

# Trimer and Tetramer Bound States in Heteronuclear Systems

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# Outline

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Efimov Effect

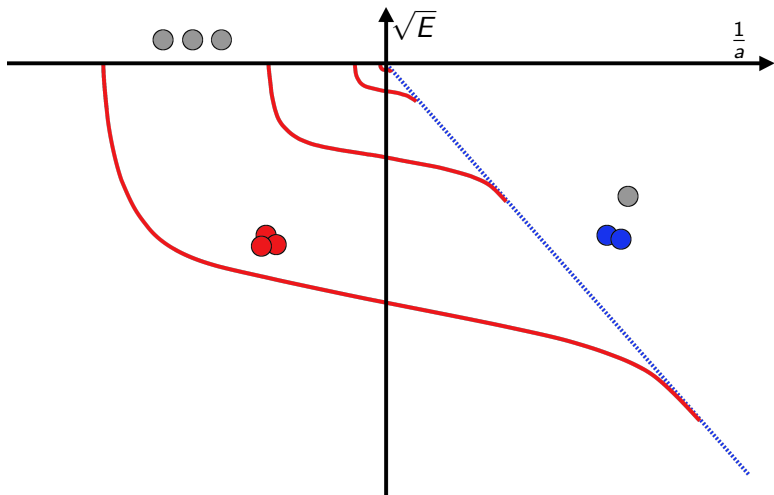
Efimov Effect for Mixtures

Gaussian Expansion Method

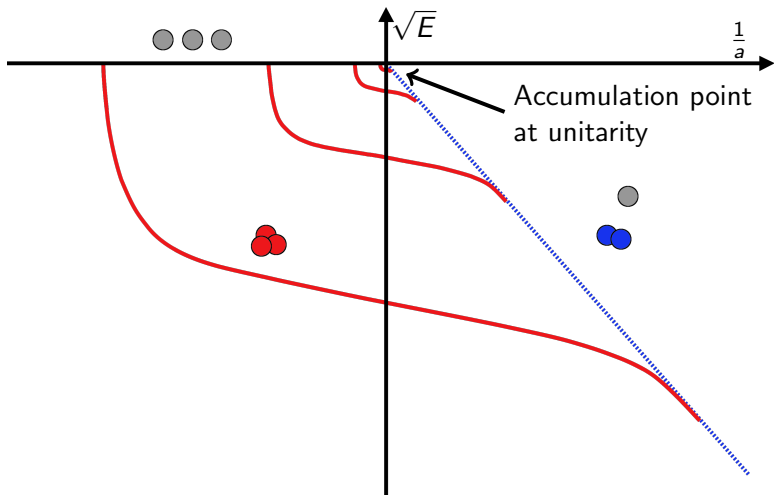
Results

Summary and Outlook

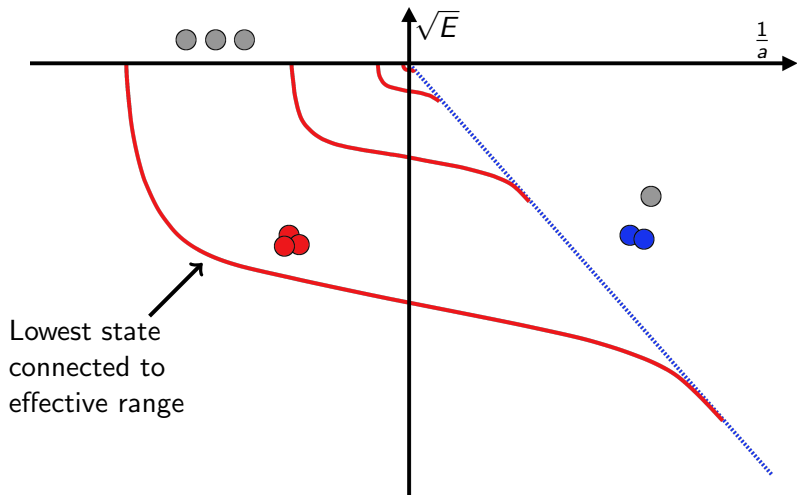
# Efimov Effect



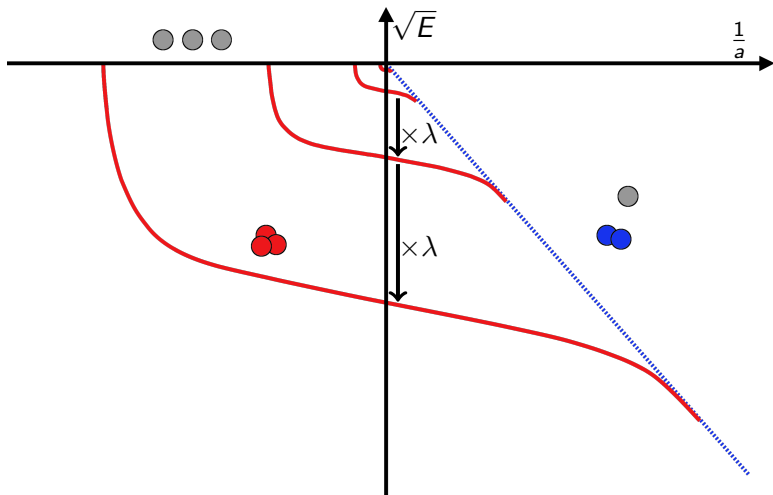
# Efimov Effect



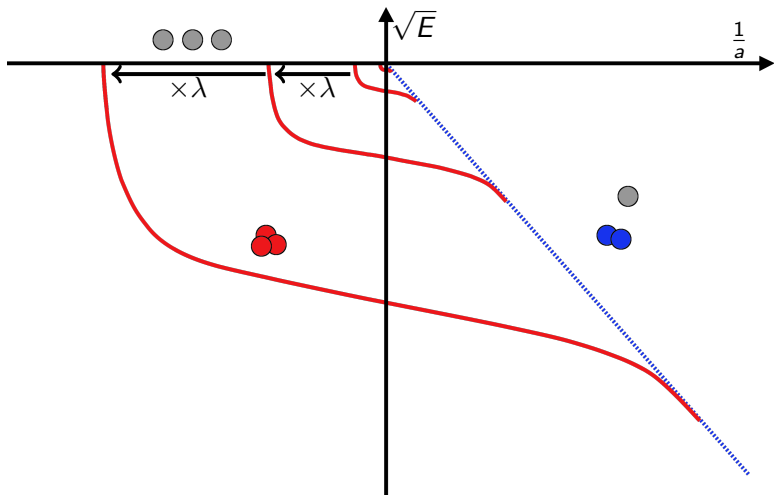
# Efimov Effect



# Efimov Effect



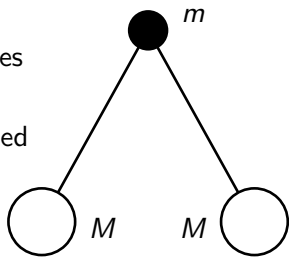
# Efimov Effect



## Efimov Effect for Mixtures

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- Similar effect appears for fermion-boson and boson-boson mixtures
- The factor between consecutive energies of states **depends on the mass ratio**
- If there is only one fermion, spin can be neglected
- We neglected interaction between atoms of the same type
- In **heavy-heavy-light** mixtures, the ratio between consecutive energies of states is smaller than for identical bosons  
→ **easier to observe** experimentally

