



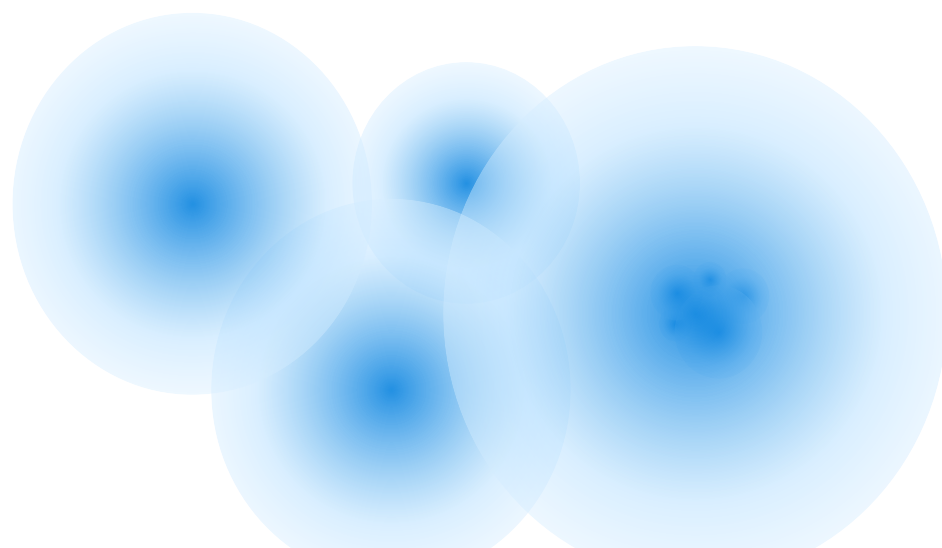
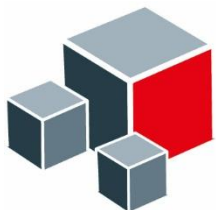
Ground state properties of weakly bound few-body systems

- collaborators -



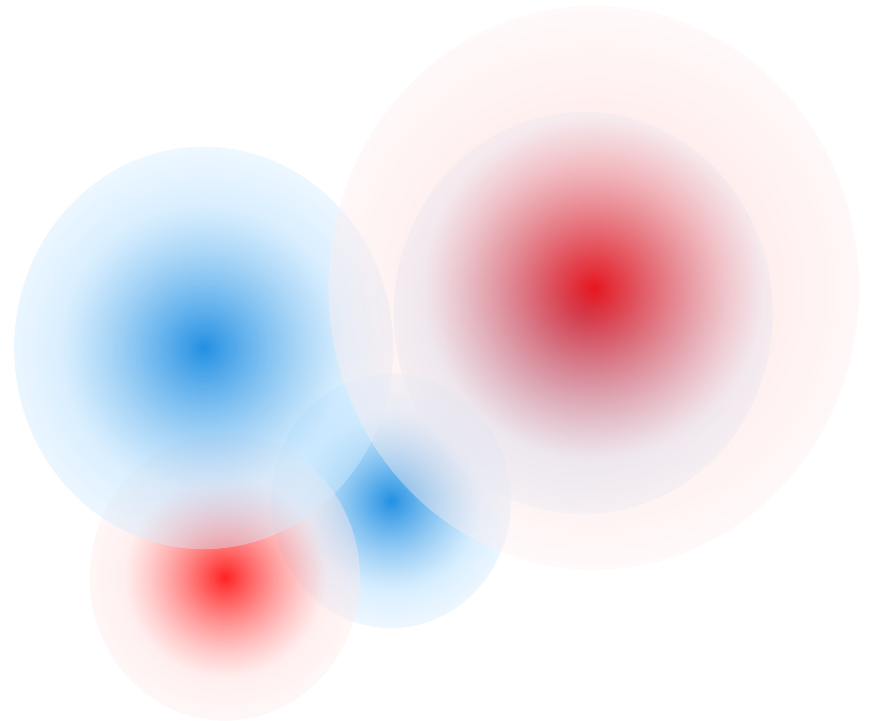
Leandra Vranješ Markić

Jordi Boronat



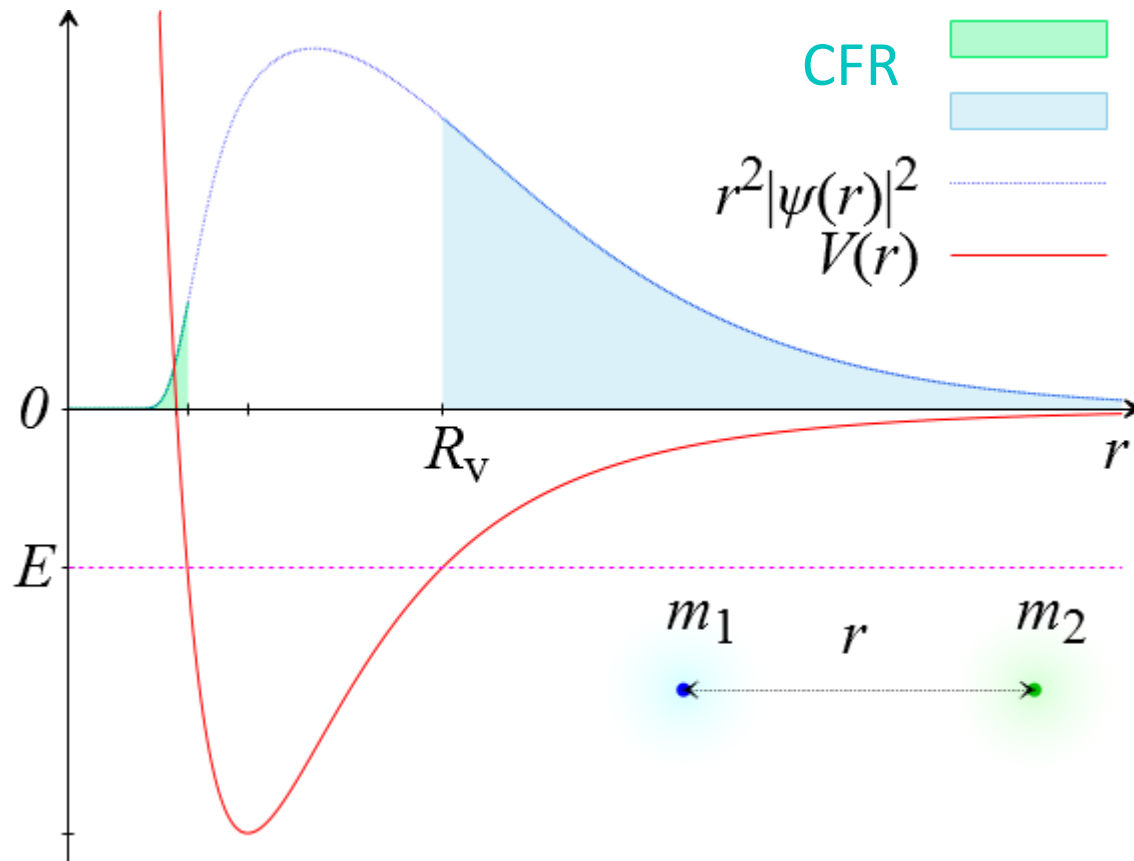
>> Outline

- researching clusters of few particles
 - dimers A_2
 - trimers $A_3 B$
 - tetramers $A_2 B_2$
 - ...
- halo state
- energy-size scaling
- universalities
- structural properties



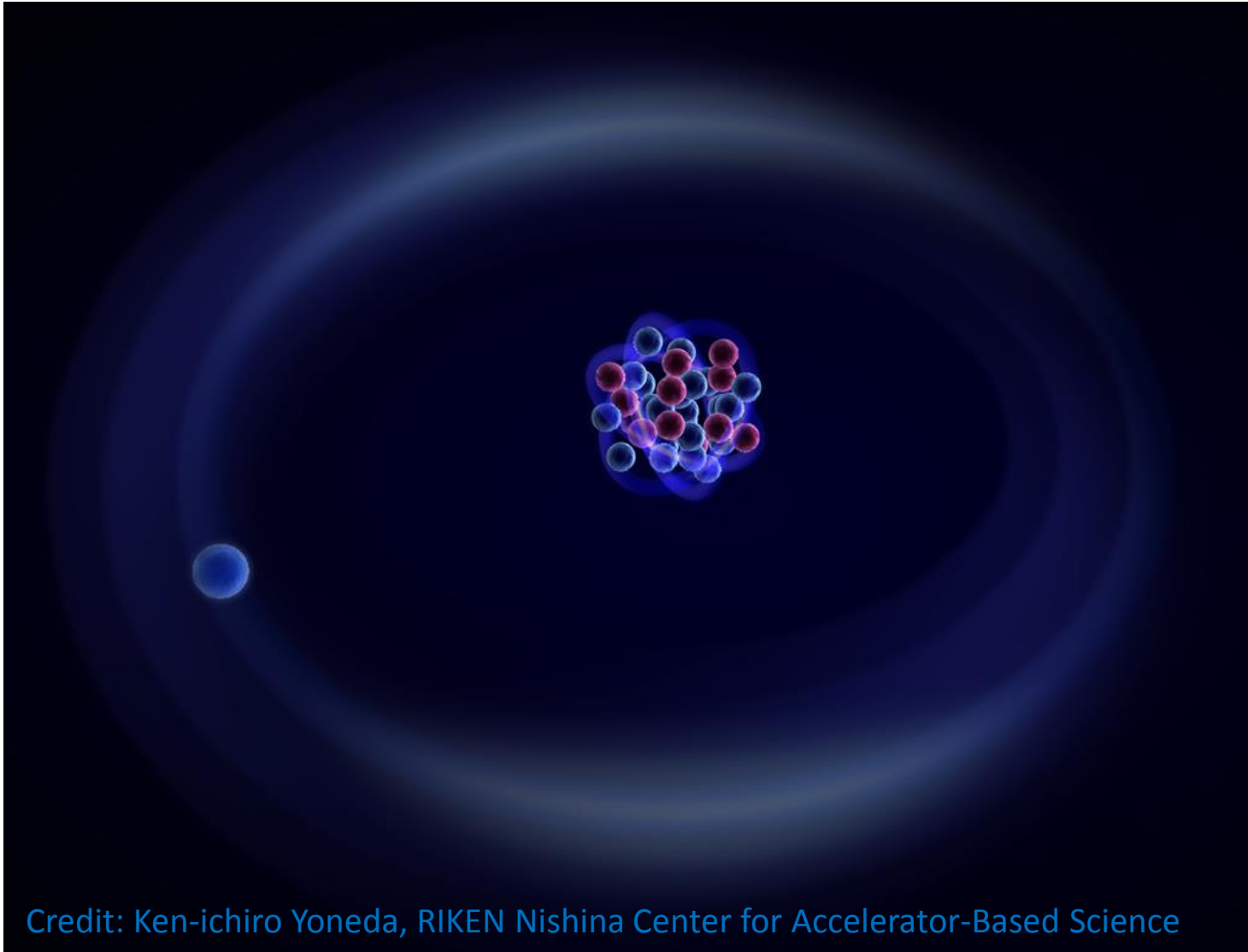
» Intro: Halo states

- quantum halo states – quantum effects very significant



»» Intro: Halo states

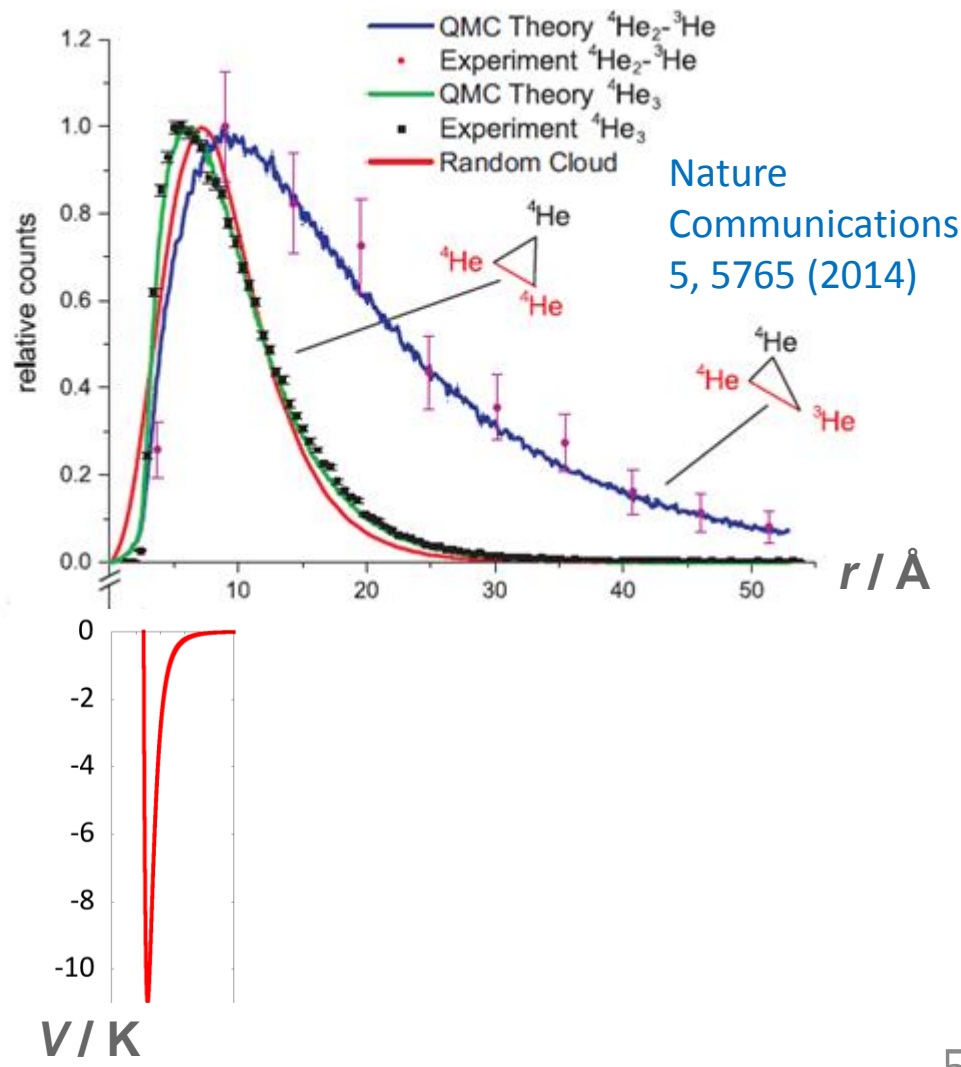
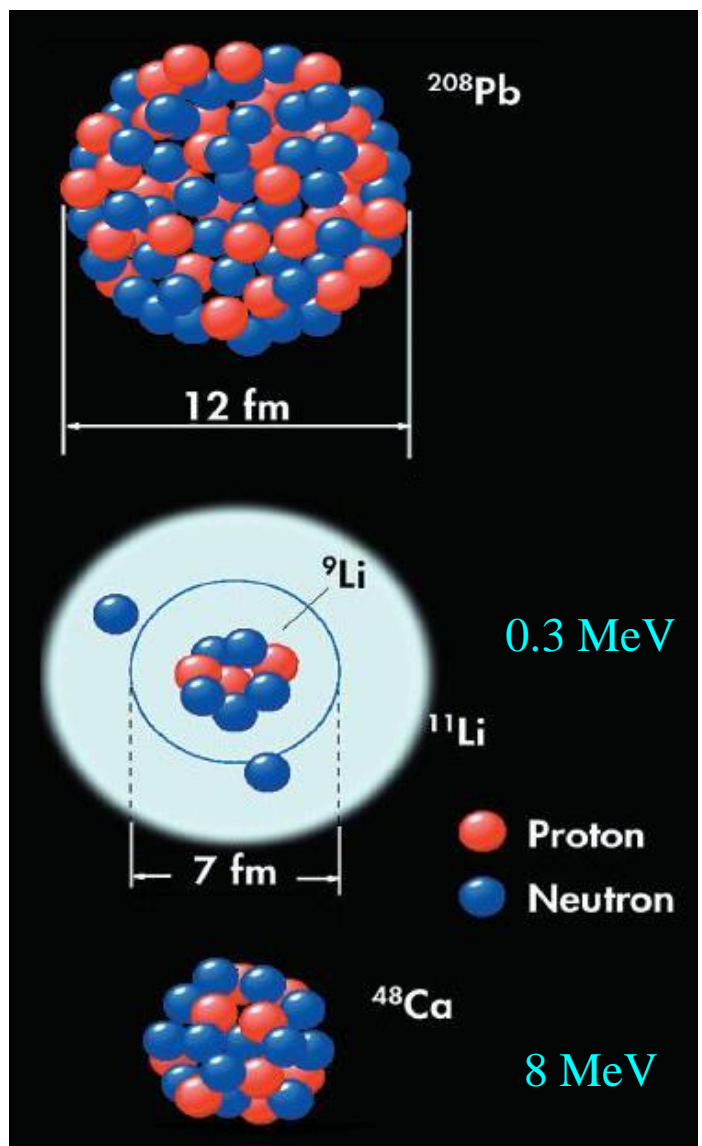
- first were noticed in nuclear physics - dimer



Credit: Ken-ichiro Yoneda, RIKEN Nishina Center for Accelerator-Based Science

» Intro: Halo states

- characteristics: weakly bound & wide systems



» Intro: Dimers

? is there unique relationship between physical properties of halo dimers

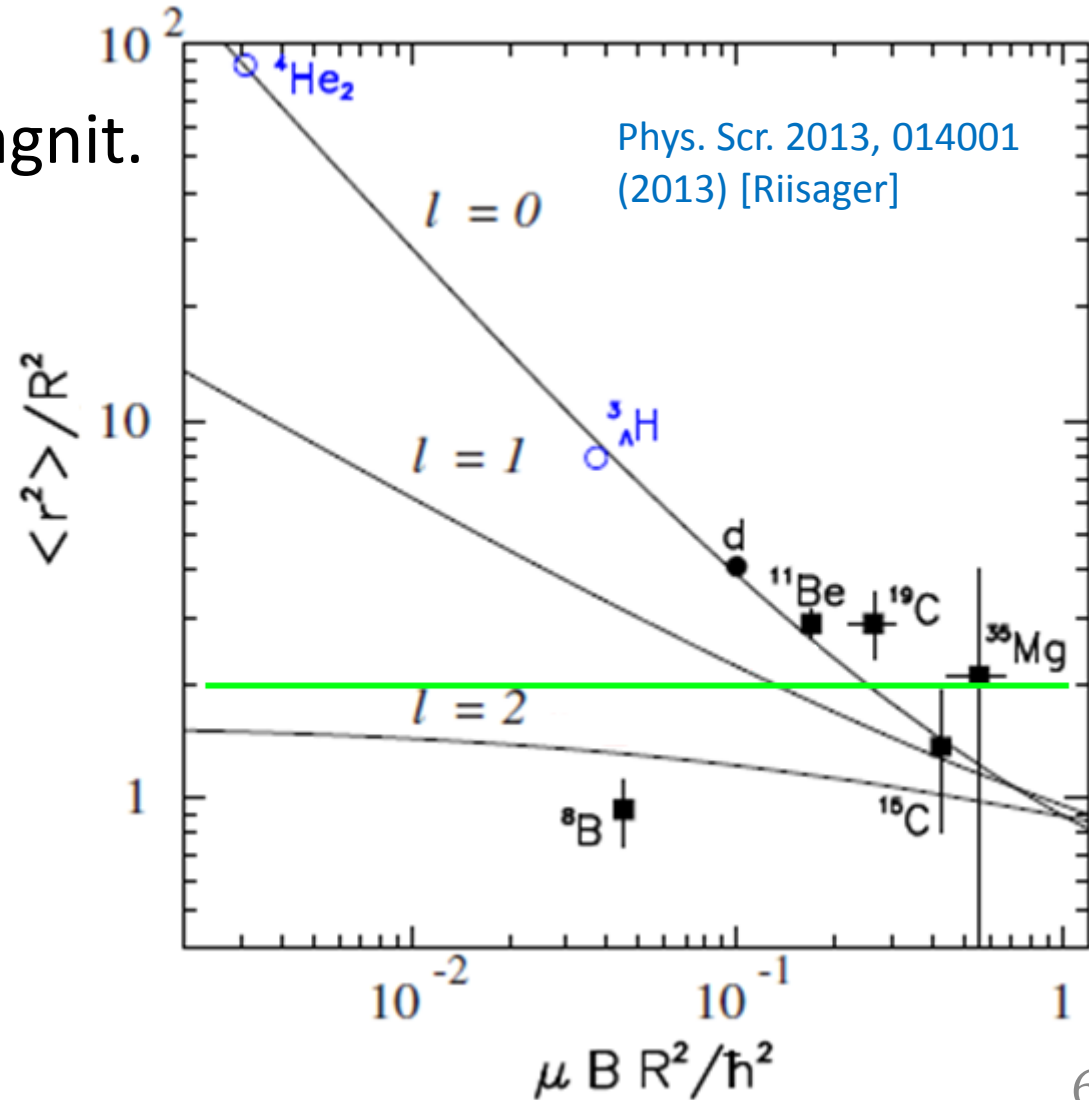
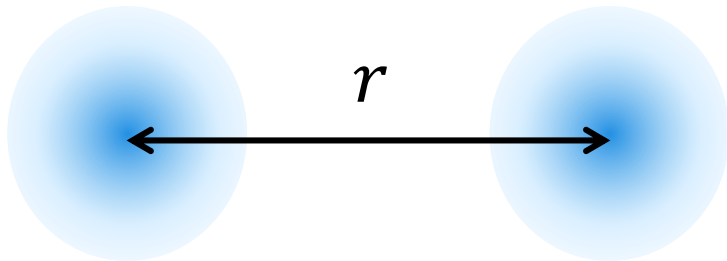
- dimers - 16 orders of magnit.
- scaled quantities (R ?)

- $x_E = \mu B R^2 / \hbar^2$

- $y_r = \langle r^2 \rangle / R^2$

- halo states

- $y_r = \langle r^2 \rangle / R^2 \geq 2$



» Intro: Trimers

- trimer properties were scaled:

- $X_E = mB\rho_0^2/\hbar^2$

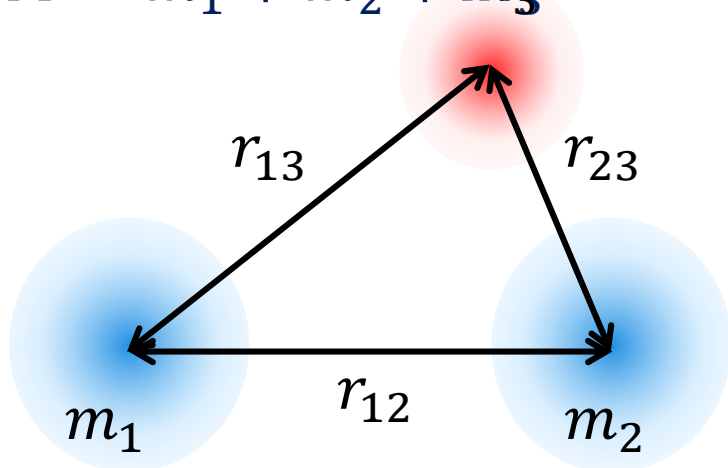
- $Y_r = \langle \rho^2 \rangle / \rho_0^2$

$$\mu r^2$$

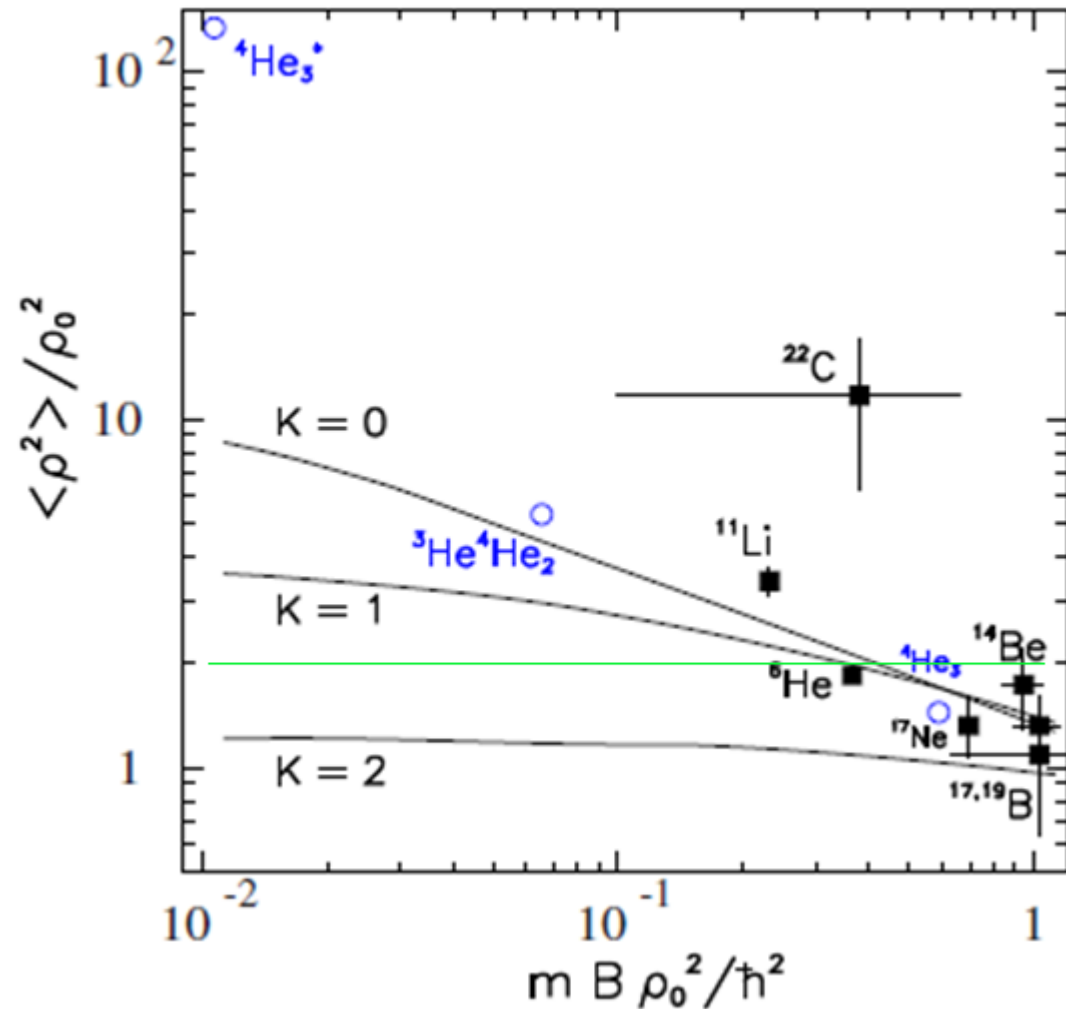
↓

$$m\rho^2 = \frac{1}{M} \sum_{i < k} m_i m_k r_{ik}^2$$

$$M = m_1 + m_2 + m_3$$



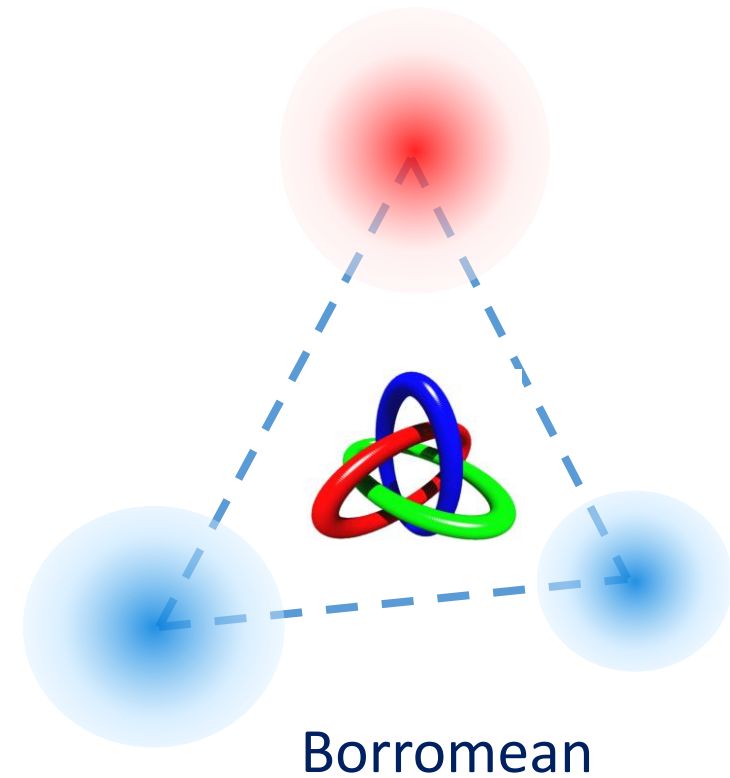
Phys. Scr. 2013, 014001 (2013) [Riisager]



