Dynamics of several ultra-cold particles in a double-well potential

Tomasz Sowinski
Institute of Physics of the Polish Academy of Sciences

References:
[1] T. Sowiński, M. Gajda, K. Rzążewski: EPL **113**, 56003 (2016)
[2] J. Dobrzyniecki, T. Sowiński: EPJ D **70**, 83 (2016)

Deterministic Preparation of a Tunable Few-Fermion System

F. Serwane,[1,2,3]*† G. Zürn,[1,2]† T. Lompe,[1,2,3] T. B. Ottenstein,[1,2,3]‡ A. N. Wenz,[1,2] S. Jochim[1,2,3]

Systems consisting of few interacting fermions are the building blocks of matter, with atoms and nuclei being the most prominent examples. We have created a few-body quantum system with complete control over its quantum state using ultracold fermionic atoms in an optical dipole trap. Ground-state systems consisting of 1 to 10 particles are prepared with fidelities of ~90%. We can tune the interparticle interactions to arbitrary values using a Feshbach resonance and have observed the interaction-induced energy shift for a pair of repulsively interacting atoms. This work is expected to enable quantum simulation of strongly correlated few-body systems.

The exploration of naturally occurring few-body quantum systems such as atoms and nuclei has been extremely successful, largely because they could be prepared in well-defined quantum states. Because these systems have limited tunability, researchers created quantum dots—"artificial atoms"—in which properties such as particle number, interaction strength, and confining potential can be tuned (1, 2). However, quantum dots are generally strongly coupled to their environment, which hindered the deterministic preparation of well-defined quantum states.

In contrast, ultracold gases provide tunable systems in a highly isolated environment (3, 4). They have been proposed as a tool for quantum simulation (5, 6), which has been realized experimentally for various many-body systems (7–10). Achieving quantum simulation of few-body systems is more challenging because it requires complete control over all degrees of freedom: the particle number, the internal and motional states of the particles, and the strength of the interparticle interactions.

One possible approach to this goal is using a Mott insulator state of atoms in an optical lattice as a starting point. In this way, systems with up to four bosons per lattice site have been prepared in their ground state (11, 12). Recently, single lattice sites have been addressed individually (13).

In single isolated trapping geometries, researchers could suppress atom number fluctuations by loading bosonic atoms into small-volume optical dipole traps (14–18). However, these experiments were not able to gain control over the system's quantum state.

We prepare few-body systems consisting of 1 to 10 fermionic atoms in a well-defined quantum state, making use of Pauli's principle, which states that each single-particle state cannot be occupied by more than one identical fermion. Therefore, the occupation probability of the lowest-energy states approaches unity for a degenerate Fermi gas, and we can control the number of particles by controlling the number of available single-particle states. We realize this by deforming the confining potential such that quantum states above a well-defined energy become unbound. This approach requires a highly degenerate Fermi gas in a trap whose depth can be controlled with a precision much higher than the separation of its energy levels.

To fulfill these requirements, we use a small-volume optical dipole trap with large level spacing. This microtrap is created by the focus of a single laser beam (Fig. 1) with a waist of $w_0 \leq 1.8$ μm and measured radial and axial trapping frequencies $(\omega_r, \omega_a) = 2\pi \times (14.0 \pm 0.1, 1.487 \pm 0.010)$ kHz (19). We load the microtrap from a reservoir of cold atoms. The reservoir consists of a two-component mixture of ^{6}Li atoms in the two lowest-energy Zeeman substates $|F = 1/2, m_F = +1/2\rangle$ and $|F = 1/2, m_F = -1/2\rangle$ (labeled state $|1\rangle$ and $|2\rangle$) in a large-volume optical dipole trap. The reservoir has a degeneracy of $T/T_F \approx 0.5$ (19), where T_F is the Fermi temperature. We superimpose the microtrap with the reservoir and transfer about 600 atoms into the microtrap. After removal of the reservoir, the degeneracy of the system is determined by $T_F \approx 3$ μK in the microtrap and the temperature $T \lesssim 250$ nK

Fig. 1. Experimental setup. Systems with up to 10 fermions are prepared with ^{6}Li atoms in a micrometer-sized optical dipole trap created by the focus of a single laser beam. The number of atoms in the samples is detected with single-atom resolution by transferring them into a compressed magneto-optical-trap (MOT) and collecting their fluorescence on a CCD camera. A Feshbach resonance allows one to tune the interaction between the particles with a magnetic offset field.

[1]Physikalisches Institut, Ruprecht-Karls-Universität Heidelberg, 69120 Heidelberg, Germany. [2]Max-Planck-Institut für Kernphysik, Saupfercheckweg 1, 69117 Heidelberg, Germany. [3]ExtreMe Matter Institute EMMI, GSI Helmholtzzentrum für Schwerionenforschung, 64291 Darmstadt, Germany.
*To whom correspondence should be addressed. E-mail: friedhelm.serwane@mpi-hd.mpg.de
†These authors contributed equally to this work.
‡Present address: DIOPTIC GmbH, Bergstraße 92A, 69469 Weinheim, Germany.

PRL 108, 075303 (2012) PHYSICAL REVIEW LETTERS week ending 17 FEBRUARY 2012

Fermionization of Two Distinguishable Fermions

G. Zürn,[1,2,*] F. Serwane,[1,2,3] T. Lompe,[1,2,3] A. N. Wenz,[1,2] M. G. Ries,[1,2] J. E. Bohn,[1,2] and S. Jochim[1,2,3]
[1]Physikalisches Institut, Ruprecht-Karls-Universität Heidelberg, Germany
[2]Max-Planck-Institut für Kernphysik, Saupfercheckweg 1, 69117 Heidelberg, Germany
[3]ExtreMe Matter Institute EMMI, GSI Helmholtzzentrum für Schwerionenforschung, Darmstadt, Germany
(Received 11 November 2011; published 16 February 2012)

We study a system of two distinguishable fermions in a 1D harmonic potential. This system has the exceptional property that there is an analytic solution for arbitrary values of the interparticle interaction. We tune the interaction strength and compare the measured properties of the system to the theoretical prediction. For diverging interaction strength, the energy and square modulus of the wave function for two distinguishable particles are the same as for a system of two noninteracting identical fermions. This is referred to as fermionization. We have observed this phenomenon by directly comparing two distinguishable fermions with diverging interaction strength with two identical fermions in the same potential. We observe good agreement between experiment and theory. By adding more particles our system can be used as a quantum simulator for more complex systems where no theoretical solution is available.

DOI: 10.1103/PhysRevLett.108.075303 PACS numbers: 67.85.Lm, 03.75.−b

PRL 111, 175302 (2013) PHYSICAL REVIEW LETTERS week ending 25 OCTOBER 2013

Pairing in Few-Fermion Systems with Attractive Interactions

G. Zürn,[1,2,*] A. N. Wenz,[1,2] S. Murmann,[1,2] A. Bergschneider,[1,2] T. Lompe,[1,2,3] and S. Jochim[1,2,3]
[1]Physikalisches Institut, Ruprecht-Karls-Universität Heidelberg, 69120 Heidelberg, Germany
[2]Max-Planck-Institut für Kernphysik, Saupfercheckweg 1, 69117 Heidelberg, Germany
[3]ExtreMe Matter Institute EMMI, GSI Helmholtzzentrum für Schwerionenforschung, 64291 Darmstadt, Germany
(Received 18 July 2013; published 22 October 2013)

We study quasi-one-dimensional few-particle systems consisting of one to six ultracold fermionic atoms in two different spin states with attractive interactions. We probe the system by deforming the trapping potential and by observing the tunneling of particles out of the trap. For even particle numbers, we observe a tunneling behavior that deviates from uncorrelated single-particle tunneling indicating the existence of pair correlations in the system. From the tunneling time scales, we infer the differences in interaction energies of systems with different number of particles, which show a strong odd-even effect, similar to the one observed for neutron separation experiments in nuclei.

DOI: 10.1103/PhysRevLett.111.175302 PACS numbers: 67.85.Lm

From Few to Many: Observing the Formation of a Fermi Sea One Atom at a Time

A. N. Wenz,[1,2]*† G. Zürn,[1,2]† S. Murmann,[1,2] I. Brouzos,[3] T. Lompe,[1,2,4] S. Jochim[1,2,4]

Knowing when a physical system has reached sufficient size for its macroscopic properties to be well described by many-body theory is difficult. We investigated the crossover from few- to many-body physics by studying quasi–one-dimensional systems of ultracold atoms consisting of a single impurity interacting with an increasing number of identical fermions. We measured the interaction energy of such a system as a function of the number of majority atoms for different strengths of the interparticle interaction. As we increased the number of majority atoms one by one, we observed fast convergence of the normalized interaction energy toward a many-body limit calculated for a single impurity immersed in a Fermi sea of majority particles.

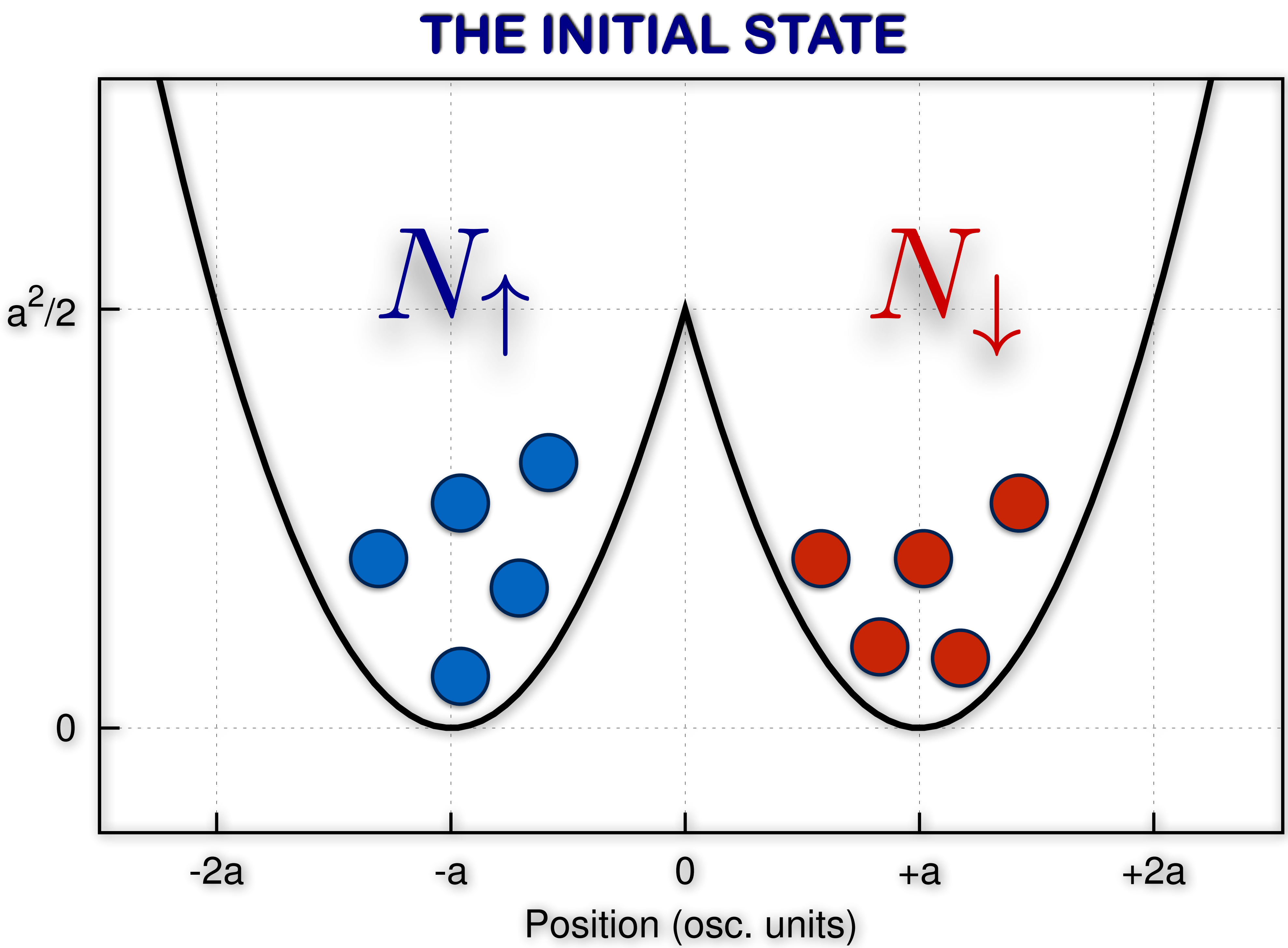
The ability to connect the macroscopic properties of a many-body system to the microscopic physics of its individual constituent particles is one of the great achievements of physics. This connection is usually made using the assumption that the number of particles tends to infinity. Then a transition from discrete to continuous variables can be made, which greatly simplifies the theoretical description of large systems. When does a system become large enough for this approximation to be valid? This is a difficult question to answer because most calculations based on a microscopic description become prohibitively complex before their predictions approach the many-body solution. Experimentally, this question has been studied in the context of helium droplets (1) and nuclear physics (2) by measuring the emergence of superfluidity for increasing system size. We addressed this question with the use of ultracold lithium atoms, which have already been used t… tems with tunable interac…

In our experiments, w… size on the single-particle le… full control over the inter… We achieve this by determi… few-particle systems of ultra… whose interparticle interaction… Feshbach resonances (7, 8). Th… plore the crossover from few- to … ics by studying the fermionic qu… problem, where a single impurity… with a number of fermionic major… majority atoms do not interact wi… because of the Pauli principle. In th… impurity acts as a test particle, whic… probe the majority component. The lin… gle majority particle (Fig. 1A) has been…

Fig. 1. From few to many. A single impurity (blu… interacting with one, few, and many fermions (green… in a harmonic trapping potential. In the many-body… case, the majority component can be described as… a Fermi sea with a Fermi energy E_F.

[1]Physikalisches Institut, Ruprecht-Karls-Universität Heidelberg, 69210 Heidelberg, Germany. [2]Max-Planck-Institut für Kernphysik, Saupfercheckweg 1, 69117 Heidelberg, Germany. [3]Institut für Quanten-Informationsverarbeitung, Universität Ulm, 89069 Ulm, Germany. [4]Extreme Matter Institute (EMMI), GSI Helmholtzzentrum für Schwerionenforschung, 64291 Darmstadt, Germany.
*Corresponding author. E-mail: wenz@physi.uni-heidelberg.de
†These authors contributed equally to this work.

