Different properties of $\bar{K}NN$ and $\bar{K}\bar{K}N$ systems

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The history

K^-pp quasi-bound state

Prediction of the existence of deep and narrow K^- pp bound state
(G-matrix calculation, complex potential): E_B = - 48 MeV, Γ = 61 MeV
T. Yamazaki and Y. Akaishi, Phys. Lett. B535 (2002) 70

Many theoretical calculations, different models and inputs
(Faddeev, variational calculations, FCA):

$$E_B \sim -14 - 80 \text{ MeV}, \quad \Gamma \sim 40 - 110 \text{ MeV}$$

FINUDA collaboration: evidence for a deeply bound state
(correlated Λ and *p*) with E_B = -115 MeV, Γ = 67 MeV:
M. Agnello et. al., Phys. Rev. Lett. 94 (2005) 212303

DISTO collaboration: E_B = -103 MeV, Γ = 118 MeV
T. Yamazaki et al. Phys. Rev. Lett. 104, (2010)

$\rightarrow$ Faddeev calculation of K^- pp quasi-bound state
with phenomenological and chiral-motivated two-body input ($\bar{K}N$)

$\bar{K}N - \pi\Sigma$ is the basic interaction

How to investigate: - fit to two-body experimental data (not enough),
- use in a few- or many-body calculation and compare with experiment.

1. Faddeev calculation of $K^- d$ scattering length with phenomenological and chirally-motivated two-body input (not directly measurable)

   Experiment: measurement of kaonic deuterium *1s* level shift and width (~~SIDDHARTA~~, SIDDHARTA-2) → calculation of the observables, corresponding to the obtained a($K^- d$)

   (plus: search of a quasi-bound state in $K^- d$ *)*

2. Calculation of elastic $K^- d$ scattering amplitudes and construction of a two-body K^- - d complex potential, based on them.

3. Use the potential for calculation of the *1s* level shift and width of kaonic deuterium (to be measured)

One more three-body system with strangeness and possible qbs: $\bar{K}\bar{K}N - \bar{K}\pi\Sigma$

$\bar{K}NN - \pi\Sigma N$ and $\bar{K}\bar{K}N - \bar{K}\pi\Sigma$ systems: coupled-channel Faddeev equations in Alt-Grassberger-Sandhas form

$$U_{ij}^{\alpha\beta} = \delta_{\alpha\beta}\left(1-\delta_{ij}\right)\left(G_0^{\alpha}\right)^{-1} + \sum_{k,\gamma=1}^{3}\left(1-\delta_{ik}\right)T_k^{\alpha\gamma}G_0^{\gamma}U_{kj}^{\gamma\beta}, \qquad i,j=1,2,3$$

$\bar{K}N$ interaction is strongly coupled with $\pi\Sigma$ $\Rightarrow$ particle channels (α):

$$\alpha=1: \left|\bar{K}_1\, N_2\, N_3\right\rangle, \quad \alpha=2: \left|\pi_1\, \Sigma_2\, N_3\right\rangle, \quad \alpha=3: \left|\pi_1\, N_2\, \Sigma_3\right\rangle \; (\bar{K}NN)$$

$$\alpha=1: \left|\bar{K}_1\, \bar{K}_2\, N_3\right\rangle, \quad \alpha=2: \left|\bar{K}_1\, \pi_2\, \Sigma_3\right\rangle, \quad \alpha=3: \left|\pi_1\, \bar{K}_2\, \Sigma_3\right\rangle \; (\bar{K}\bar{K}N)$$

<u>Quantum numbers</u>: spin $S=0\,(K^- pp)$, $S=1\,(K^- d)$ or $S=1/2\,(K^- K^- p$ system),
orbital momentum $L=0$, isospin $I=1/2$

Two identical nucleons - antisymmetrization,
two identical antikaons - symmetrization $\Rightarrow$ system of 10 integral equations

Coupled-channel $\bar{K}N - \pi\Sigma - \pi\Lambda$ interaction

Experimental data

- $1s$ level shift and width of kaonic hydrogen (by SIDDHARTA)

  $\Delta_{1s}^{SIDD} = -283 \pm 36 \pm 6\,\text{eV}, \quad \Gamma_{1s}^{SIDD} = 541 \pm 89 \pm 22\,\text{eV}$
- Cross-sections of $K^- p \to K^- p$ and $K^- p \to MB$ reactions
- Threshold branching ratios γ, R_c and R_n
- ◦ $\Lambda(1405)$ with one- or two-pole structure