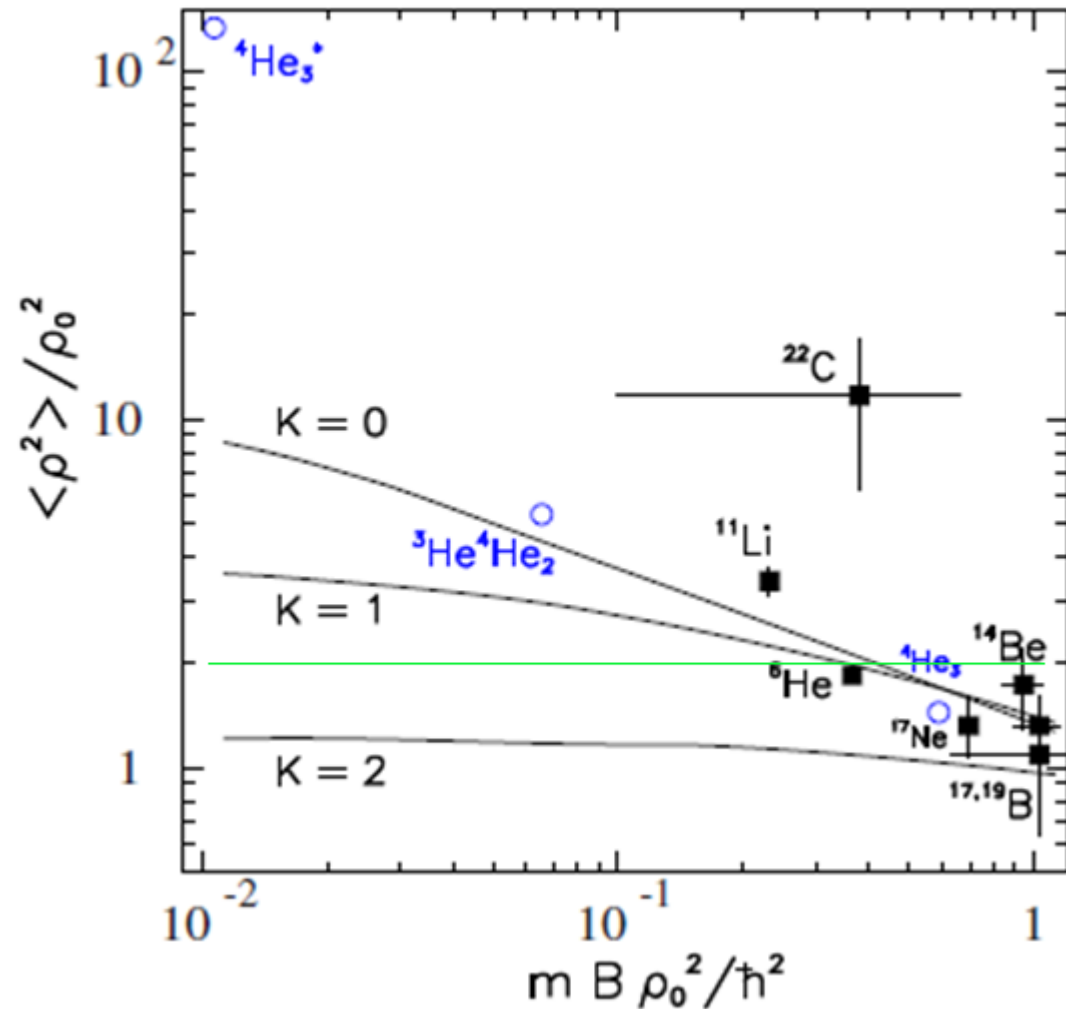
» Intro: Trimers

- larger number of atomic quantum clusters is missing

? are different-type 3mer
properties related



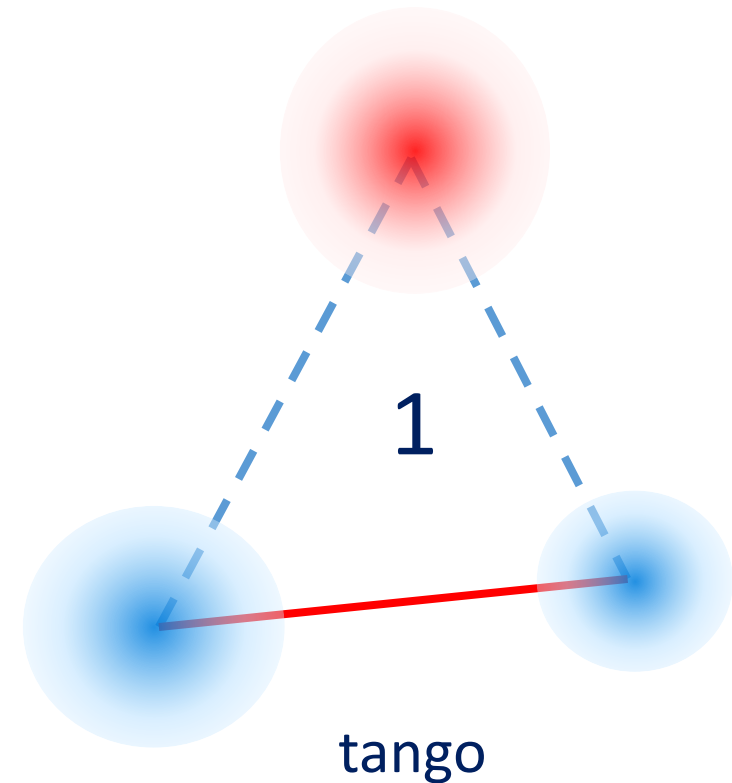
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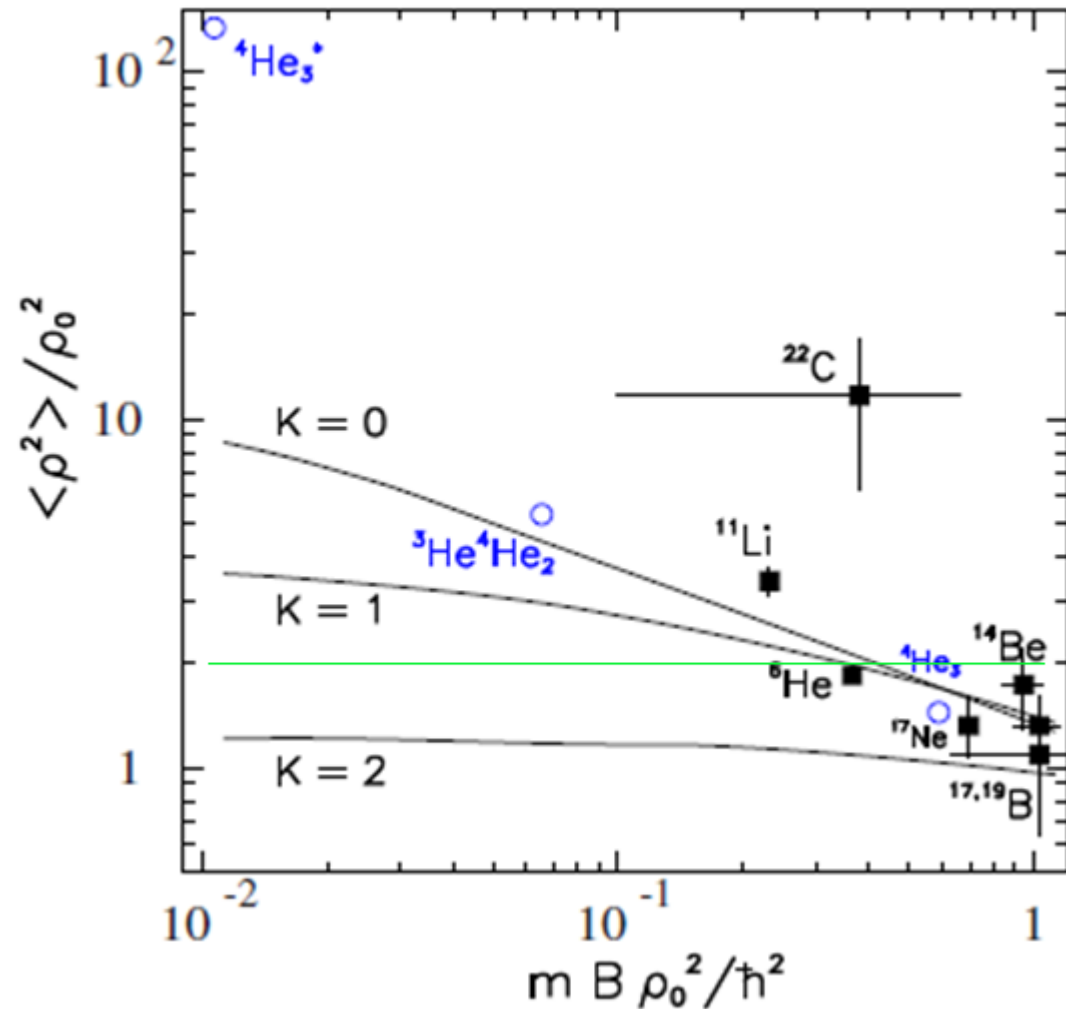
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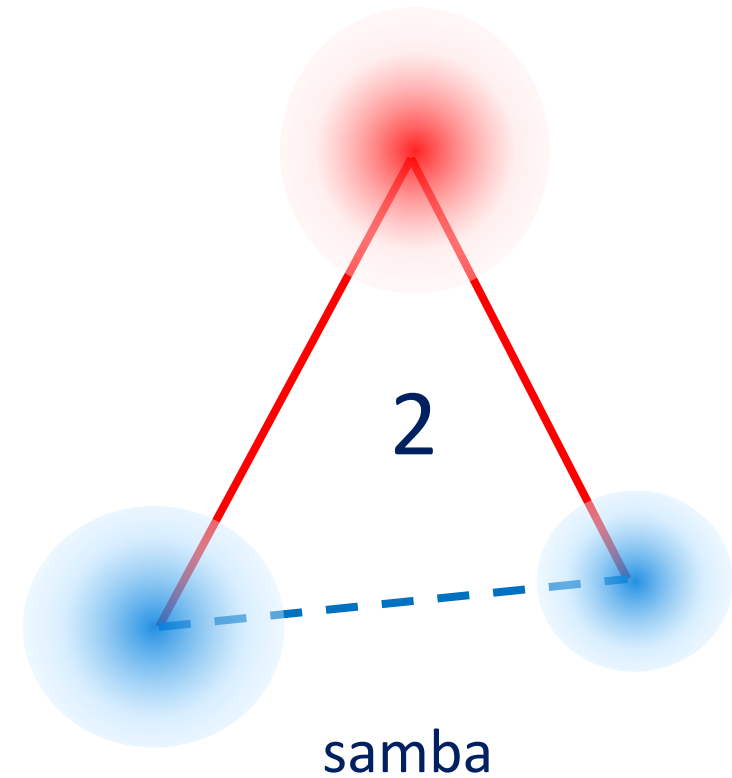
Phys. Scr. 2013, 014001 (2013) [Riisager]



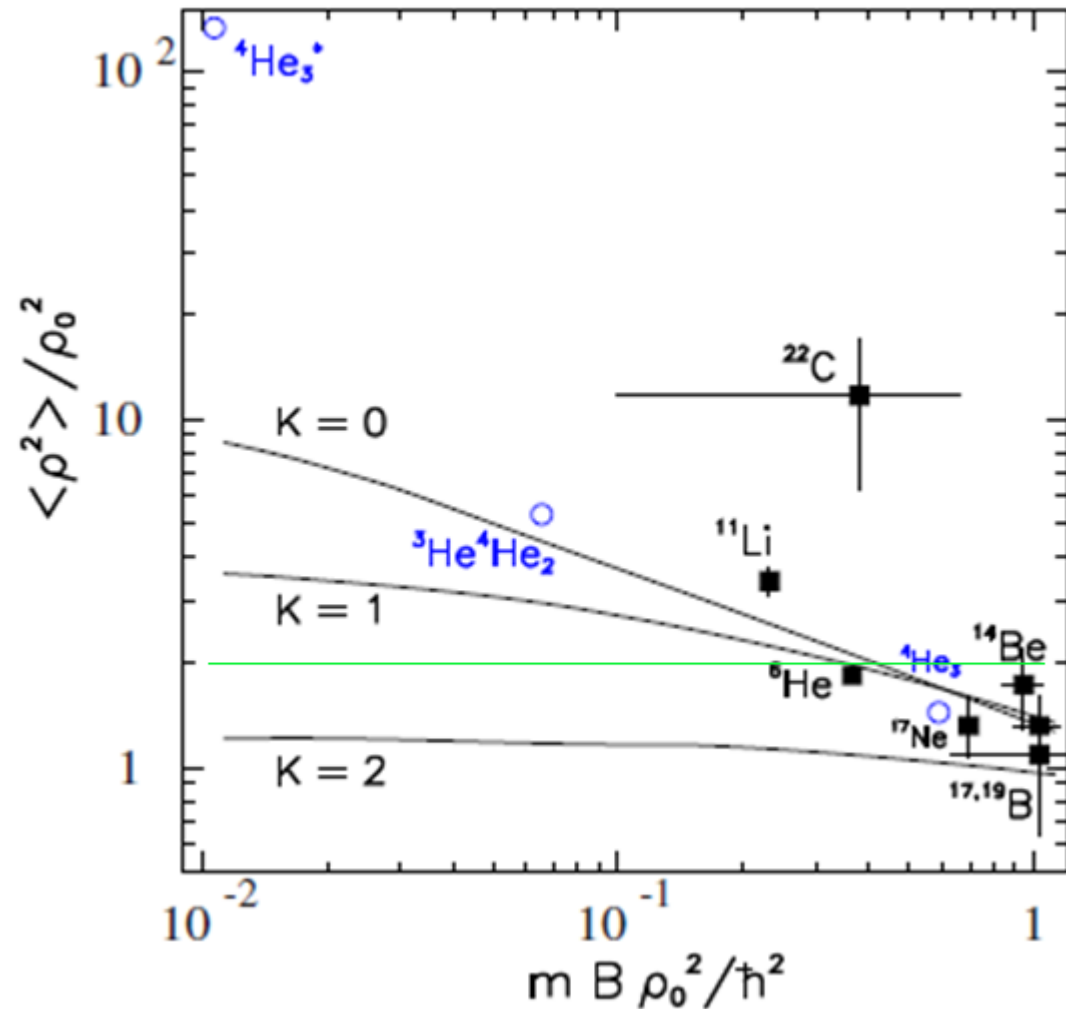
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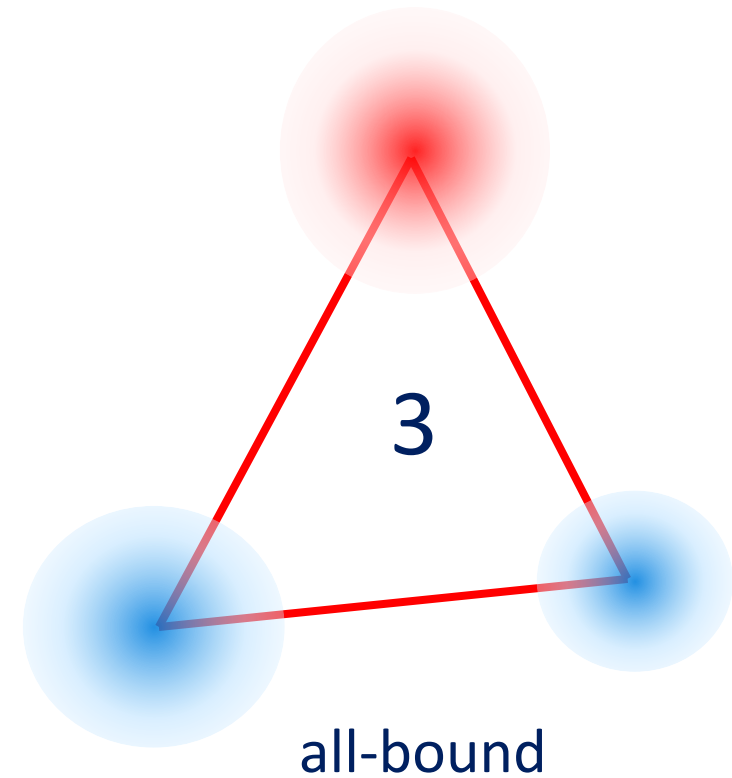
Phys. Scr. 2013, 014001 (2013) [Riisager]



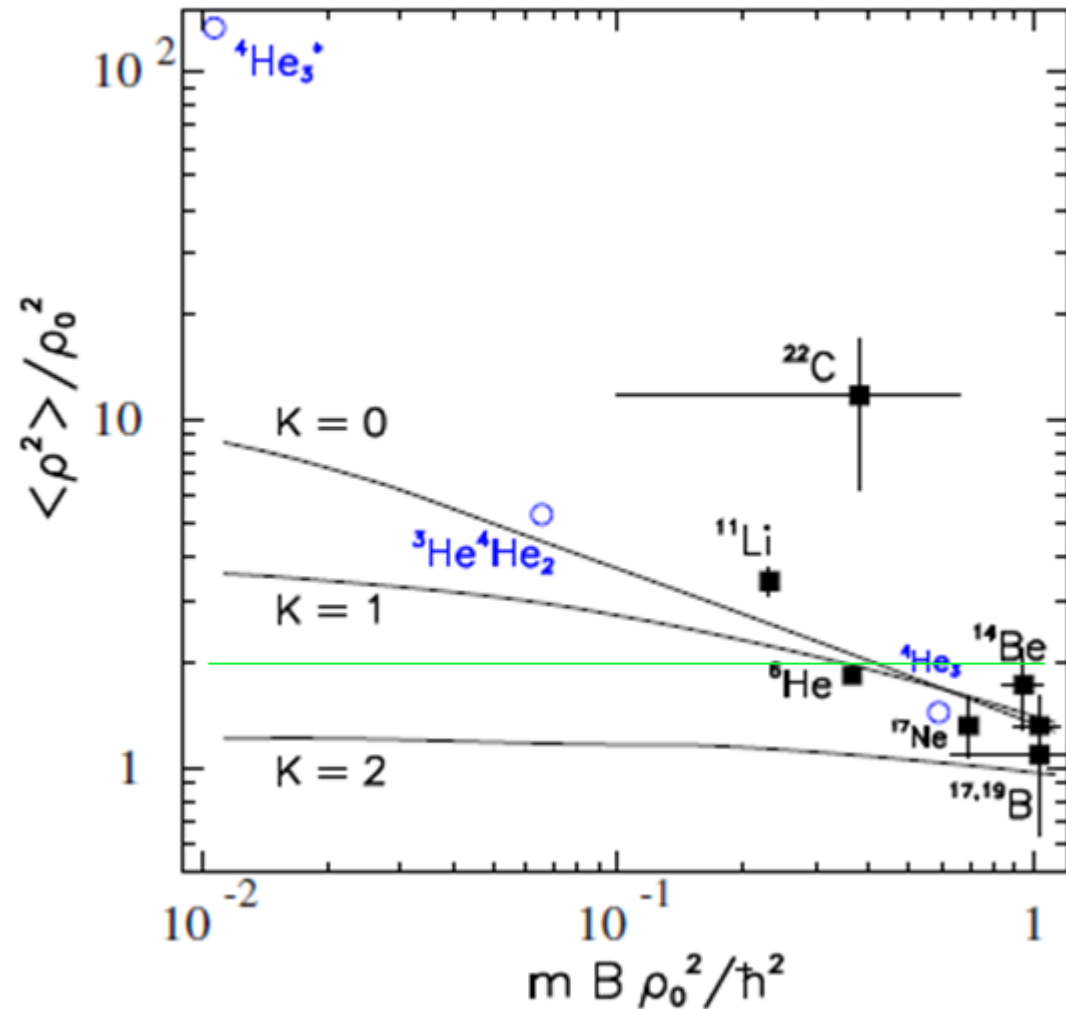
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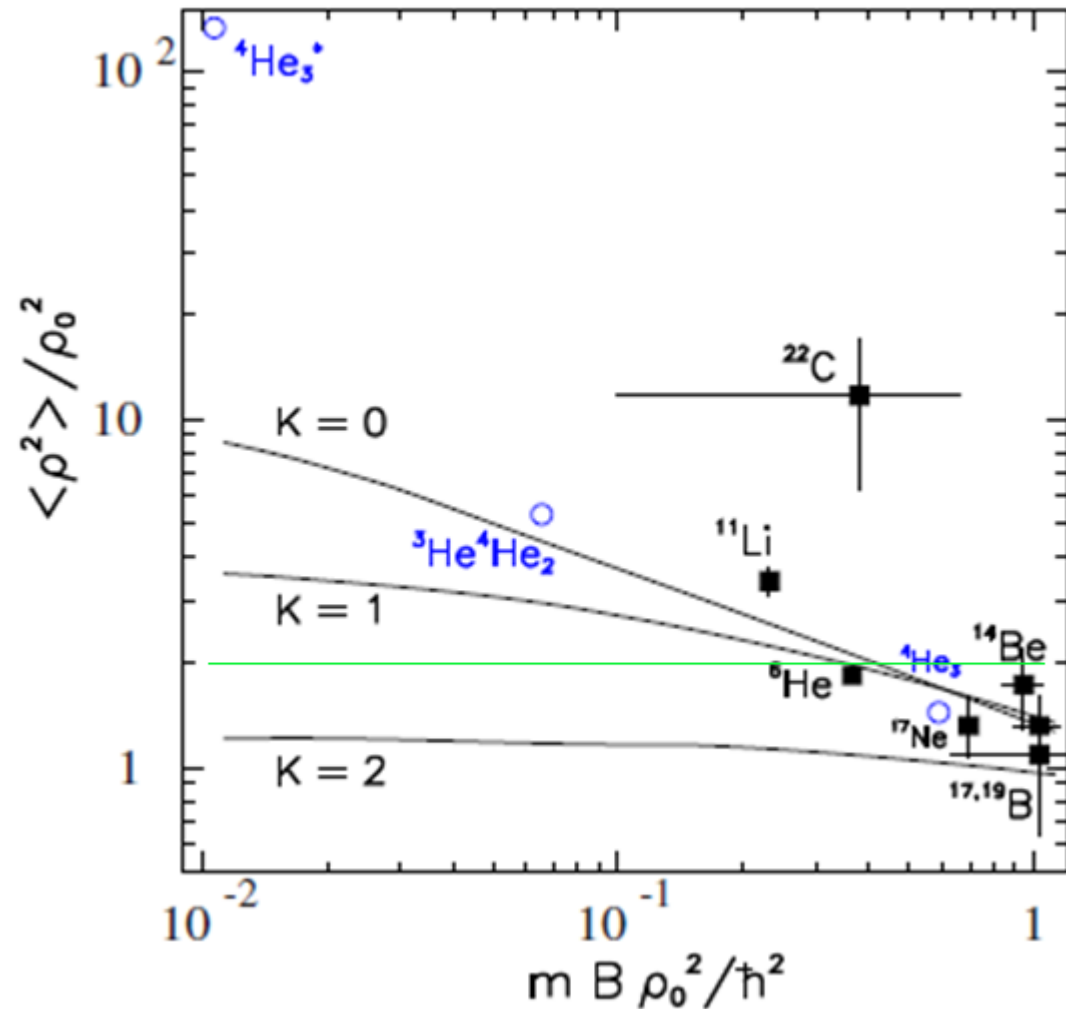
- larger number of atomic quantum clusters is missing

? are different-type 3mer
properties related

- objective: clarify ratio of
scaled size and
scaled energy

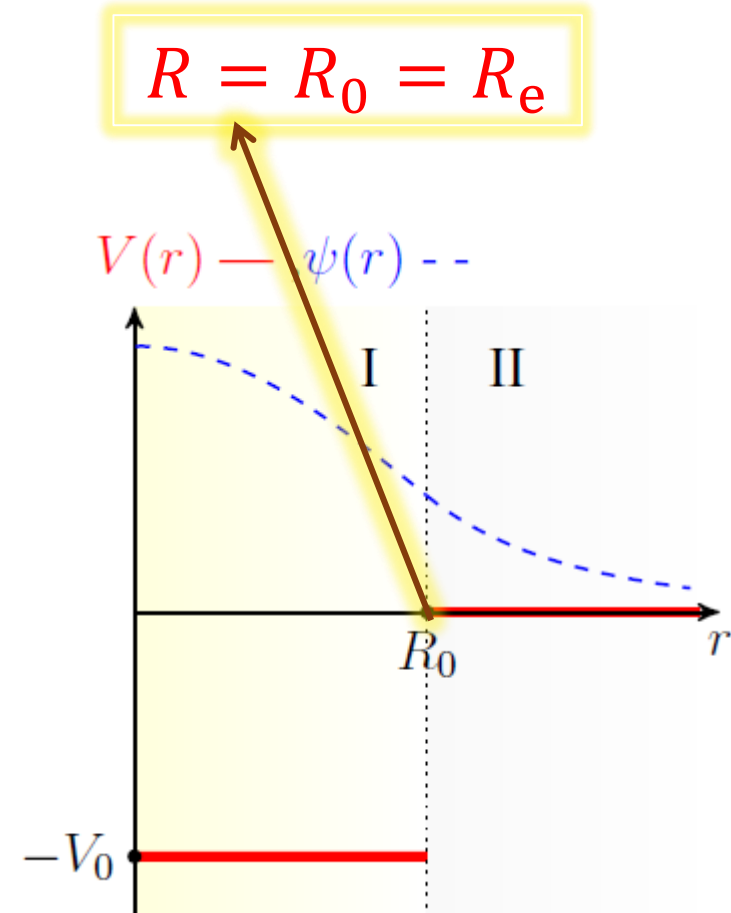
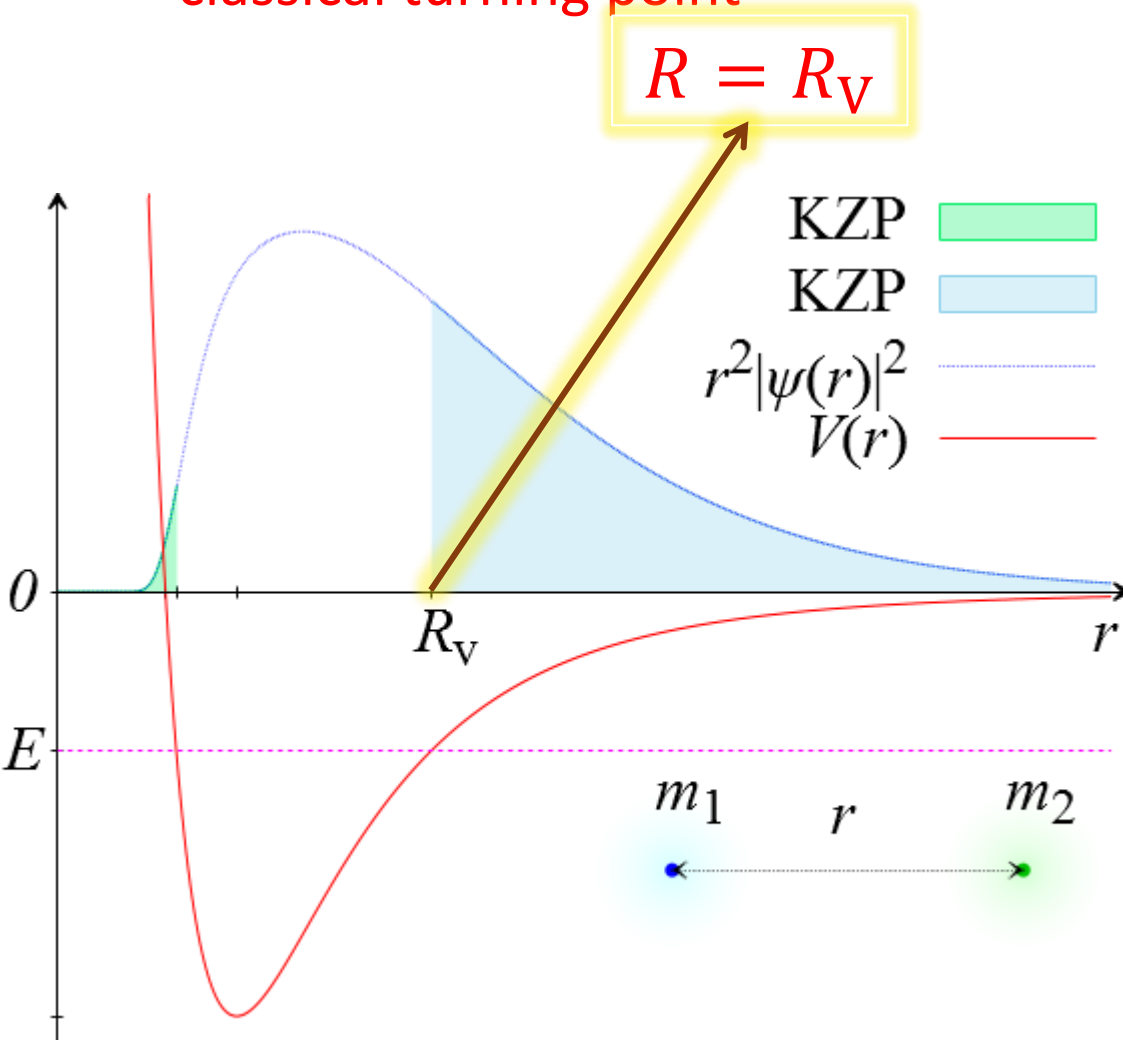
- very important
 - × accuracy of V
 - ✓ precise $E, \langle r_{ij}^2 \rangle$
- method
 - ✓ DMC

Phys. Scr. 2013, 014001 (2013) [Riisager]



» METHOD: Scaling

- e.g. 2 definitions of R for scaling: $\langle r^2 \rangle / R^2$; $\mu B R^2 / \hbar^2$
 - classical turning point



» Dimer scaling

\times ${}^4\text{He}_2$

[a] Rev. Mod. Phys. 76, 215 (2004) [Jensen et al.]

