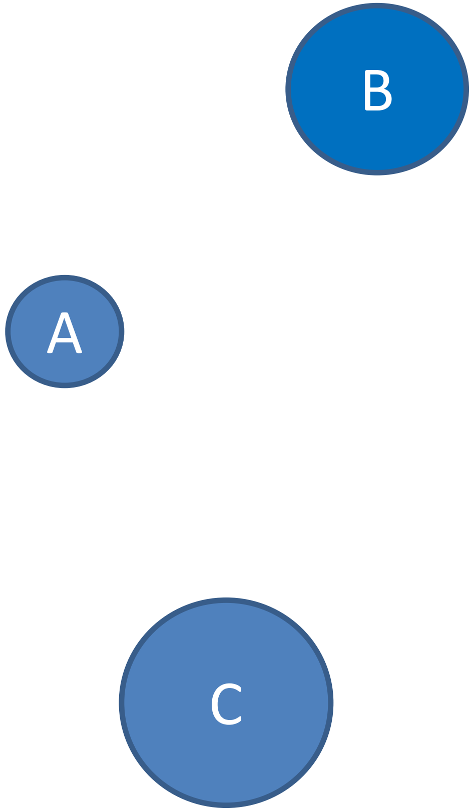
Many-body effects in three-body systems: a case of (d,p) reactions

N.K. Timofeyuk

UNIVERSITY OF SURREY



Three-body Schrodinger equation:

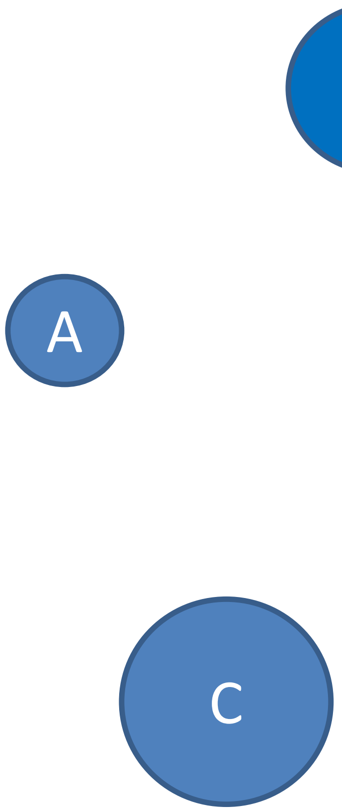
$$\left(T + V_{AB} + V_{BC} + V_{AC} - E\right) = 0$$

Available methods to solve three-body Schrodinger equation:

- Faddeev
- Hyperspherical harmonics expansion
- CDCC
- Integral relations (Kievsky)
- Scattering theory on the momentum lattice (Rubtsova,Kukulin)
- Lagrange basis expansion (D. Baye)
- Weinberg basis expansion (Johnson-Tandy)
- R-matrix
- Eikonal (high energies)

…

Three-body Schrodinger equation:



$$\left(T + V_{AB} + V_{BC} + V_{AC} - E\right) = 0$$

What are V_{AB}, V_{BC} and V_{AC}?

If *A*, *B* and *C* are nucleons then we use that describes NN phase shifts and deuteron properties.

What if *A*, *B* and *C* have complex internal structure?

- Bound *A*+*B*+*C* states : V_{AB}, V_{BC} and V_{AC} describe *A*-*B*, B-C and *A*-*C* bound subsystems
- *A*+(*B*+*C*) scattering: V_{BC} describes B-C bound subsystem, V_{AB} and V_{AC} describe *A*-*B* and *A*-*C* scattering

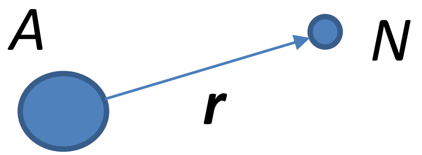
Transfer reactions A(d,p)B

n-A and p-A optical potentials are energy-dependent.

How should this dependence be included into three-body Hamiltonian?

