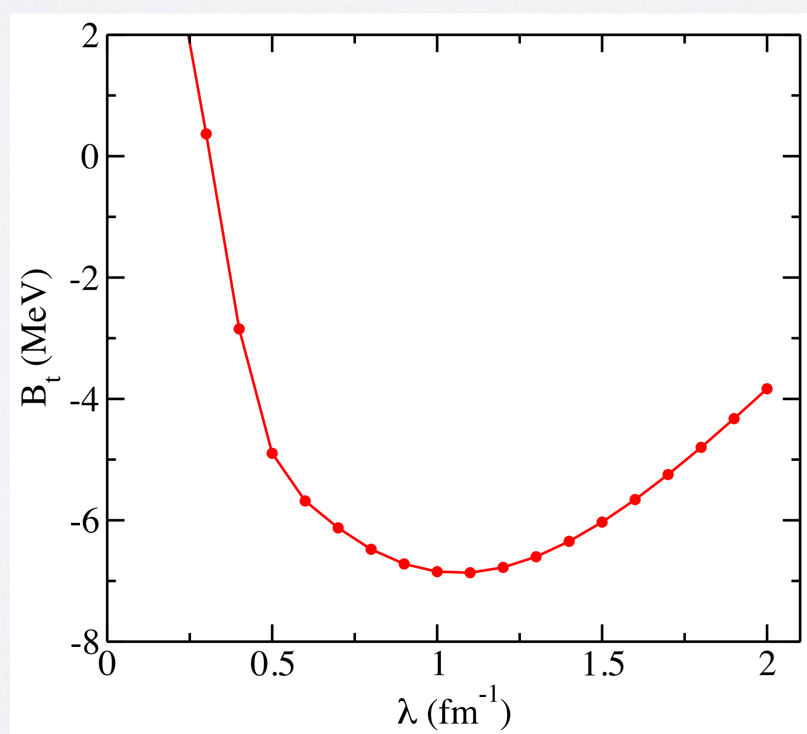


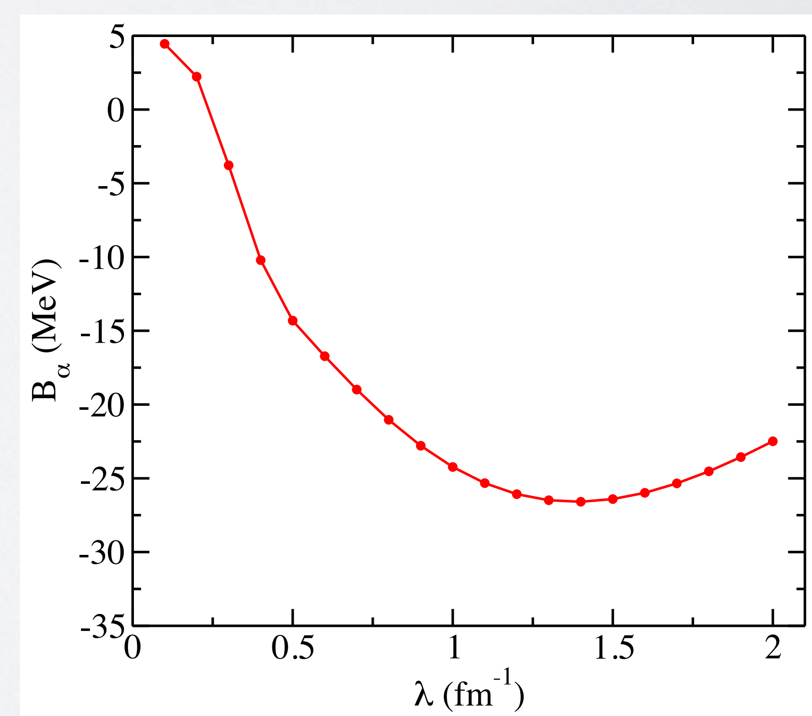




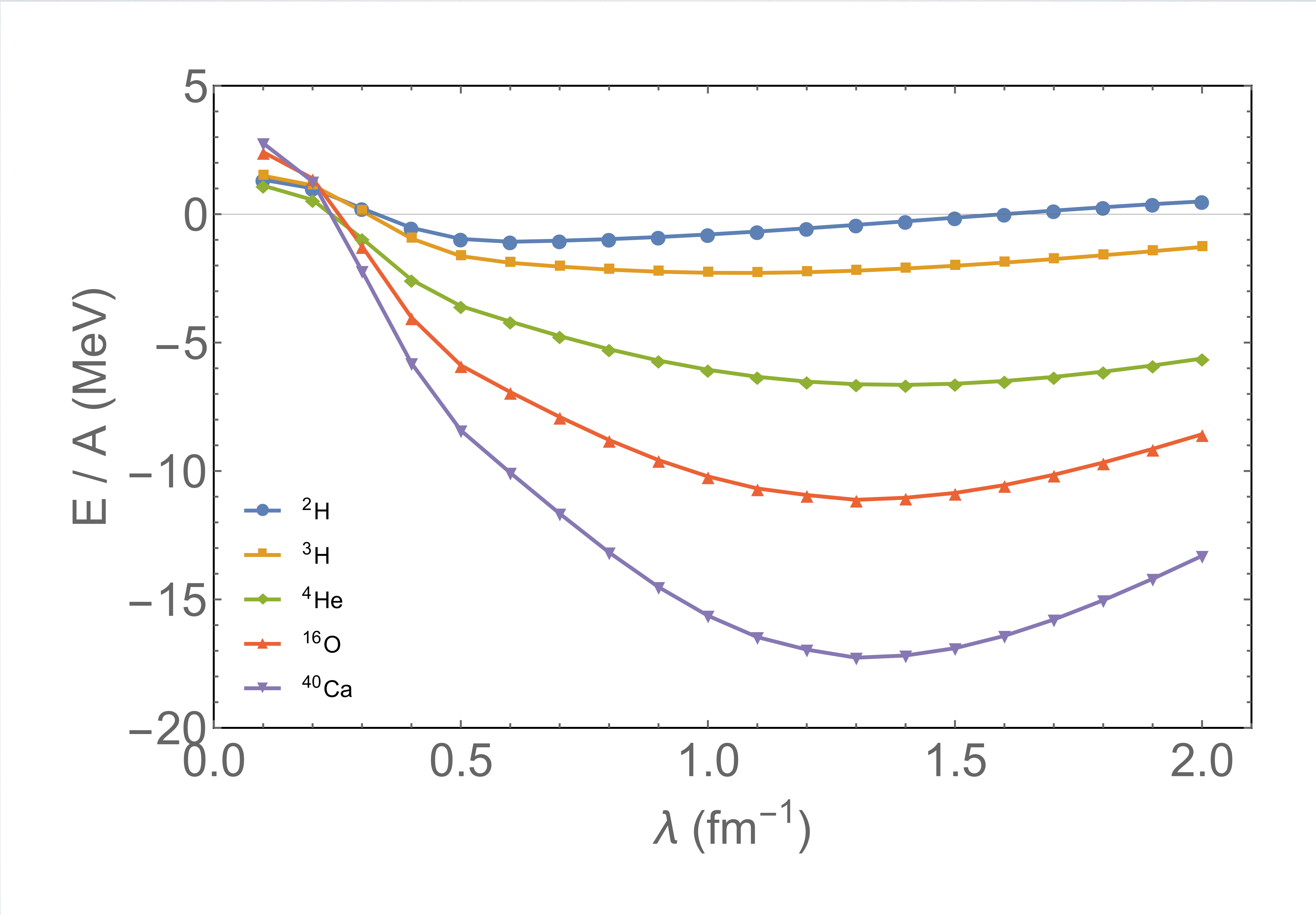