The question

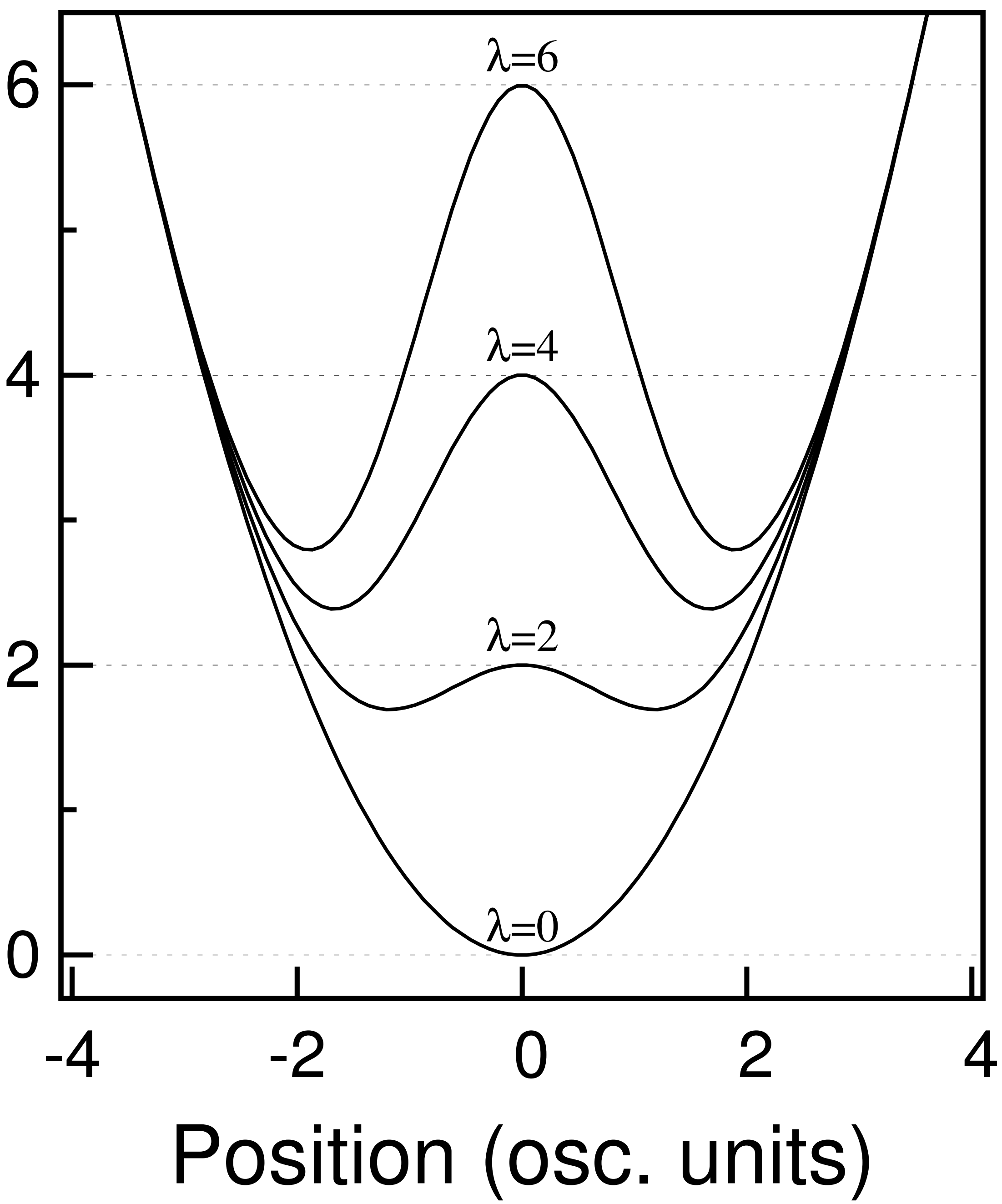


What one can say about the evolution governed by the many-body Hamiltonian without any approximations???

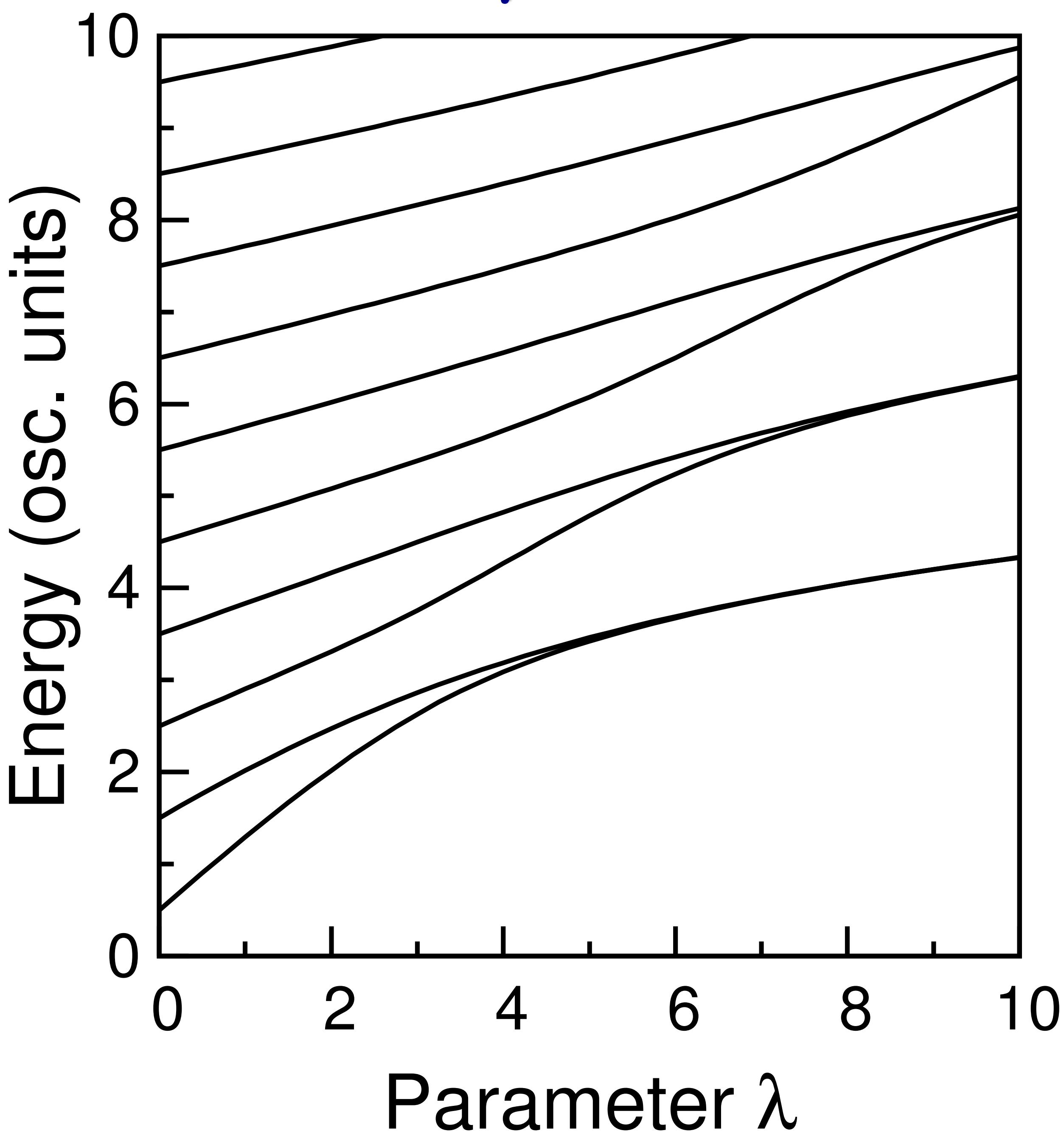
Double-well models

$$\hat{\mathcal{H}}_0 = -\frac{\hbar^2}{2m}\frac{\mathrm{d}^2}{\mathrm{d}x^2} + \frac{m\Omega^2}{2}x^2 + \lambda \exp\left(-\frac{m\Omega}{2\hbar}x^2\right)$$

shape of a potential



spectrum

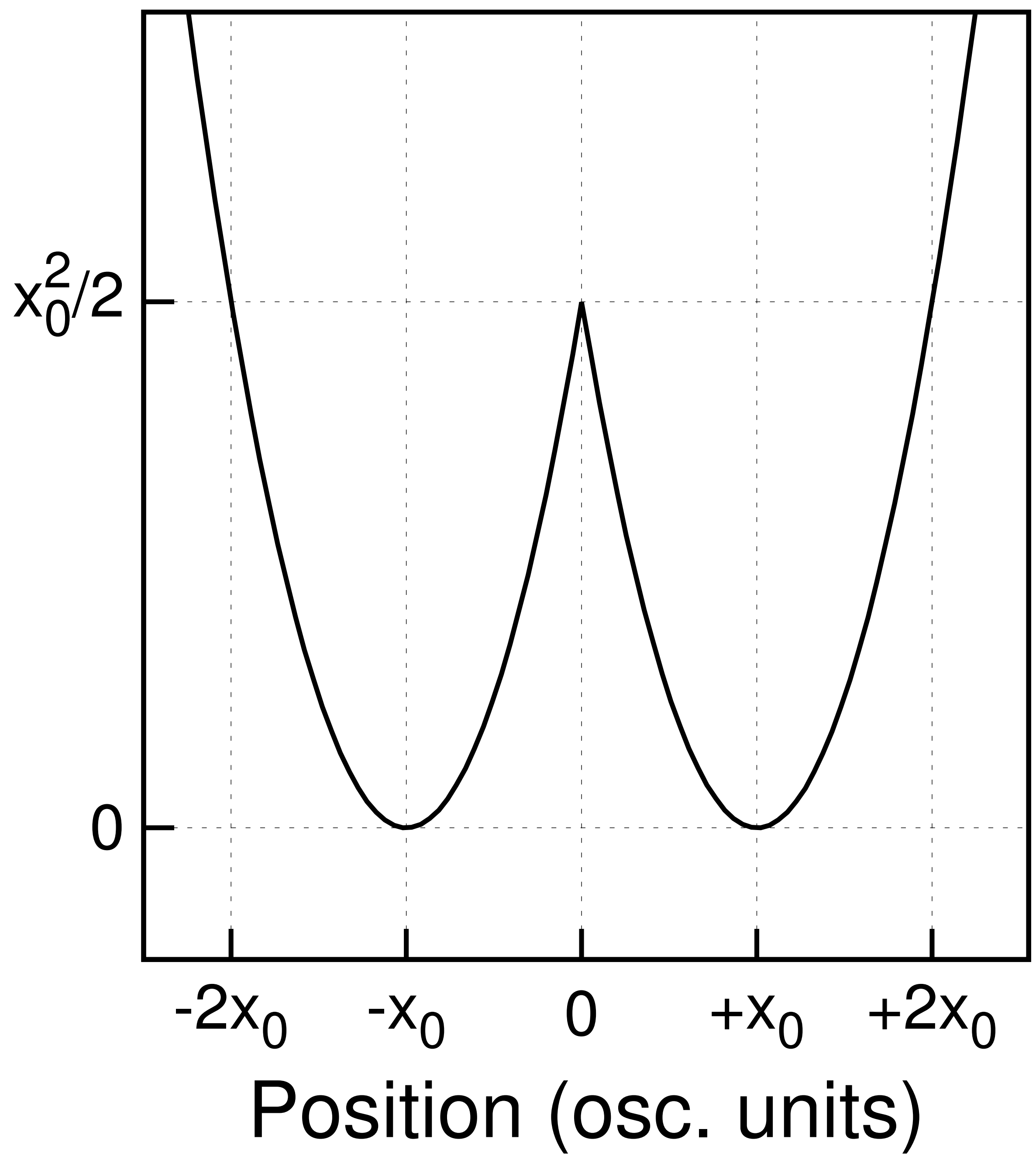


x

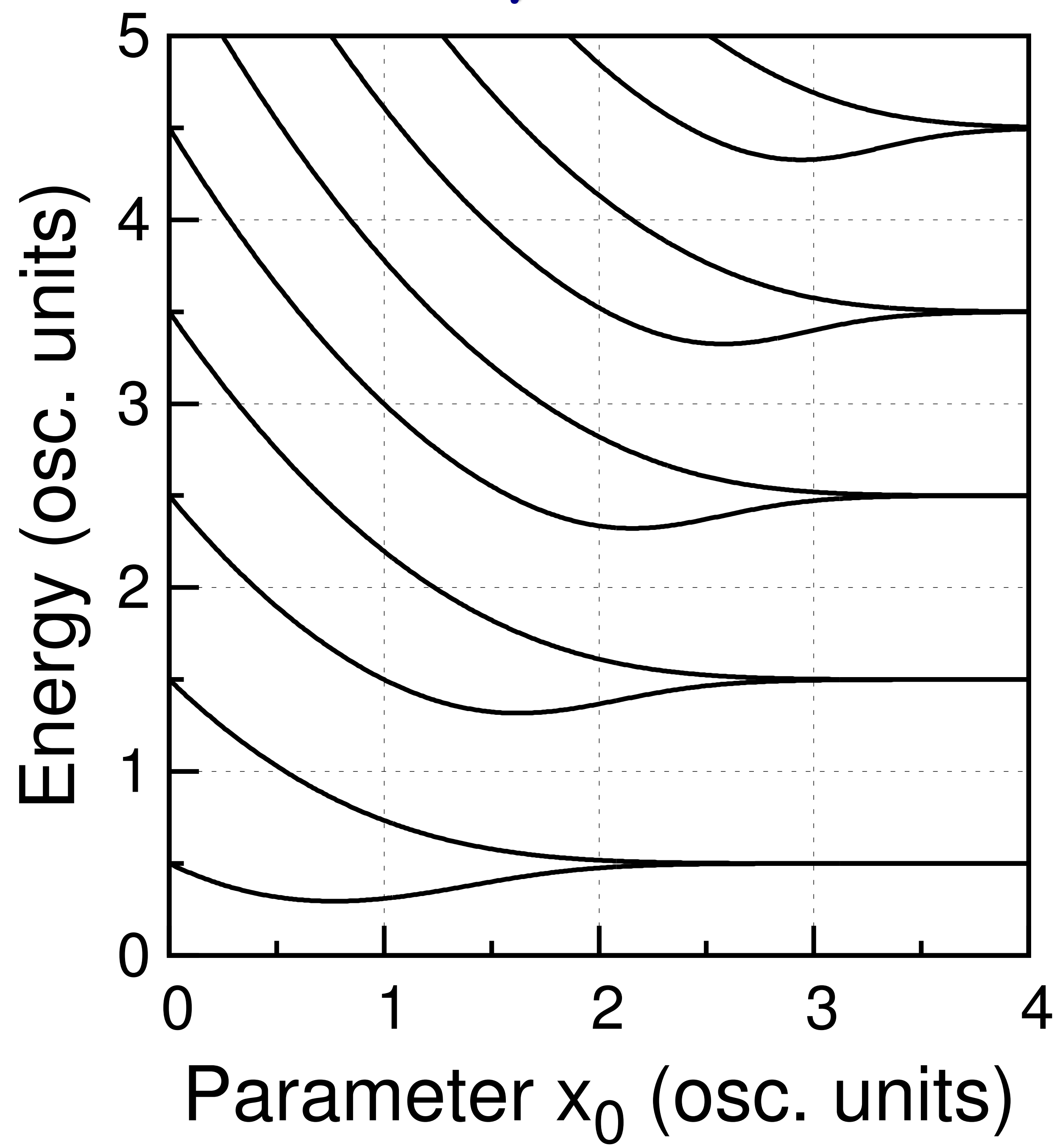
Double-well models

$$\hat{\mathcal{H}}_0 = -\frac{\hbar^2}{2m}\frac{\mathrm{d}^2}{\mathrm{d}x^2} + \frac{m\Omega^2}{2}(|x| - x_0)^2$$