Two-body scattering of complex nuclei : Feshbach formalism

Nucleon scattering:



$$\Psi = \underbrace{\phi_{g.s.}\chi_0(\mathbf{r})}_{\Psi_P = P\Psi} + \underbrace{\sum_{i\neq 0}\phi_i\chi_i(\mathbf{r})}_{\Psi_Q = Q\Psi}$$

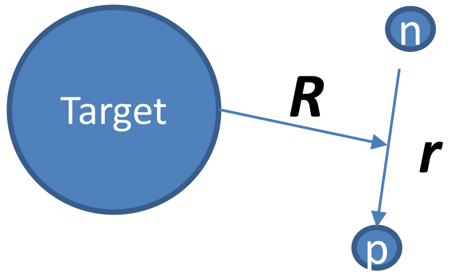
$$Q = \sum_{i\neq 0}\left|\phi_i\right\rangle\left\langle\phi_i\right|$$

χ_0 is found from the two-body equation: $(T + V_{opt} - E)\chi_0 = 0$

$$V_{opt} = \left\langle \phi_{g.s.} \right| \upsilon_{NA} + \upsilon_{NA} Q \frac{1}{E_N - Q\upsilon_{NA}Q} Q \left| \phi_{g.s.} \right\rangle \qquad \upsilon_{NA} = \sum_{i=1}^{A} \upsilon_{Ni}$$

All core excitations are hidden into energy-dependent non-local non-hermitian optical potential.

Optical potentials in the A + n + p three-body model



$$\Psi = \phi_{g.s.}\chi_0(\mathbf{r},\mathbf{R}) + \sum_{i\neq 0}\phi_i\chi_i(\mathbf{r},\mathbf{R})$$

$$Q = \sum_{i\neq 0}\left|\phi_i\right\rangle\left\langle\phi_i\right|$$

$$\Psi_P = P\Psi \qquad \Psi_Q = Q\Psi$$

Ground-state channel function can be found from three-body model

$$(T_3 + V_{np} + \left\langle\phi_{g.s.}\middle|V_{opt}\middle|\phi_{g.s.}\right\rangle - E_3)\chi_0 = 0$$

$$V_{opt} = U_{nA} + U_{pA} + U_{nA}\frac{Q}{e}U_{pA} + U_{pA}\frac{Q}{e}U_{nA} + \ldots$$

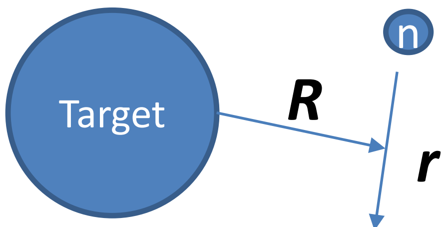
$$U_{NA} = \upsilon_{NA} + \upsilon_{NA}\frac{Q}{e}U_{NA}$$

Optical potential for 3-body system has two-body and three-body terms

R.C. Johnson and N.K. Timofeyuk, PRC 89, 024605 (2014)

Two-body force in a three-body system

R.C. Johnson and N.K. Timofeyuk, PRC 89, 024605 (2014)

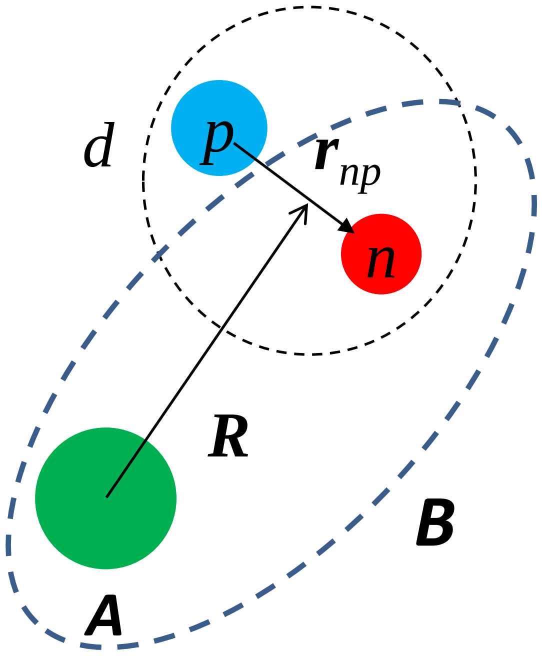


$$(T_3 + V_{np} + \langle \varphi_{g.s.} | U_{nA} + U_{pA} | \varphi_{g.s.} \rangle - E_3)\chi_0 = 0$$

$$U_{NA} = \upsilon_{NA} + \upsilon_{NA} \frac{Q}{E_3 + i0 - T_3 - V_{np} - (H_A - E_A)} U_{NA}$$

Two-body N-A optical potential: $$U_{NA} = \upsilon_{NA} + \upsilon_{NA} \frac{Q}{E_N + i0 - T_{NA} - (H_A - E_A)} U_{NA}$$