N3LO





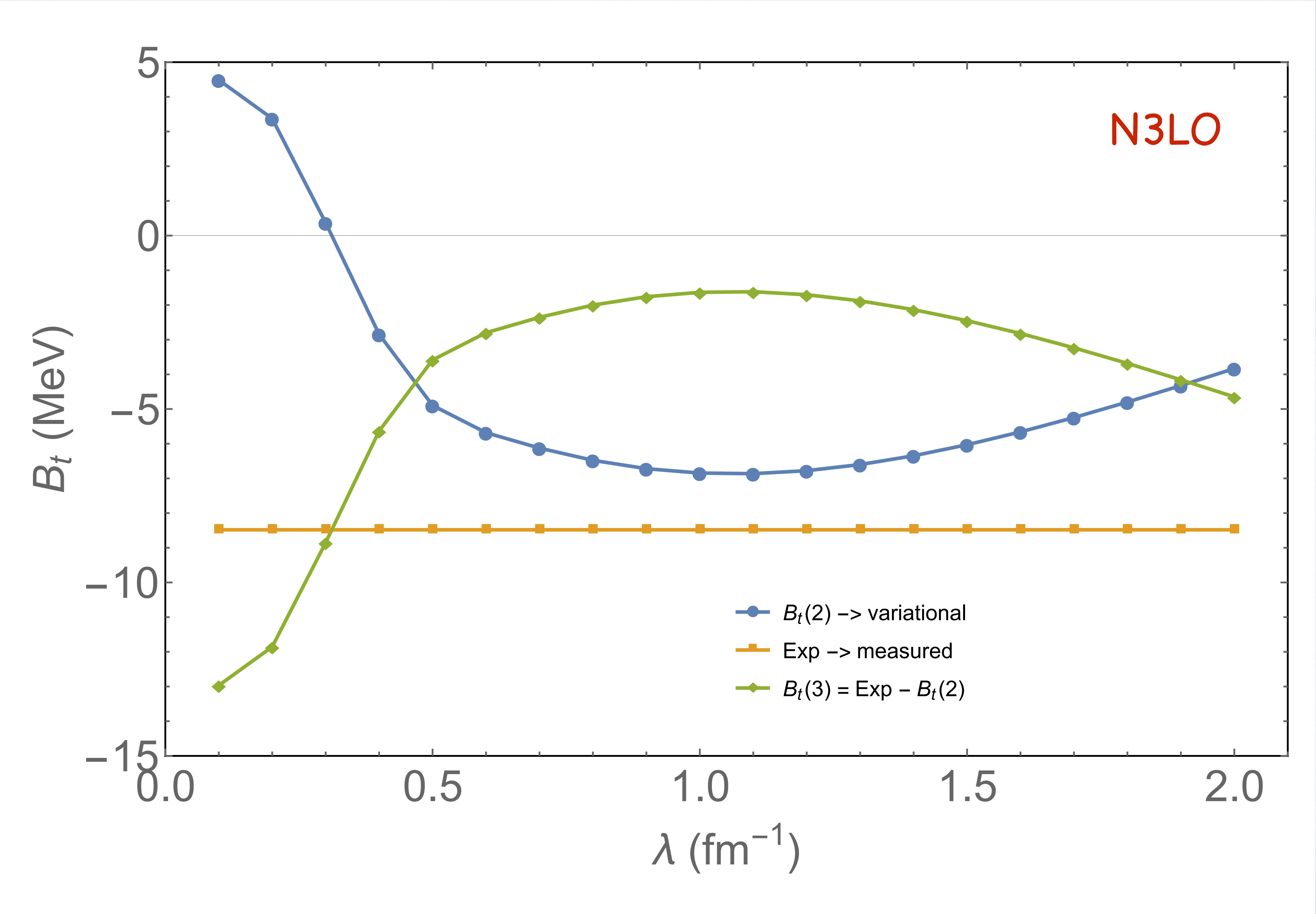


Binding energies per nucleon (N3LO)