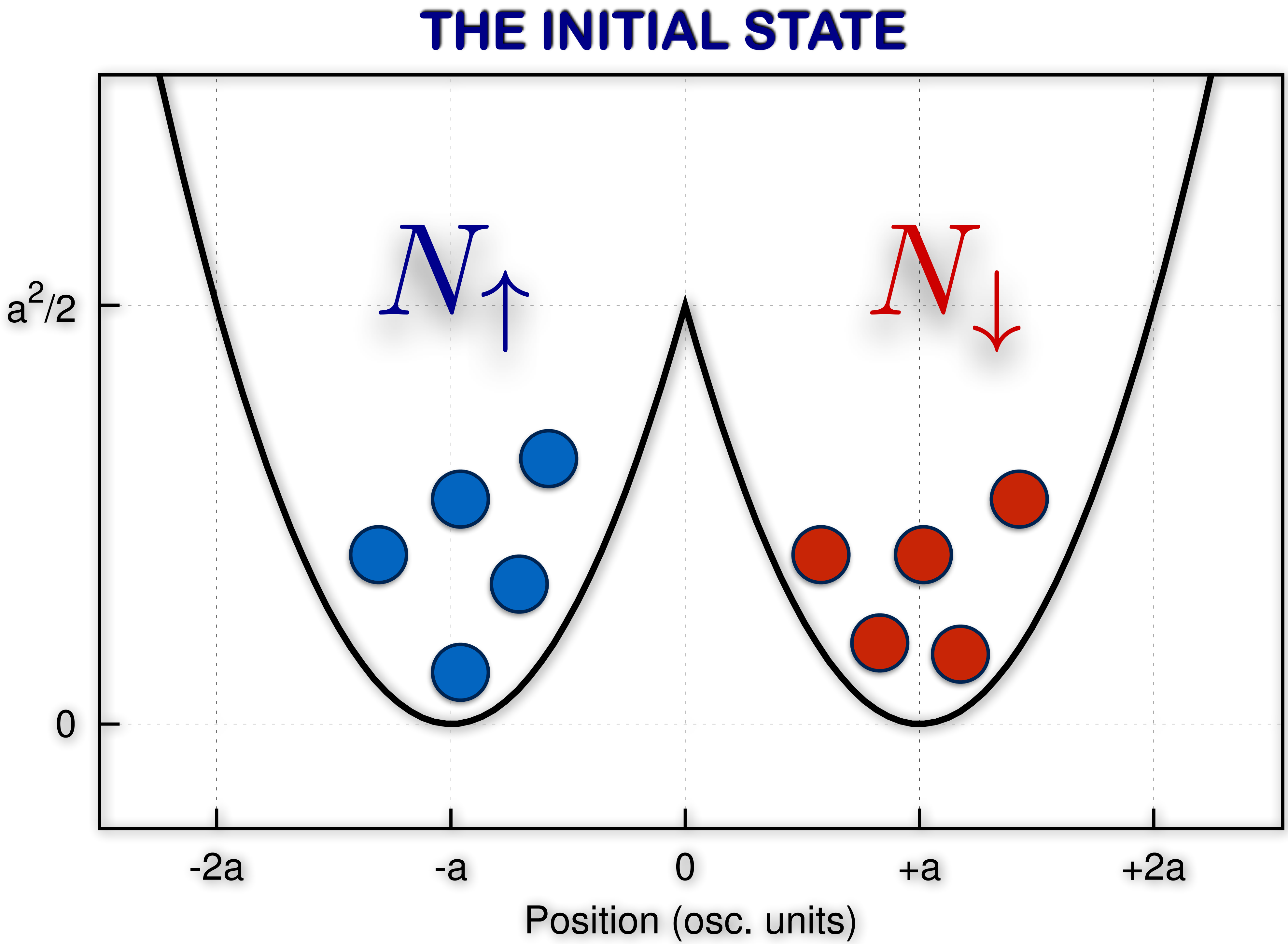
shape of a potential



spectrum

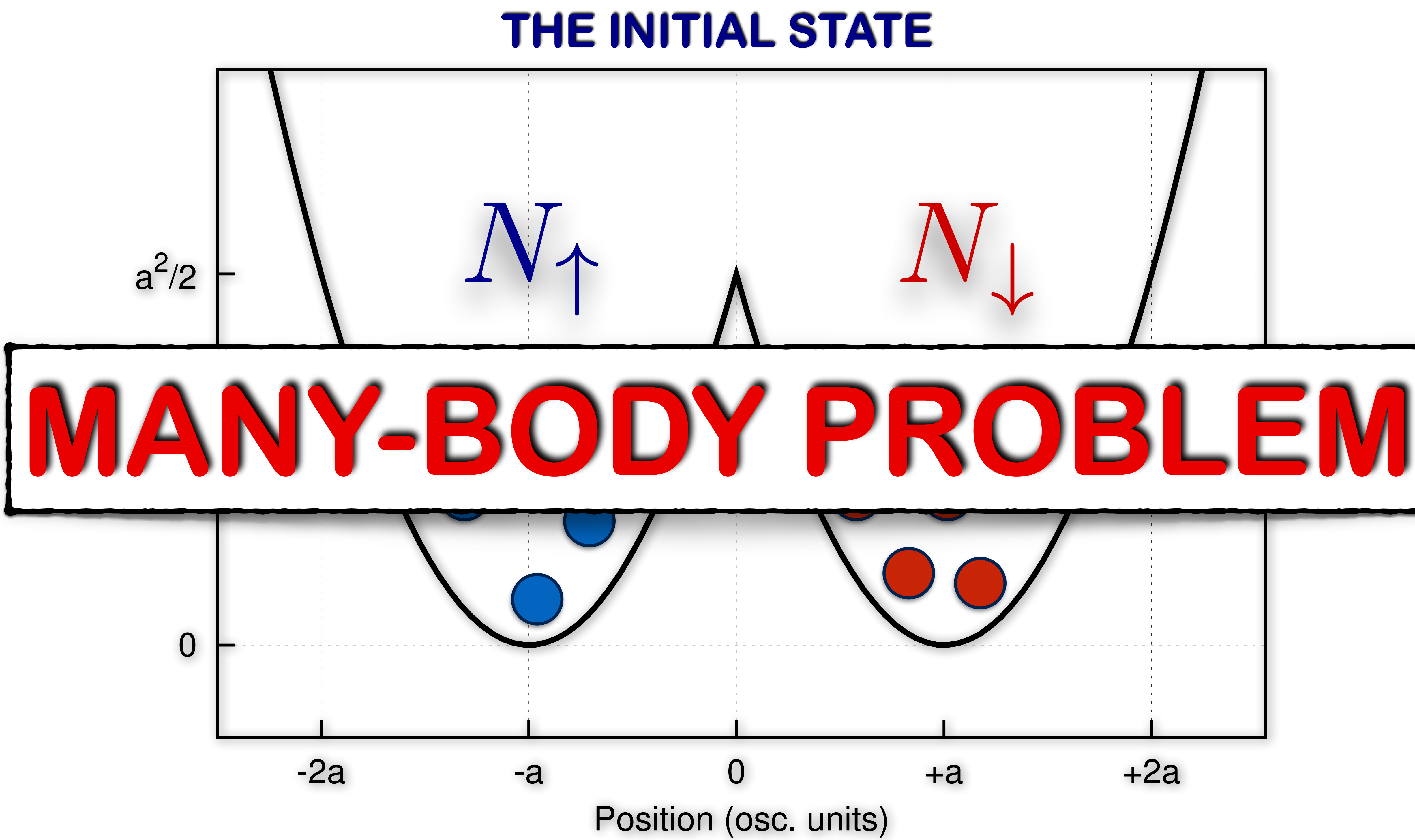


The question



What one can say about the evolution governed by the many-body Hamiltonian without any approximations???

The question



What one can say about the evolution governed by the many-body Hamiltonian without any approximations???

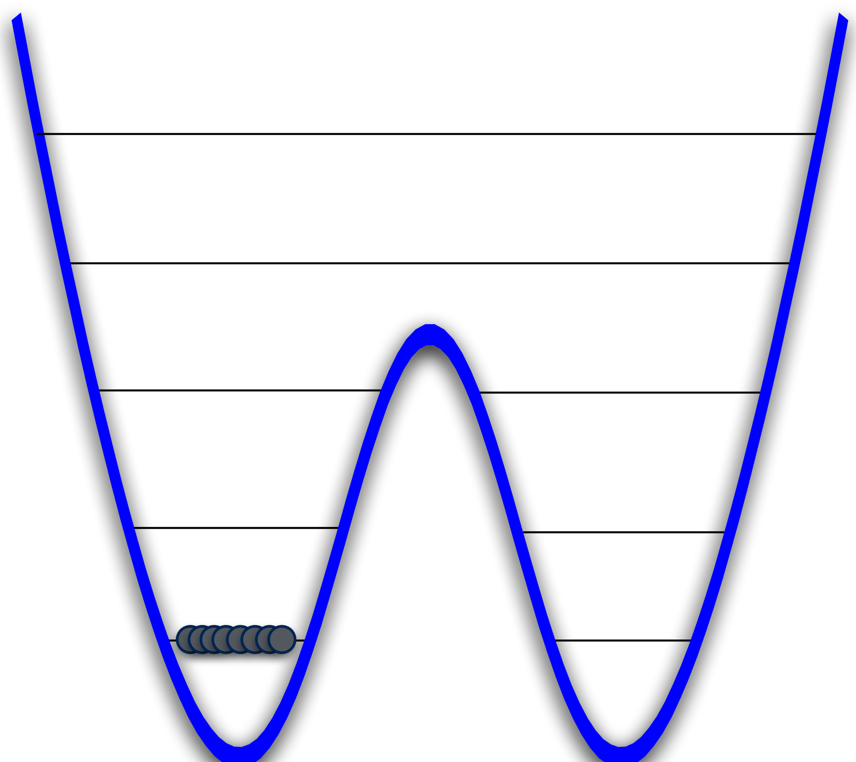
MANY-BODY PROBLEM

BOSONS

$$\hat{\mathcal{H}} = \int \mathrm{d}x\, \hat{\mathbf{\Psi}}^{\dagger}(x)\mathcal{H}_0 \hat{\mathbf{\Psi}}(x) + \frac{g}{2}\int \mathrm{d}x\, \hat{\mathbf{\Psi}}^{\dagger}(x)\hat{\mathbf{\Psi}}^{\dagger}(x)\hat{\mathbf{\Psi}}(x)\hat{\mathbf{\Psi}}(x)$$

standard commutation relations

$$\left[\hat{\mathbf{\Psi}}(x), \hat{\mathbf{\Psi}}^{\dagger}(x')\right] = \delta(x - x')$$



MANY-BODY PROBLEM

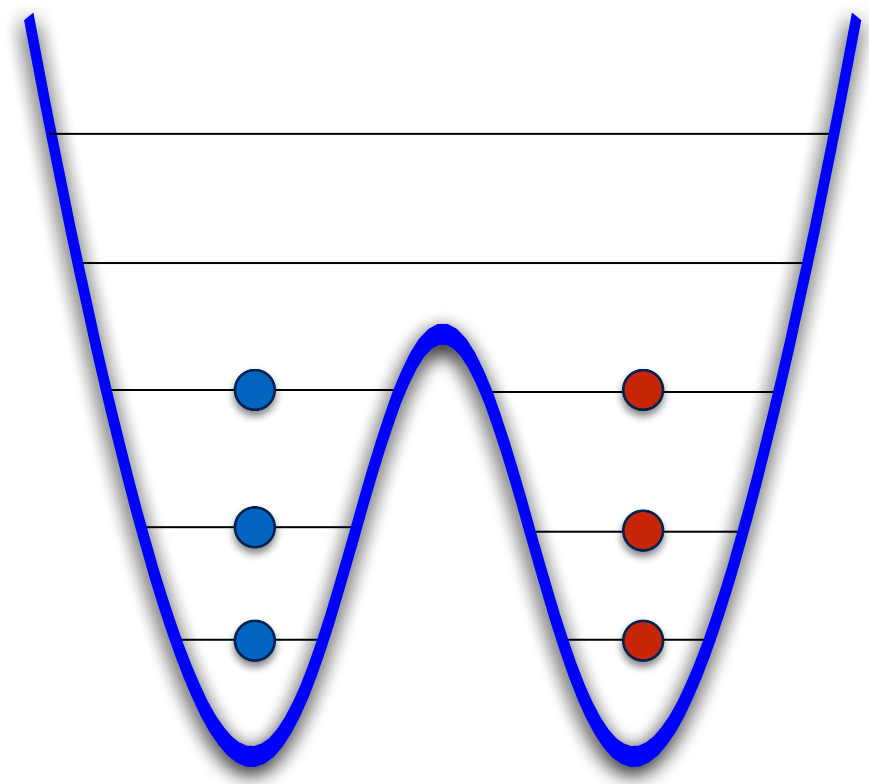
FERMIONS

$$\hat{\mathcal{H}} = \sum_{\sigma} \int \mathrm{d}x \, \hat{\mathbf{\Psi}}^{\dagger}_{\sigma}(x) \mathcal{H}_0 \hat{\mathbf{\Psi}}_{\sigma}(x) + g \int \mathrm{d}x \, \hat{\mathbf{\Psi}}^{\dagger}_{\downarrow}(x) \hat{\mathbf{\Psi}}^{\dagger}_{\uparrow}(x) \hat{\mathbf{\Psi}}_{\uparrow}(x) \hat{\mathbf{\Psi}}_{\downarrow}(x)$$

anticommutation relations

$$\left\{ \hat{\mathbf{\Psi}}_{\sigma}(x), \hat{\mathbf{\Psi}}^{\dagger}_{\sigma'}(x') \right\} = \delta(x - x') \delta_{\sigma\sigma'}$$

$$\left\{ \hat{\mathbf{\Psi}}_{\sigma}(x), \hat{\mathbf{\Psi}}_{\sigma'}(x') \right\} = 0$$