  $M_{\Lambda(1405)}^{PDG} = 1405.1^{+1.3}_{-1.0}\,\text{MeV}, \quad \Gamma_{\Lambda(1405)}^{PDG} = 50.5 \pm 2.0\,\text{MeV}$

Phenomenological and "chirally-motivated" potentials
with isospin-breaking effects:

1. Kaonic hydrogen: direct inclusion of *Coulomb interaction*
2. Use of the *physical masses*: $m_{K^-}, m_{\bar{K}^0}, m_p, m_n$ instead of $m_{\bar{K}}, m_N$

Phenomenological $\bar{K}N - \pi\Sigma$ potentials with one- or two-pole structure of the Λ(1405) resonance, equally properly reproducing all experimental data, except πΛ channel (taken into account indirectly)

$$V_I^{\alpha\beta}(\vec{k}^{\alpha}, \vec{k}'^{\beta}) = g_I^{\alpha}(\vec{k}^{\alpha})\, \lambda_{\mathrm{I}}^{\alpha\beta}\, g_I^{\beta}(\vec{k}'^{\beta}), \quad \text{form-factors:}$$

- 1-pole Λ(1405):

$$g_{I,1pole}^{\alpha}(k^{\alpha}) = \frac{1}{(k^{\alpha})^2 + (\beta_{\mathrm{I}}^{\alpha})^2}$$

for $\alpha = K$ ($\bar{K}N$ channel) or π ($\pi\Sigma$ channel)

- 2-pole Λ(1405):

$$g_{I,1pole}^{\alpha}(k^{\alpha}) = \frac{1}{(k^{\alpha})^2 + (\beta_{\mathrm{I}}^{\alpha})^2} \qquad \text{for } \alpha = K \ (\bar{K}N \text{ channel})$$

$$g_{I,2pole}^{\alpha}(k^{\alpha}) = \frac{1}{(k^{\alpha})^2 + (\beta_{\mathrm{I}}^{\alpha})^2} + \frac{s\,(\beta_{\mathrm{I}}^{\alpha})^2}{[\,(k^{\alpha})^2 + (\beta_{\mathrm{I}}^{\alpha})^2\,]^2} \qquad \text{for } \alpha = \pi \ (\pi\Sigma \text{ channel}).$$

"Chirally-motivated" $\bar{K}N-\pi\Sigma-\pi\Lambda$ potential, properly reproducing all experimental data (two-pole $\Lambda(1405)$ resonance structure)

$$V_I^{\alpha\beta}(\vec{k}^{\,\alpha},\vec{k}'^{\,\beta};\sqrt{s}\,)=\sqrt{\frac{M_\alpha}{2\omega_\alpha E_\alpha}}\,g_I^\alpha(\vec{k}^{\,\alpha})\;\frac{C_I^{\alpha\beta}(\sqrt{s}\,)}{(2\pi)^3 f_\alpha f_\beta}\;\sqrt{\frac{M_\beta}{2\omega_\beta E_\beta}}\;g_I^\beta(\vec{k}'^{\,\beta}),$$

Yamaguchi form-factors: $$g_I^\alpha(k^\alpha)=\frac{(\beta_{\mathrm{I}}^\alpha)^2}{(k^\alpha)^2+(\beta_{\mathrm{I}}^\alpha)^2}$$