3N force in the limit $\lambda \to 0$: Triton

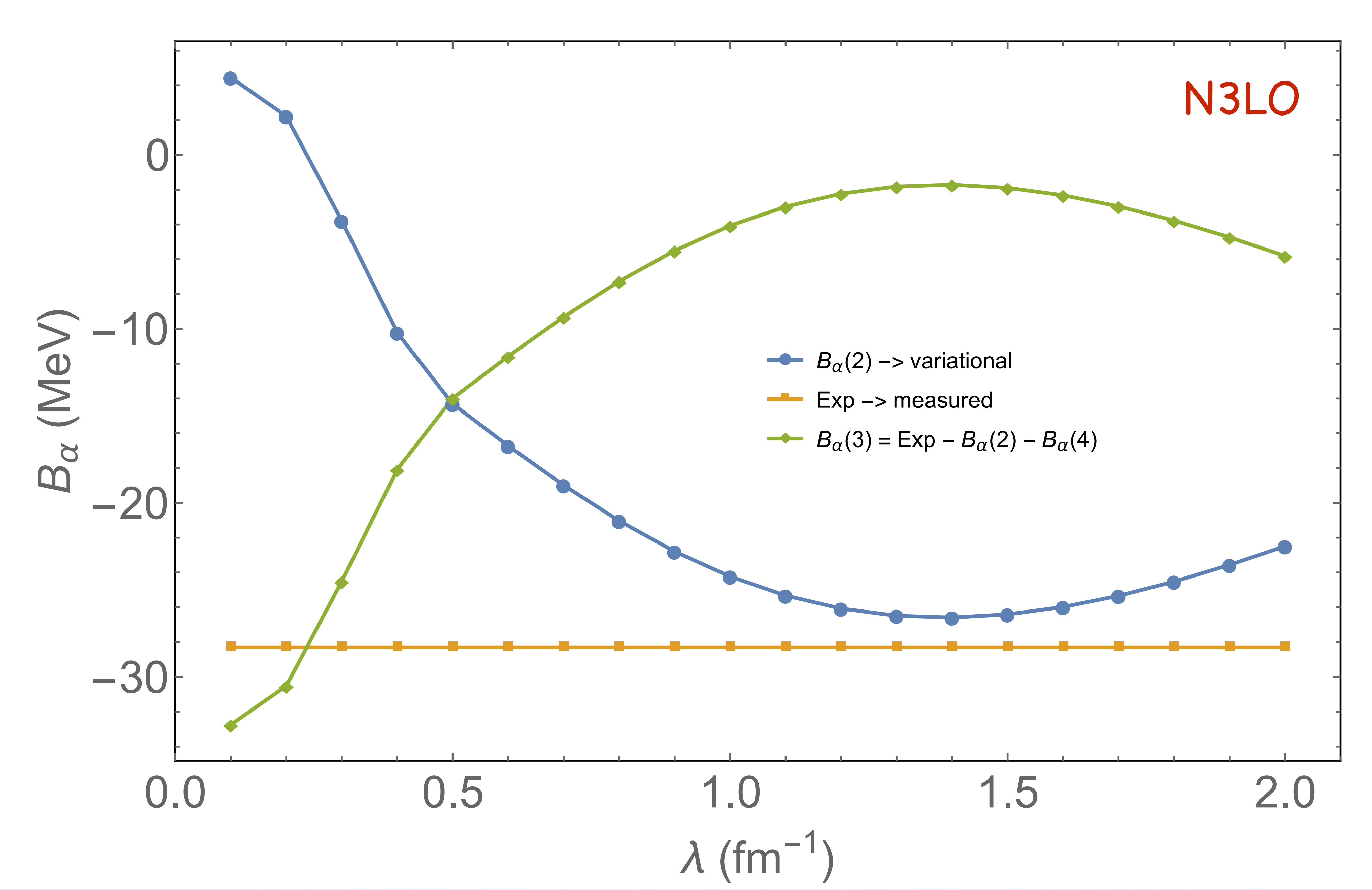
$$B_t^\lambda(3) = B_t(\exp) - B_t^\lambda(2)$$



3N force in the limit $\lambda \to 0$: Helium

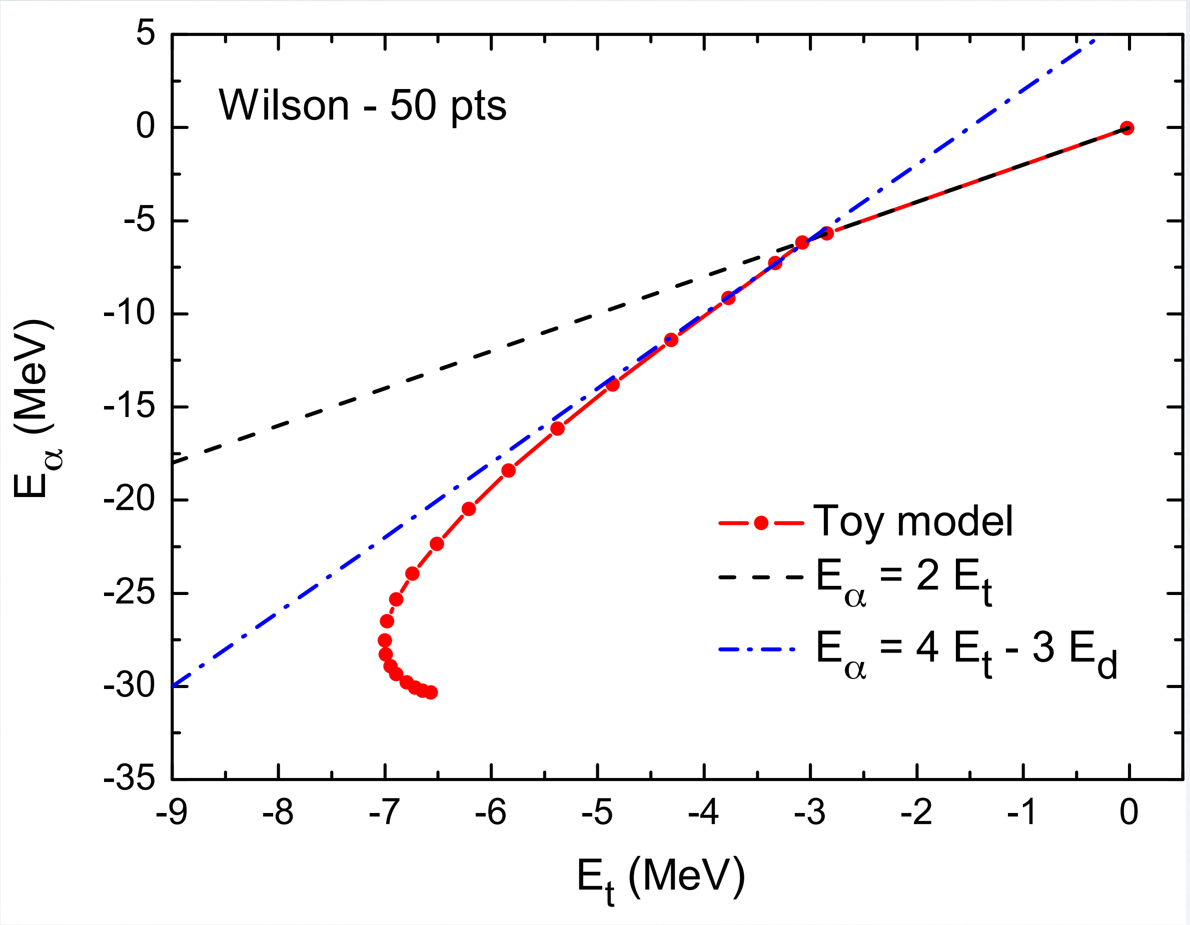
$$B^{\lambda}_{\alpha}(3) = B_{\alpha}(\mathrm{exp}) - B^{\lambda}_{\alpha}(2) - B^{\lambda}_{\alpha}(4)$$