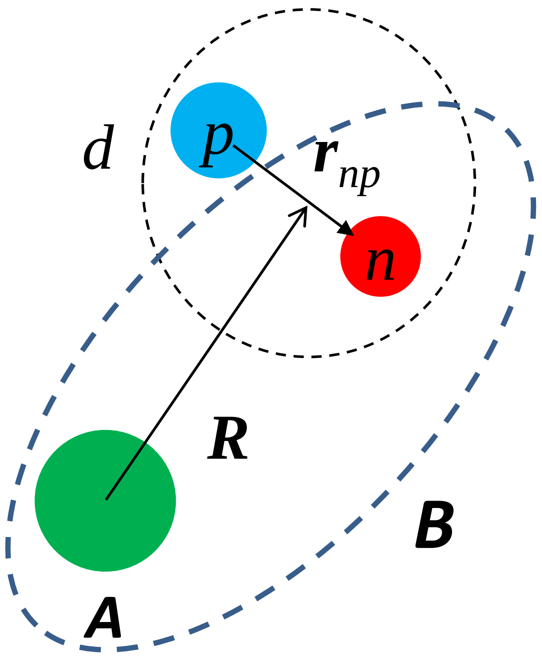
Two-body force in three-body system differs from two-body optical potential!



Many-body amplitude of transfer reaction A(d,p)B with three-body final state wave function

$$T_{(d,p)} = \left\langle \chi_{pB}^{(-)} \psi_p \psi_B \middle| V_{np}(\boldsymbol{r}_{np}) \middle| \underbrace{\psi_{npA}^{(+)}(\boldsymbol{R}, \boldsymbol{r}_{np}) \psi_A}_{\Psi_P = P\Psi} \right\rangle$$

N-A two-body force in A(d,p)B reactions



$$T_{(d,p)} = \left\langle \chi_{pB}^{(-)} \psi_p \psi_B \middle| V_{np}(\boldsymbol{r}_{np}) \middle| \psi_{npA}^{(+)}(\boldsymbol{R},\boldsymbol{r}_{np}) \psi_A \right\rangle$$

- The most important contribution to the (d,p) reaction comes from small n-p separations.
- Adiabatic approximation can be made by retaining only first Weinberg component

$$\psi_{npA}^{(+)}(\boldsymbol{R},\boldsymbol{r}_{np}) \approx \psi_{dA}^{(+)}(\boldsymbol{R}) \varphi_d(\boldsymbol{r}_{np})$$

- In the adiabatic approximation the distorted wave function satisfies the equation:

$$\left(T_R + \left\langle \varphi_d \varphi_{g.s.}^A \middle| V_{np}(U_{nA} + U_{pA}) \middle| \varphi_d \varphi_{g.s.}^A \right\rangle - E_d \right) \chi_{dA}^{(+)}(\boldsymbol{R}) = 0$$

What is needed for transfer reactions in the N-A force averaged over V_{np}

Averaging procedure gives

$$\left\langle \varphi_d \varphi_{g.s.}^A \middle| V_{np} U_{NA} \middle| \varphi_d \varphi_{g.s.}^A \right\rangle \approx \left\langle \varphi_{g.s.}^A \middle| \upsilon_{NA} + \upsilon_{NA} \frac{Q}{E_{eff} + i0 - T_N - (H_A - E_A)} U_{NA} \middle| \varphi_{g.s.}^A \right\rangle$$

Two-body N-A potential for (d,p) reaction is equal to the non-local energy-dependent complex optical N-A potential taken at the fixed effective energy

$$E_{eff} = \frac{1}{2} E_d + \frac{1}{2} \frac{\left\langle \varphi_d \middle| V_{np} T_{np} \middle| \varphi_d \right\rangle}{\left\langle \varphi_d \middle| V_{np} \middle| \varphi_d \right\rangle}$$