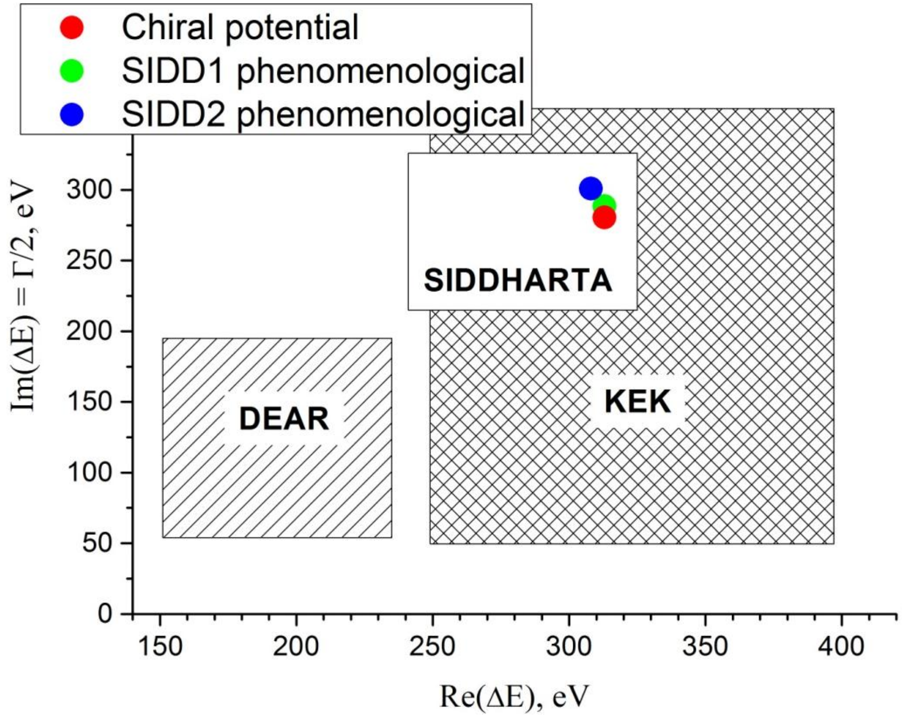
The leading-order Weinberg-Tomozawa term

$$C_I^{\alpha\beta}(\sqrt{s}\,)=-C^{WT}(2\sqrt{s}-M_\alpha-M_\beta)$$

Parameters: $f_K,\ f_\pi$ (pseudoscalar meson decay constants)

β_I^α (range parameters)

Experimental and theoretical
1s level shifts and widths
of kaonic hydrogen

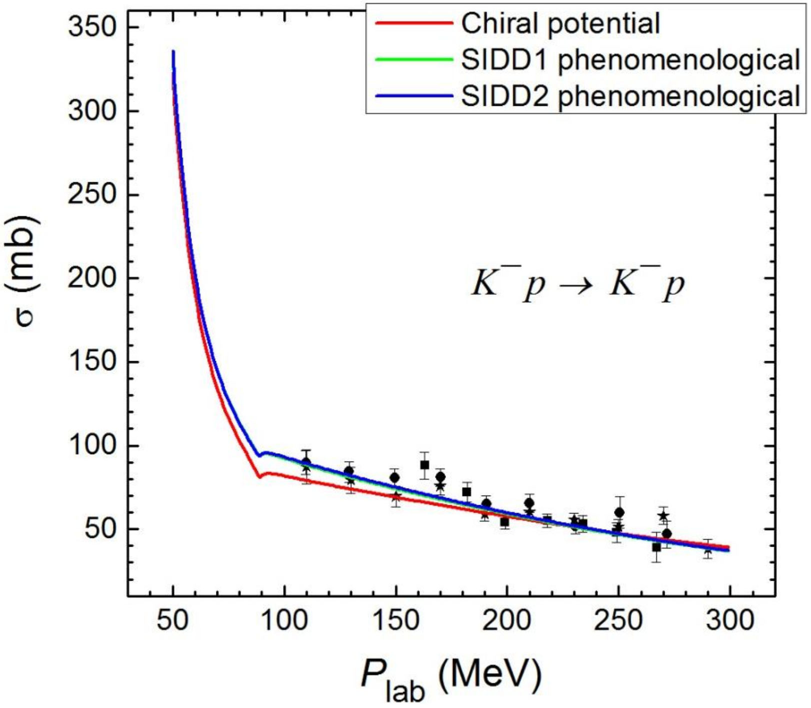


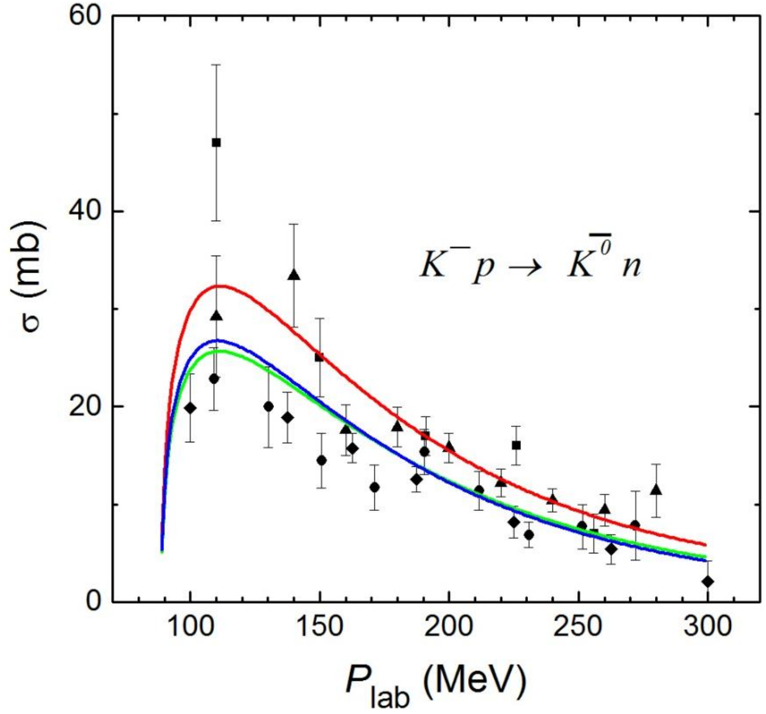
Strong pole positions (MeV)