if $B^{\lambda}_{\alpha}(4) \sim 0$ $\longrightarrow$ $B^{\lambda}_{\alpha}(3) = B_{\alpha}(\mathrm{exp}) - B^{\lambda}_{\alpha}(2)$



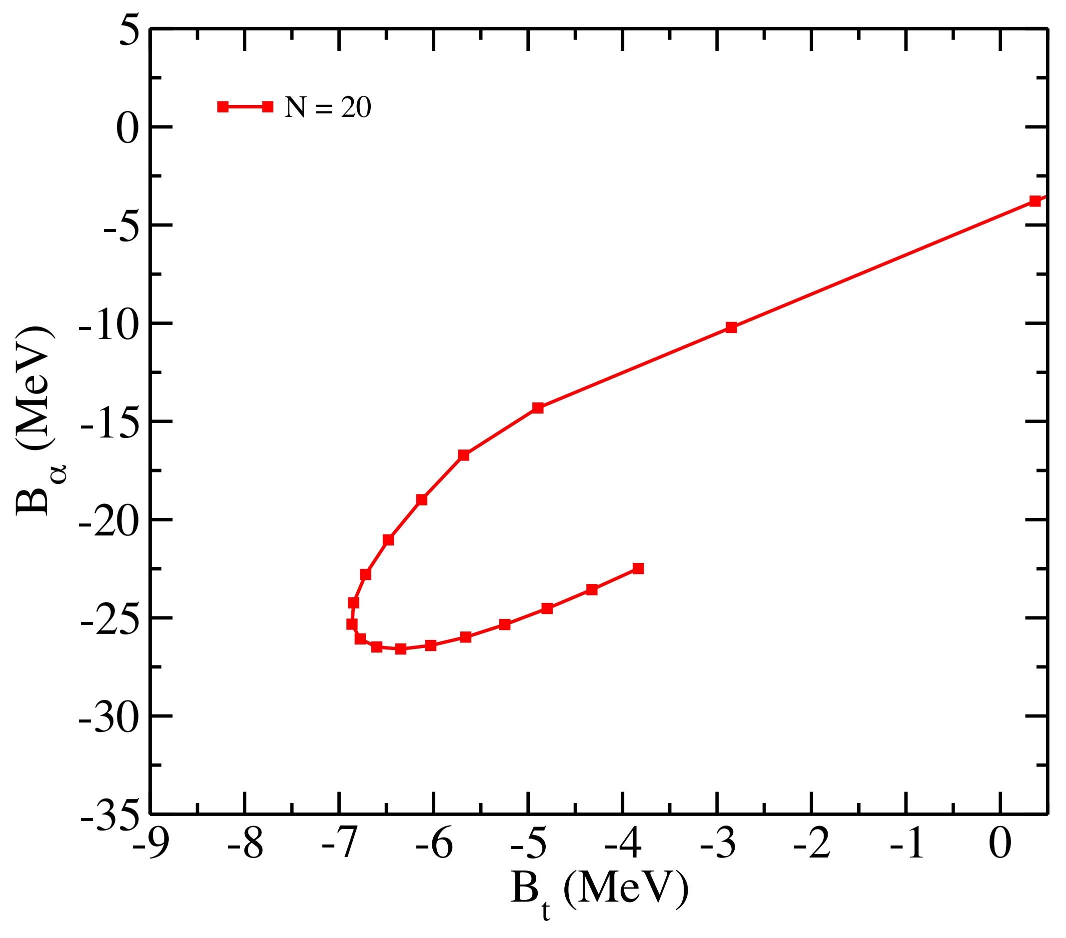
Tjon line

Toy model



Only S-waves, no repulsion

Chiral N3LO



Up to G-waves

But how can we quantify off-shellness ?

The Frobenius norm:

$$\phi = ||V_\lambda|| = \sqrt{\text{Tr } V_\lambda^2}$$

$$V_\lambda^2 = \frac{2}{\pi} \int_0^\infty dq \; q^2 \; V_\lambda(p,q) \; V_\lambda(q,p')$$

Order parameter:

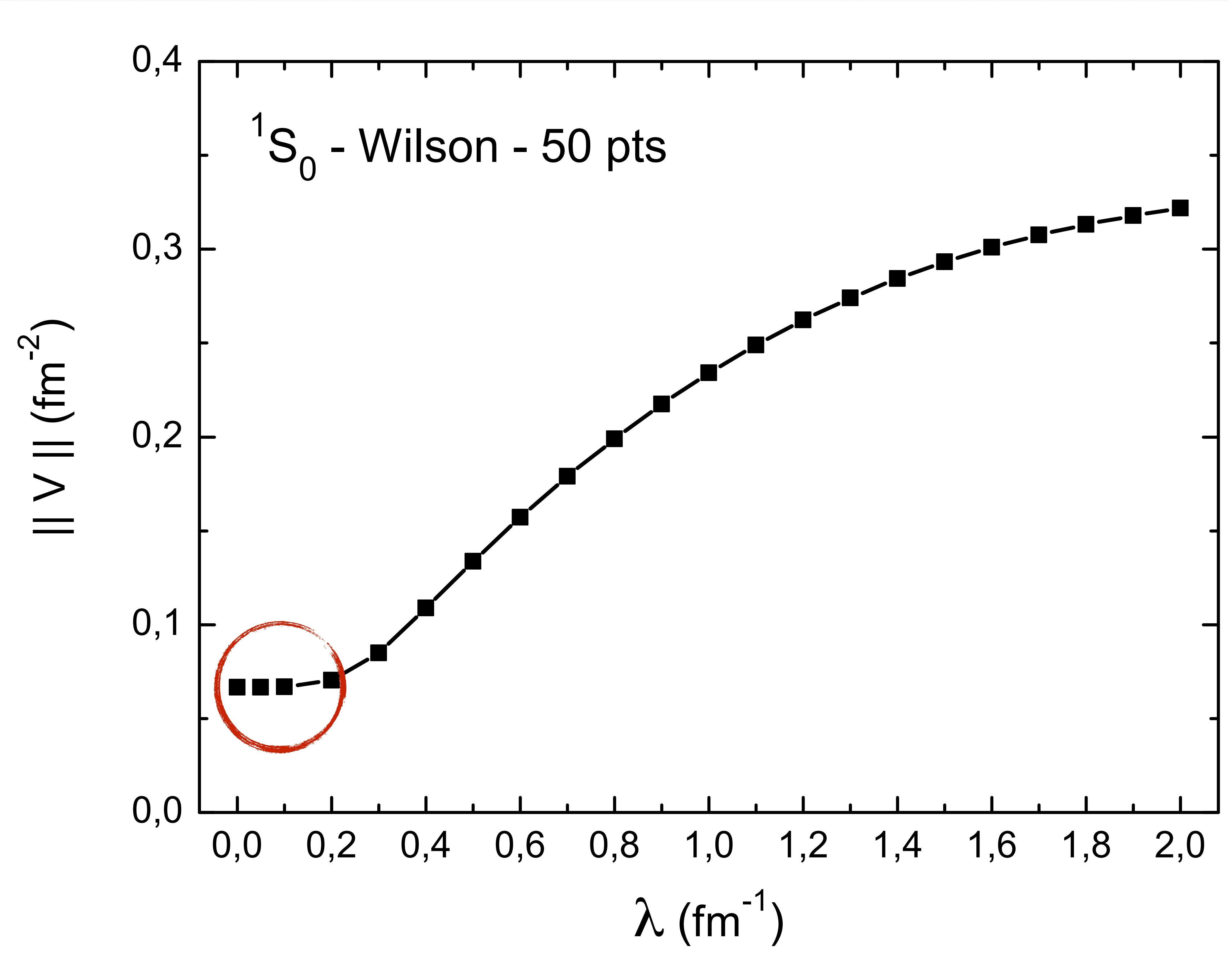
$$\beta = \frac{d\phi}{d\lambda}$$

Similarity susceptibility:

$$\eta = \frac{d\beta}{d\lambda}$$

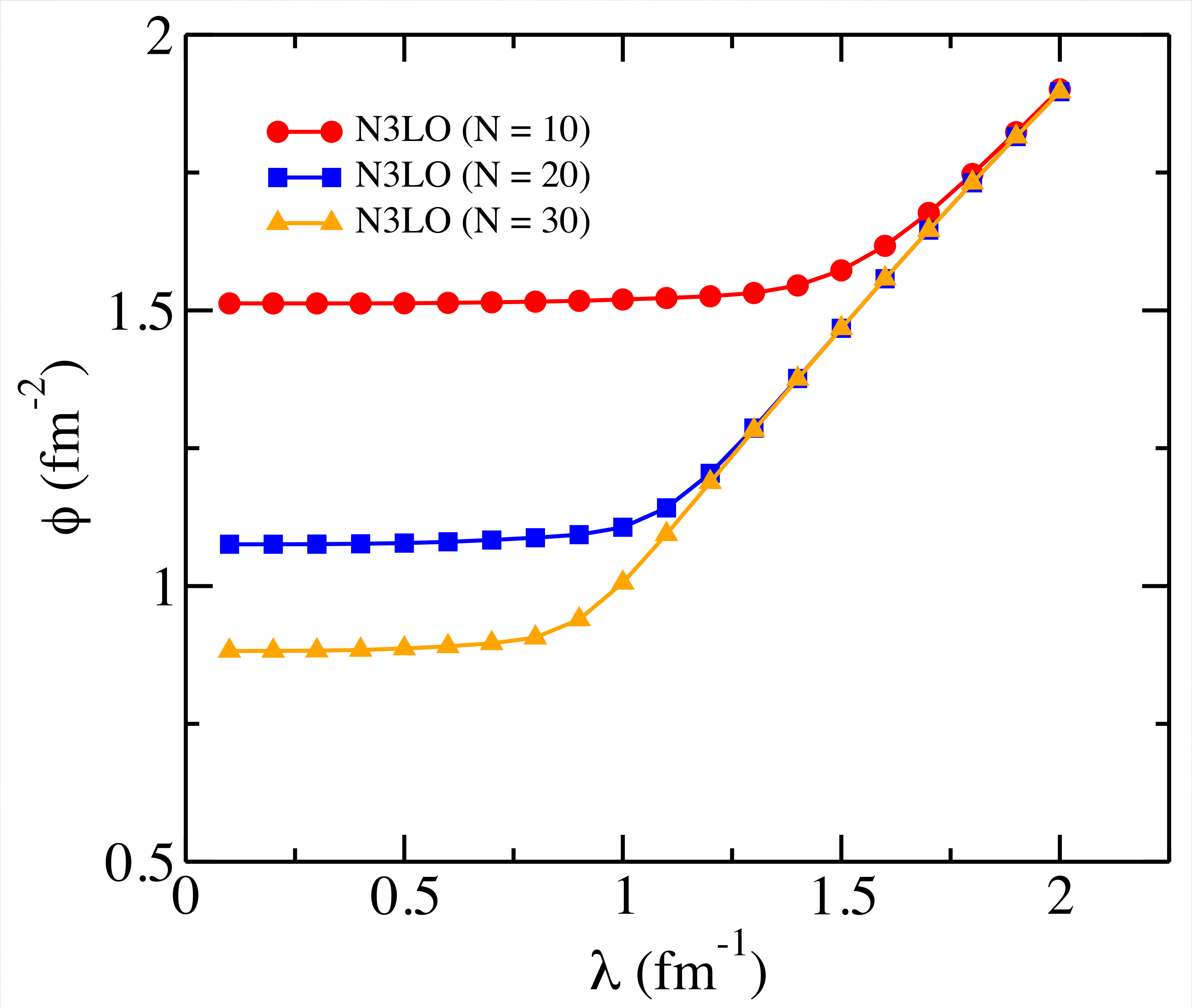
The Frobenius norm

Toy model - 1S0



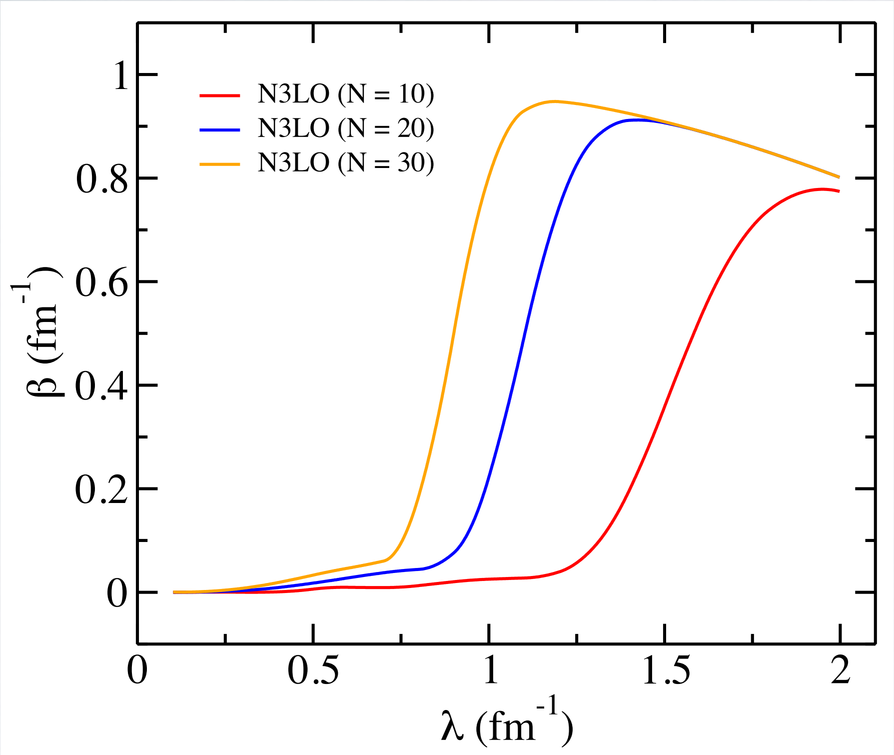
The Frobenius norm

Chiral N3LO - 1S0

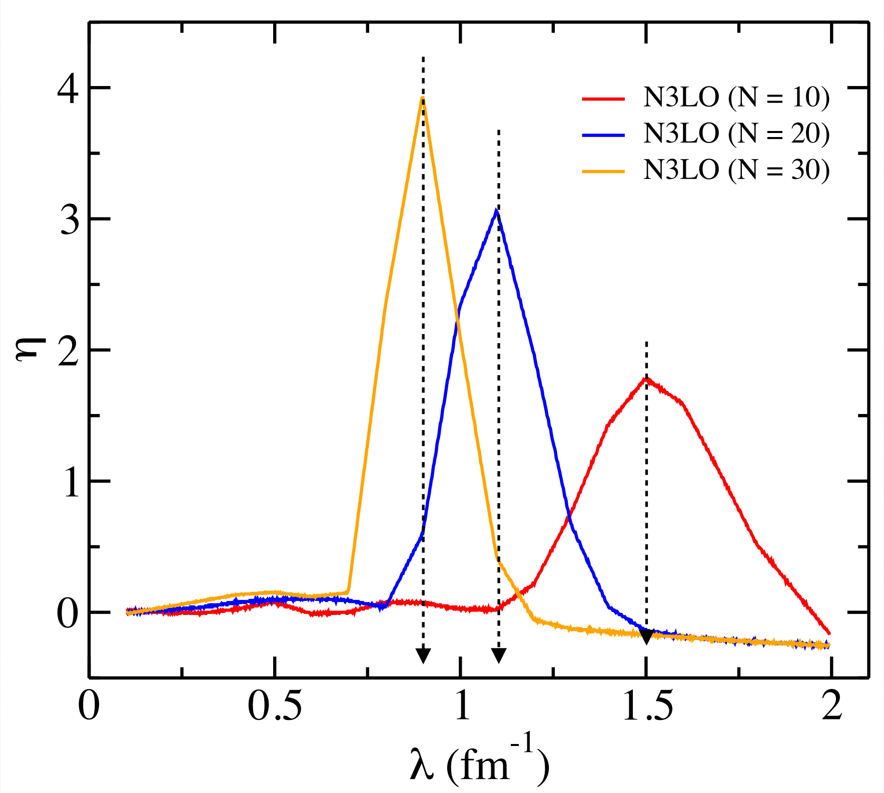


The on-shell transition - N3LO

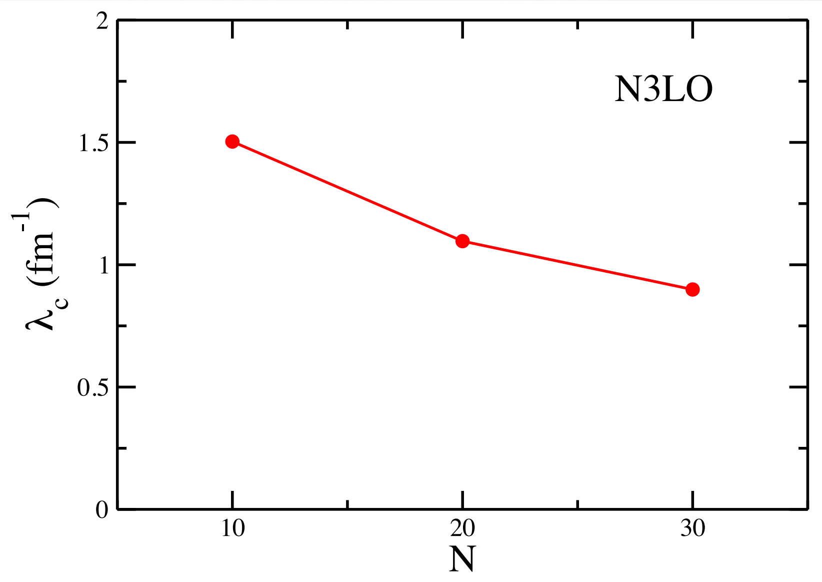