[a] Phys. Scr. 2013, 014001 (2013) [Riisager]

$y_r = r^2 R^{-2}$

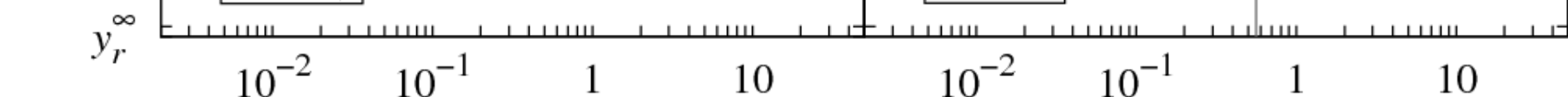
T [a] \times
E [a] \bullet

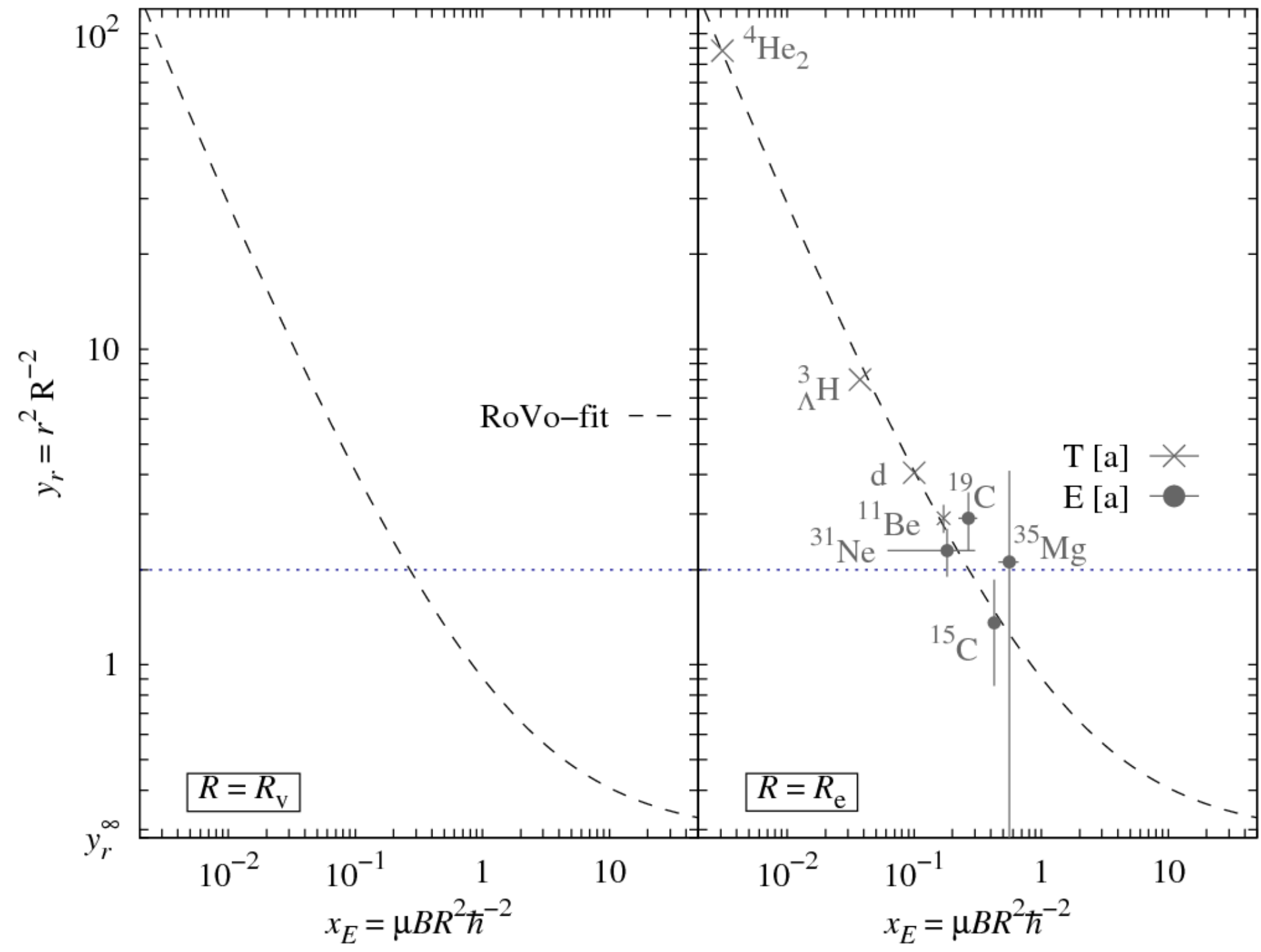
$R = R_v$

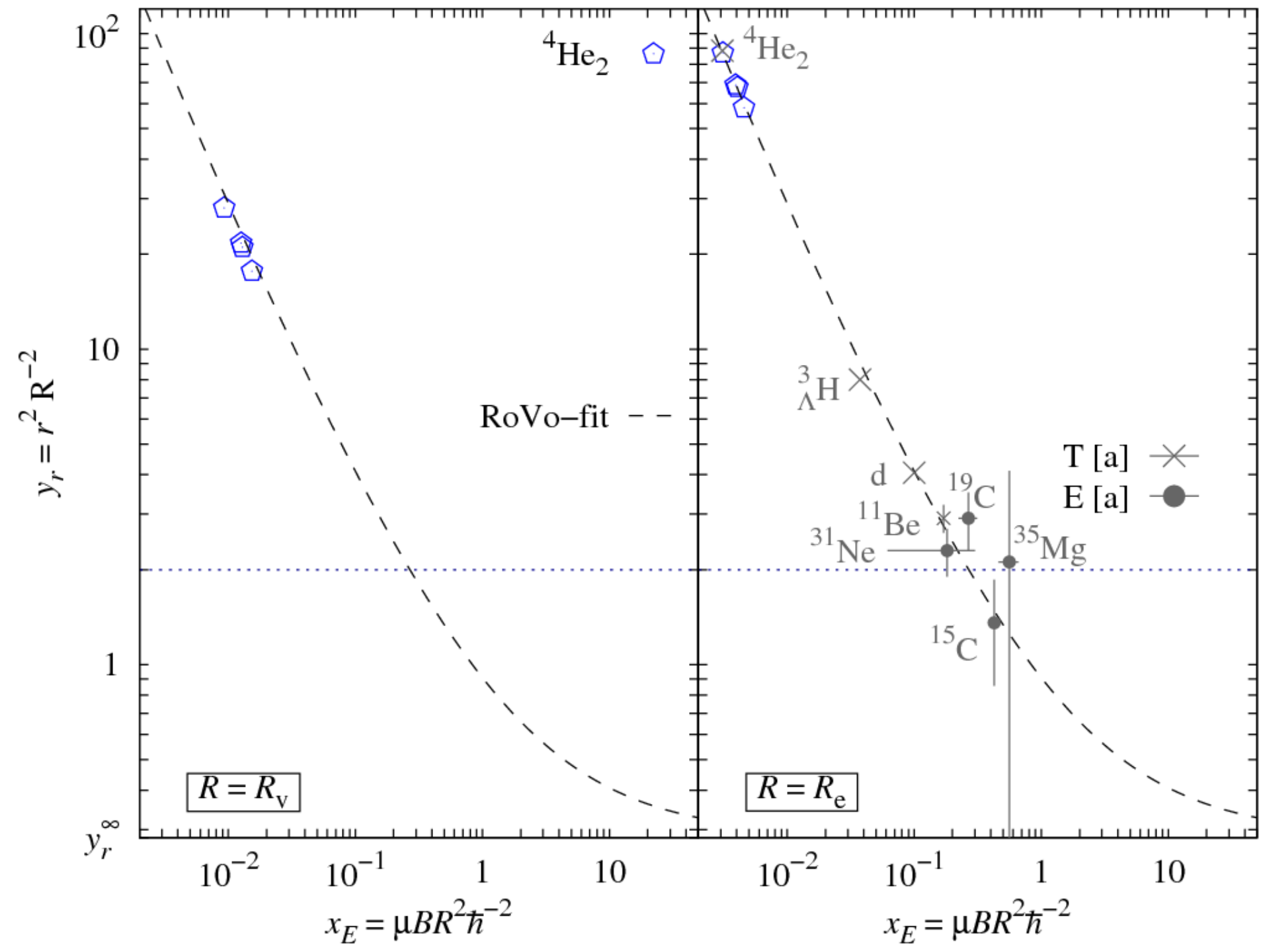
$R = R_e$

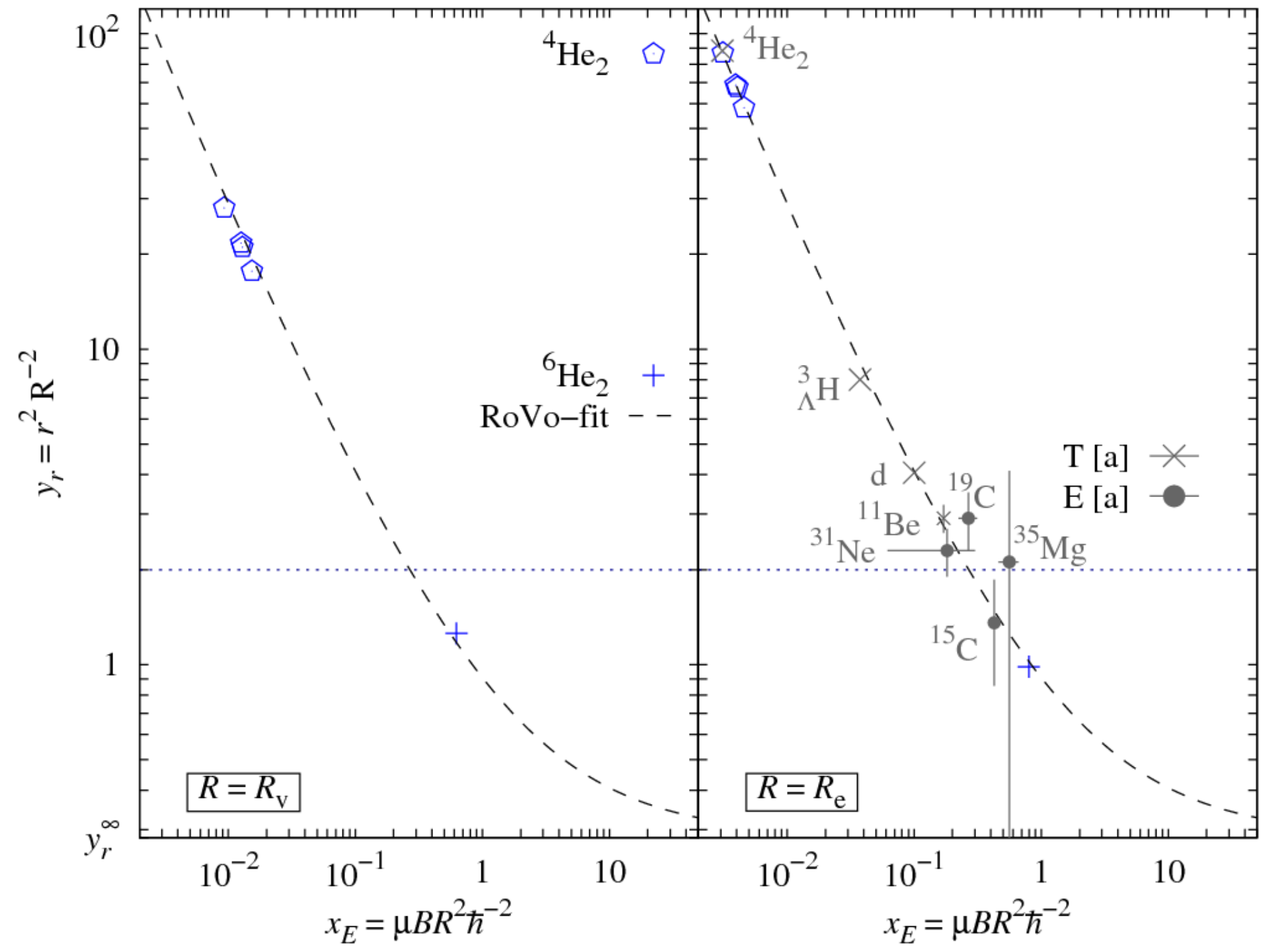
$x_E = \mu BR^2 \hbar^{-2}$

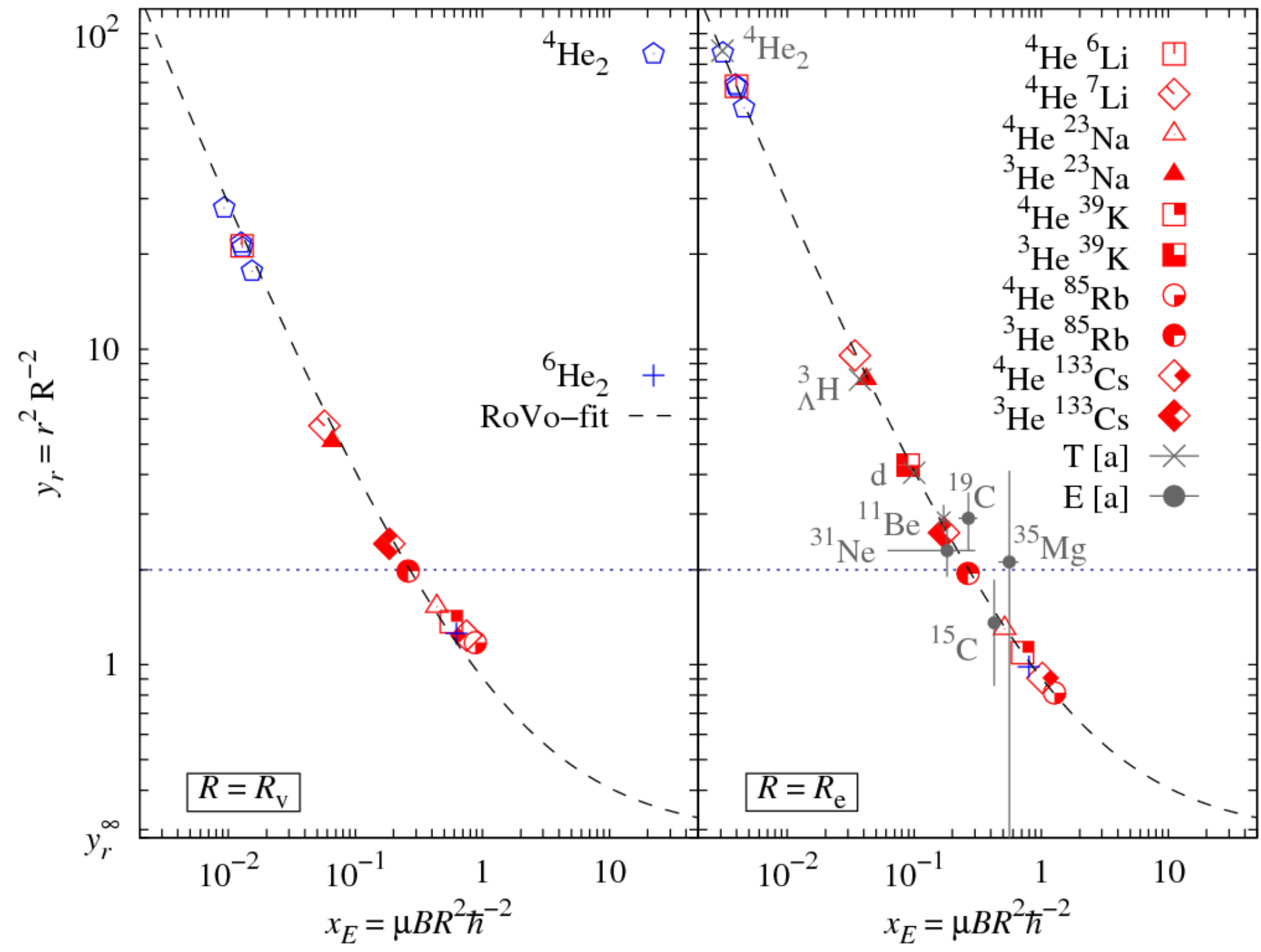
$x_E = \mu BR^2 \hbar^{-2}$

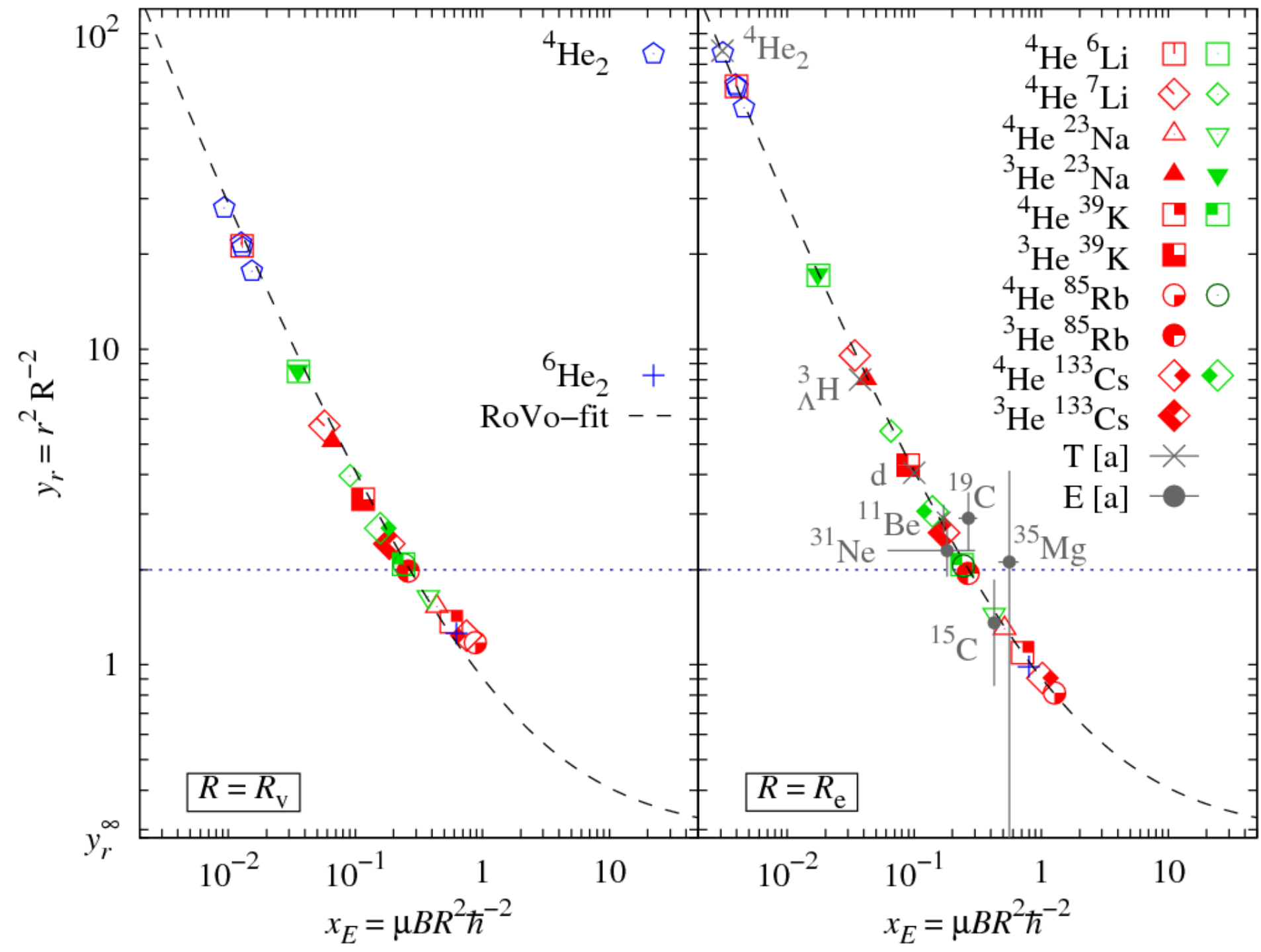


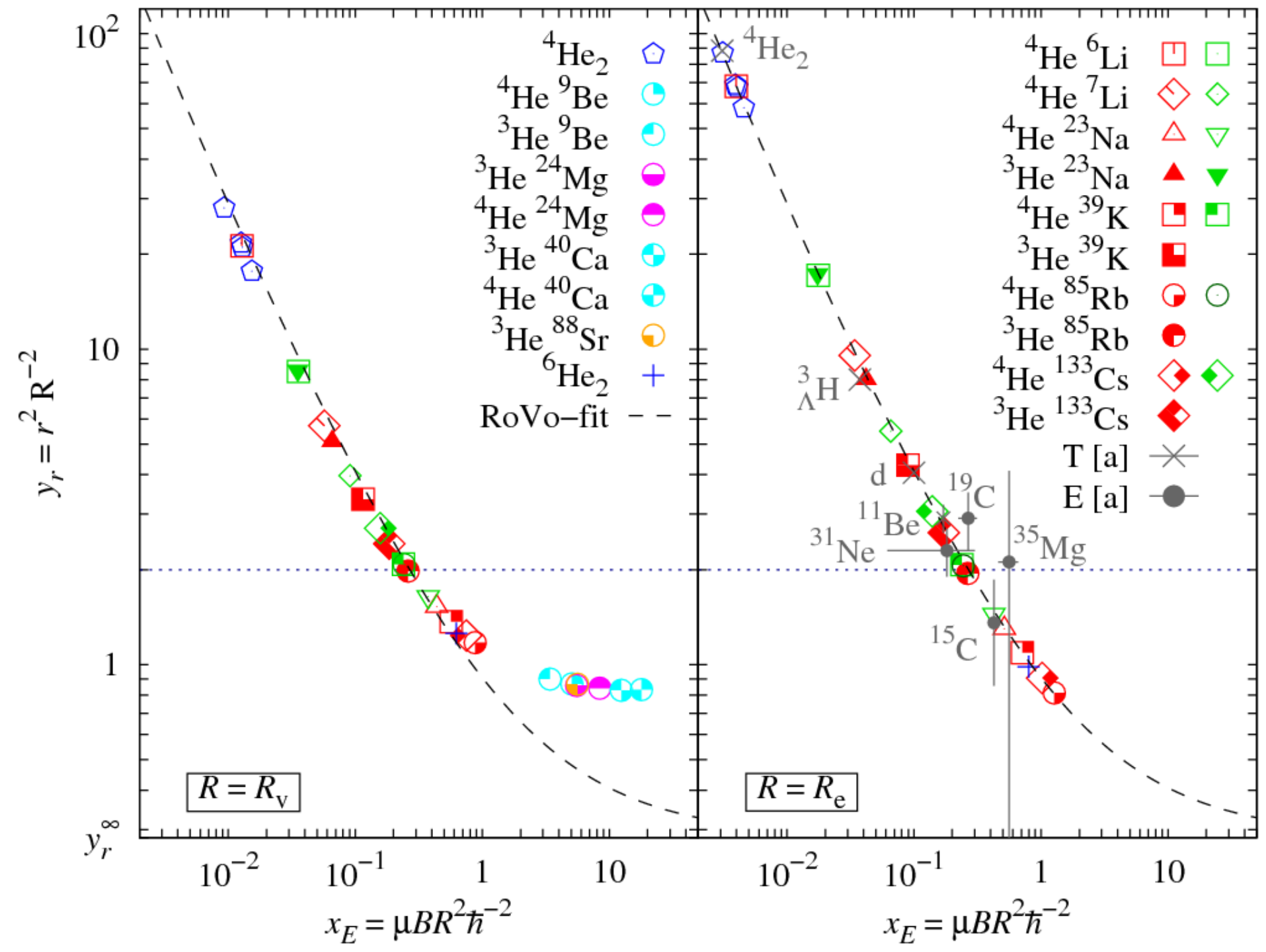




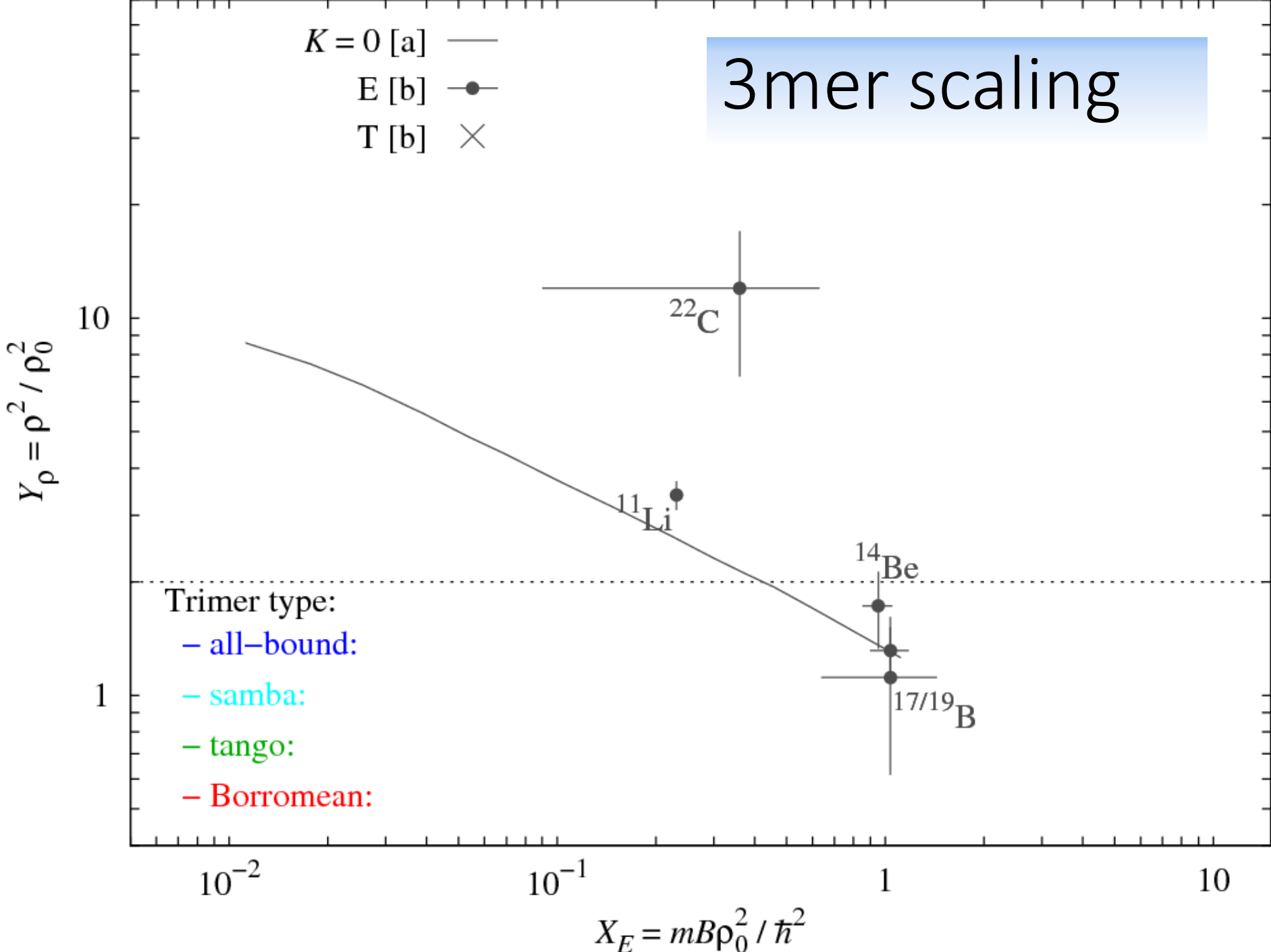


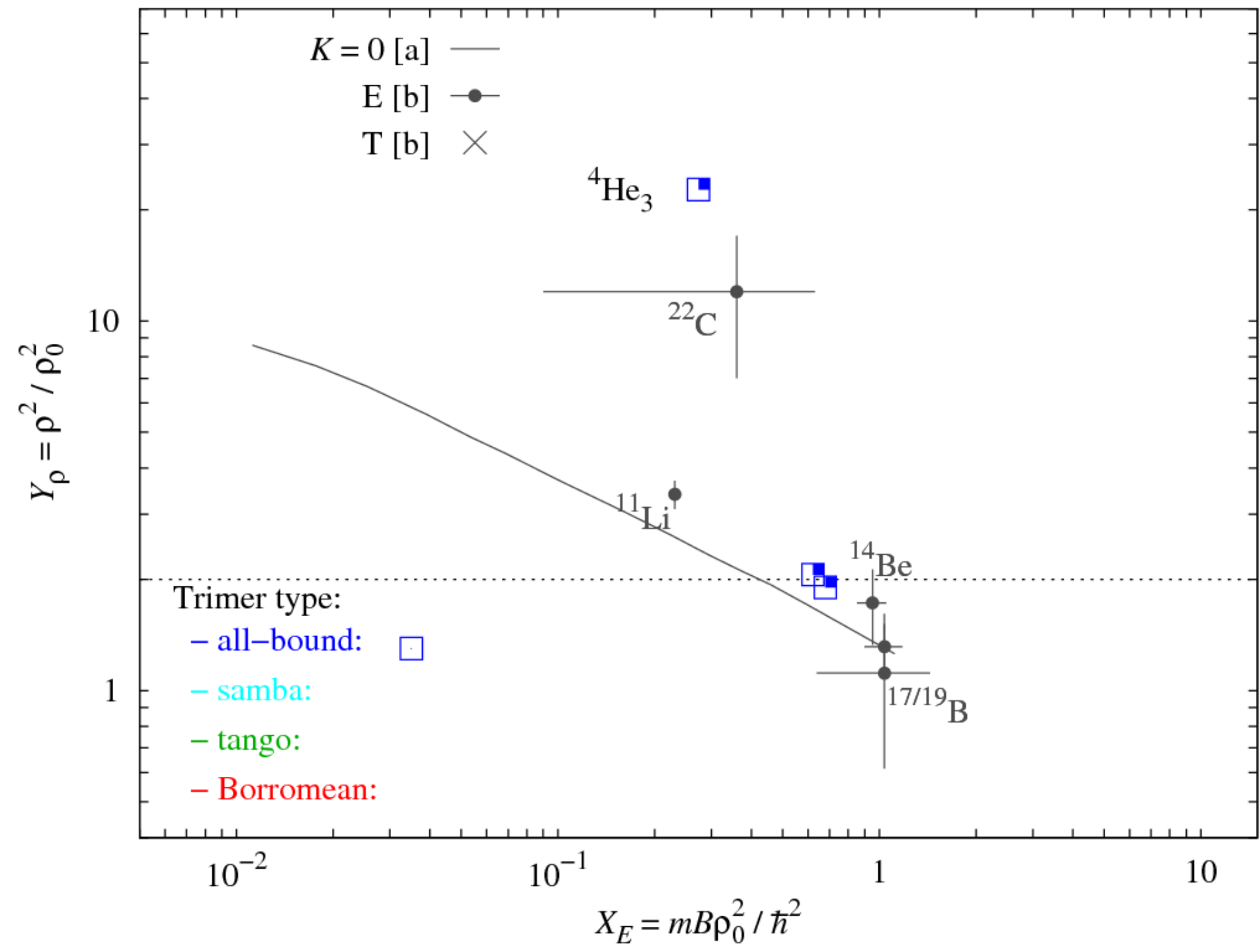


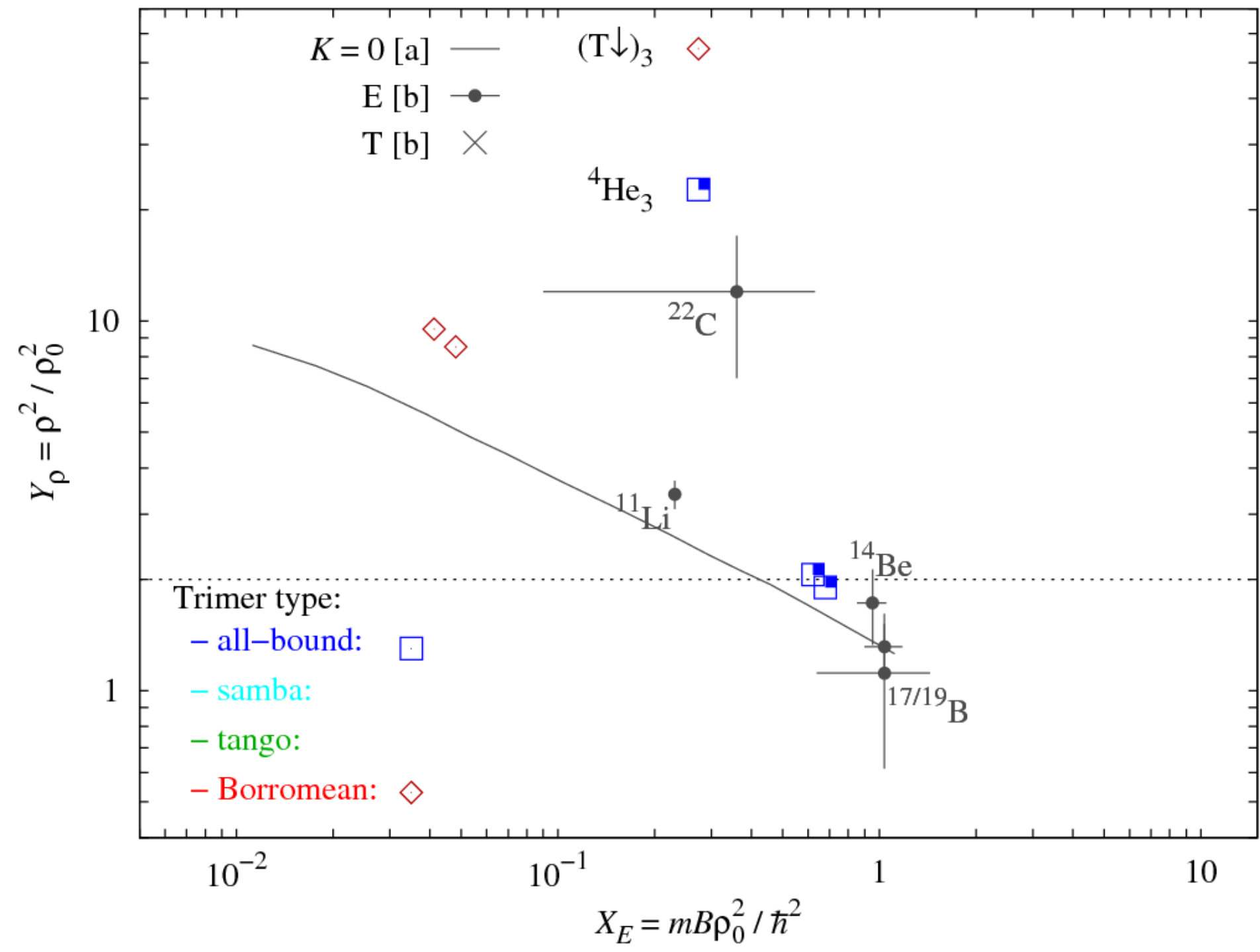


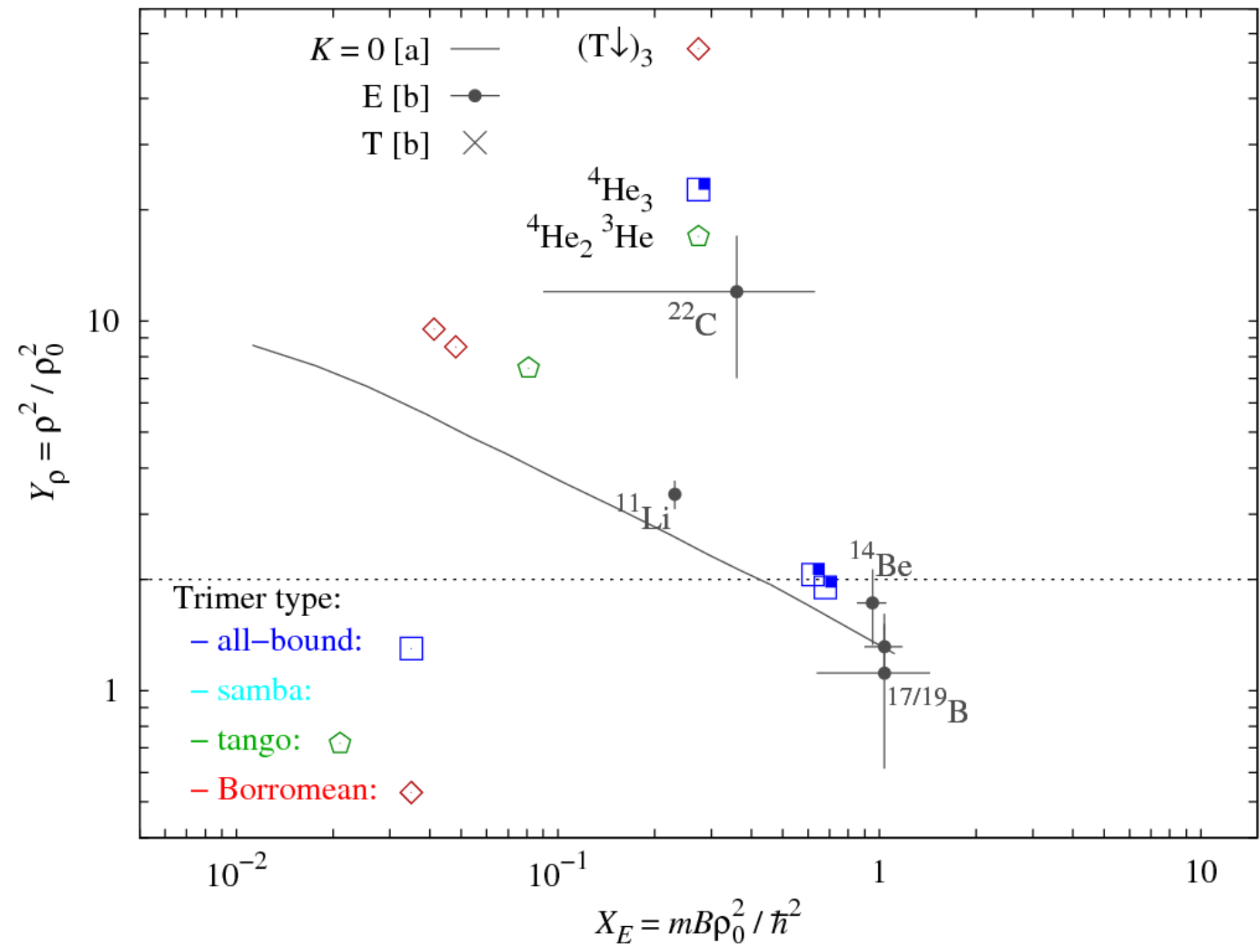


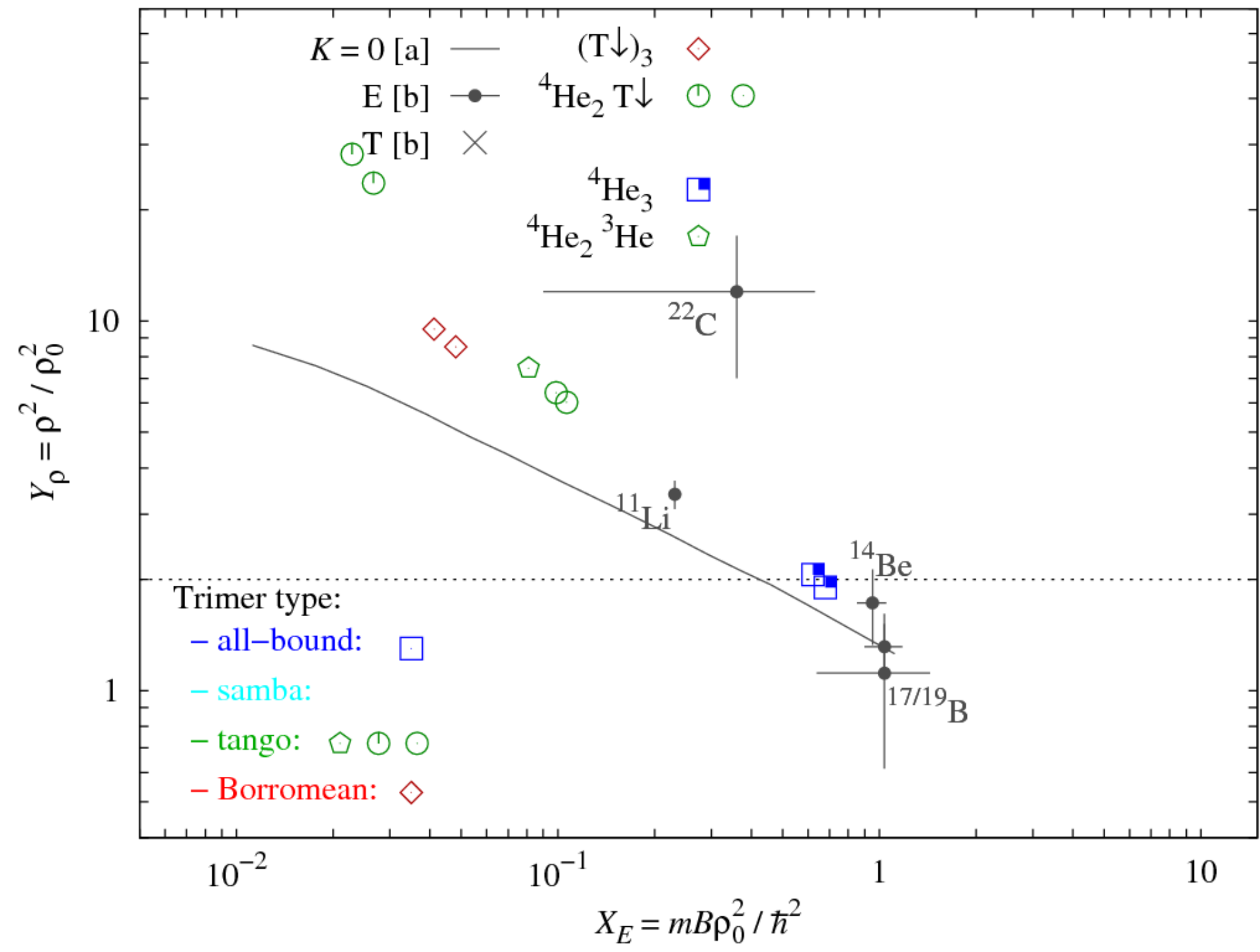