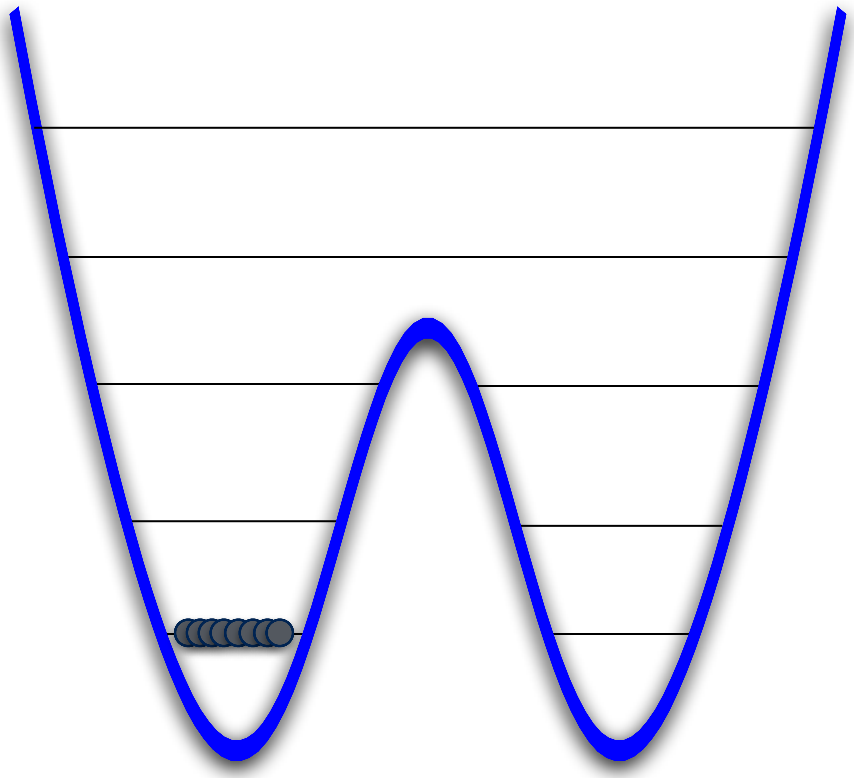
MANY-BODY PROBLEM

BOSONS

$$\hat{\mathcal{H}} = \int \mathrm{d}x\, \hat{\mathbf{\Psi}}^{\dagger}(x)\mathcal{H}_0\hat{\mathbf{\Psi}}(x) + \frac{g}{2}\int \mathrm{d}x\, \hat{\mathbf{\Psi}}^{\dagger}(x)\hat{\mathbf{\Psi}}^{\dagger}(x)\hat{\mathbf{\Psi}}(x)\hat{\mathbf{\Psi}}(x)$$

standard commutation relations

$$\left[\hat{\mathbf{\Psi}}(x), \hat{\mathbf{\Psi}}^{\dagger}(x')\right] = \delta(x - x')$$



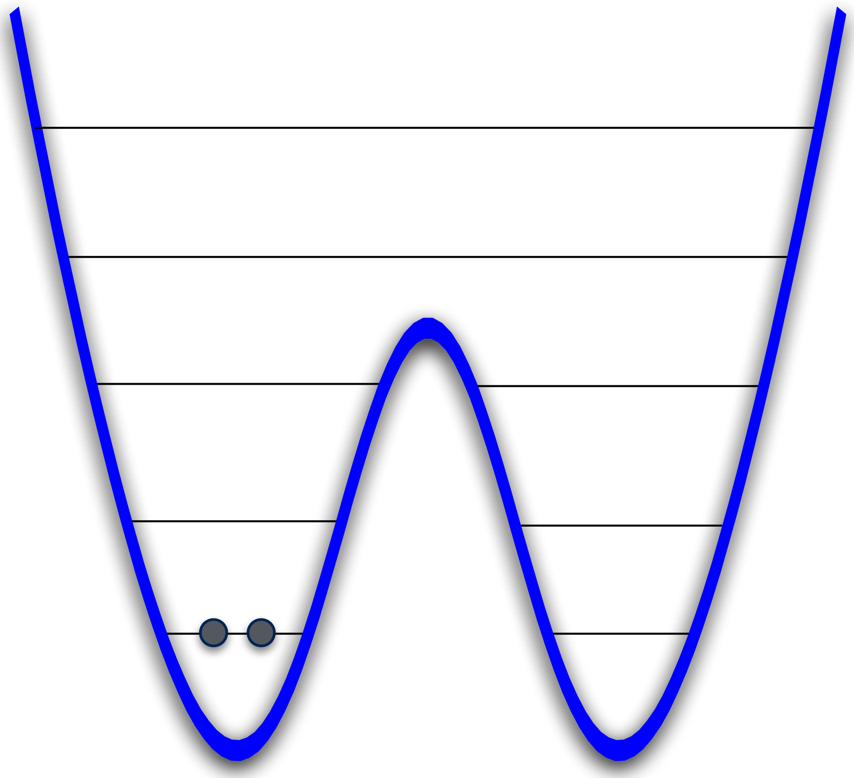
MANY-BODY PROBLEM

BOSONS

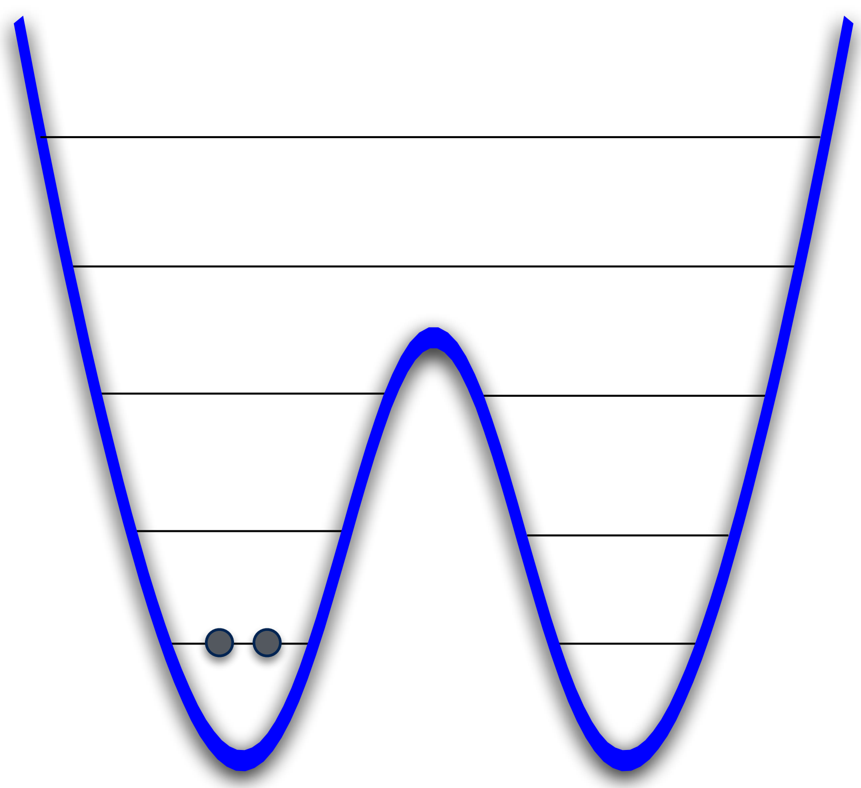
$$\hat{\mathcal{H}} = \int \mathrm{d}x\, \hat{\boldsymbol{\Psi}}^{\dagger}(x)\mathcal{H}_0\hat{\boldsymbol{\Psi}}(x) + \frac{g}{2}\int \mathrm{d}x\, \hat{\boldsymbol{\Psi}}^{\dagger}(x)\hat{\boldsymbol{\Psi}}^{\dagger}(x)\hat{\boldsymbol{\Psi}}(x)\hat{\boldsymbol{\Psi}}(x)$$

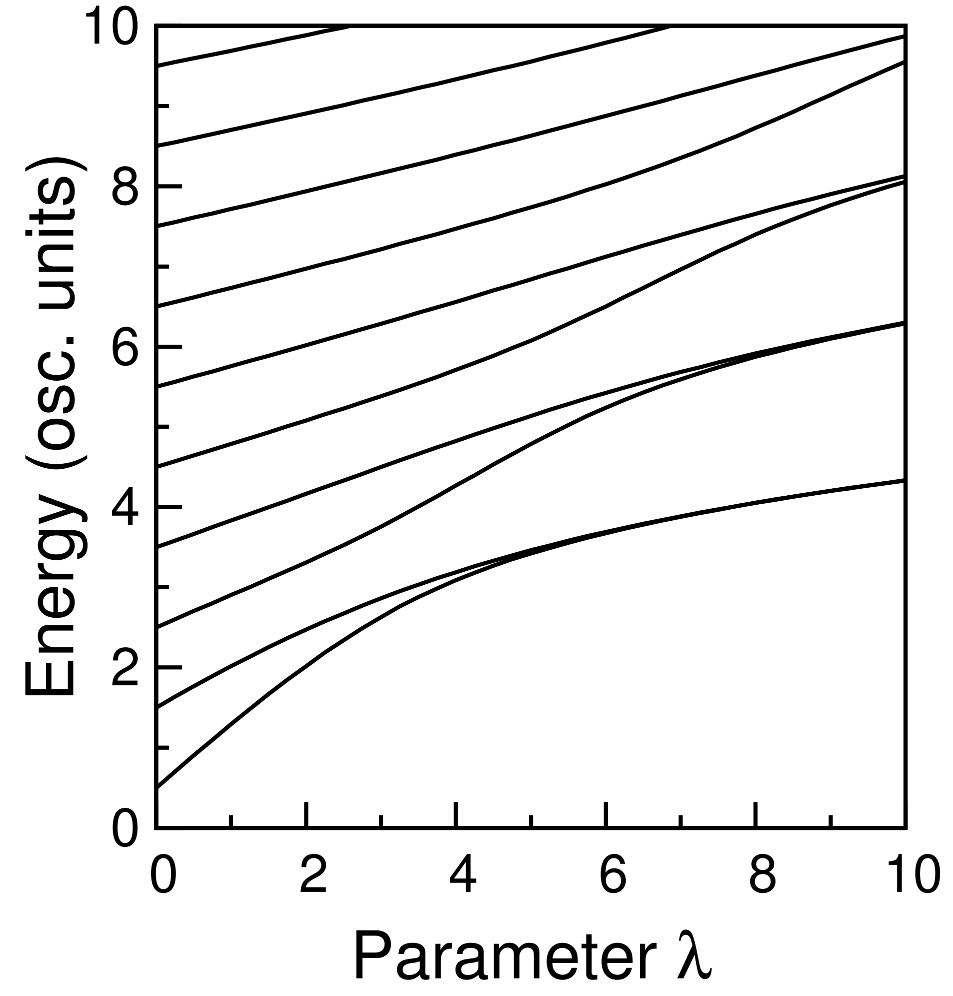
standard commutation relations

$$\left[\hat{\boldsymbol{\Psi}}(x), \hat{\boldsymbol{\Psi}}^{\dagger}(x')\right] = \delta(x - x')$$