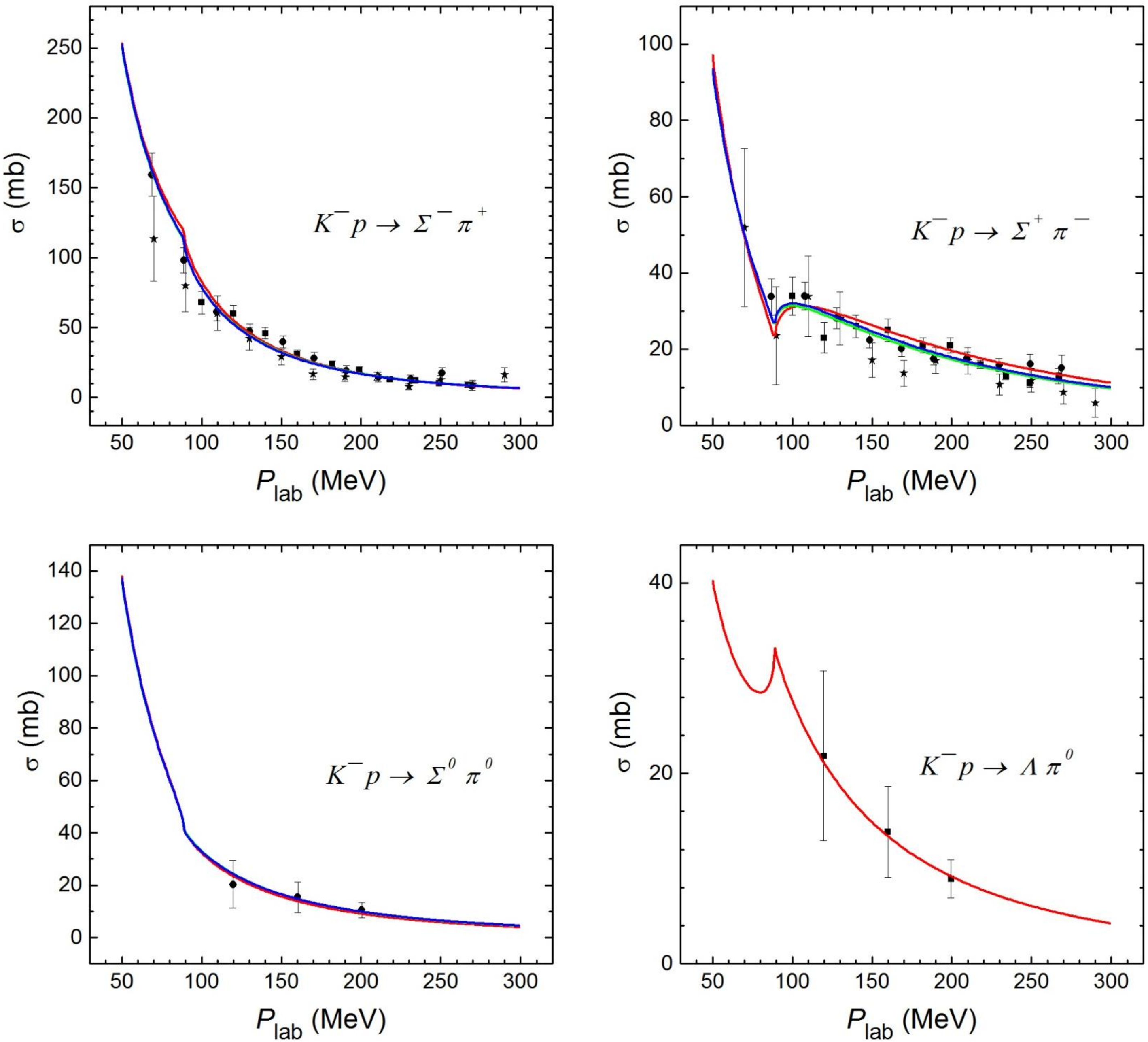
1-pole phenom. $V(\overline{K}N)$	2-pole phenom. $V(\overline{K}N)$	Chirally motiv. $V(\overline{K}N)$
1426 – *i* 48	1414 – *i* 58 1386 – *i* 104	1417 – *i* 33 1406 – *i* 89