3mer scaling

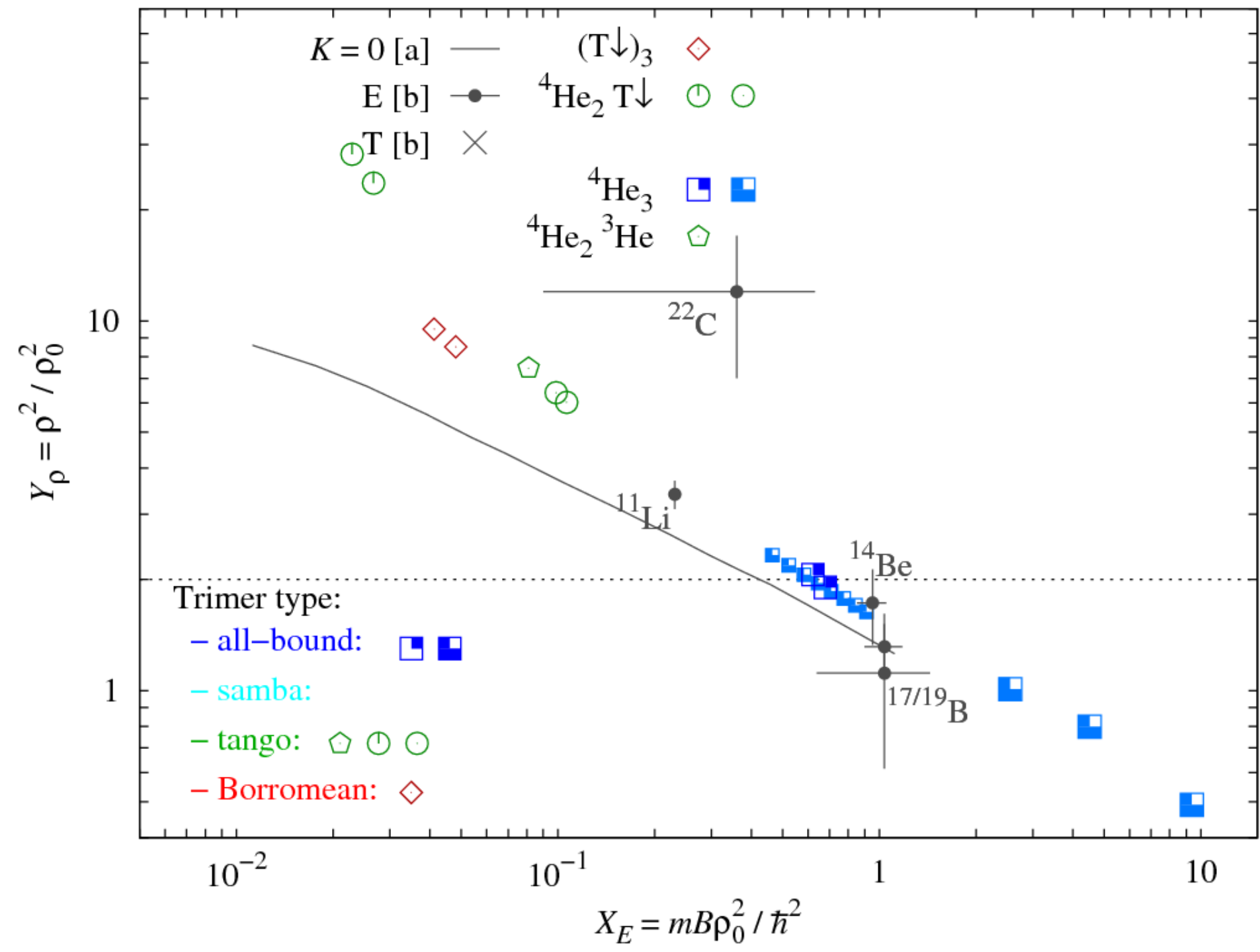


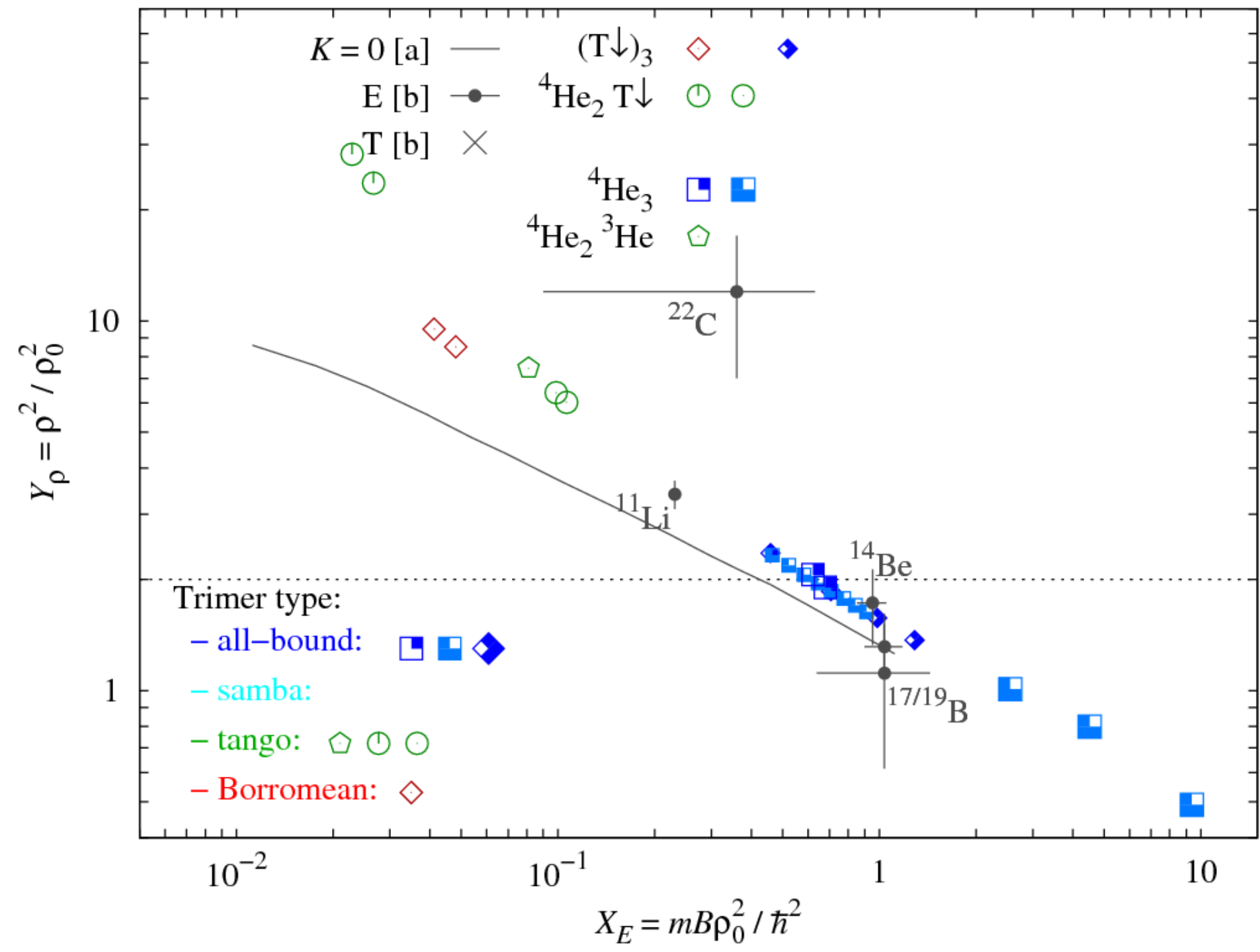


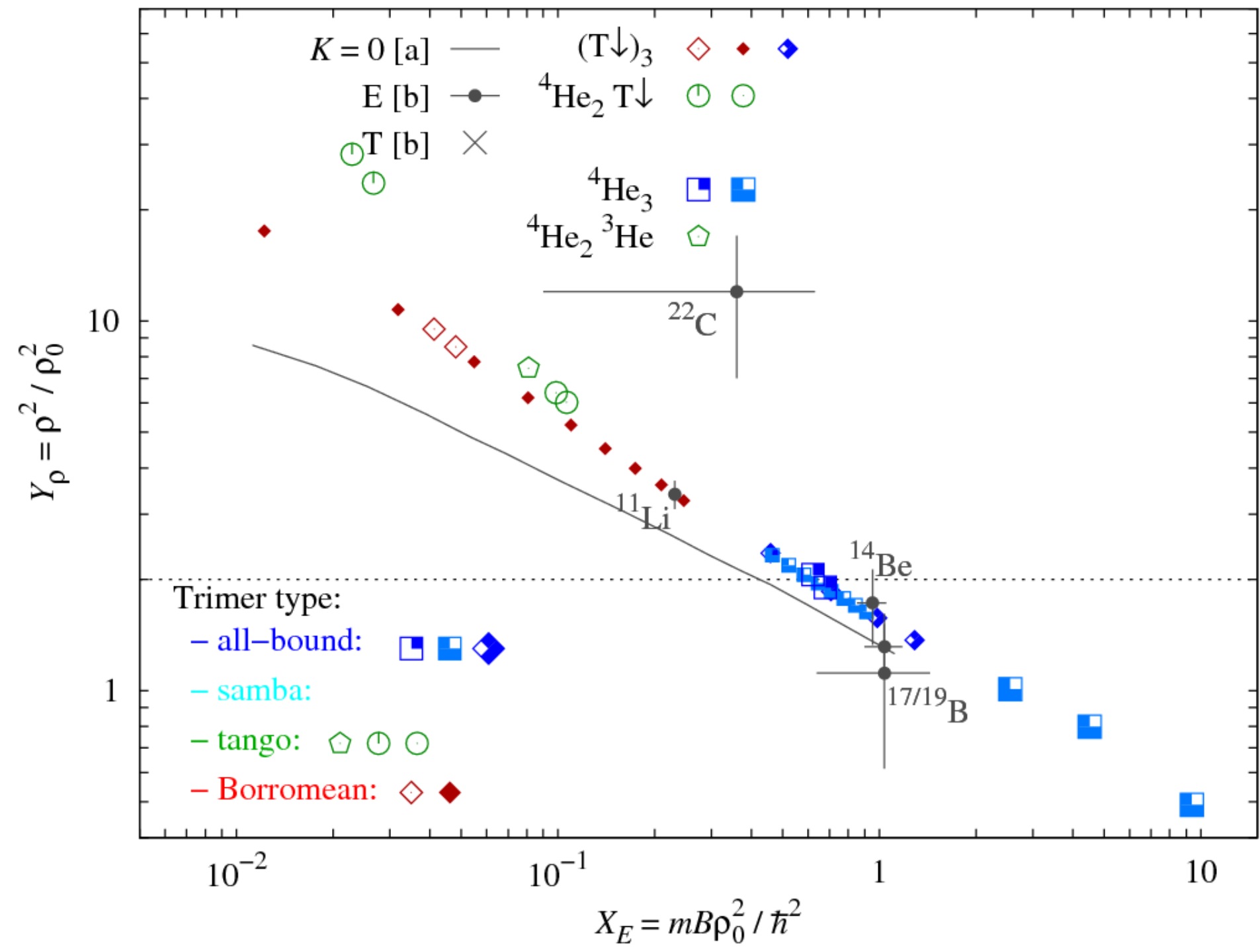


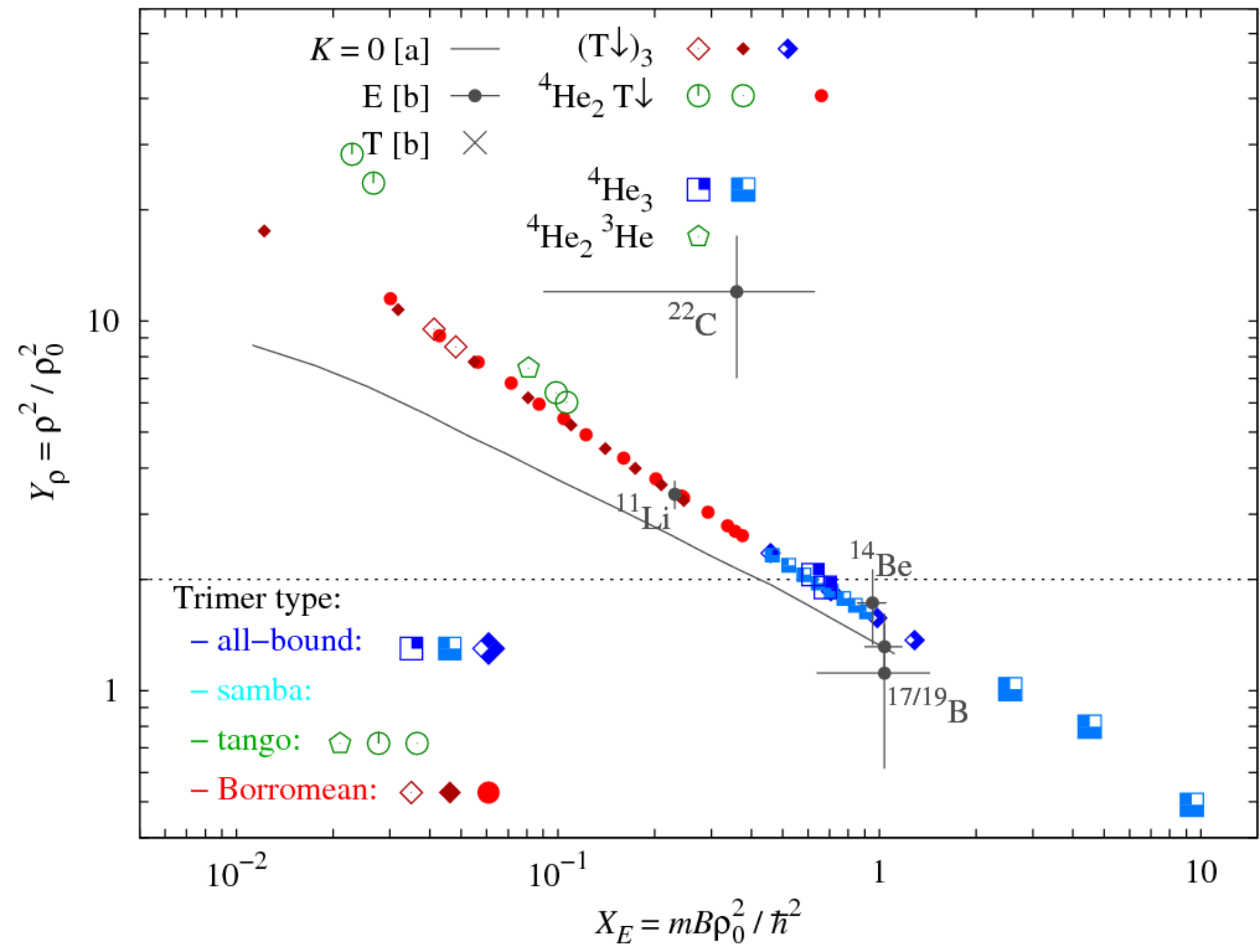


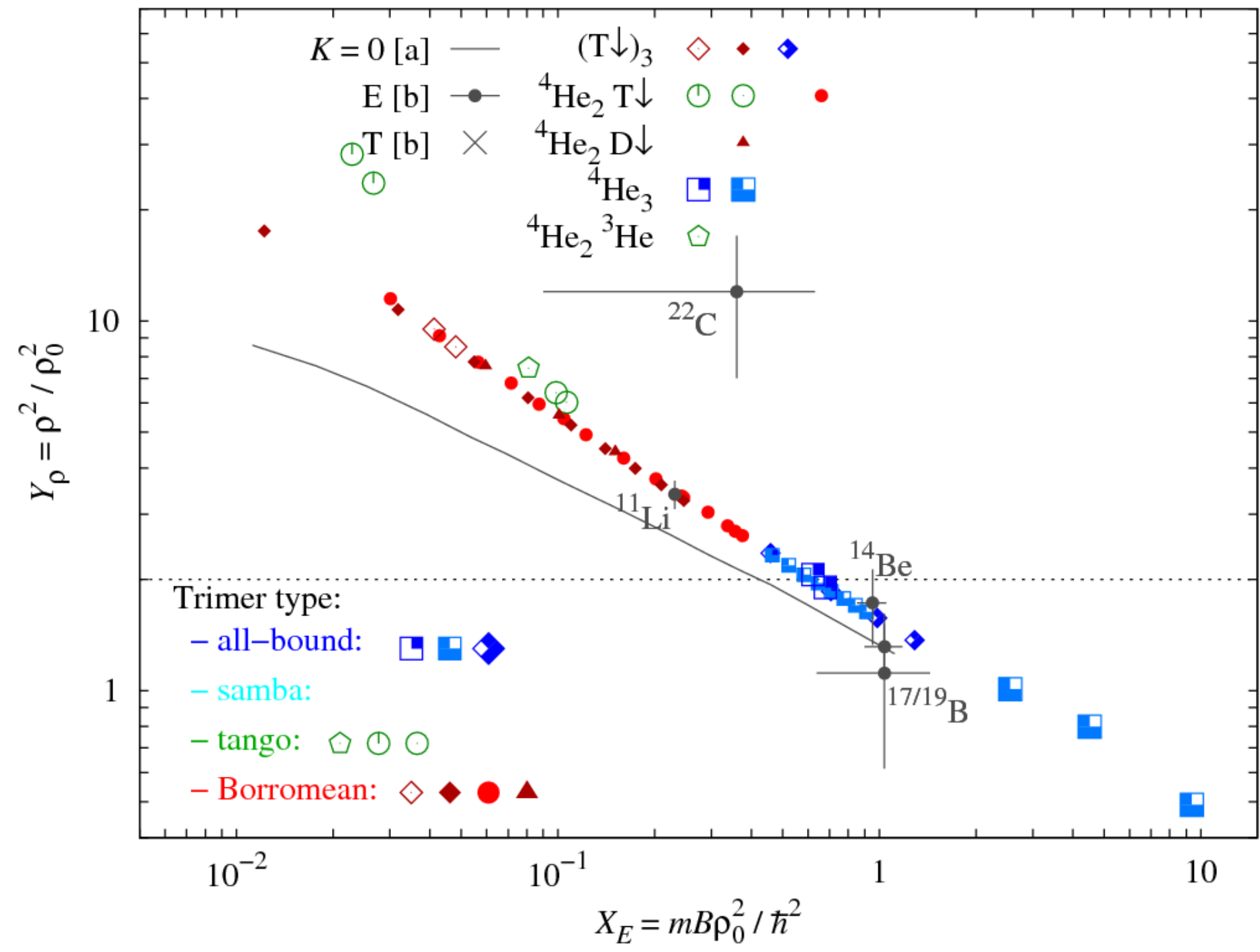


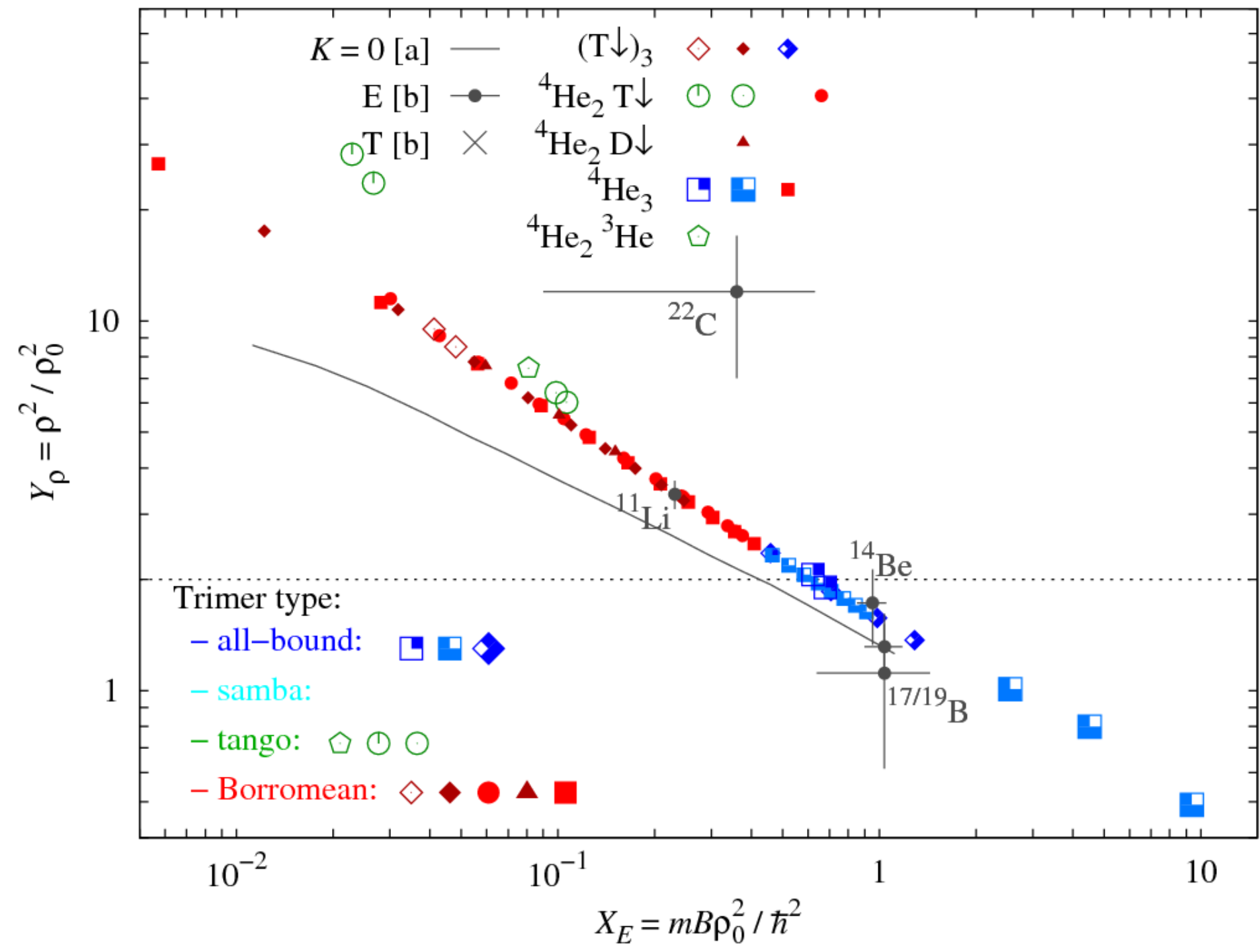


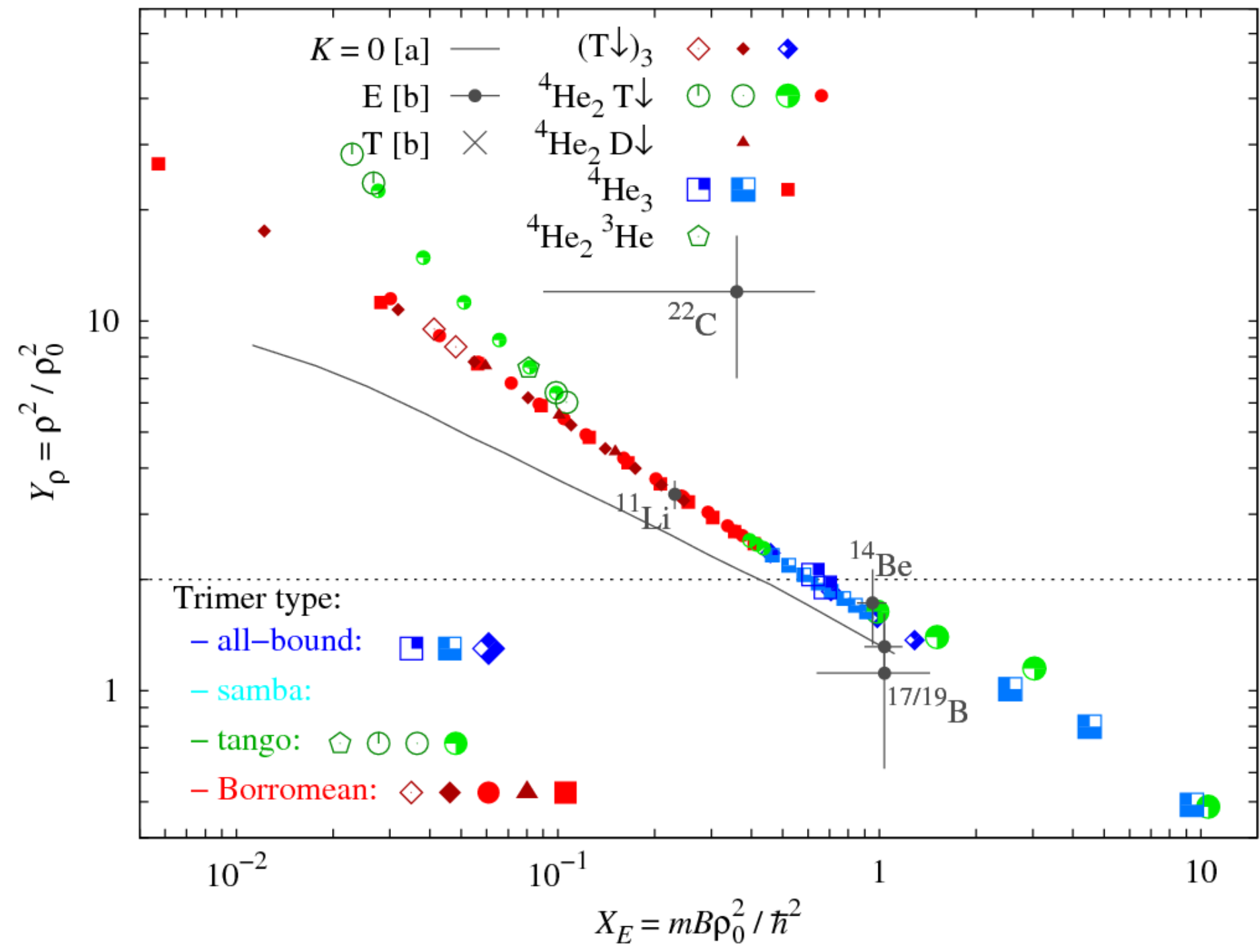


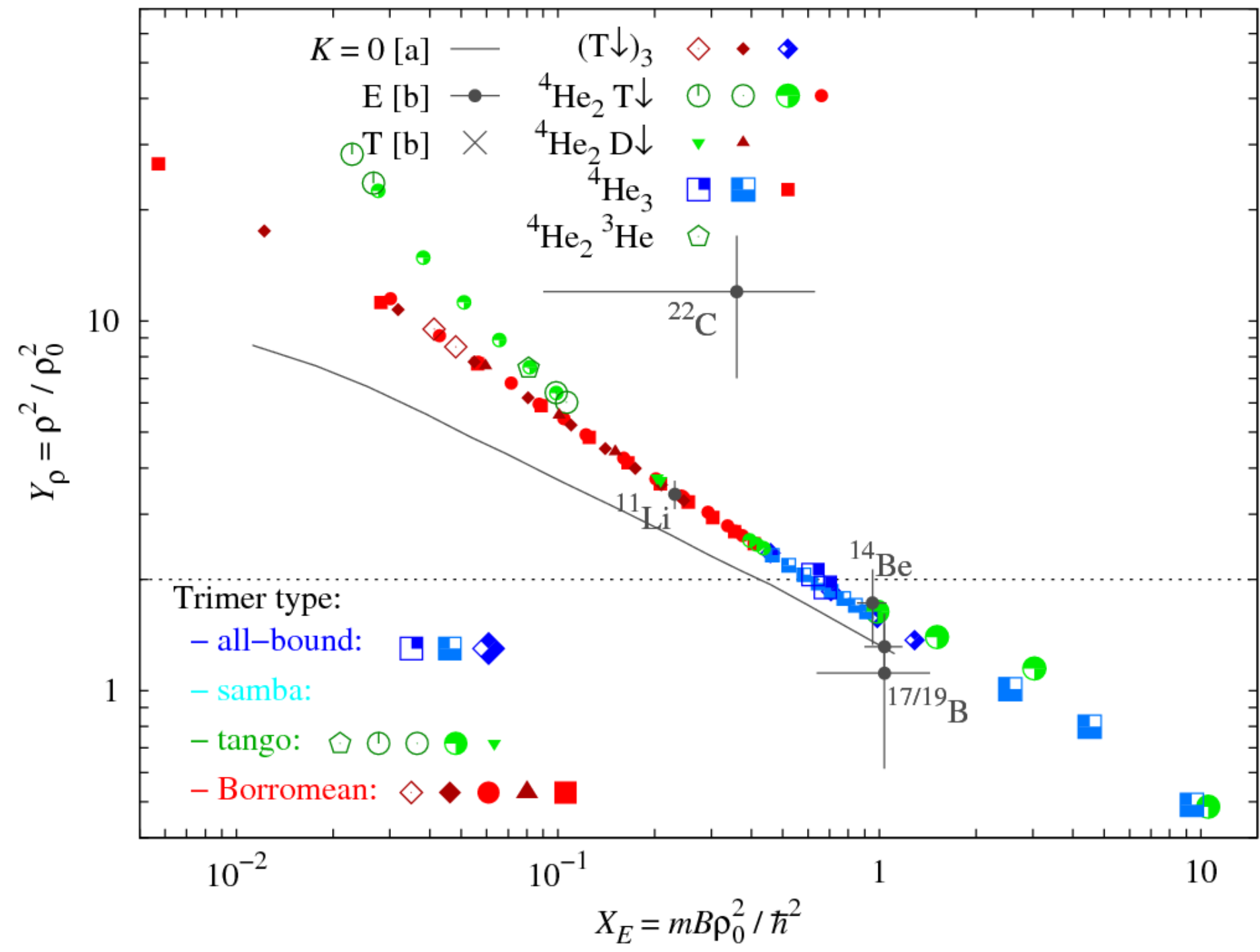


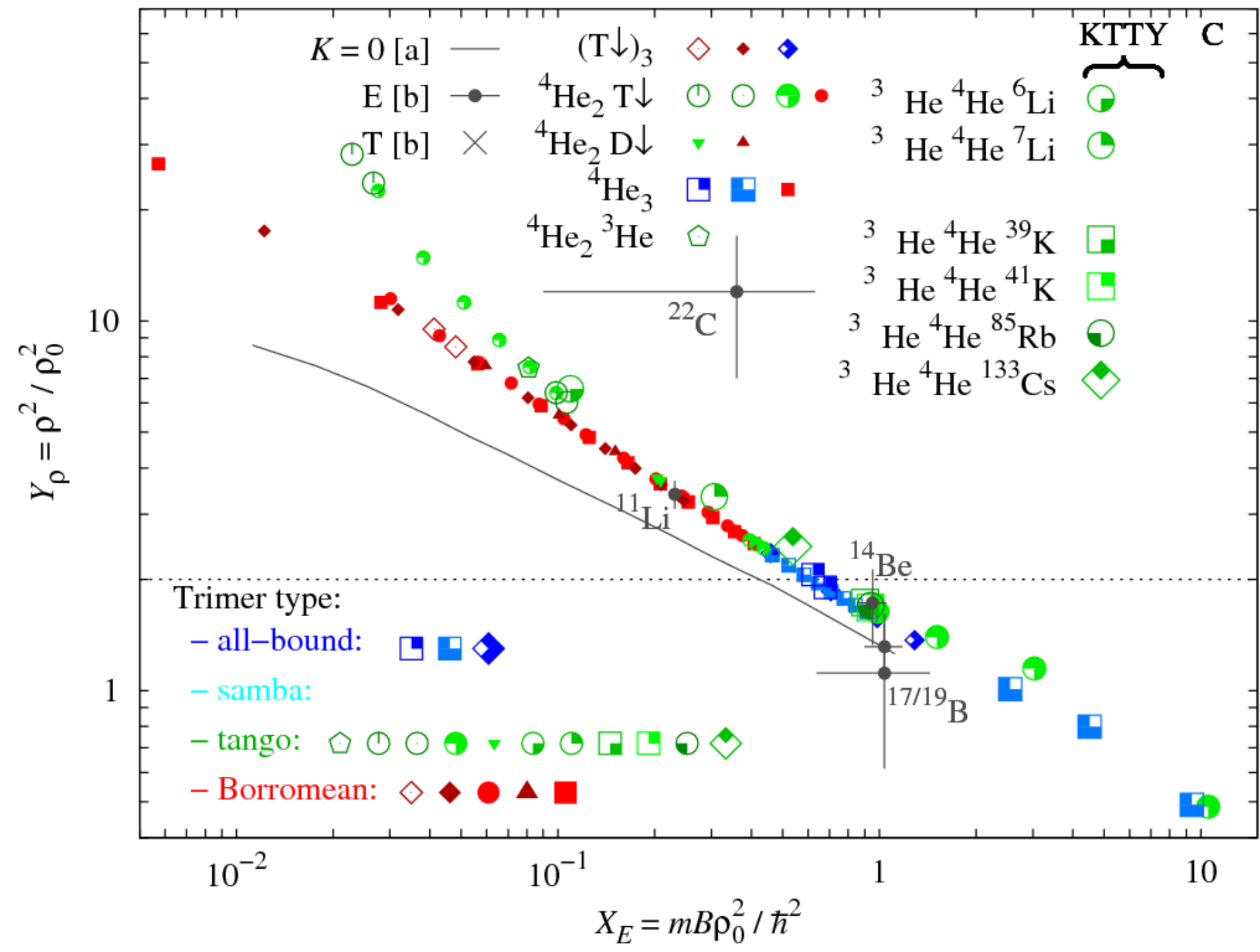


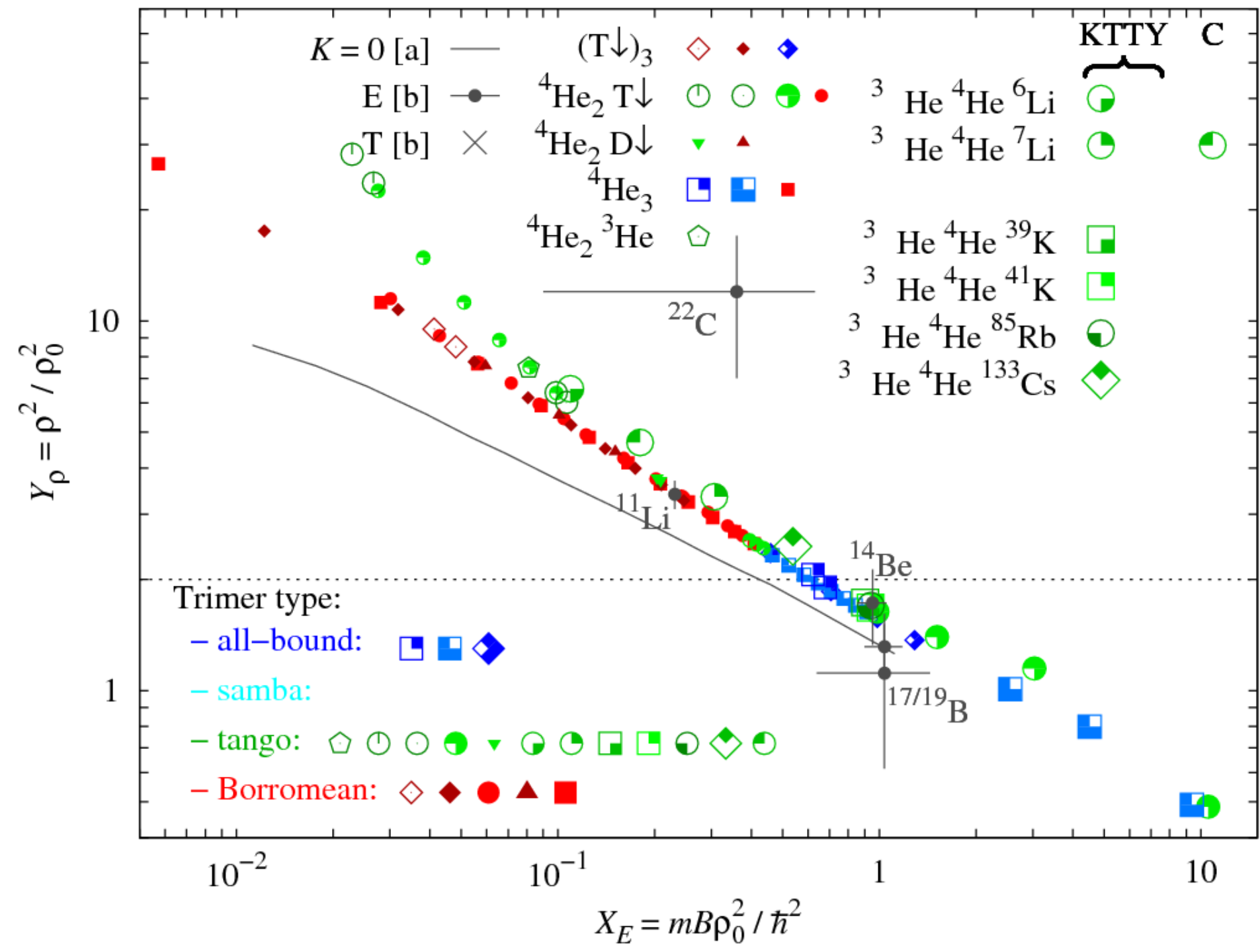


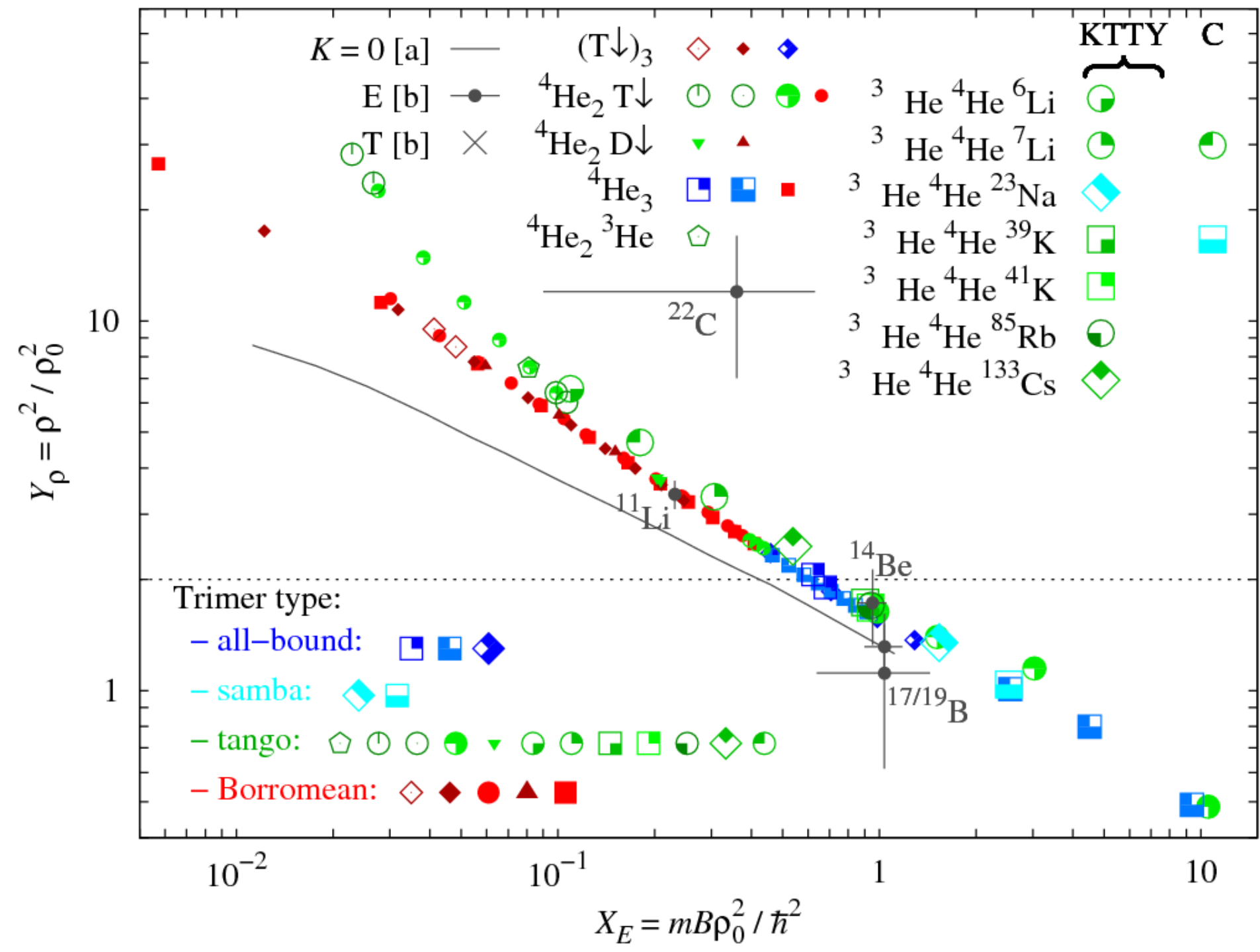


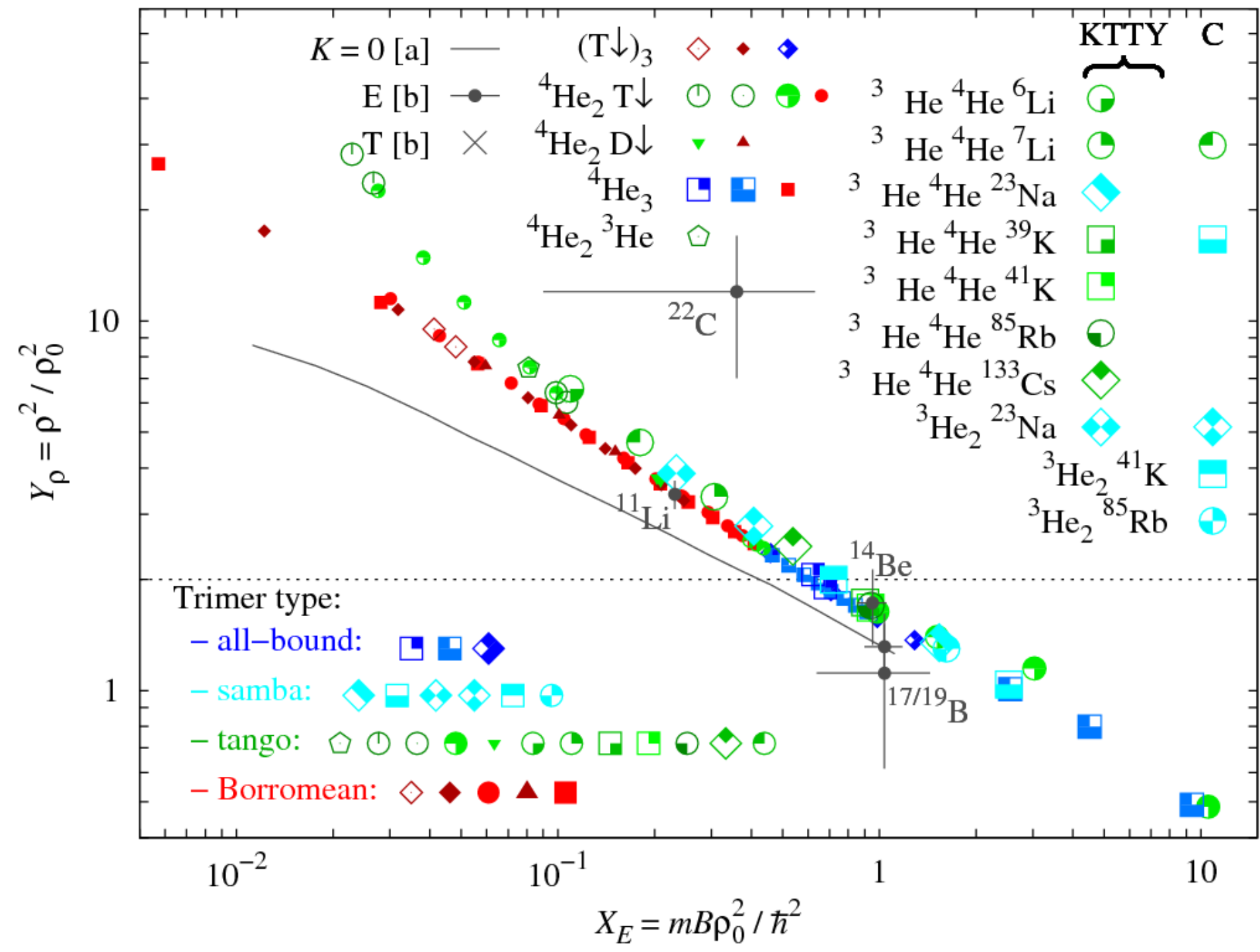


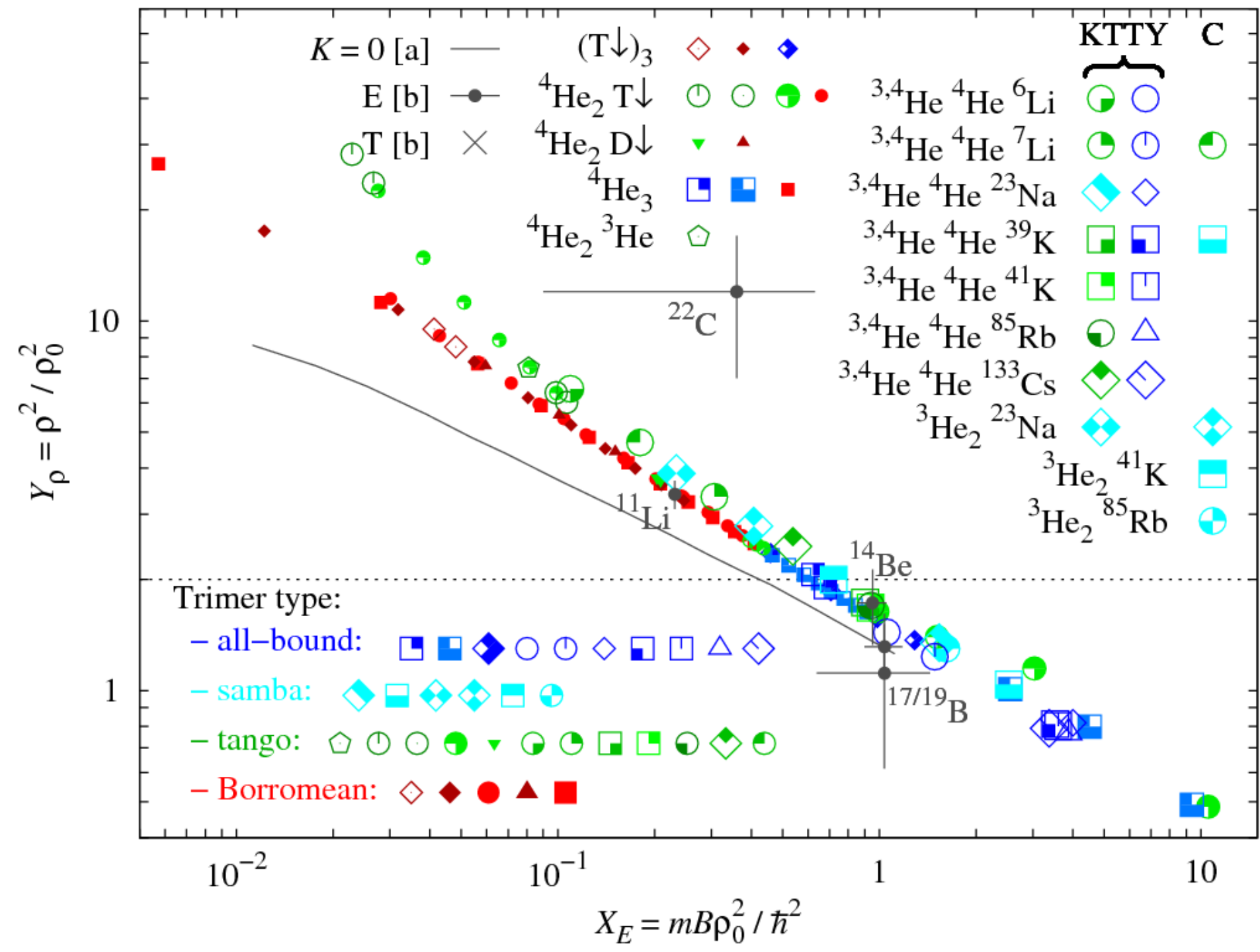


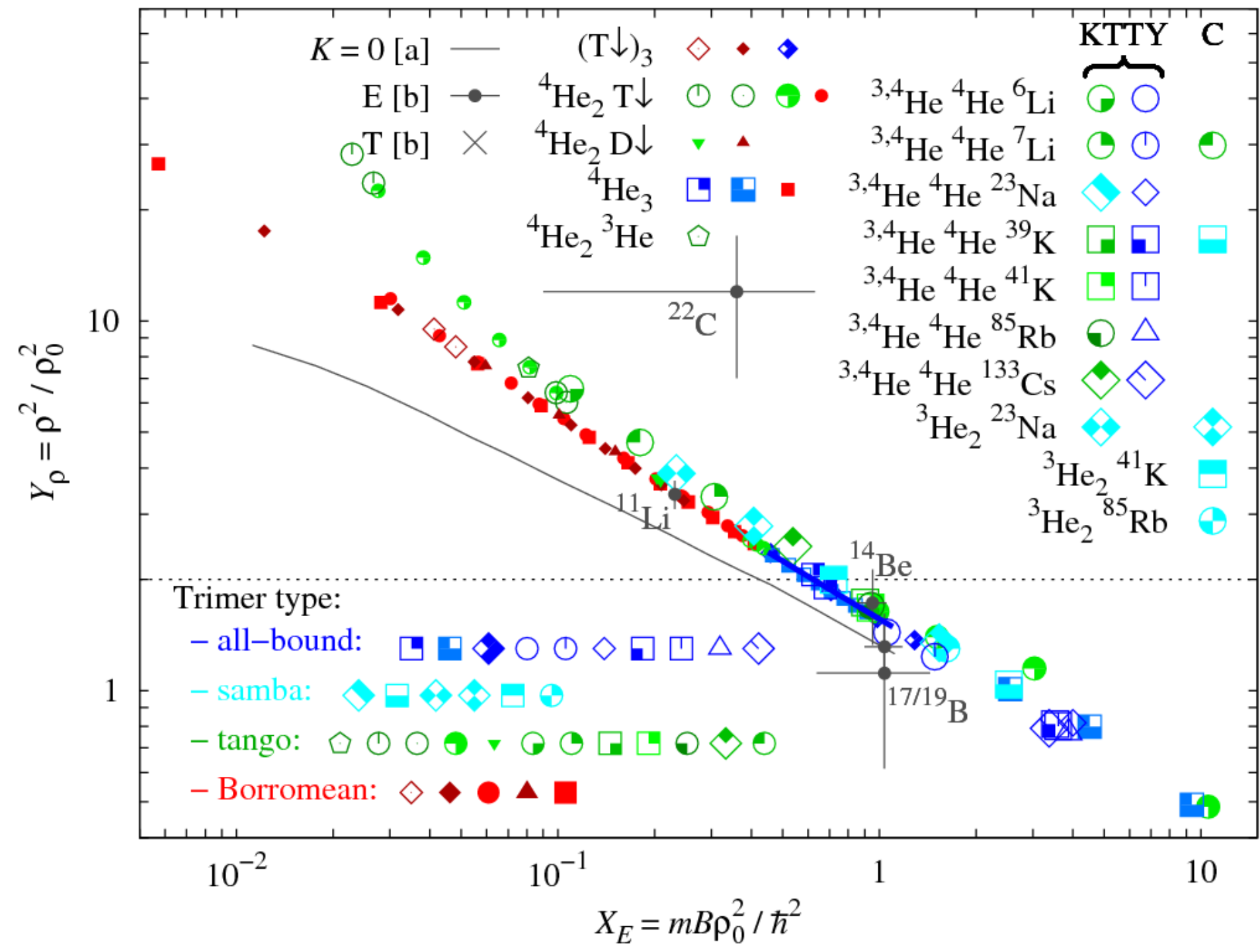


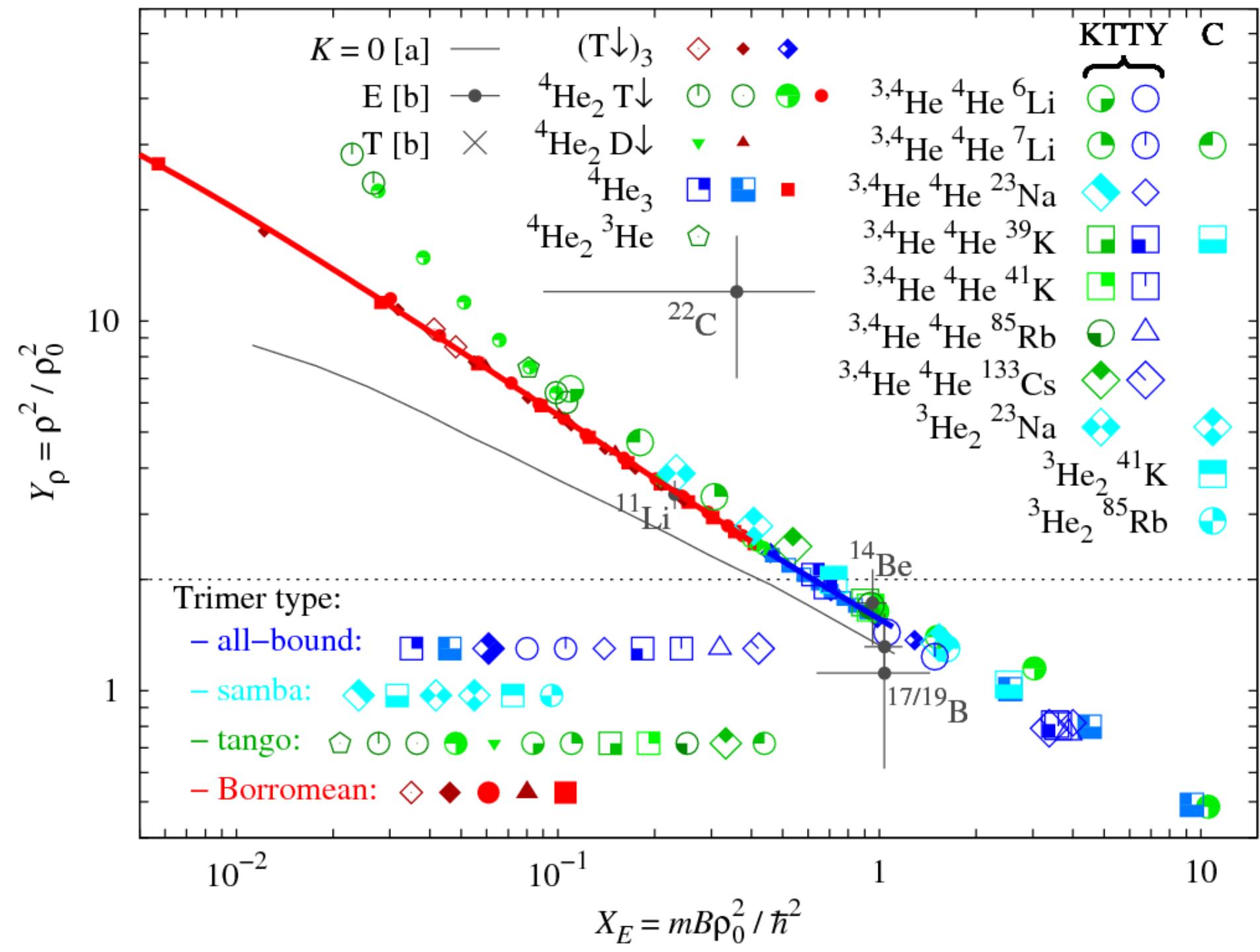