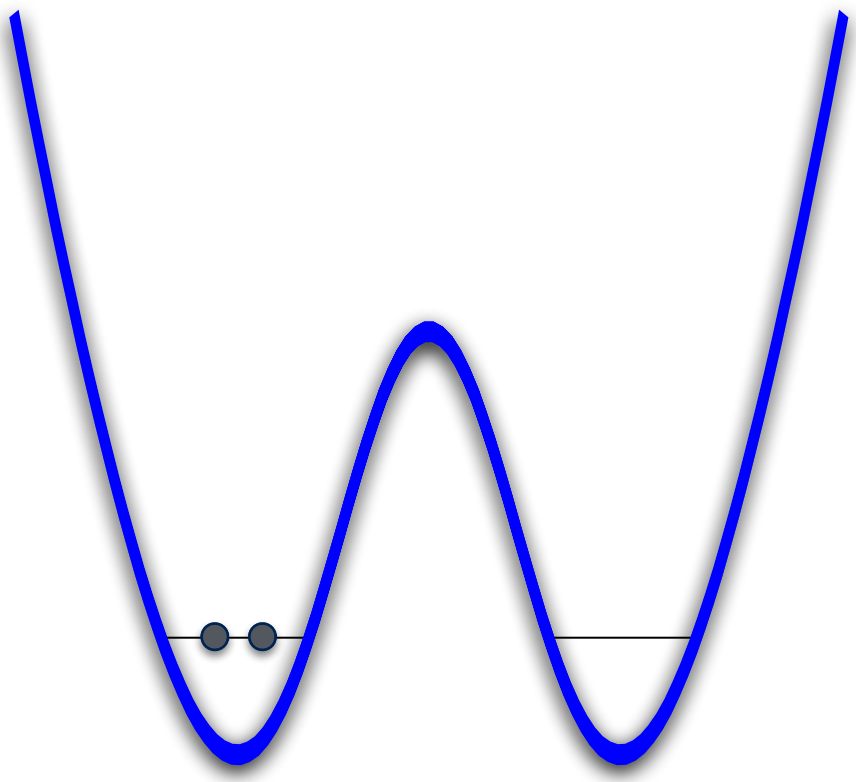


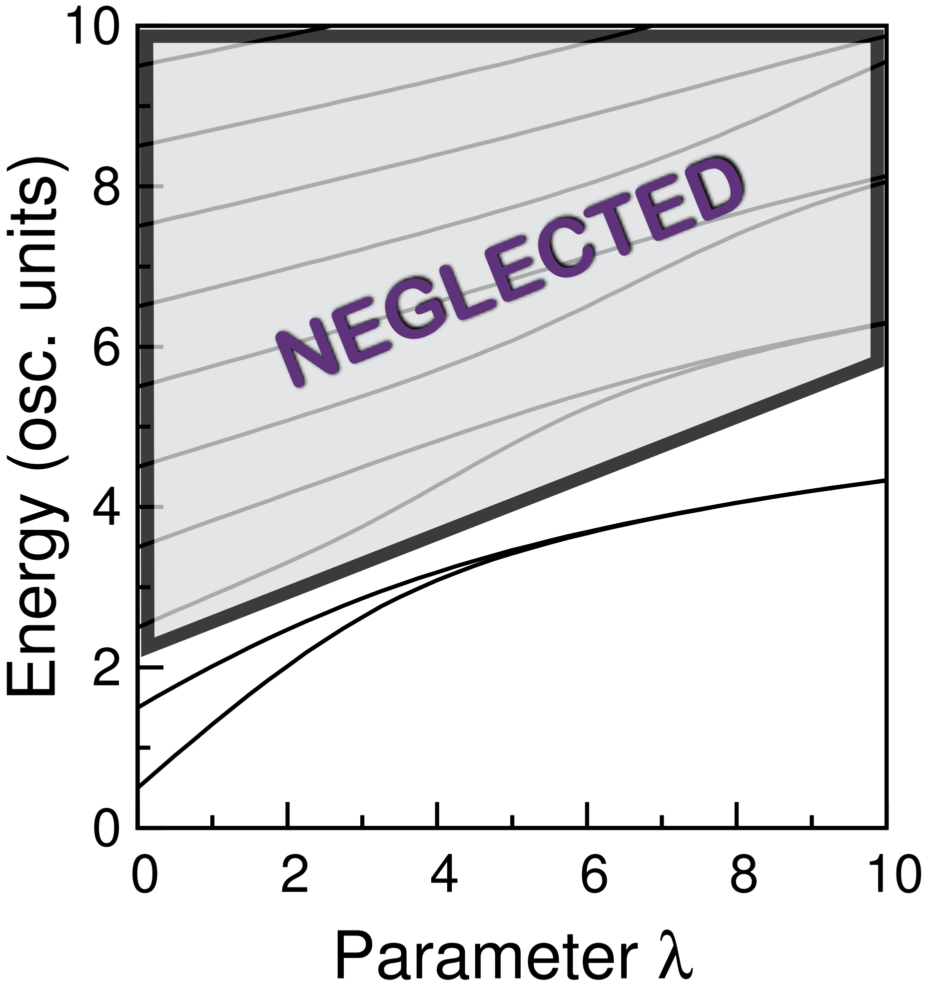
Two-mode models





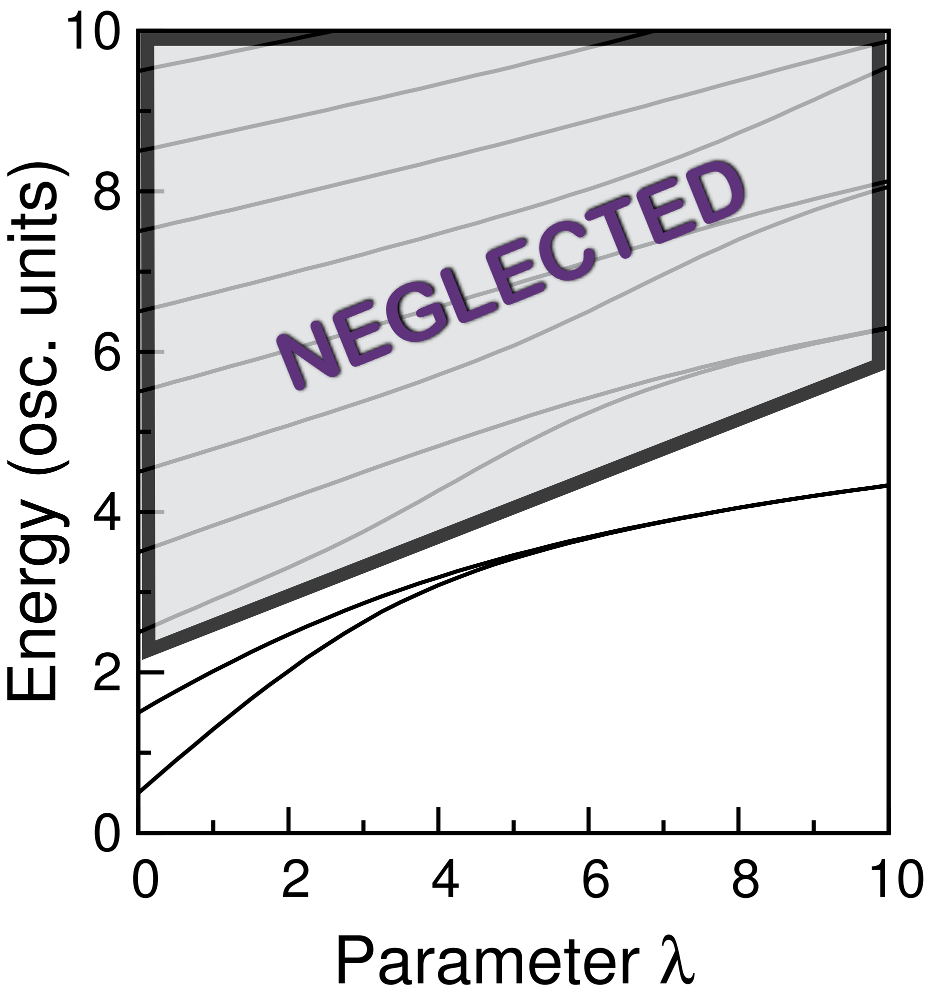
Two-mode models



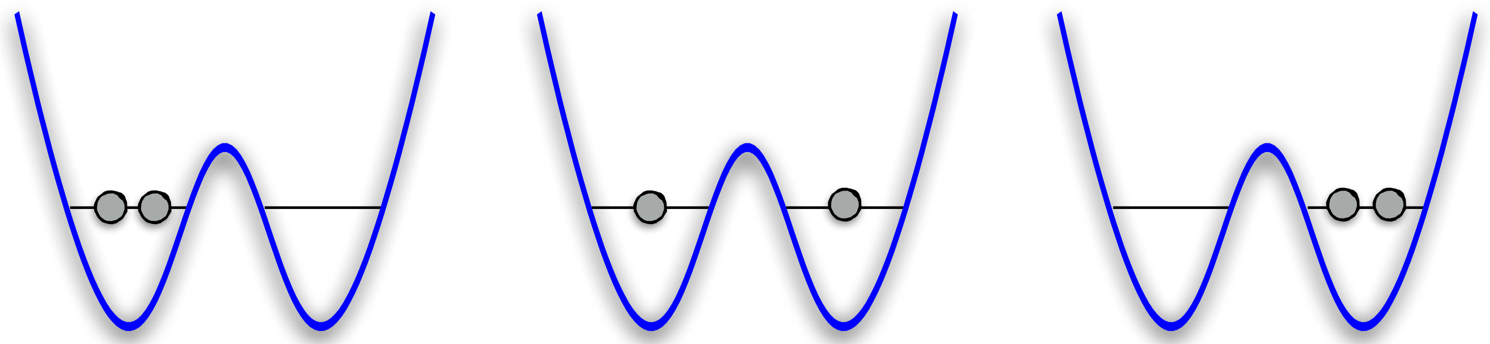


Two-mode models

- **Two-mode model appears in consequence of neglecting higher single-particle levels**



- **Then for TWO bosons only three states are relevant:**



- **The Hamiltonian**

$$\hat{\mathcal{H}}_{\mathtt{2Mode}} = \begin{pmatrix} U & \sqrt{2}(T-J) & V/2 \\ \sqrt{2}(T-J) & V & \sqrt{2}(T-J) \\ V/2 & \sqrt{2}(T-J) & U \end{pmatrix}$$

Two-mode models

$$\hat{\mathcal{H}}_{\mathtt{2Mode}} = \begin{pmatrix} U & \sqrt{2}(T-J) & V/2 \\ \sqrt{2}(T-J) & V & \sqrt{2}(T-J) \\ V/2 & \sqrt{2}(T-J) & U \end{pmatrix}$$

For short-range interactions one can anticipate that some terms can be neglected

$$T \ll U \qquad V \ll U$$

Then we obtain two-site version of the standard Bose-Hubbard model

$$\hat{\mathcal{H}}_{\mathtt{BH}} = \begin{pmatrix} U & -\sqrt{2}J & 0 \\ -\sqrt{2}J & 0 & -\sqrt{2}J \\ 0 & -\sqrt{2}J & U \end{pmatrix}$$

Two-mode models

- **Exact model**

$$\hat{\mathcal{H}} = \int \mathrm{d}x\, \hat{\boldsymbol{\Psi}}^\dagger(x)\mathcal{H}_0\hat{\boldsymbol{\Psi}}(x) + \frac{g}{2}\int \mathrm{d}x\, \hat{\boldsymbol{\Psi}}^\dagger(x)\hat{\boldsymbol{\Psi}}^\dagger(x)\hat{\boldsymbol{\Psi}}(x)\hat{\boldsymbol{\Psi}}(x)$$

- **Two-mode approximation**

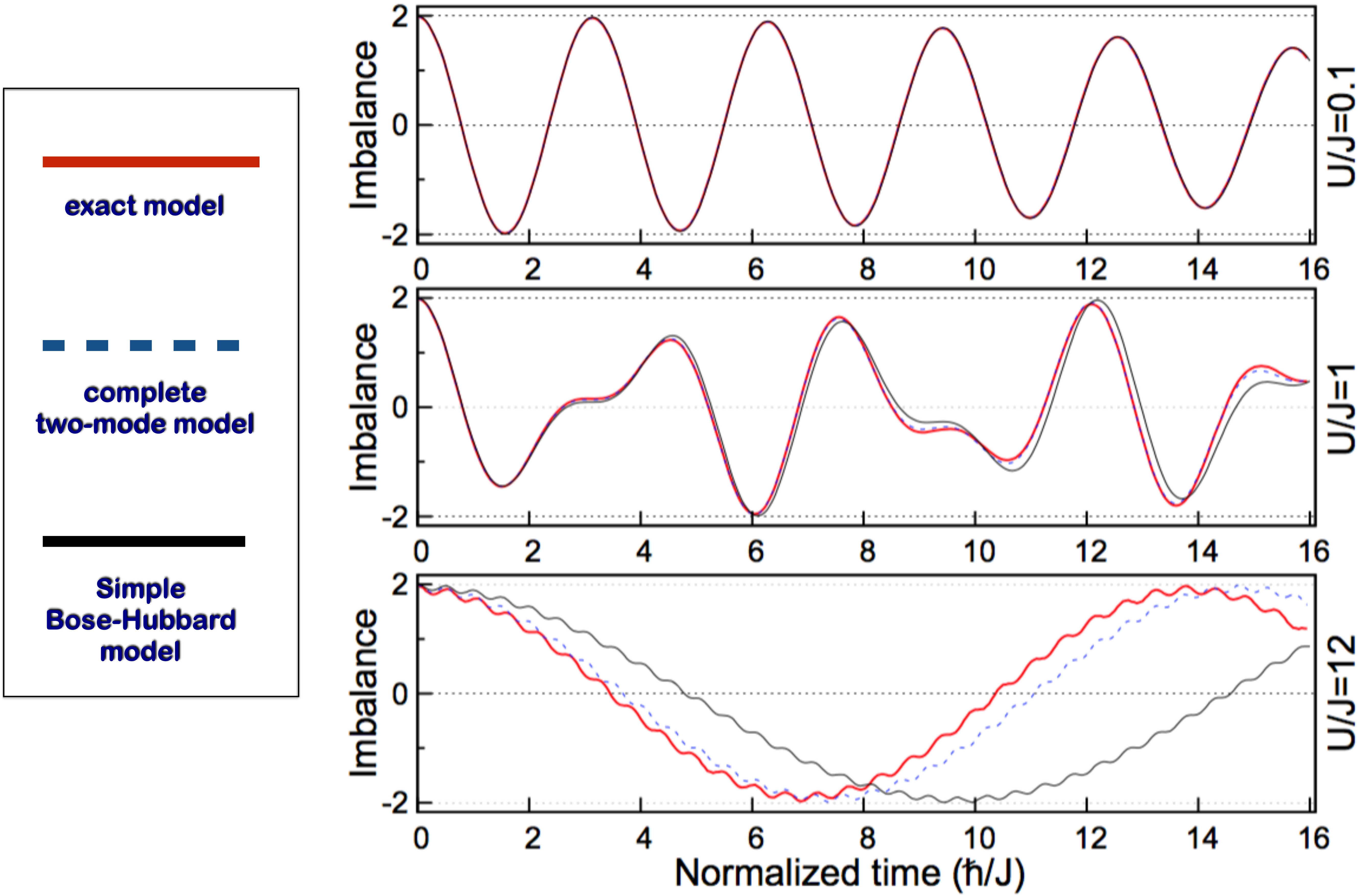
$$\hat{\mathcal{H}}_{\mathtt{2Mode}} = \begin{pmatrix} U & \sqrt{2}(T-J) & V/2 \\ \sqrt{2}(T-J) & V & \sqrt{2}(T-J) \\ V/2 & \sqrt{2}(T-J) & U \end{pmatrix}$$

- **Hubbard-like description**

$$\hat{\mathcal{H}}_{\mathtt{BH}} = \begin{pmatrix} U & -\sqrt{2}J & 0 \\ -\sqrt{2}J & 0 & -\sqrt{2}J \\ 0 & -\sqrt{2}J & U \end{pmatrix}$$