Order parameter



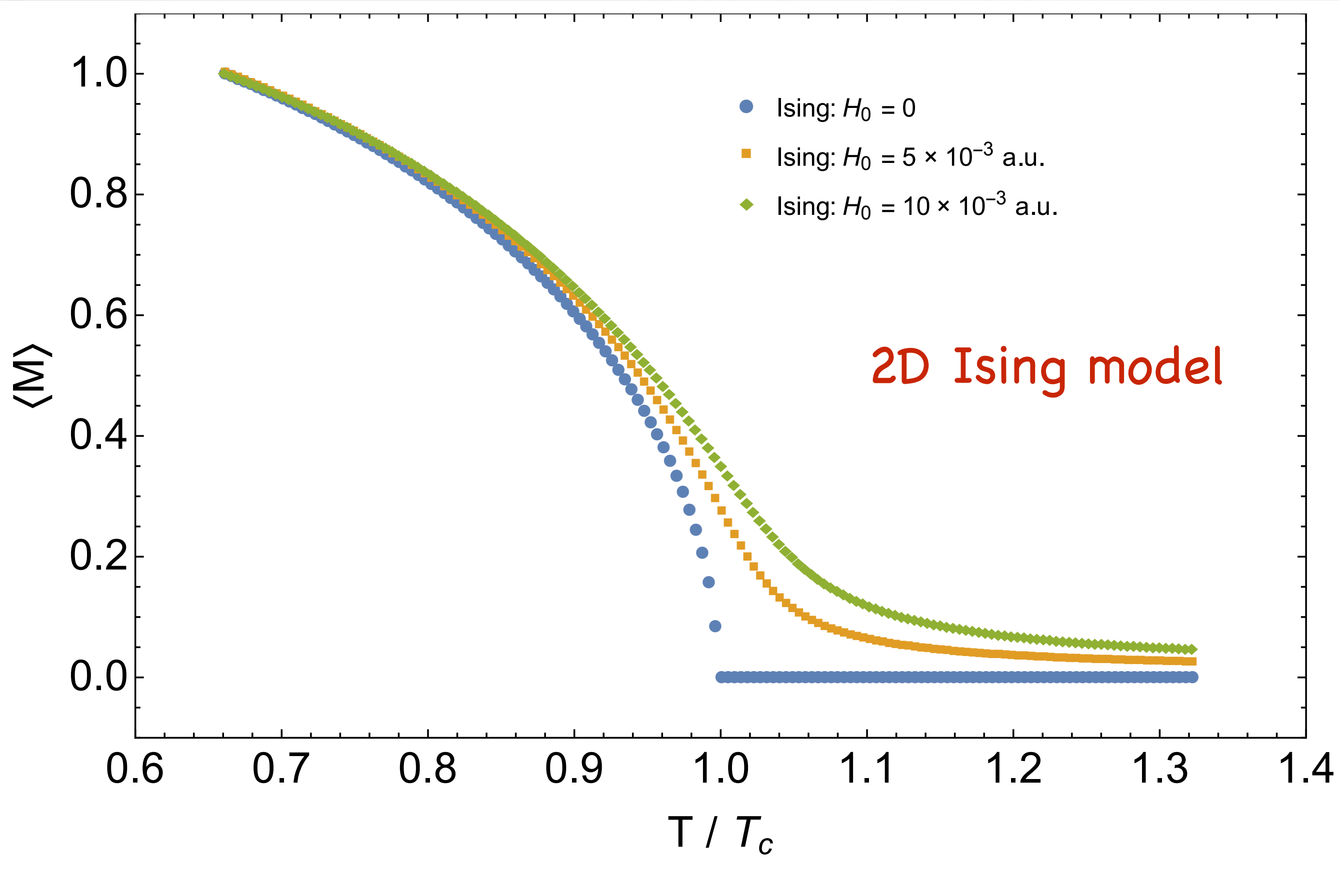
Similarity susceptibility

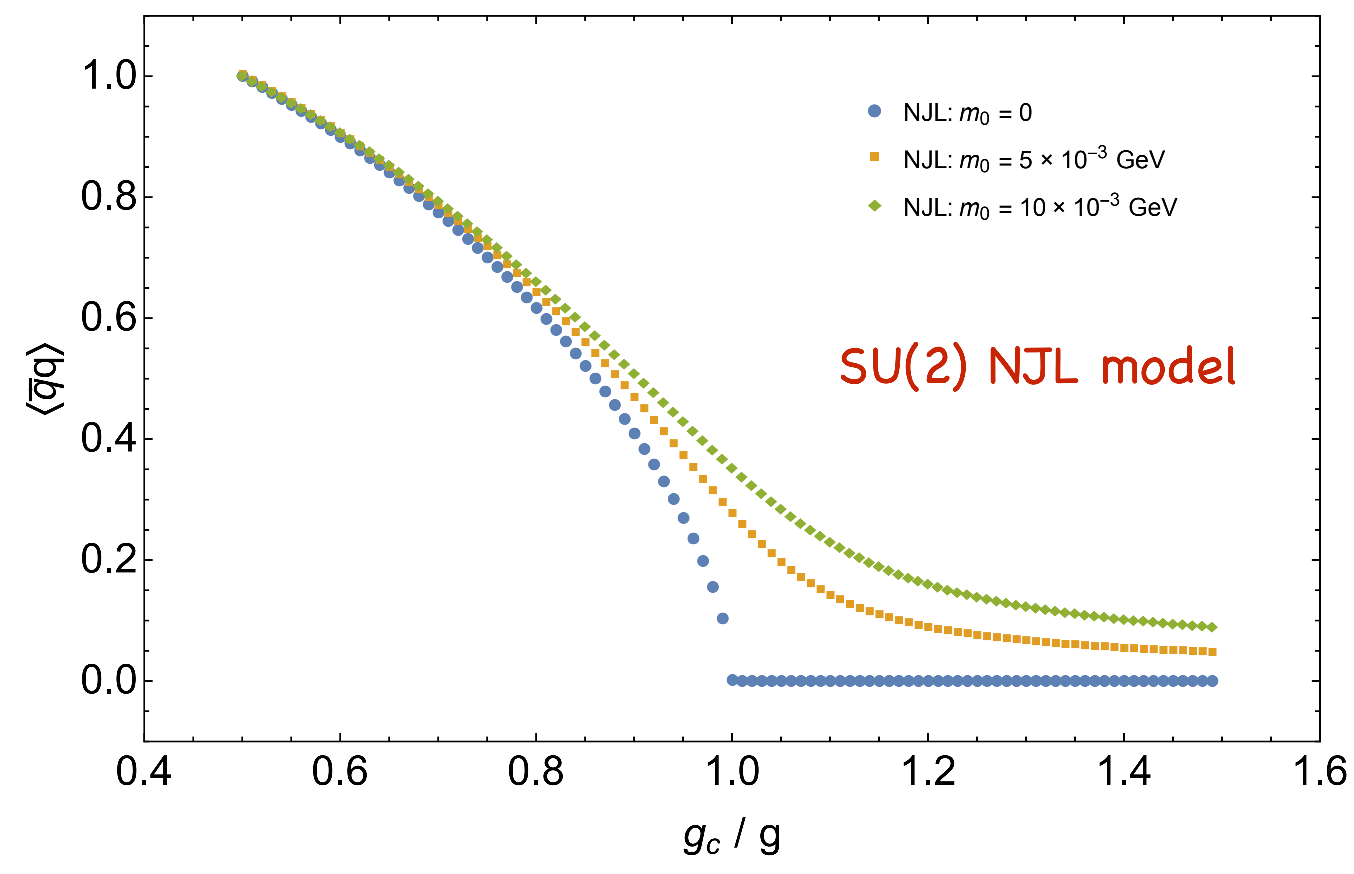


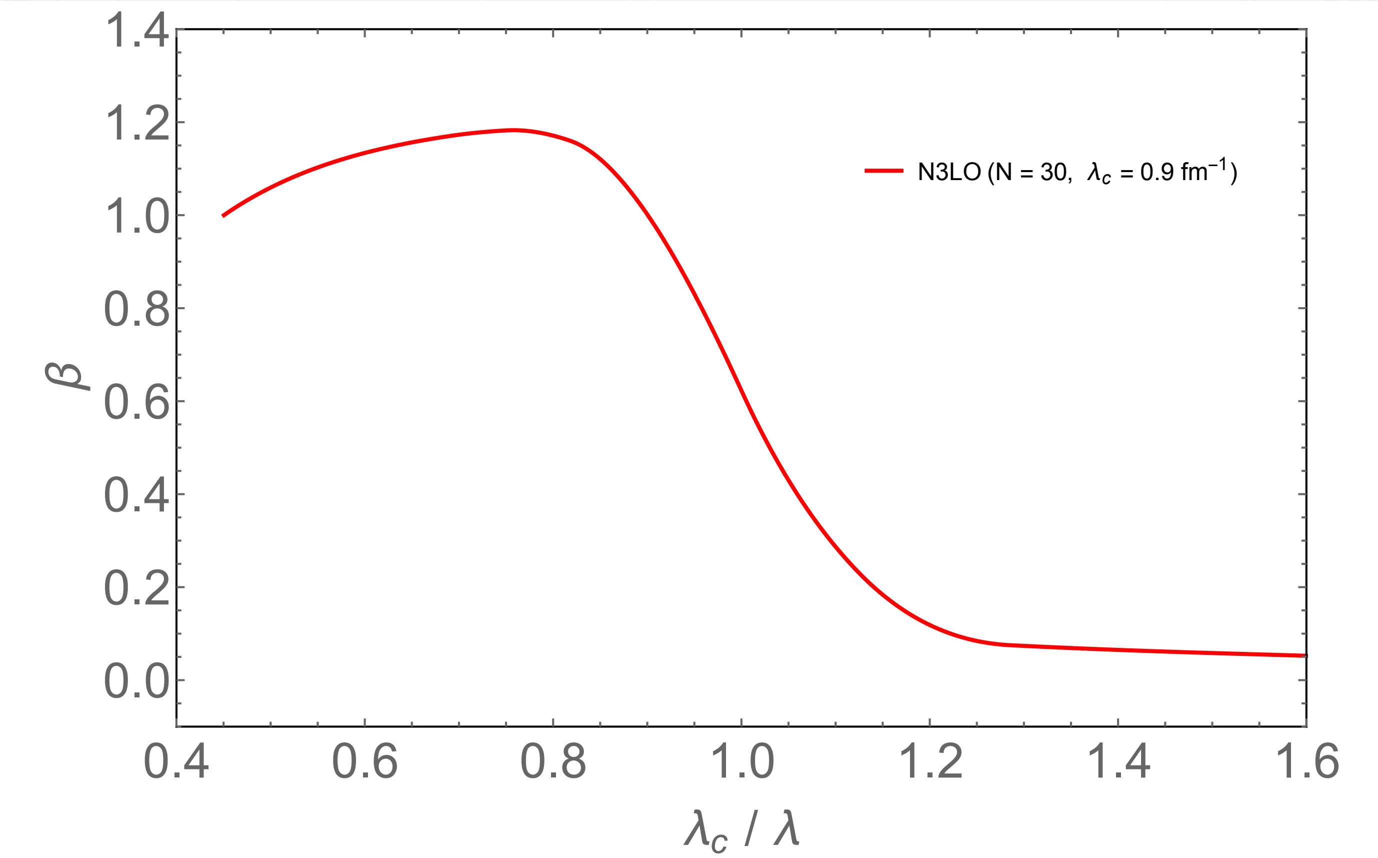
Critical similarity cutoff



β reminds <M> and <q̄q>







FINAL REMARKS

- The Toy model allowed us to explore the fixed points of the SRG for different generators with the evolution up to $\lambda \to 0$
- In the infrared limit, 2N forces are small and 3N forces are large
- Evolution of Chiral N3LO interaction towards the infrared region
- Phase transition in the SRG flow at about $\lambda_c = 0.9 \text{ fm}^{-1}$
- Interactions at small λ are universal