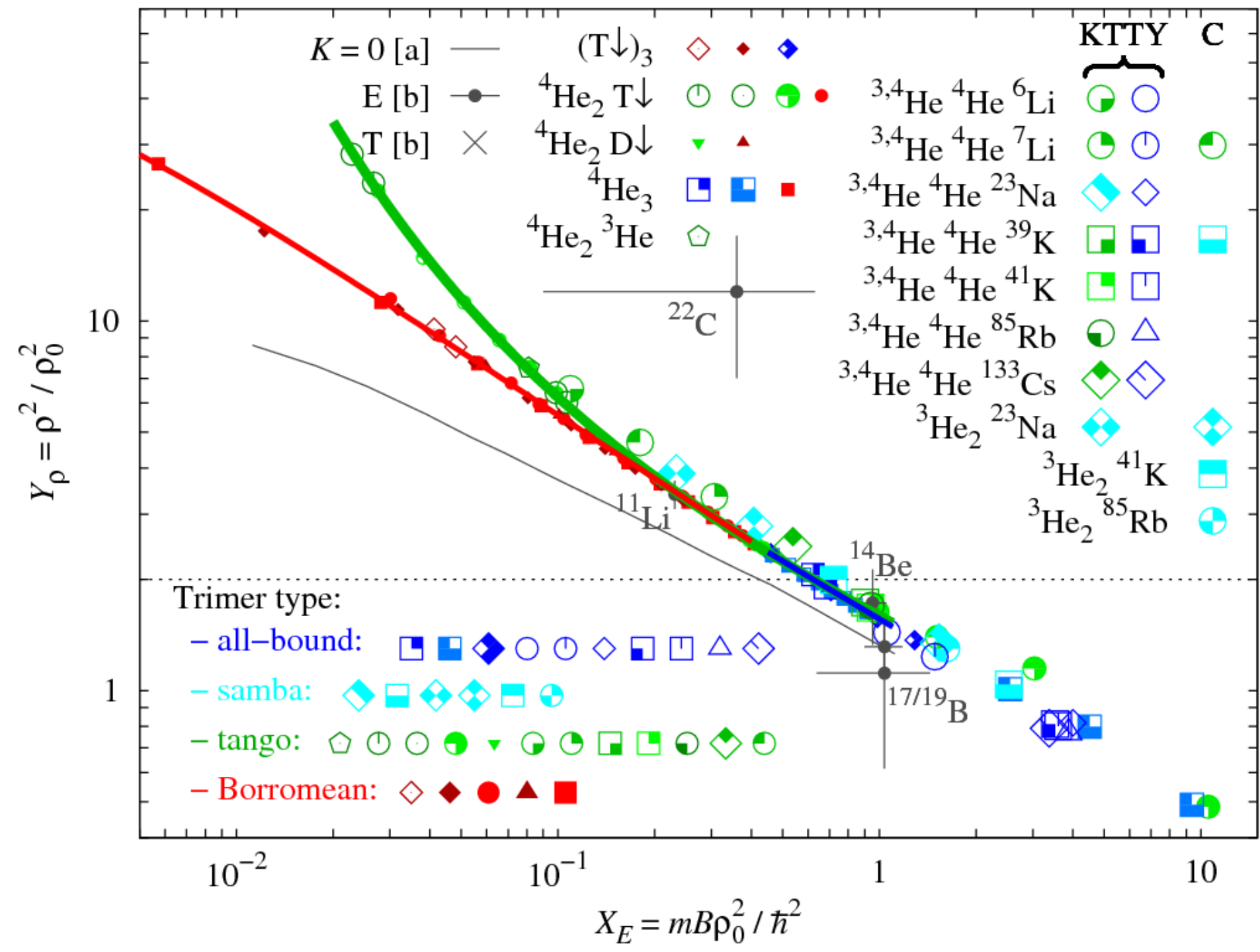


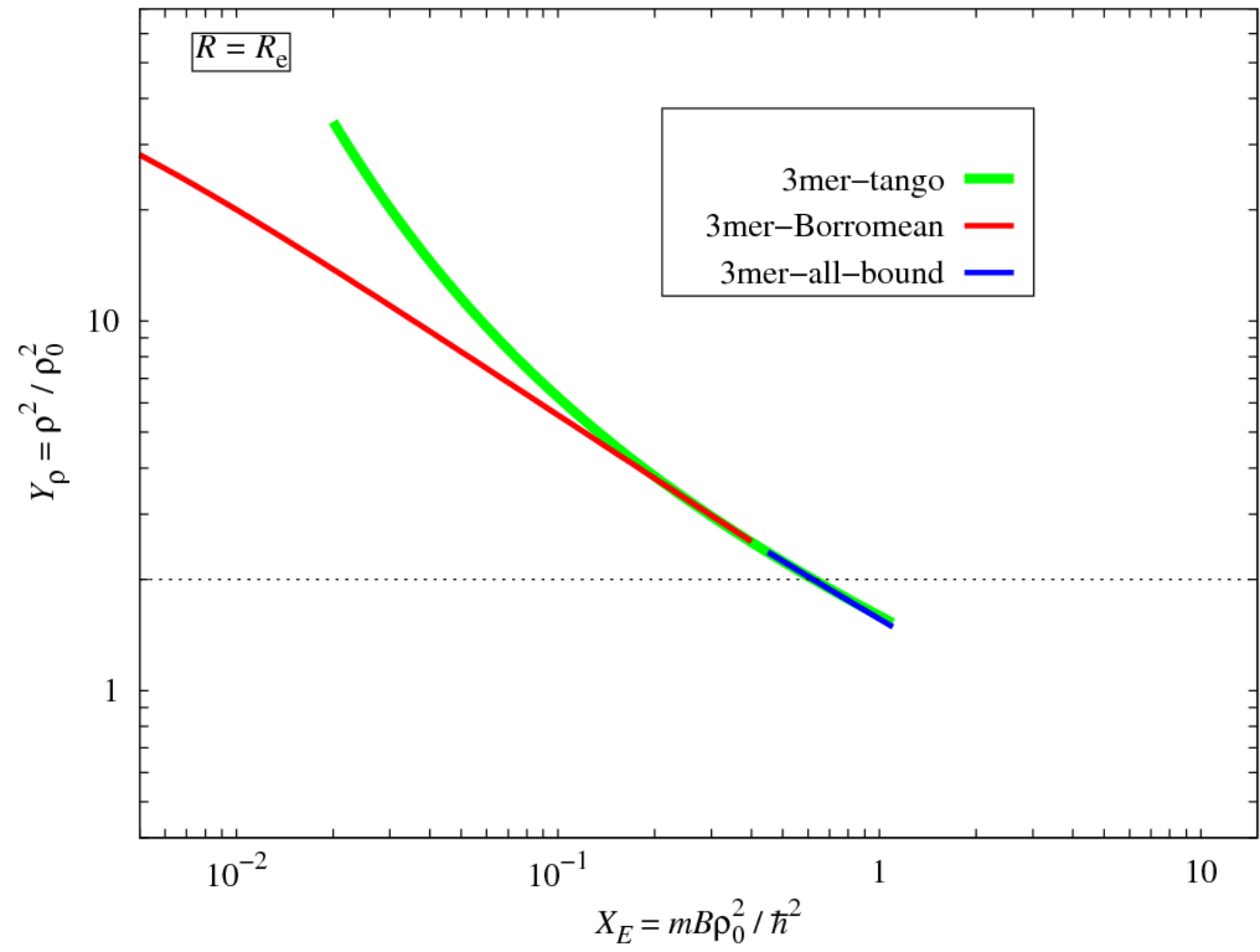


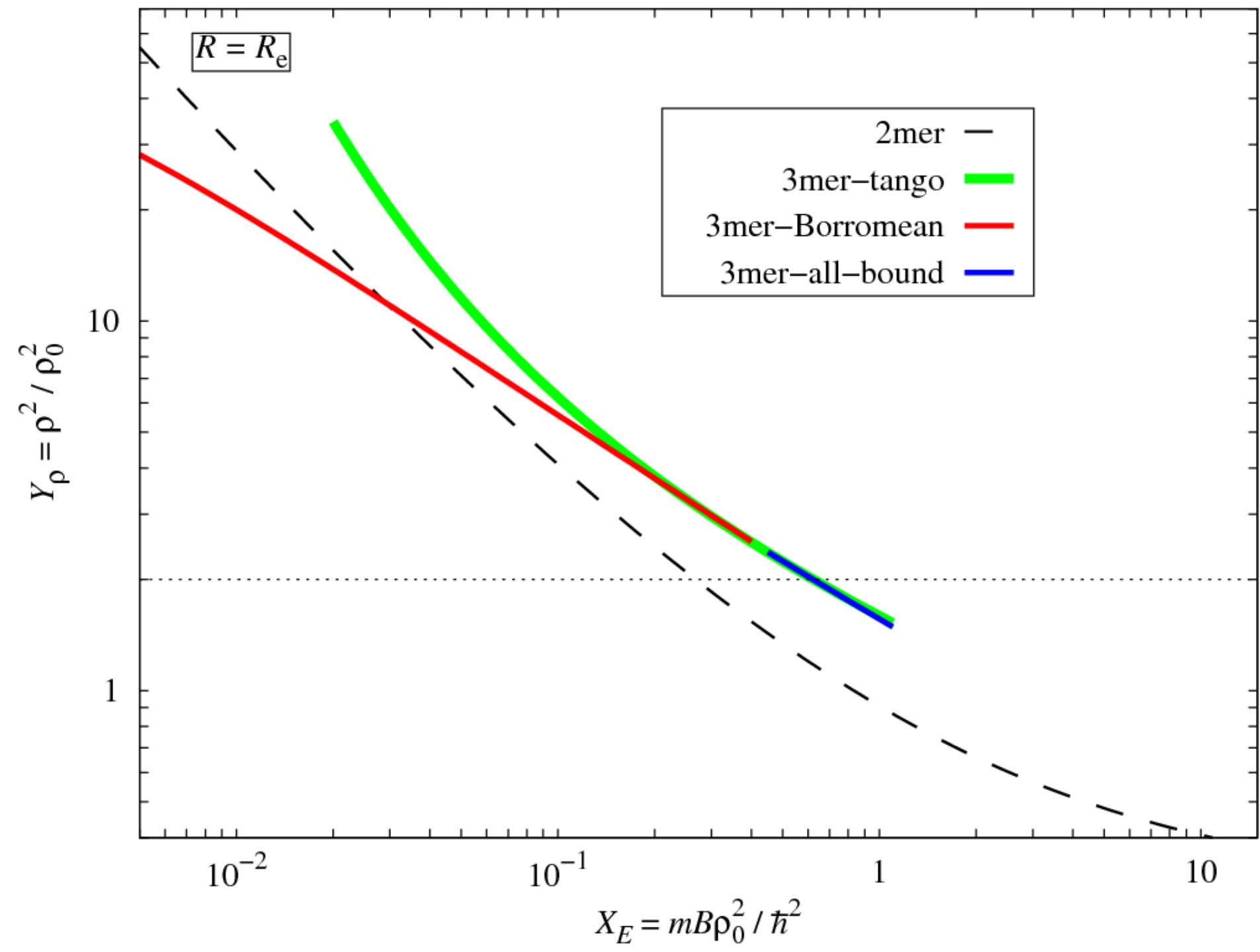


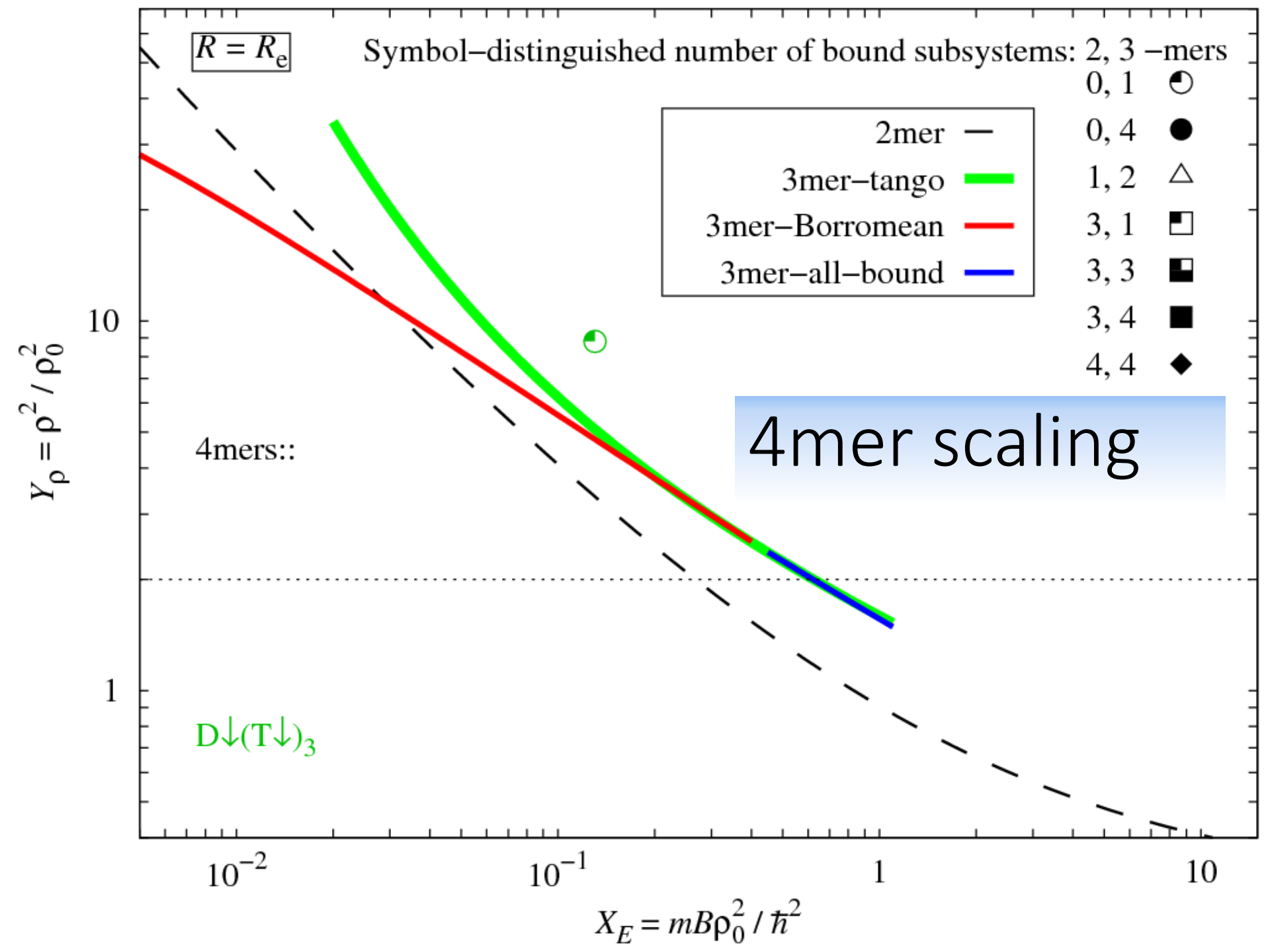


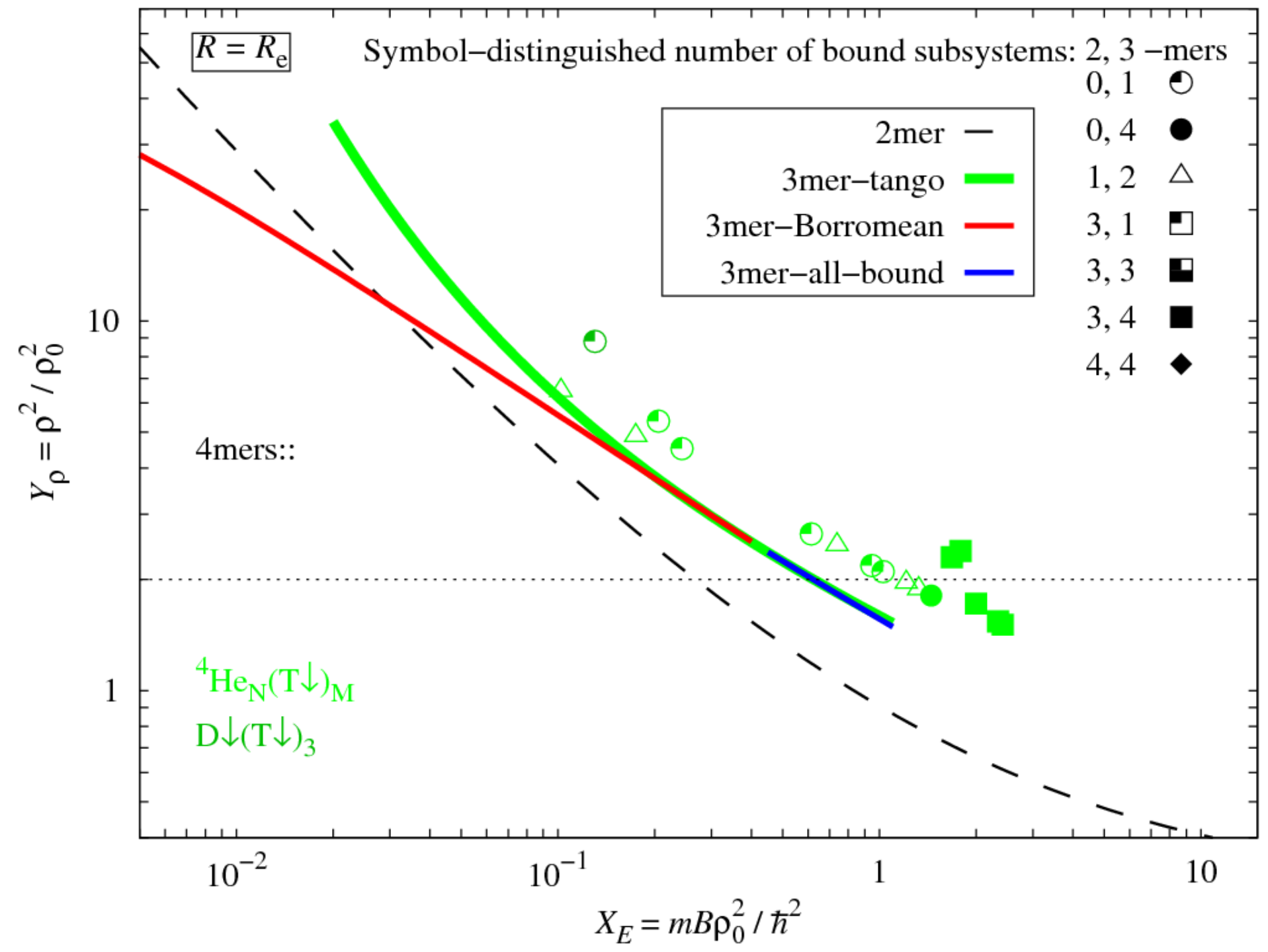


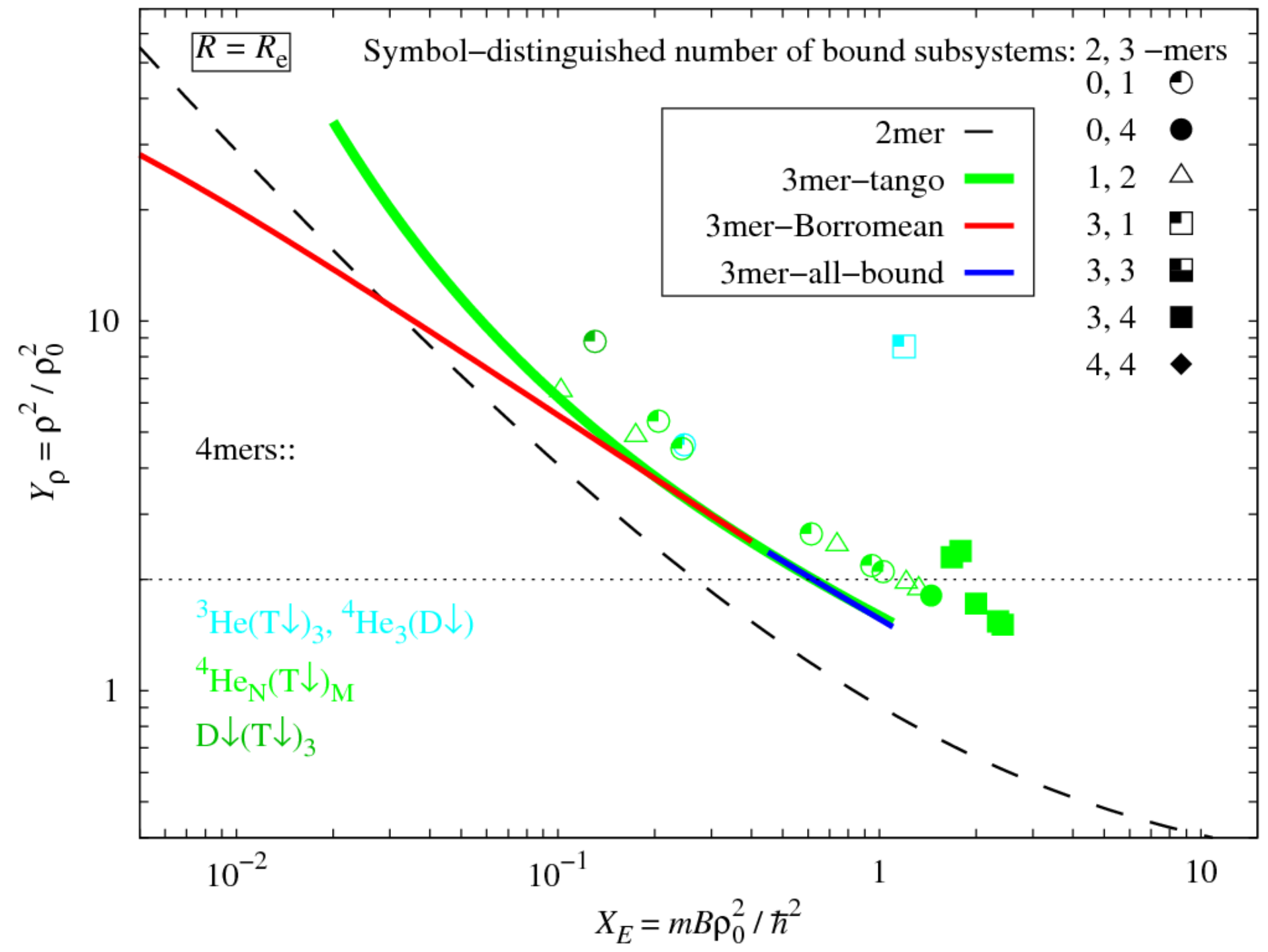


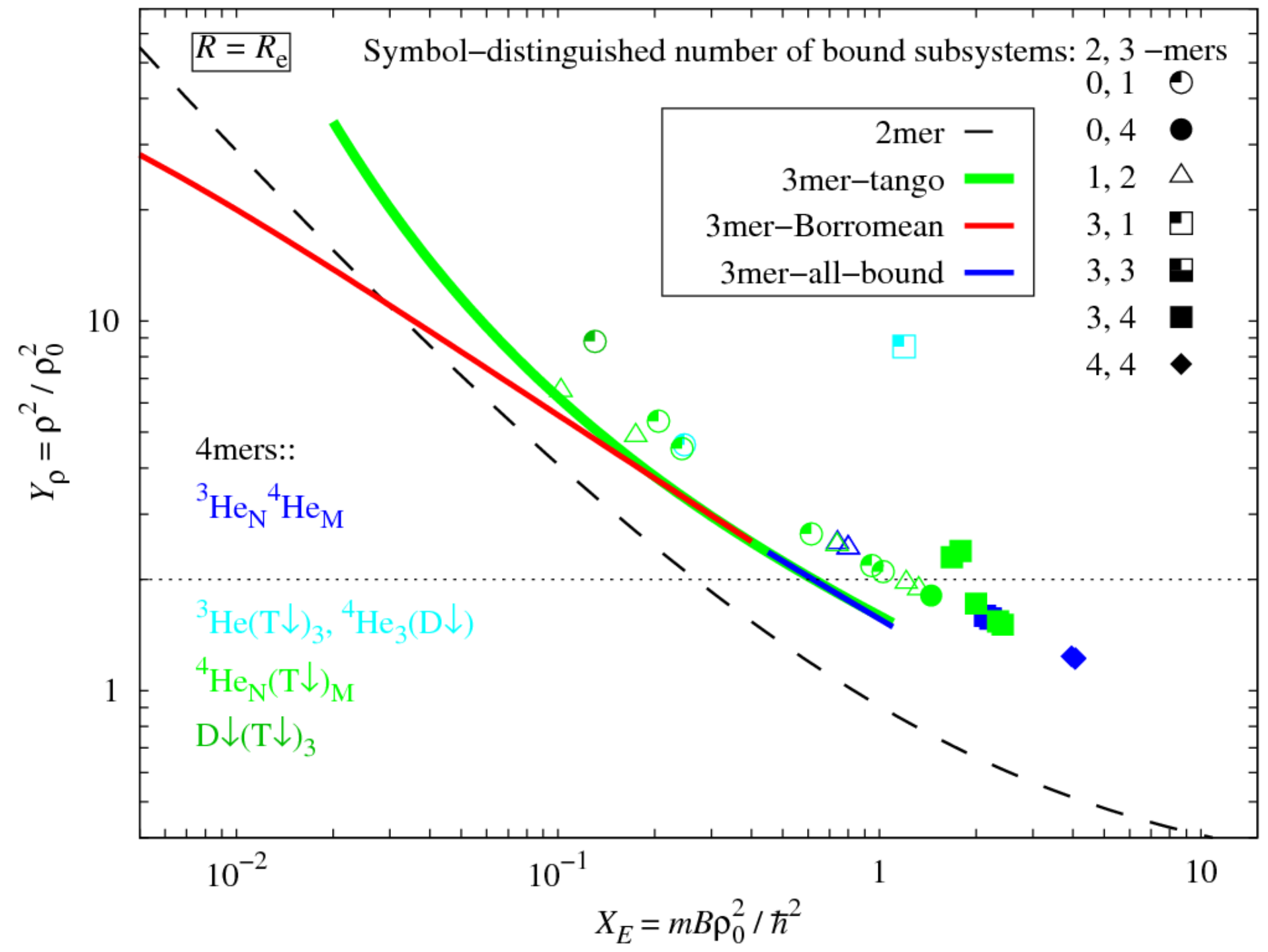


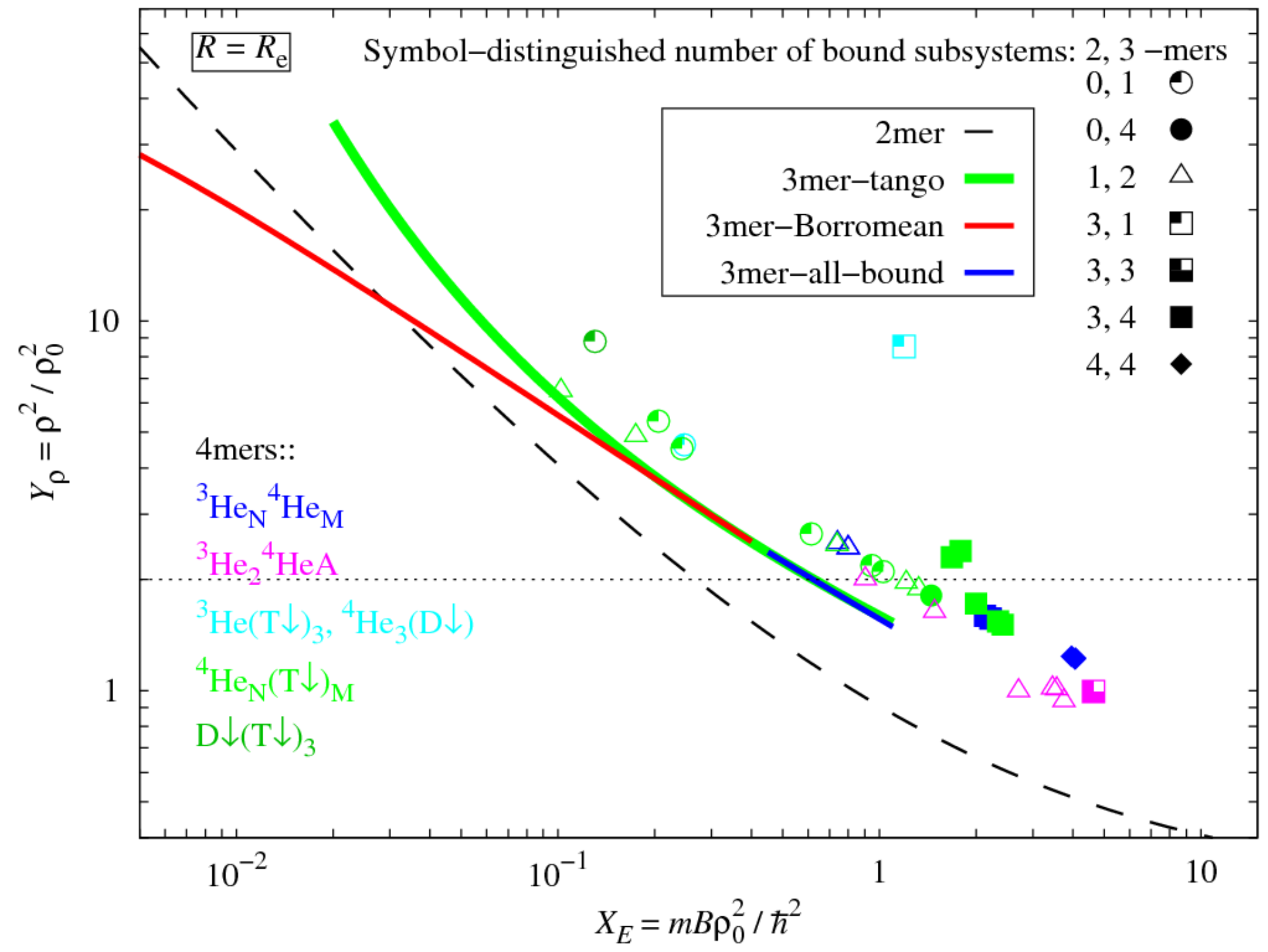


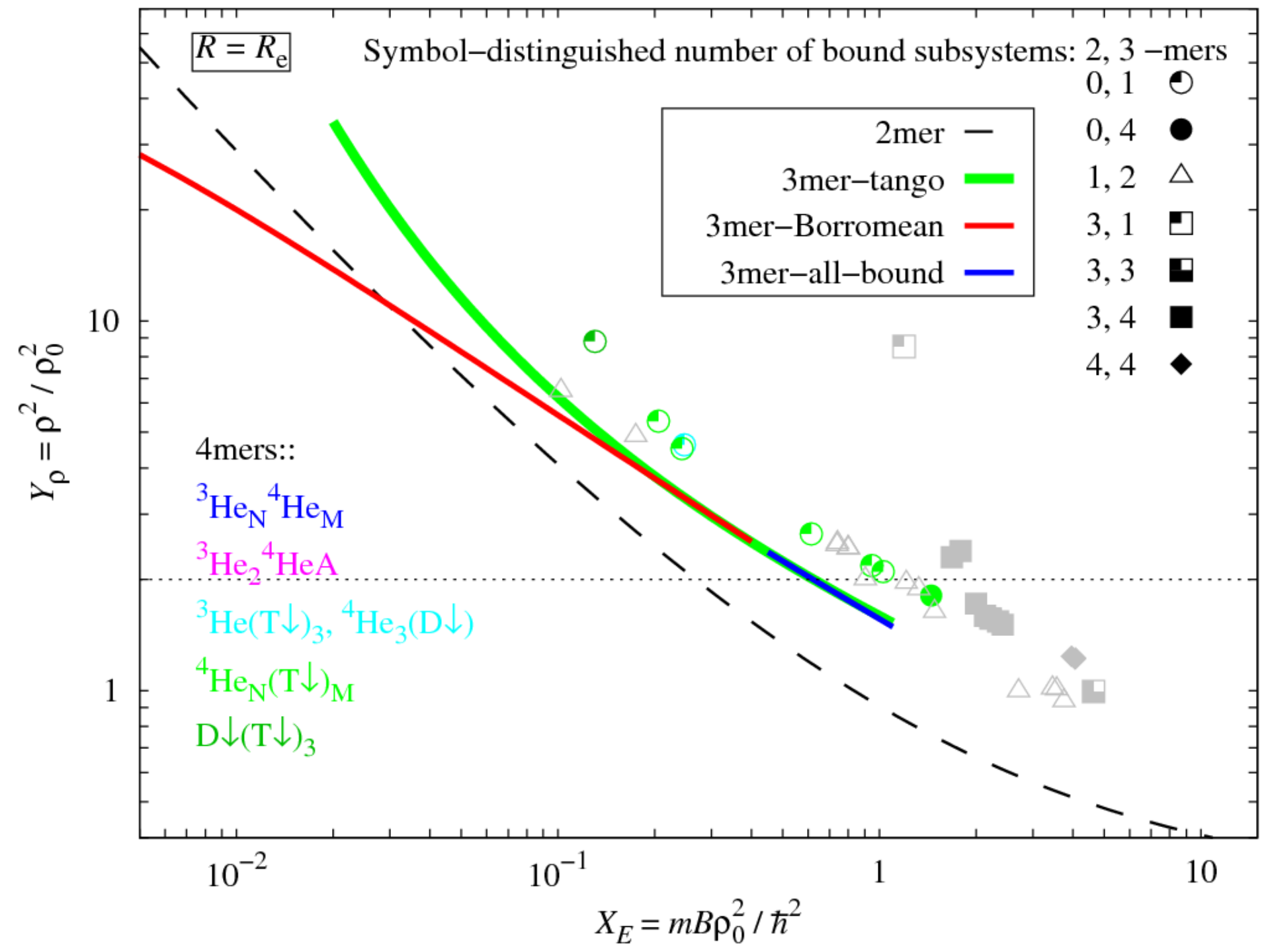


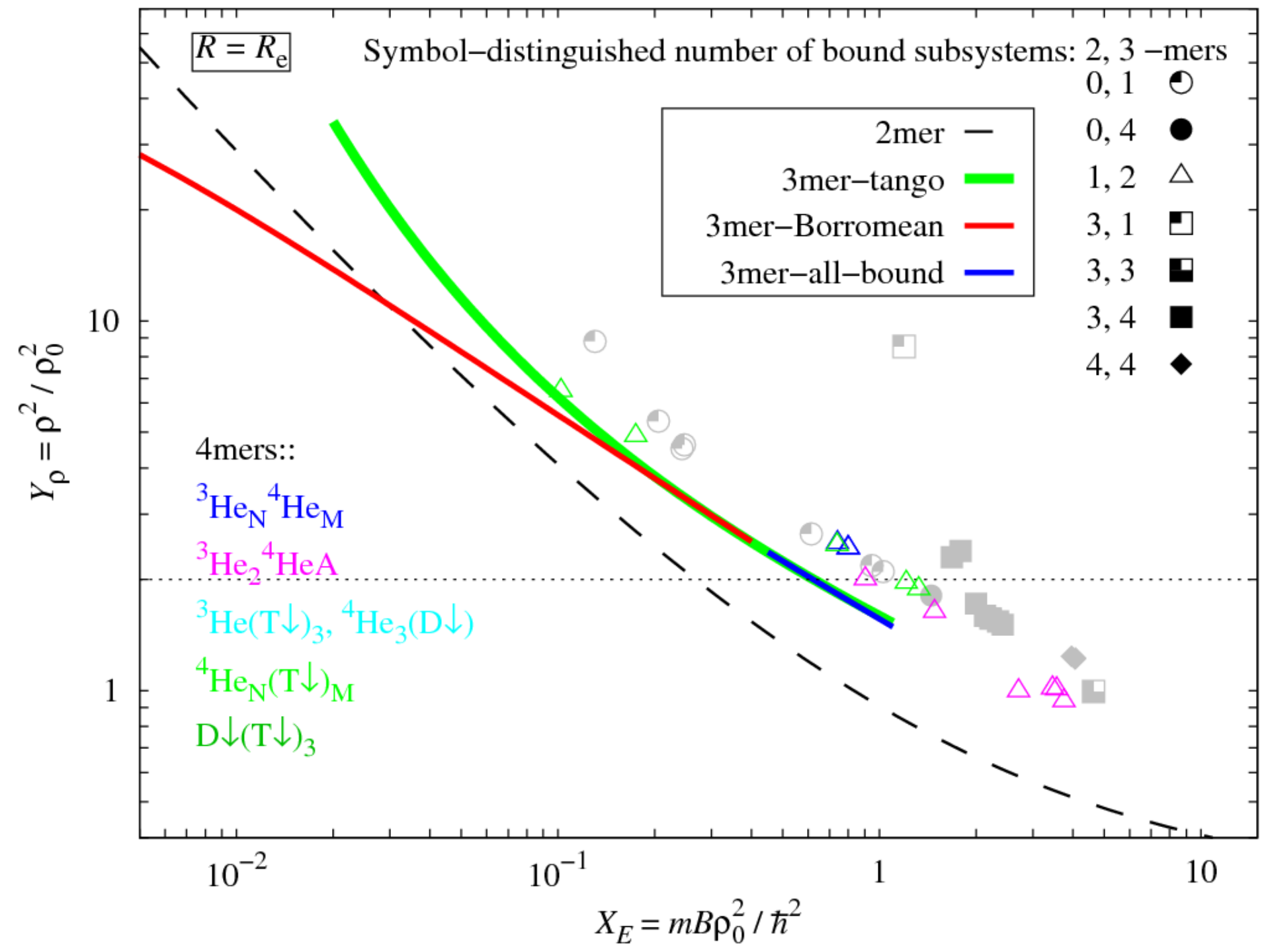


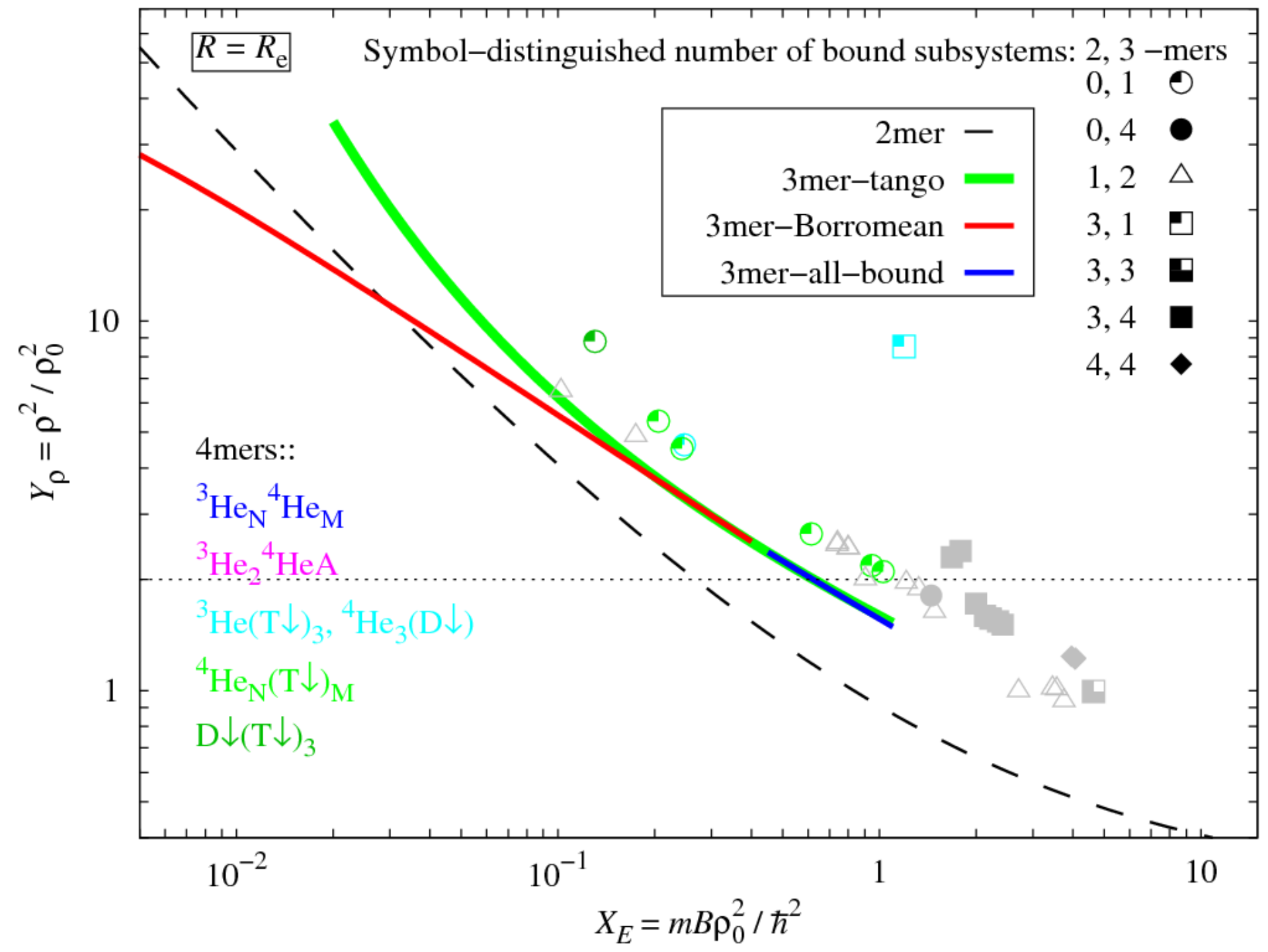


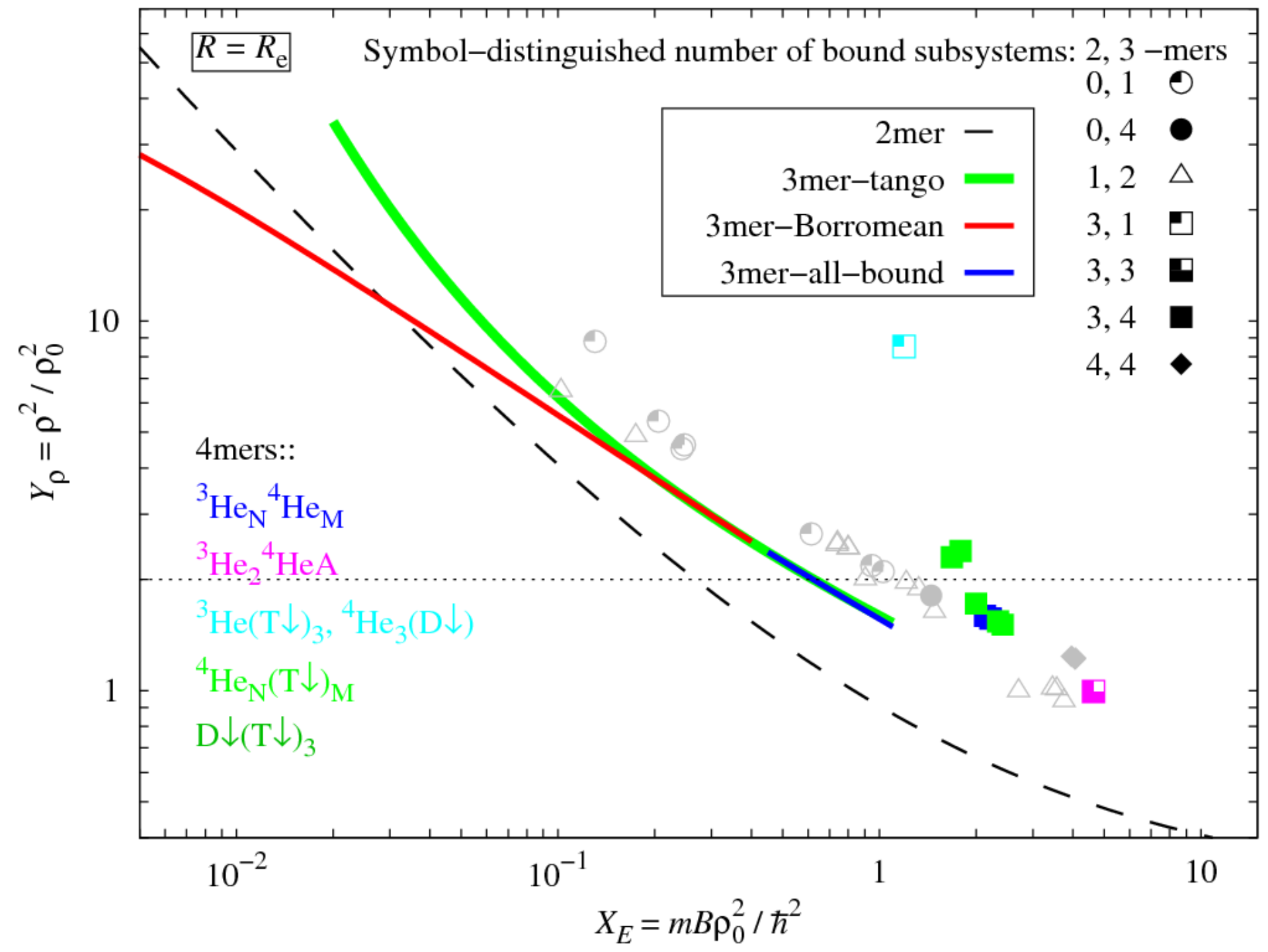


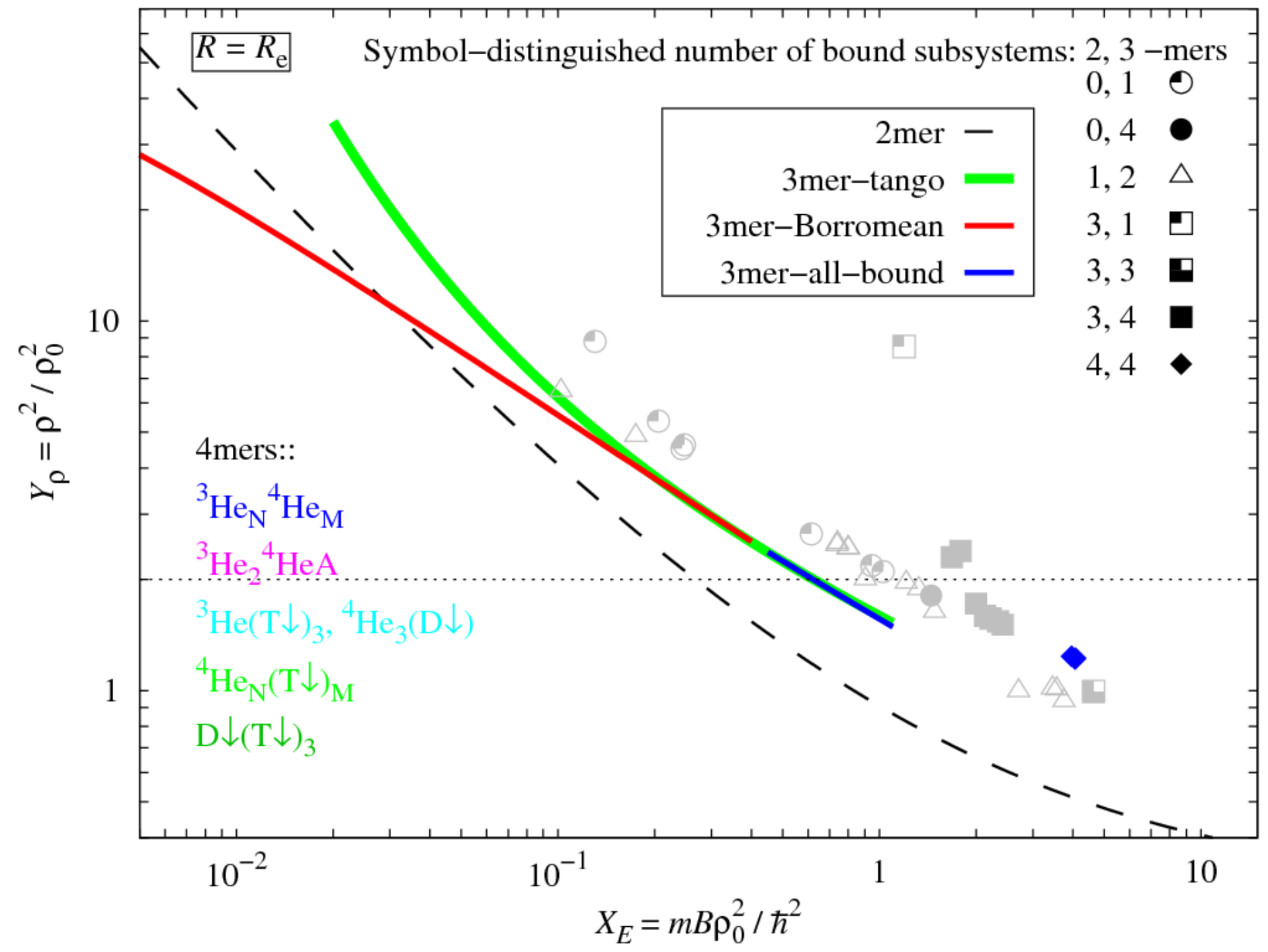


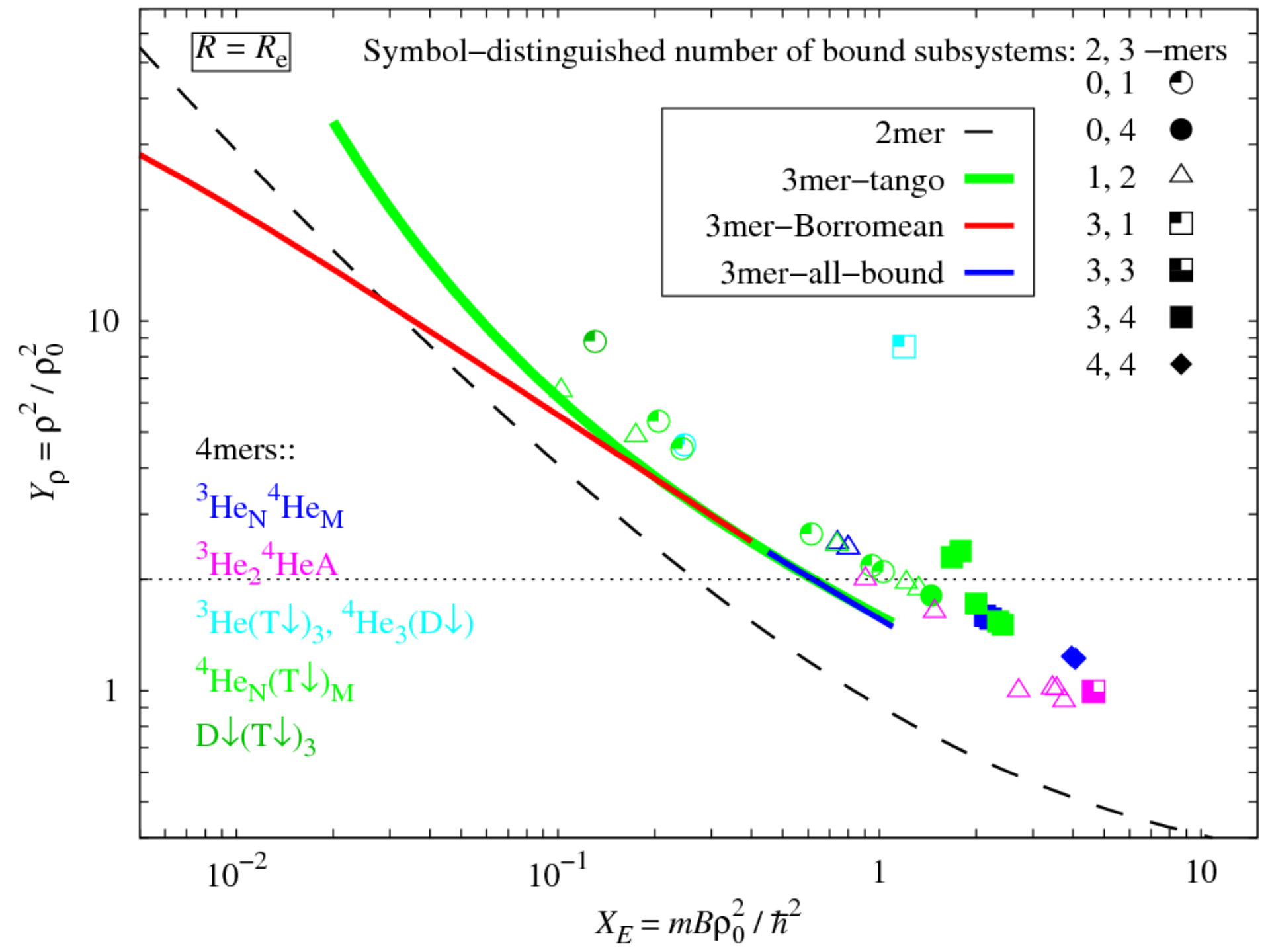






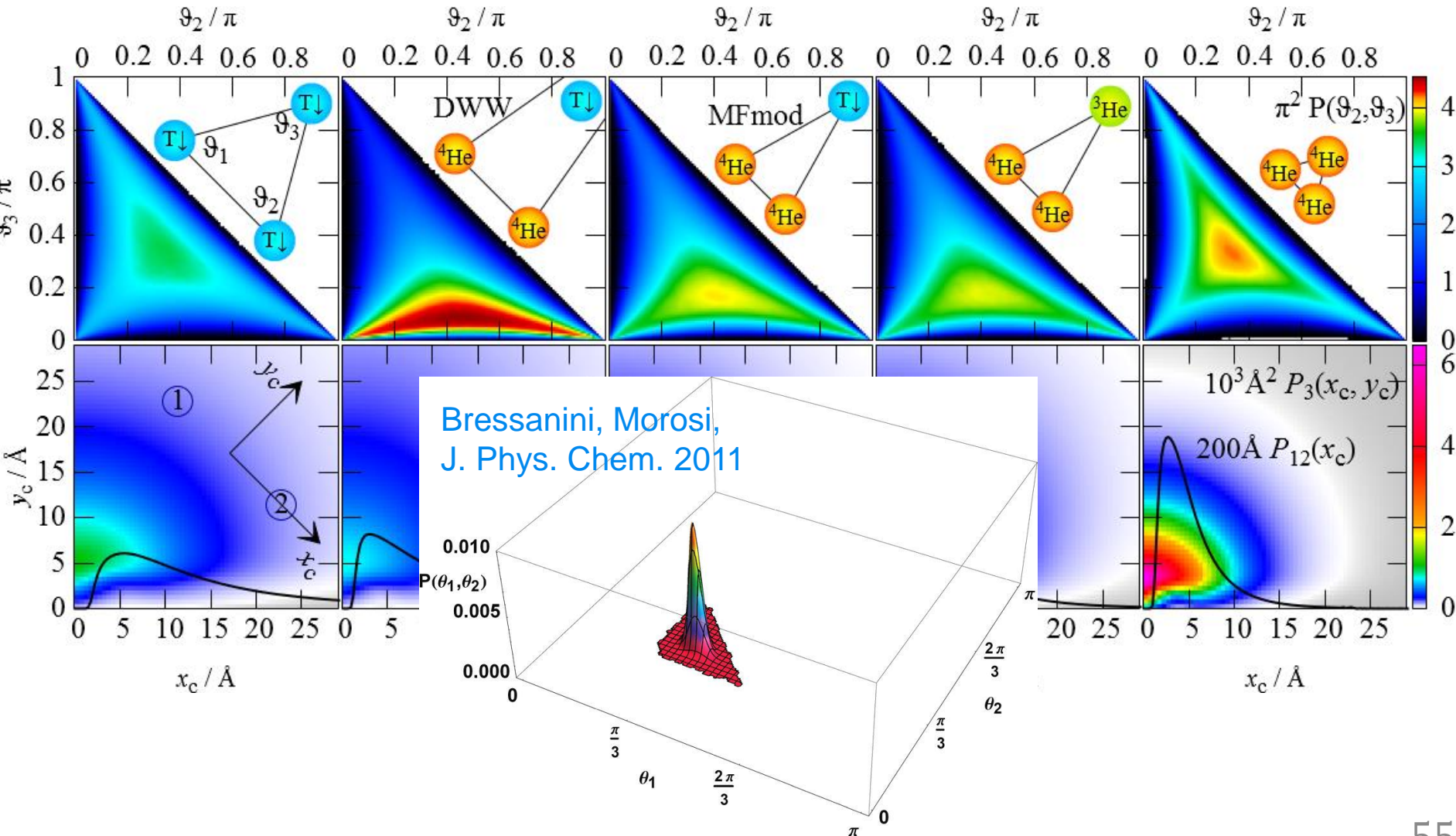