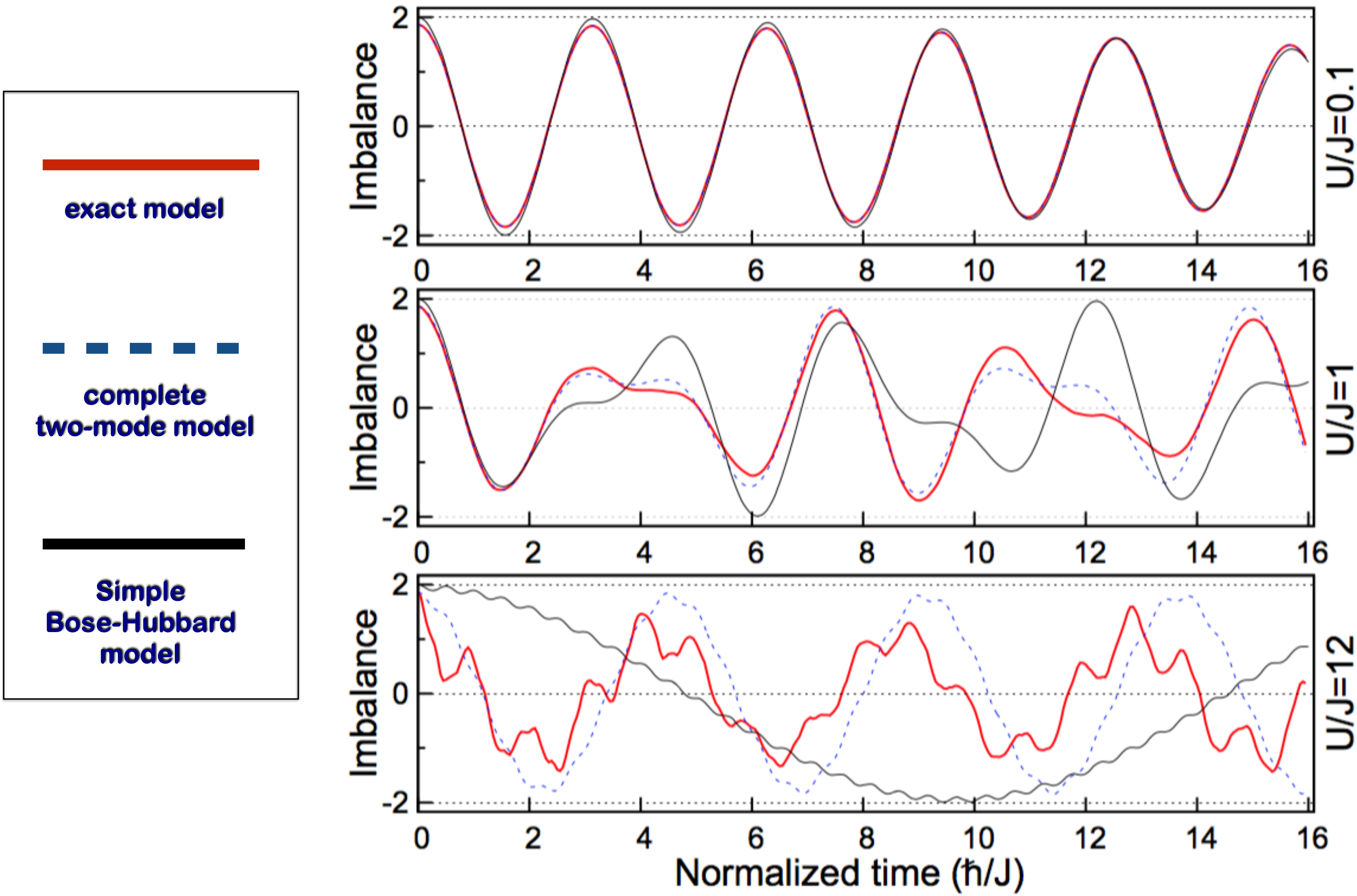
Very high barrier

$$\mathcal{I}(t) = \langle\langle \Psi(t) \| \hat{N}_L - \hat{N}_R \| \Psi(t) \rangle\rangle$$



Shallow barrier

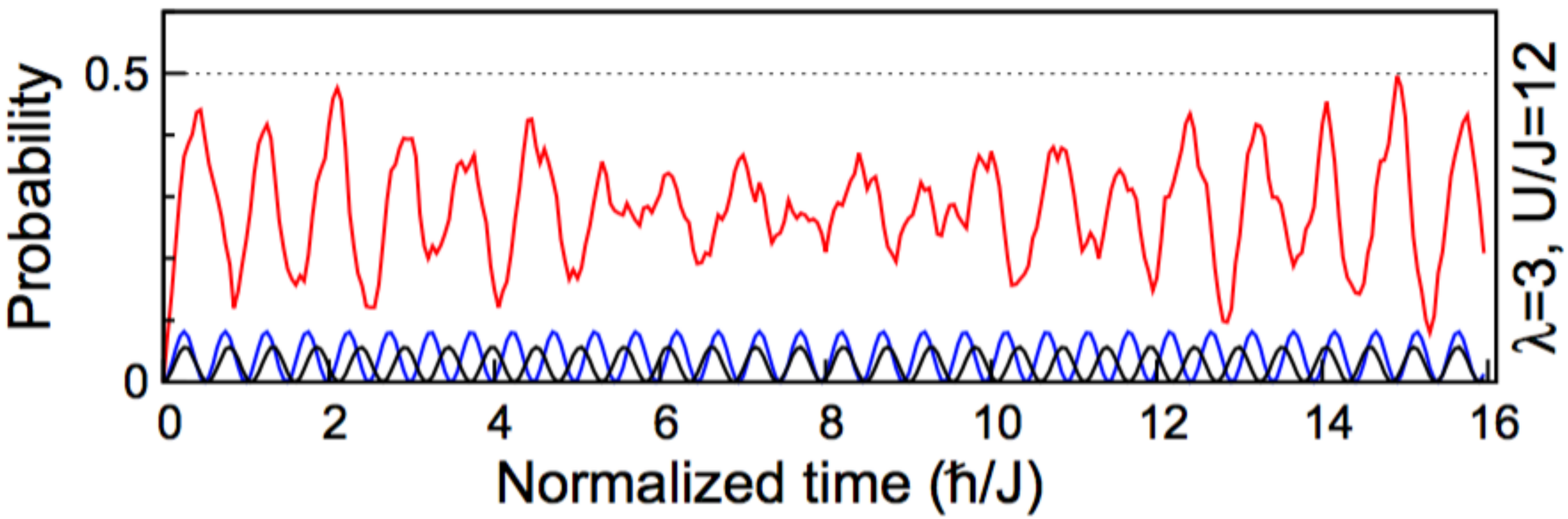
$$\mathcal{I}(t) = \langle\!\langle \Psi(t) \| \hat{N}_L - \hat{N}_R \| \Psi(t) \rangle\!\rangle$$



Inter-particle correlations

probability that bosons occupy different wells

$$\mathcal{P}(t) = 2\int_0^{\infty} \mathrm{d}x_1 \int_{-\infty}^{0} \mathrm{d}x_2 \, \langle\langle \Psi(t) \| \hat{\mathbf{\Psi}}^\dagger(x_1) \hat{\mathbf{\Psi}}^\dagger(x_2) \hat{\mathbf{\Psi}}(x_2) \hat{\mathbf{\Psi}}(x_1) \| \Psi(t) \rangle\rangle$$



Two-mode description is insufficient to describe inter-particle correlations correctly!

MANY-BODY PROBLEM

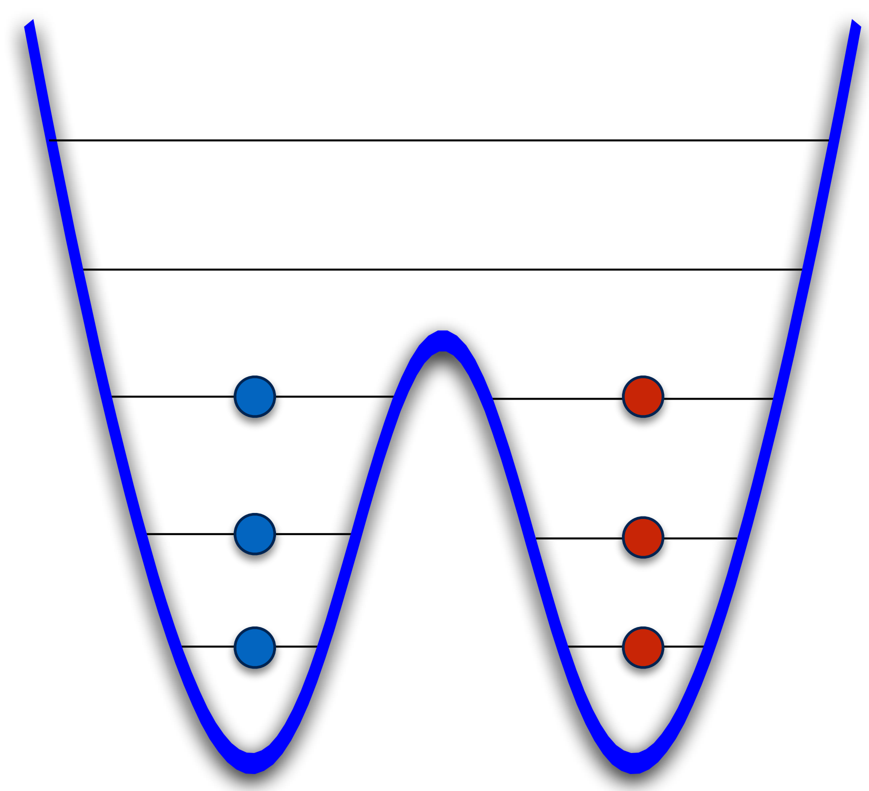
FERMIONS

$$\hat{\mathcal{H}} = \sum_\sigma \int \mathrm{d}x\, \hat{\boldsymbol{\Psi}}^\dagger_\sigma(x)\mathcal{H}_0\hat{\boldsymbol{\Psi}}_\sigma(x) + g\int \mathrm{d}x\, \hat{\boldsymbol{\Psi}}^\dagger_\downarrow(x)\hat{\boldsymbol{\Psi}}^\dagger_\uparrow(x)\hat{\boldsymbol{\Psi}}_\uparrow(x)\hat{\boldsymbol{\Psi}}_\downarrow(x)$$

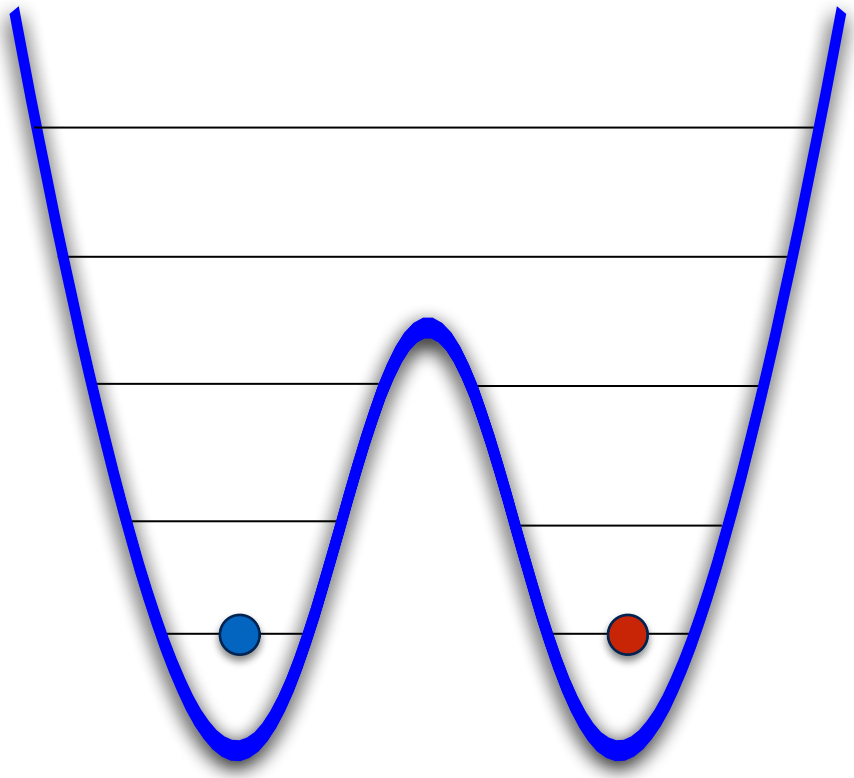
anticommutation relations

$$\left\{\hat{\boldsymbol{\Psi}}_\sigma(x), \hat{\boldsymbol{\Psi}}^\dagger_{\sigma'}(x')\right\} = \delta(x-x')\delta_{\sigma\sigma'}$$

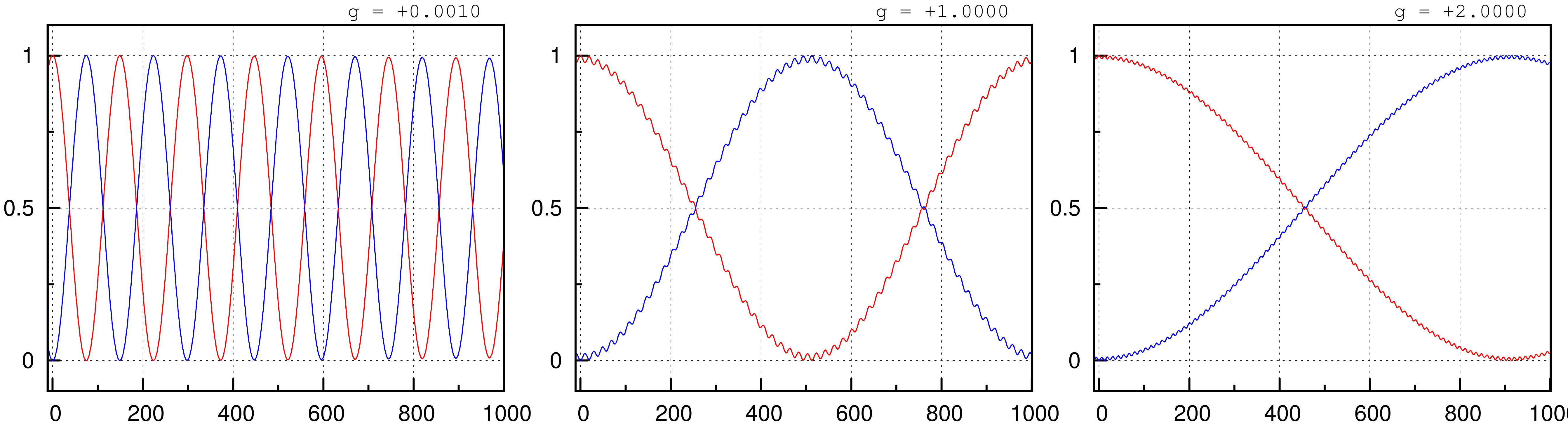
$$\left\{\hat{\boldsymbol{\Psi}}_\sigma(x), \hat{\boldsymbol{\Psi}}_{\sigma'}(x')\right\} = 0$$