Comparison with experimental data on $K^- p$ cross-sections

phenomenological and chiral-motivated potentials







(continuation)

Three-body quasi-bound states

$\bar{K}NN\,(K^{-}pp)$ pole positions (MeV)

1-pole phenom. $V(\bar{K}N)$	2-pole phenom. $V(\bar{K}N)$	Chirally motiv. $V(\bar{K}N)$
$-53.3 - i\,32.4$	$-47.4 - i\,24.9$	$-32.2 - i\,24.3$

No pole positions were found in the $K^{-}d$ system

$\bar{K}\bar{K}N$ pole positions (MeV)

1-pole phenom. $V(\bar{K}N)$	2-pole phenom. $V(\bar{K}N)$	Chirally motiv. $V(\bar{K}N)$
$-19.5 - i\,51.0$	$-25.9 - i\,42.3$	$-16.1 - i\,30.7$

Strong dependence on the $\bar{K}N$ model

$K^- d$ scattering

$K^- d$ scattering length (fm)

1-pole phenom. $V(\bar{K}N)$	2-pole phenom. $V(\bar{K}N)$	Chirally motiv. $V(\bar{K}N)$
$-1.49 + i\,1.24$	$-1.51 + i\,1.25$	$-1.59 + i\,1.32$

Weak dependence on the $\bar{K}N$ model

Near-threshold $K^- d$ scattering amplitudes $\longrightarrow$
approximate evaluation of *1s* level shift and width of kaonic deuterium:

$$K^- d \text{ system} = K^- + \text{point-like deuteron}$$

1s level shift and width of kaonic deuterium

1. Two-body strong K^- - d $V^S_{K^-d}$ potential:
reproduces the $K^- d$ scattering amplitudes

2. Energy of the *1s* level of kaonic deuterium:
Lippmann-Schinger equation with the strong and Coulomb potentials

$$H = H_0 + V^S_{K^-d}(k,k') + V^{Coul} \quad \Rightarrow \quad E^{S+Coul}_{1s}$$

3. *1s* level shift caused by strong interaction: $\Delta E_{1s} = E^{Coul}_{1s} - \mathrm{Re}(E^{S+Coul}_{1s})$

1s level shift $\Delta E^{K^-d}_{1s}$ (eV) and width $\Gamma^{K^-d}_{1s}$ (eV) of kaonic deuterium

1-pole phenom. $V(\overline{K}N)$		2-pole phenom. $V(\overline{K}N)$		Chirally motiv. $V(\overline{K}N)$	
$\Delta E^{K^-d}_{1s}$	$\Gamma^{K^-d}_{1s}$	$\Delta E^{K^-d}_{1s}$	$\Gamma^{K^-d}_{1s}$	$\Delta E^{K^-d}_{1s}$	$\Gamma^{K^-d}_{1s}$
– 785	1018	– 797	1025	– 828	1055

Weak dependence on the $\overline{K}N$ model

Conclusions

The constructed:

- phenomenological $\bar{K}N - \pi\Sigma$ with one-pole $\Lambda(1405)$ resonance
- phenomenological $\bar{K}N - \pi\Sigma$ with two-pole $\Lambda(1405)$ resonance
- chirally motivated $\bar{K}N - \pi\Sigma - \pi\Lambda$ potentials

reproduce SIDDHARTA data on kaonic hydrogen *1s* level shift and width and the scattering $K^- p$ data with the same level of accuracy

Used in the three-body calculations:

- The pole position of the K^-pp and K^-K^-p quasi-bound state strongly depends on the $\bar{K}N$ model
- Dependence of the K^-d scattering length and *1s* level shift and width of kaonic deuterium on the form of the $\bar{K}N$ potential is weak.