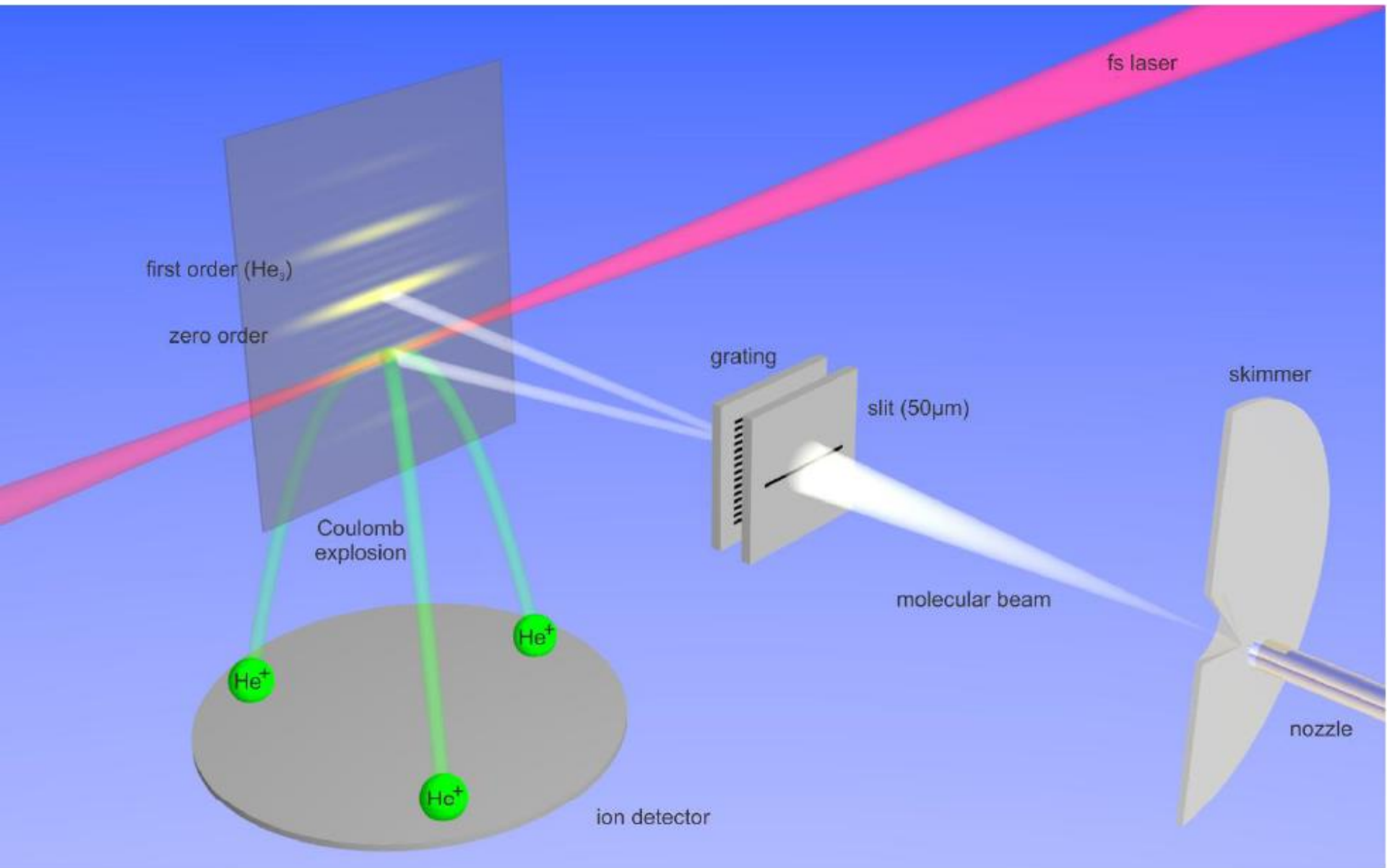


» Structure – DMC – pure estimators

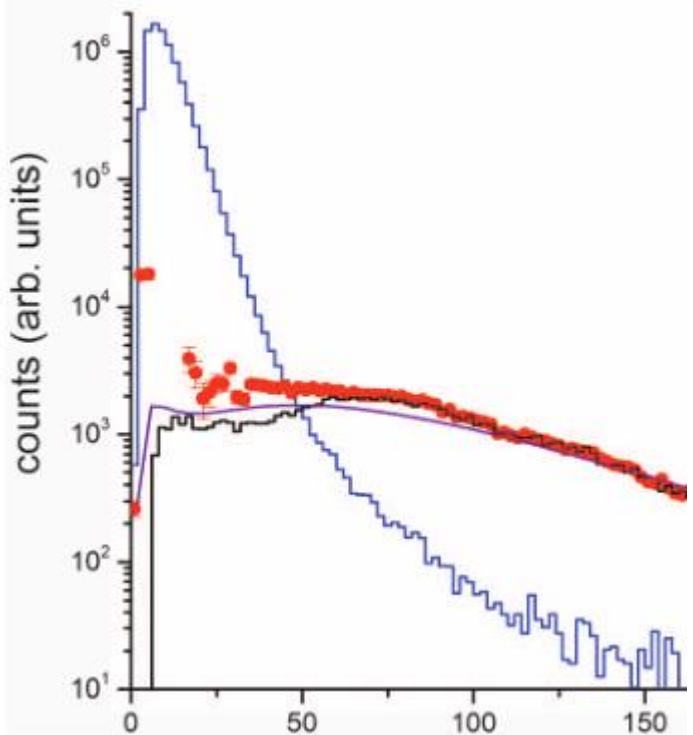
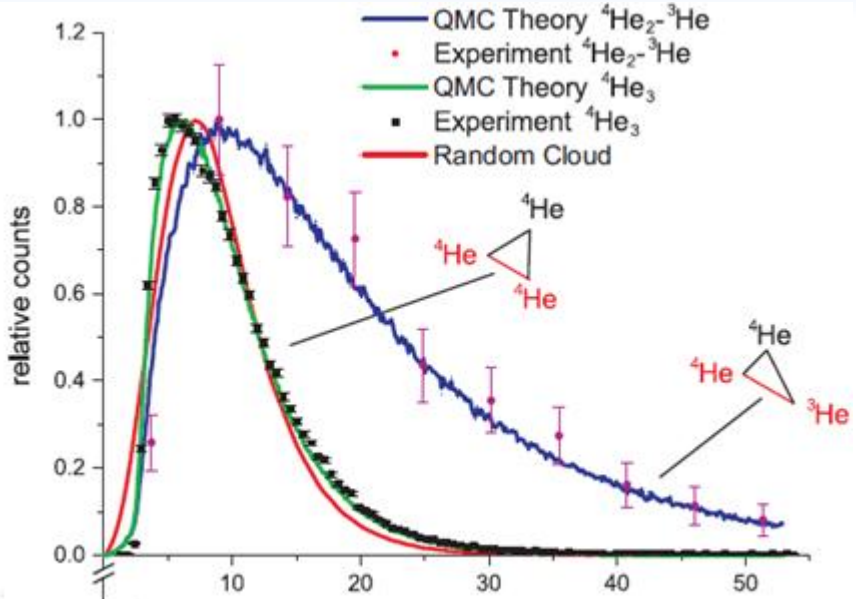
- different distribution functions



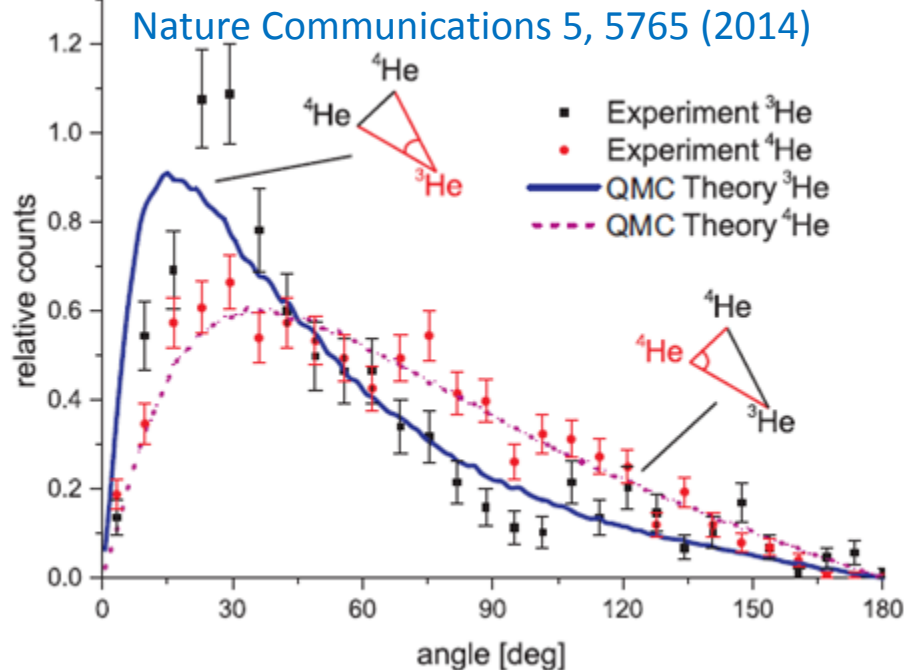
» Structure – experiments



» Structure – experiments



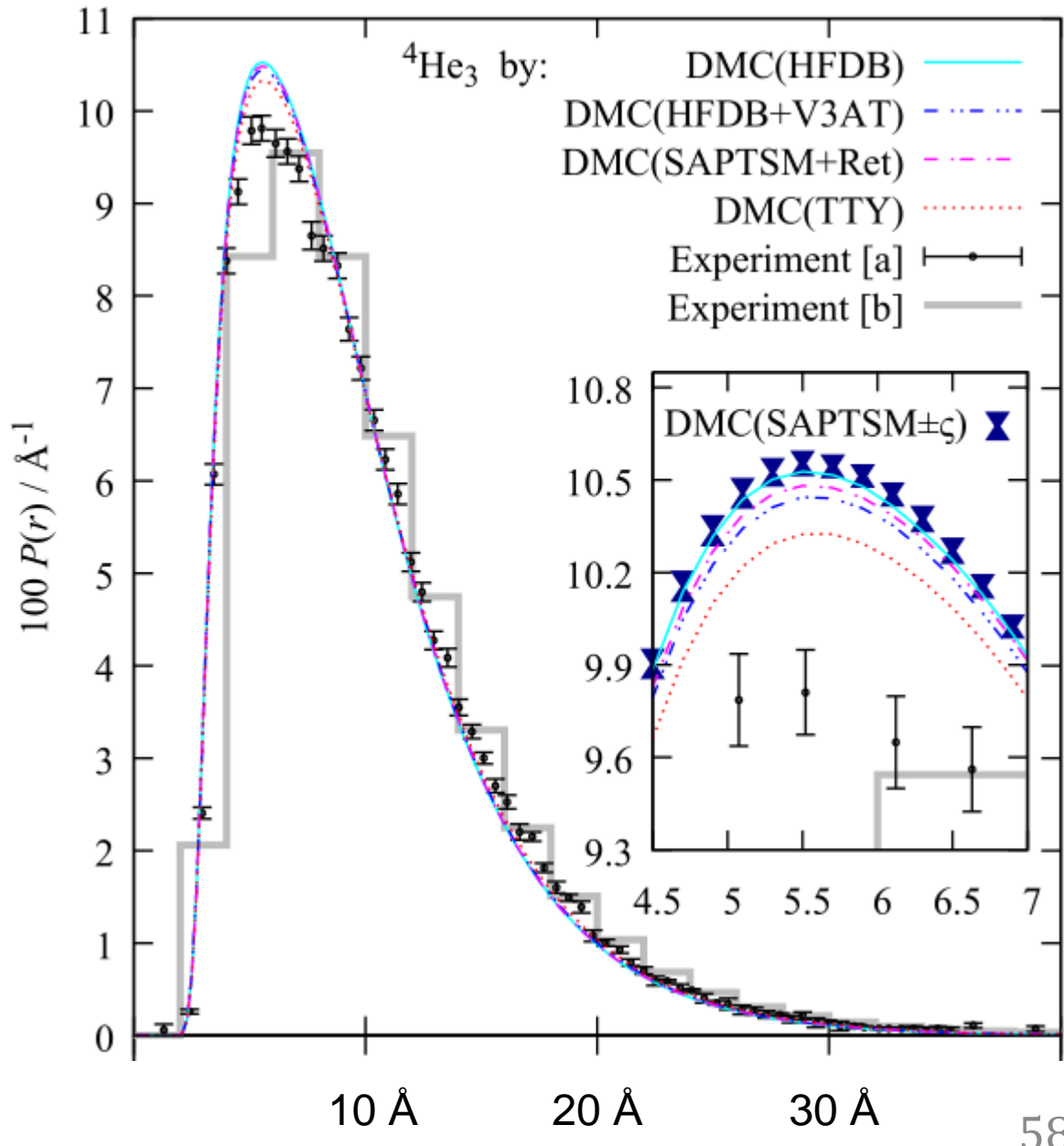
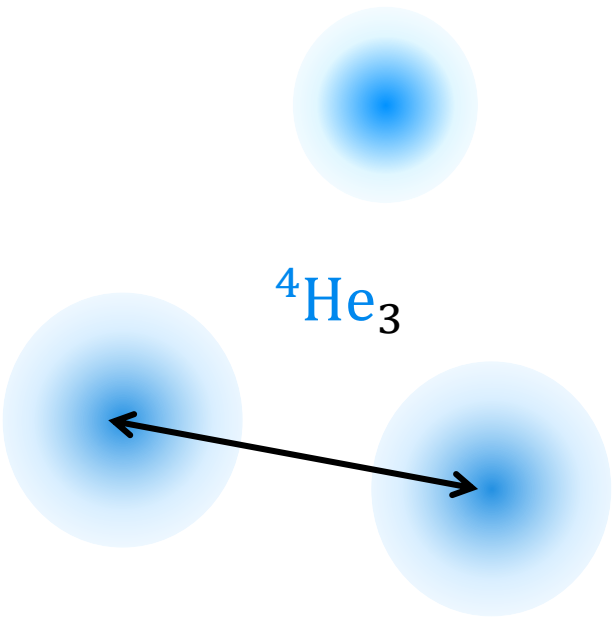
Maksim Kunitski *et al.*, Science **348**, 551 (2015)



>> Structure – DMC(V) vs experiment

[a] J. Voigtsberger *et al.*, Nature Comm. 5, 5765 (2014)

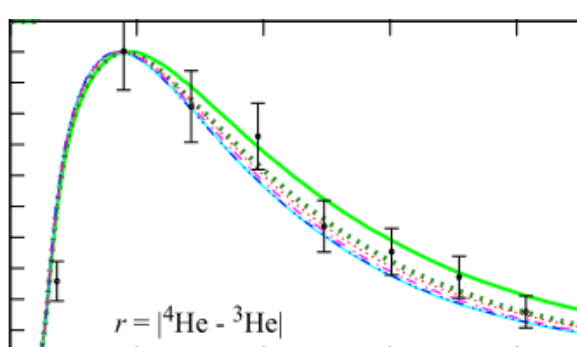
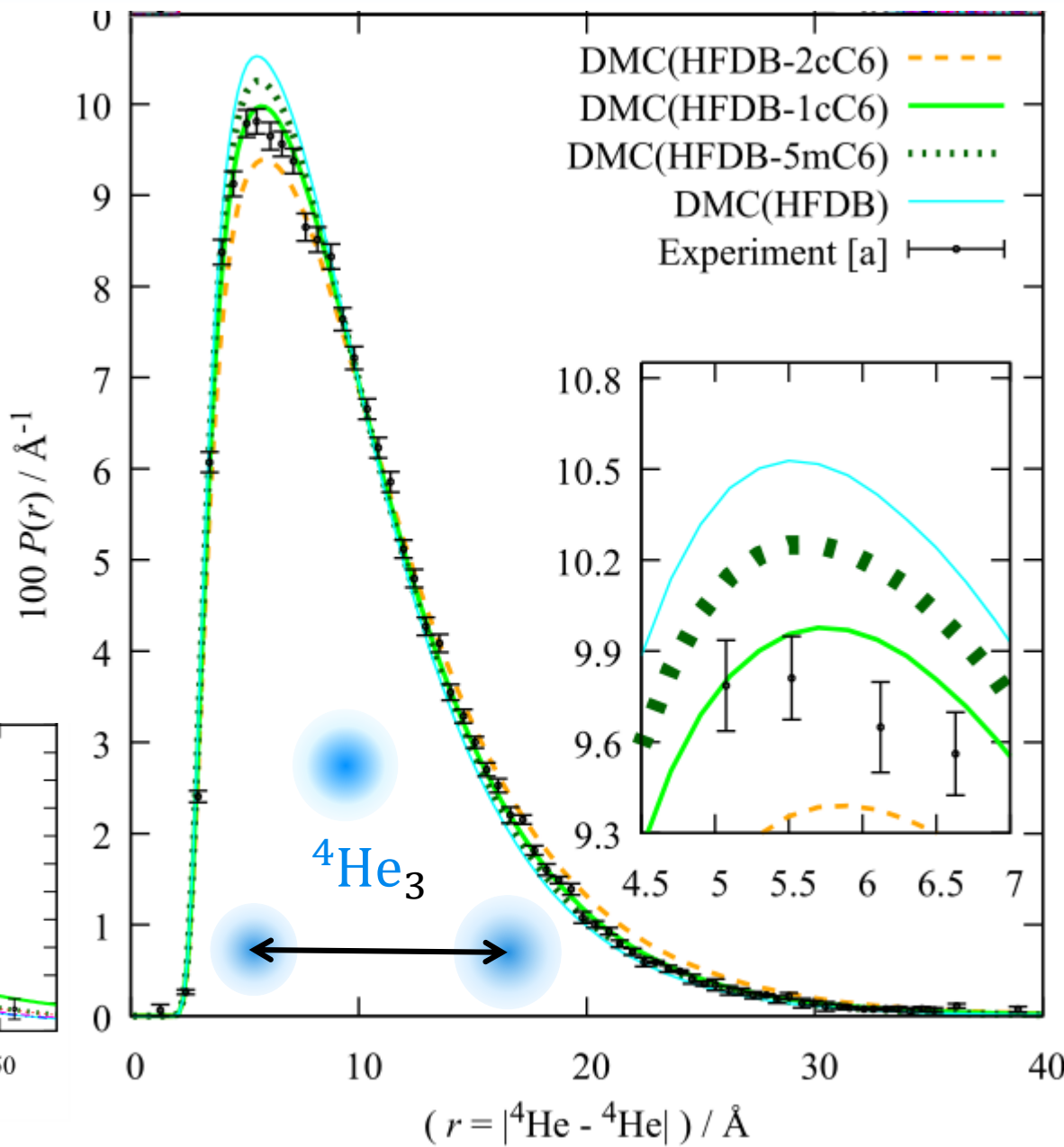
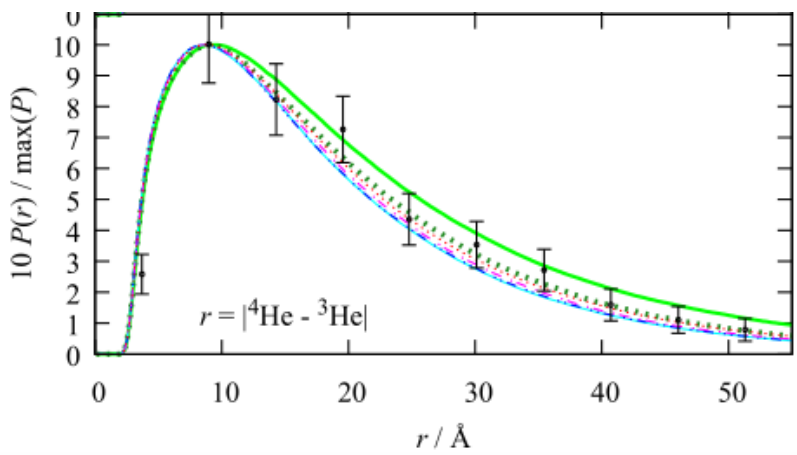