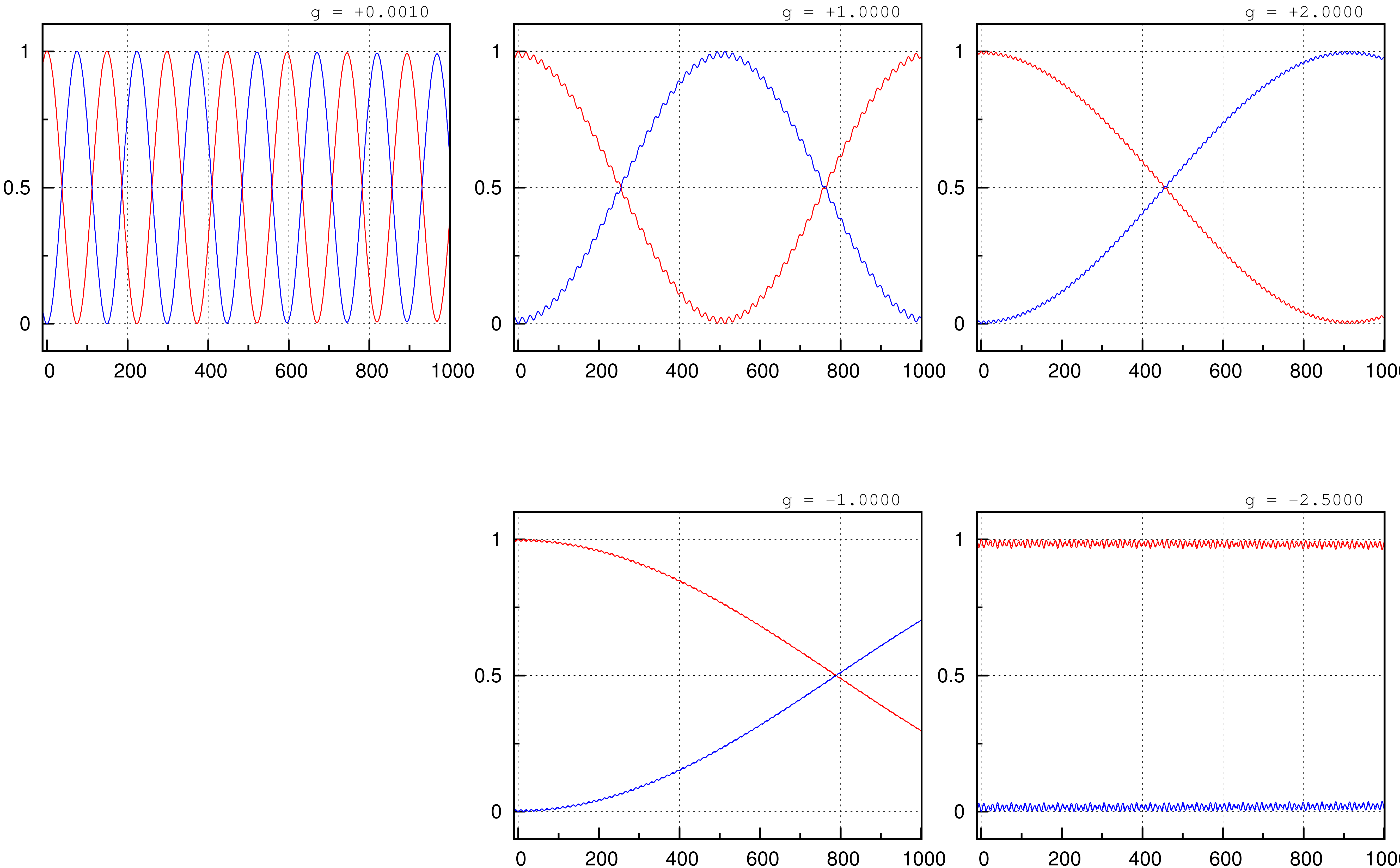
two distinguishable particles



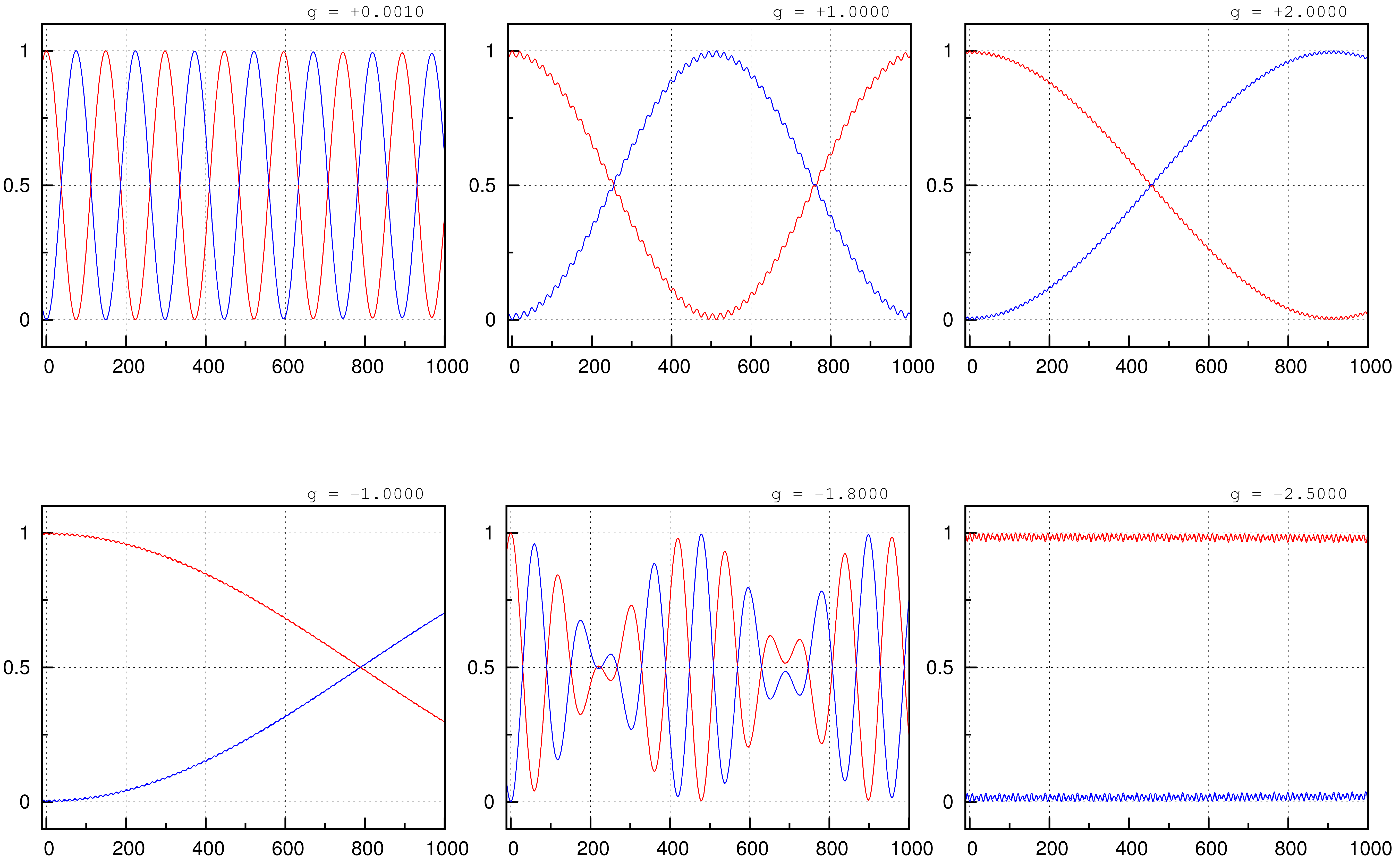
Evolution of the densities



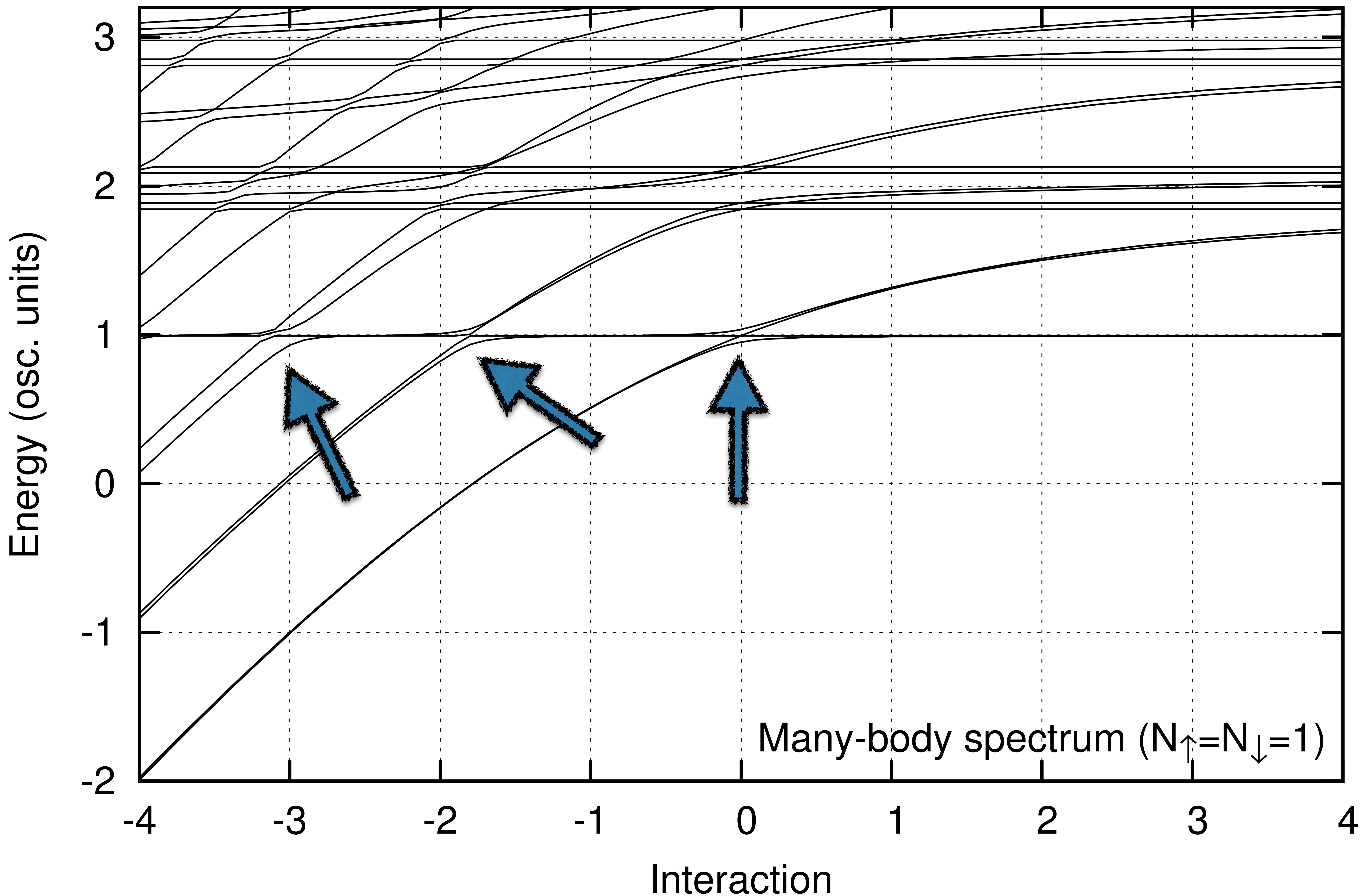
Evolution of the densities



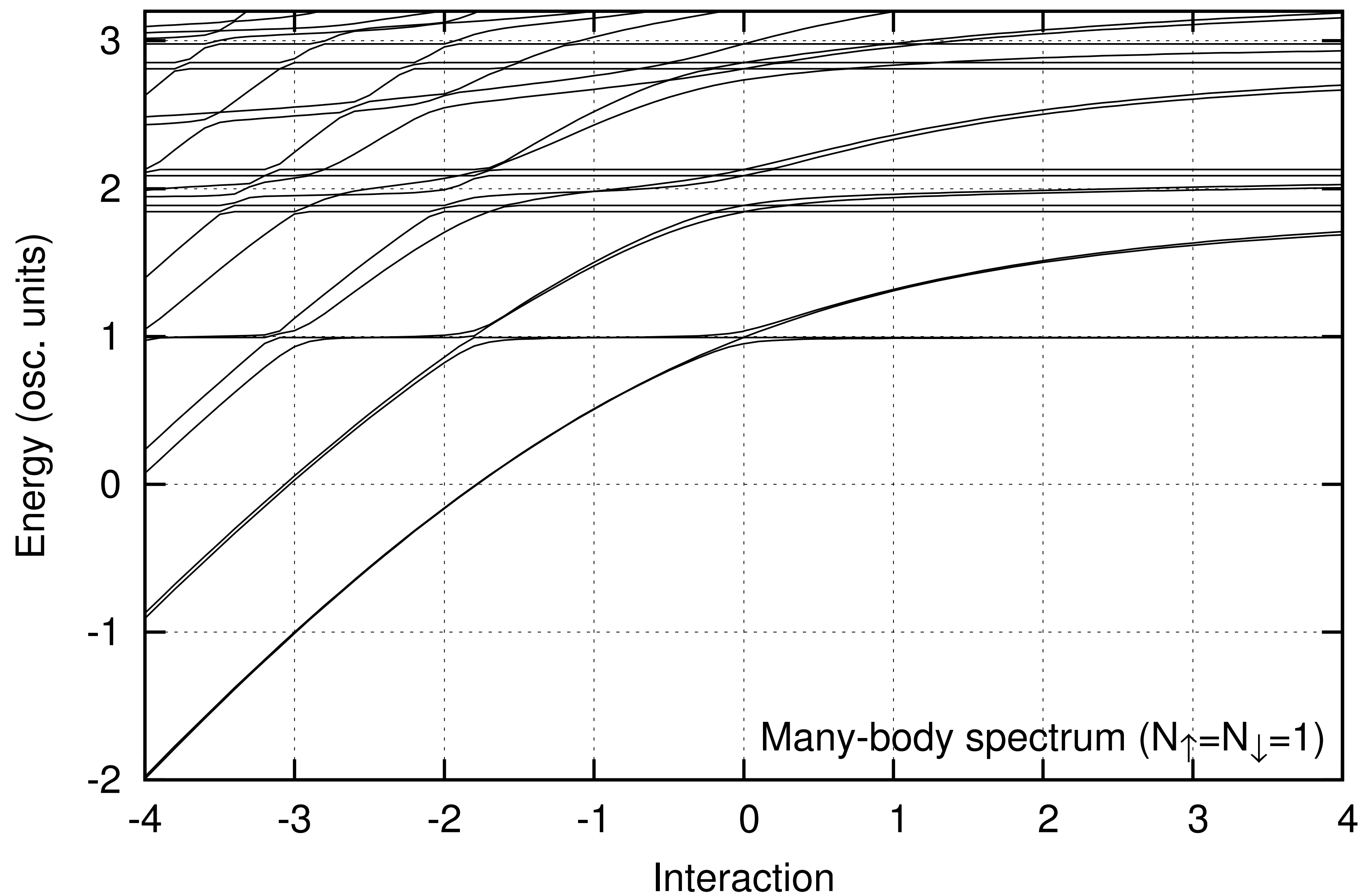
Evolution of the densities

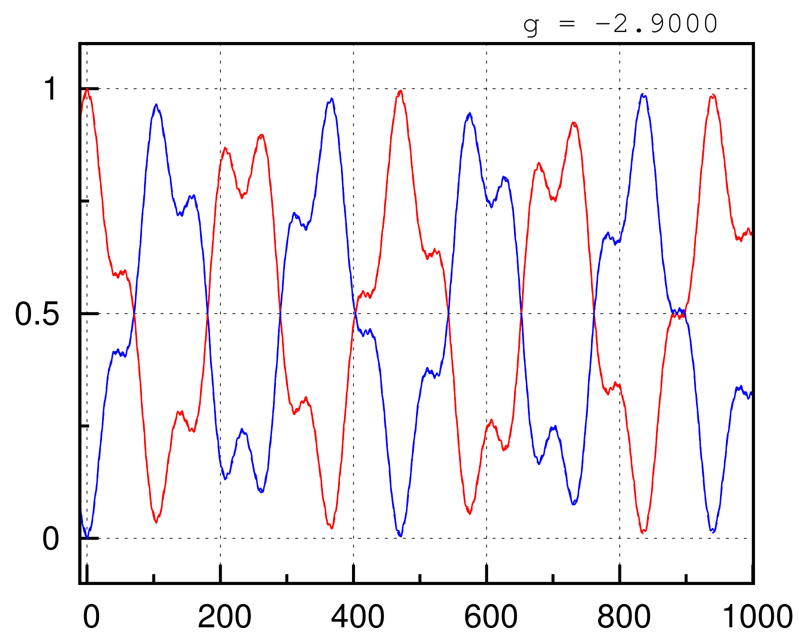


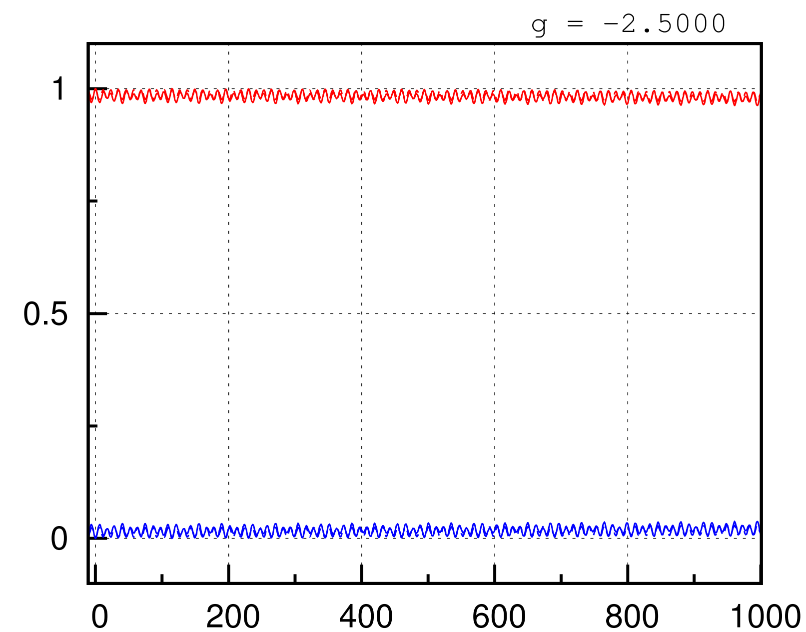
Many-body spectrum

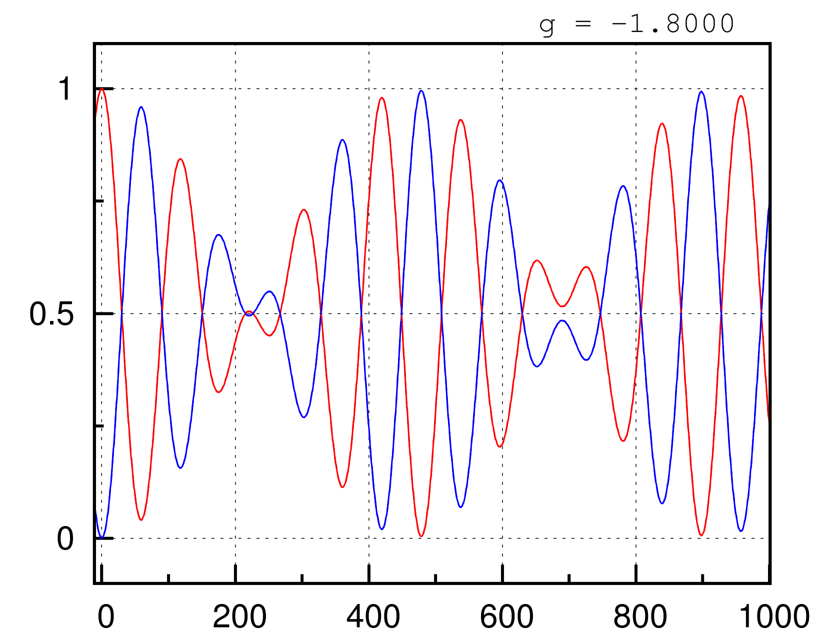