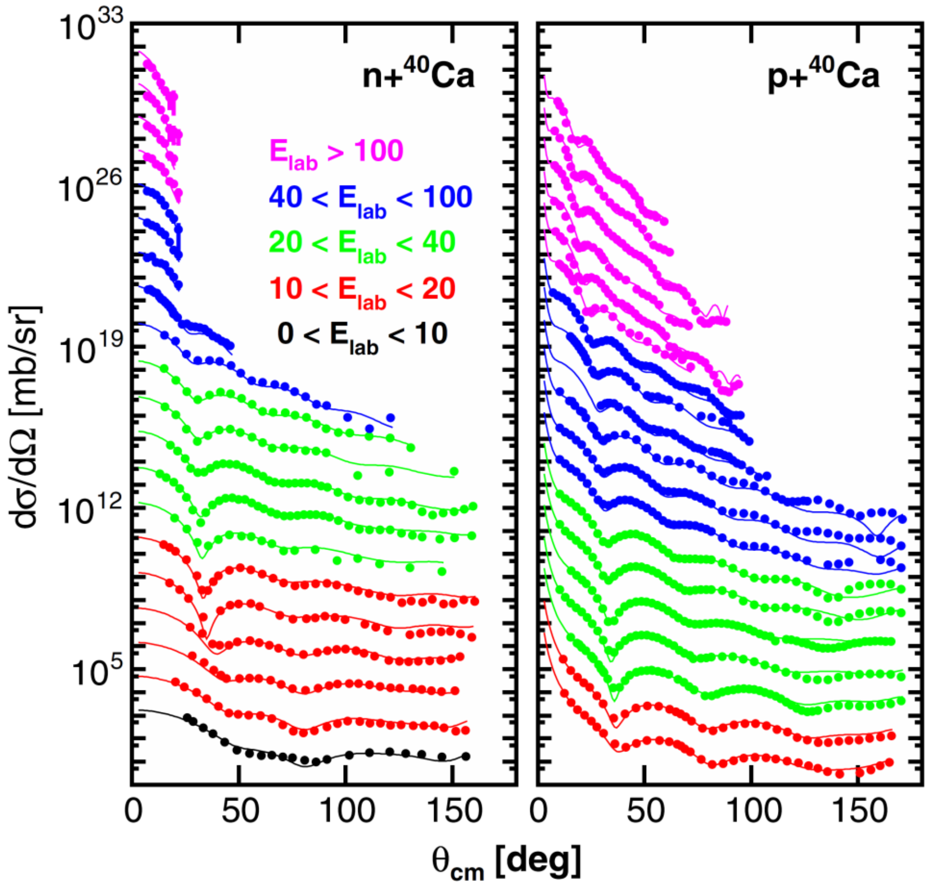
n-p kinetic energy in deuteron equal approximately 57 MeV

Three-body problem for (d,p) reactions should be solved with energy-independent nonlocal nucleon potentials taken at effective energy equal to half the deuteron energy plus a shift.

Application: ^{40}Ca(d,p)^{41}Ca reaction

Non-local energy-dependent dispersive optical potential for *N*+^{40}Ca

M.H. Mahzoon et al, Phys. Rev. Lett. 112, 162503 (2014)