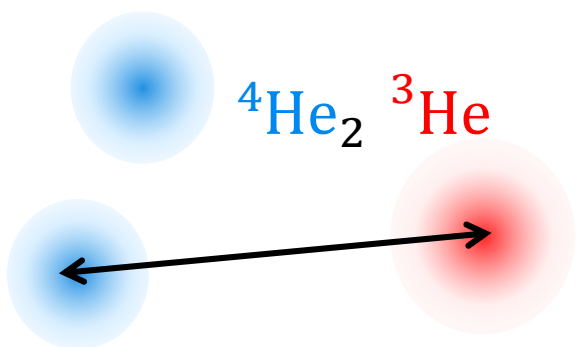
[b] Maksim Kunitski *et al.*, Science 348, 551 (2015)



» Structure – DMC(V) vs experiment

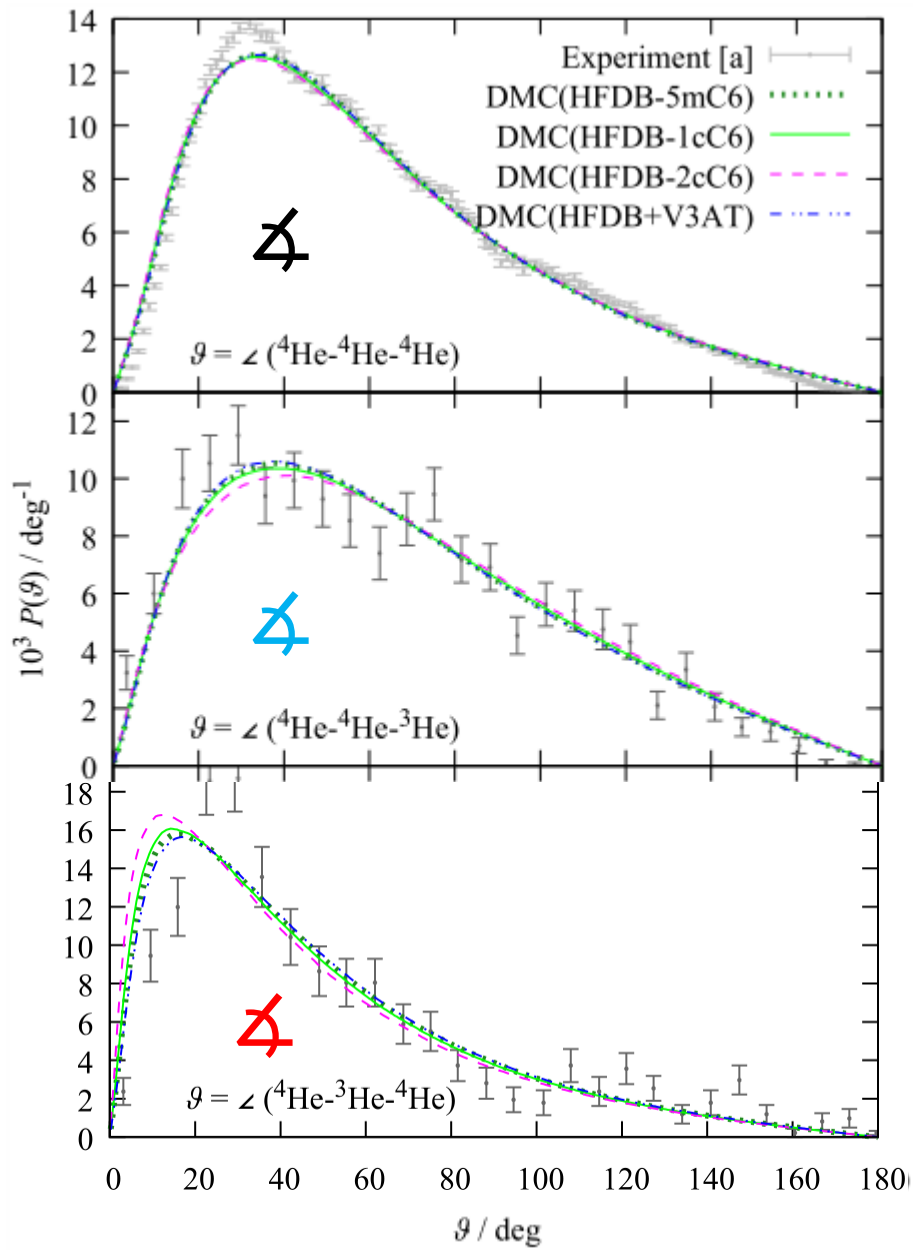
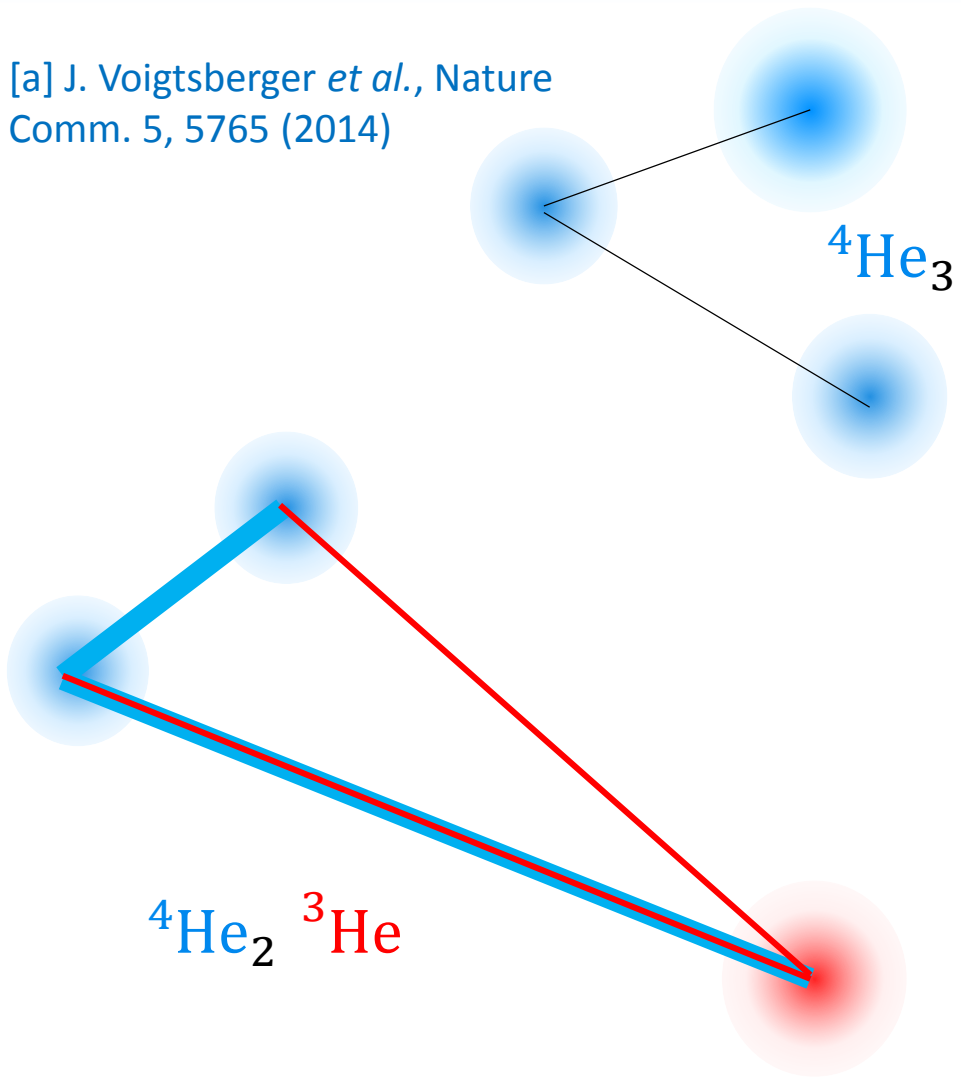
[a] J. Voigtsberger *et al.*, Nature Comm. 5, 5765 (2014)

[b] Maksim Kunitski *et al.*, Science 348, 551 (2015)



>> Structure – DMC(V) vs experiment

[a] J. Voigtsberger *et al.*, Nature Comm. 5, 5765 (2014)



❖ Summary

- Different scaling lengths were tested.
- Dimer, Borromean-trimer, all-bound trimer and HeHe-tango-trimer universal lines of scaled size-energy ratio were found.
- Clusters close to each other on universal line have similar structural properties.
- TTY+Ret potential trimer-distribution-predictions are in the best agreement with the experimental He-trimer data.
- For tetramers indication of different lines that depend on the number of bound subsystems of dimers was noticed.

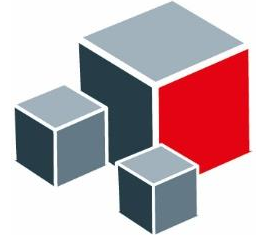
❖ Acknowledgments

- Financial support:

- ✓ Croatian Science Foundation (IP-2014-09-2452)

- ❖ Any opinions, findings, and conclusions or recommendations expressed in this material are those of the author(s) and do not necessarily reflect the views of Croatian Science Foundation.

- ✓ MICINN-Spain (FIS2014-56257-C2-1-P)



- Computational support:



Thank you for attention! Questions?