Many-body spectrum

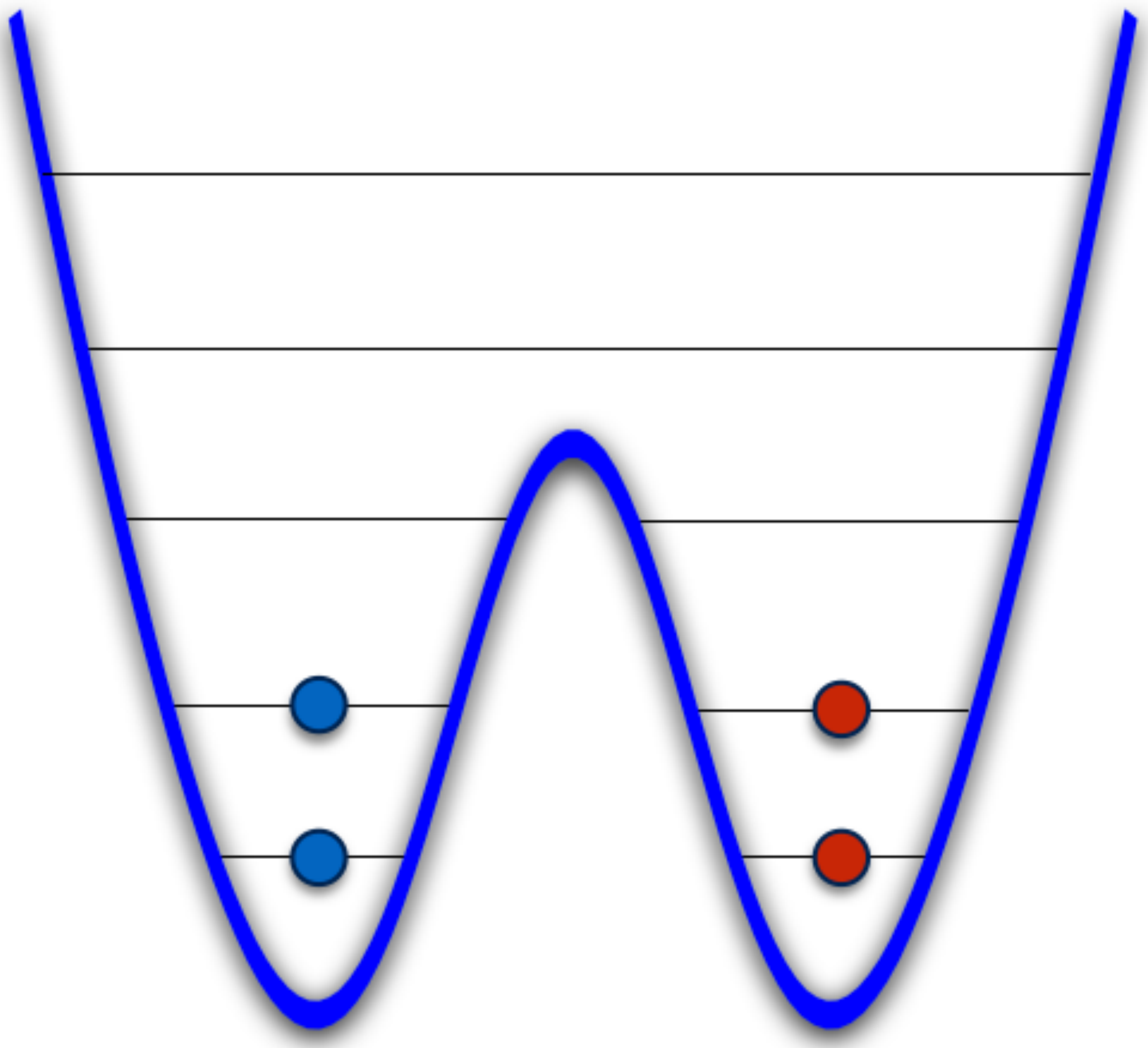




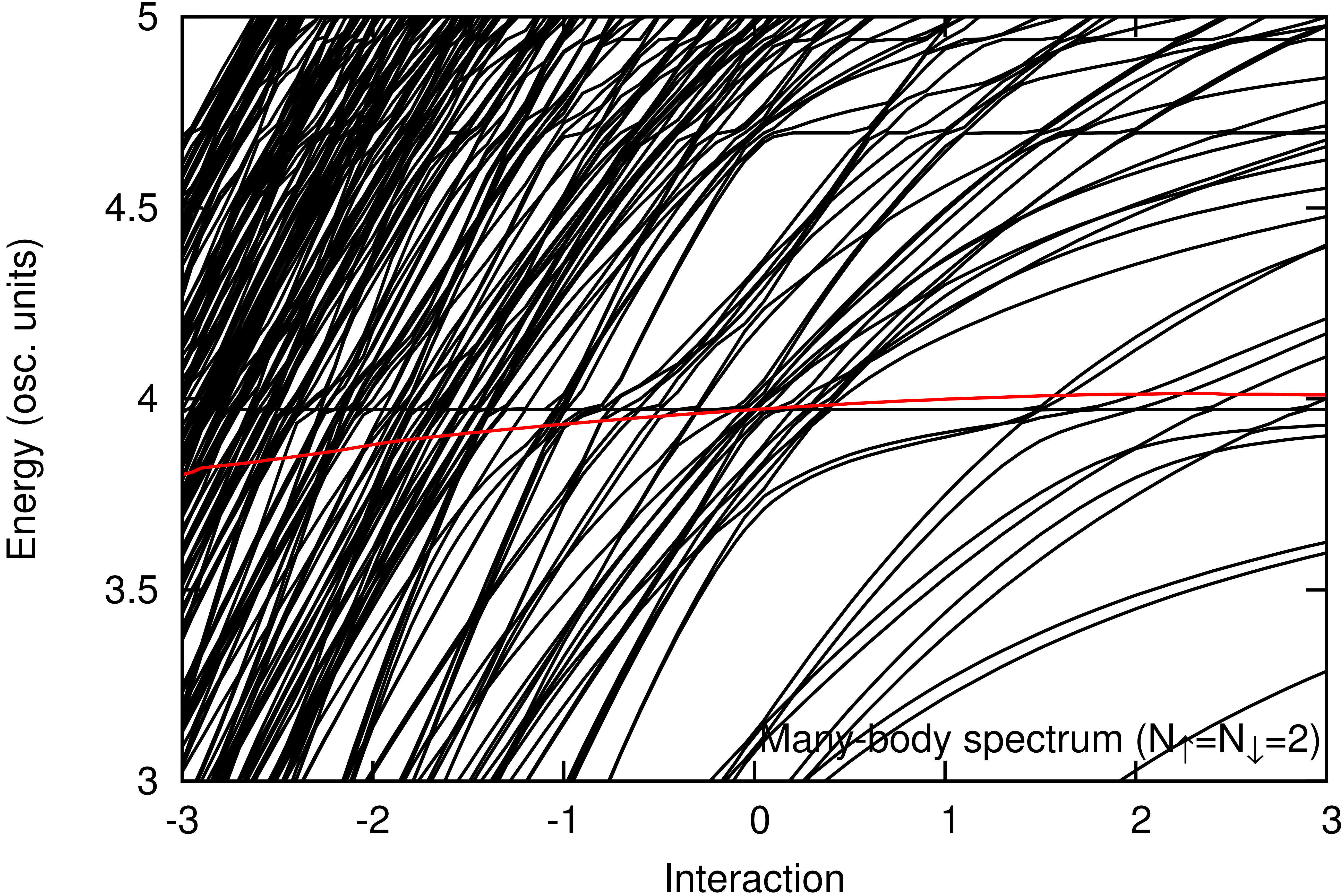




two fermionic cloudlets



Spectrum of the Hamiltonian



Dynamics of several ultra-cold particles in a double-well potential

Tomasz Sowinski
Institute of Physics of the Polish Academy of Sciences

References:

[1] T. Sowiński, M. Gajda, K. Rzążewski: EPL **113**, 56003 (2016)
[2] J. Dobrzyniecki, T. Sowiński: EPJ D **70**, 83 (2016)