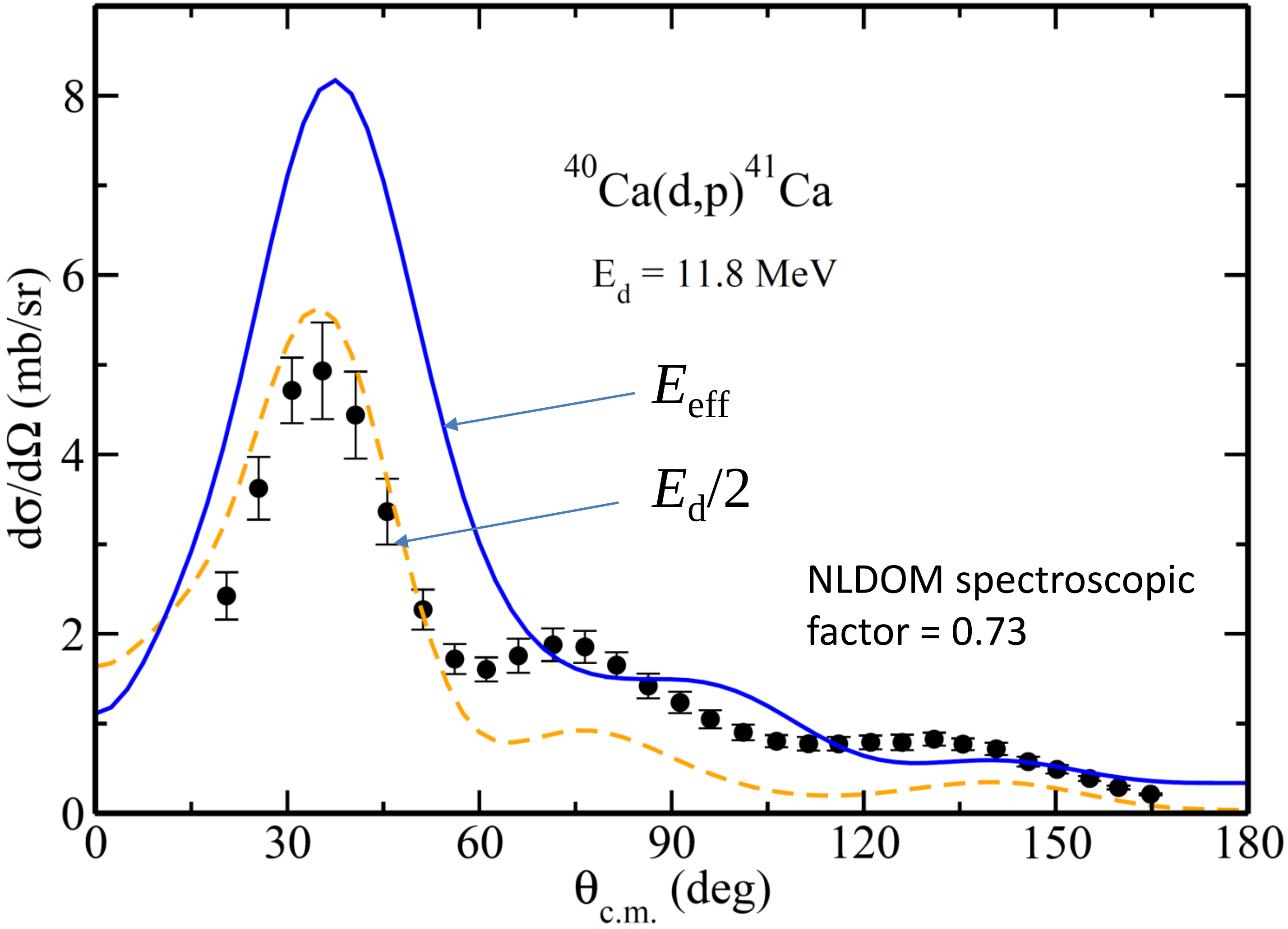
$$
\begin{aligned}
\Sigma(\boldsymbol{r},\boldsymbol{r}';E) = & \, U_{HF}^{vol1}(\tilde{r})H(x;\beta_{vol_1}) \\
& + U_{HF}^{vol2}(\tilde{r})H(x;\beta_{vol_2}) \\
& + U_{HF}^{wb}(\tilde{r})H(x;\beta_{wb}) \\
& + U_{dy}^{sur+}(\tilde{r};E)H(x;\beta_{sur+}) \\
& + U_{dy}^{sur-}(\tilde{r};E)H(x;\beta_{sur-}) \\
& + U_{dy}^{vol+}(\tilde{r};E)H(x;\beta_{vol+}) \\
& + U_{dy}^{vol-}(\tilde{r};E)H(x;\beta_{vol-}) \\
& + U_{so}(r;E)
\end{aligned}
$$

$$\tilde{r} = |\boldsymbol{r}+\boldsymbol{r}'|/2 \text{ and } \boldsymbol{x} = \boldsymbol{r}' - \boldsymbol{r}$$

$$H(\boldsymbol{x};\beta) = \exp(-\boldsymbol{x}^2/\beta^2)/(\pi^{3/2}\beta^3)$$

This potential simultaneously reproduces the data above and below the Fermi energy

Fixing single-particle physics from Nonlocal Dispersive Optical Model (NLDOM)



S.J Waldecker and N.K. Timofeyuk, Phys. Rev. C, in press (2016)

What is missing?
Main assumptions:

- The (d,p) amplitude contains projection to A+n+p channel. Projections onto all excited states are neglected.
- Only p-A and n-A potentials were used to calculate d+A potential. Multiple scattering effects (A+n+p force) are neglected.
- (d,p) amplitude contains V_{np} only.
- Adiabatic approximation is used.

Conclusions:

- Application of few-body models for complex systems has to be justified from many-body point of view.

- The three-body model for the A+n+p system contains two-body and three-body contribution. The latter describes interaction of n and p through intermediate recoil excitation of the target.

- Two-body n-A and p-A potentials in the three-body A+n+p system differ from those in free n-A and p-A systems.

- For A(d,p)B reaction one needs to know a non-local energy dependent N-A potential at the energy equal to half the deuteron incident energy shifted by the n-p kinetic energy averaged over the short-ranged n-p interaction.

- Application to $^{40}Ca(d,p)^{41}Ca$ reaction using NLDOM potential shows that some physics (most likely many-body one) is missing.