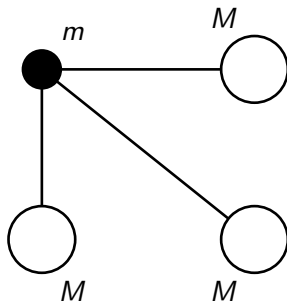




## Efimov Effect for Mixtures II

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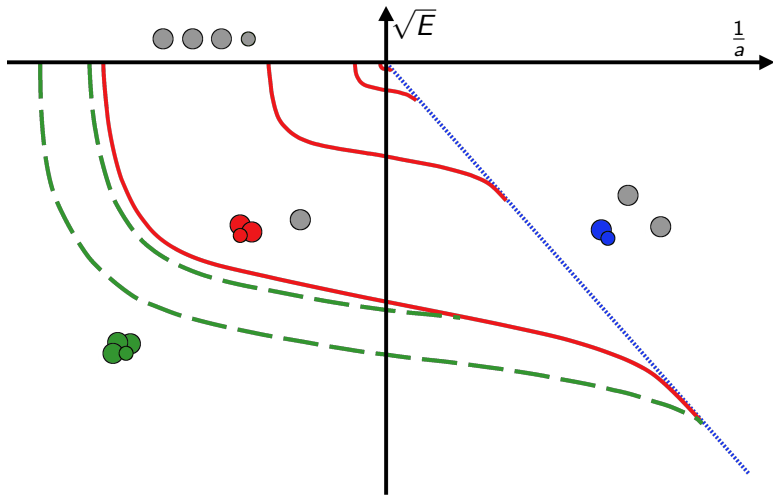
- Four-body states exist as well



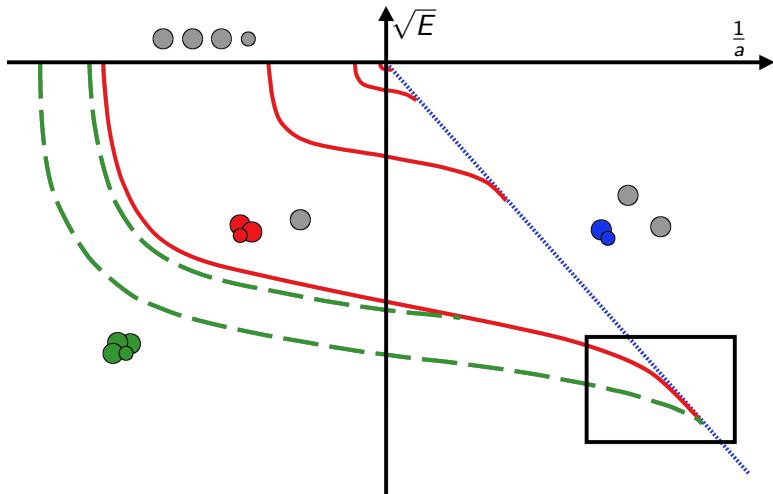
- As with identical bosons, there is typically a ground and an excited state at unitarity

Blume, Yan *Phys. Rev. Lett.* 113 (2014),213201

# Efimov Effect in the Heteronuclear Four-Body System



# Efimov Effect in the Heteronuclear Four-Body System



# Gaussian Expansion Method

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Implementation by Hiyama, Kino, Kamimura *PPNP* 51 (2003),223

- Rayleigh-Ritz Variational Method
- Base functions are selected via geometric progression between a minimum and a maximum range
- Number of parameters used:

System	Parameters
Dimer	3
Trimer	18
Tetramer	45
- Parameter space increases rapidly
- Used a mixture of systematic scanning of parameter subspaces and random sampling within relatively broad boundaries to find optimized base functions

# Interaction

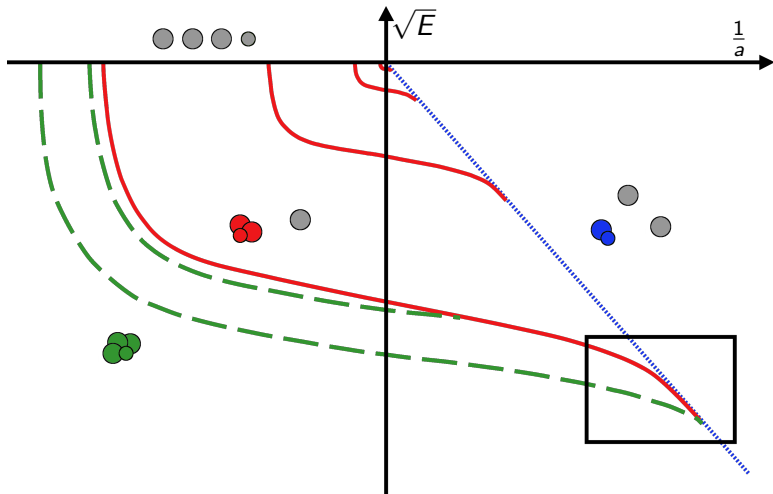
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- Use **effective** potentials for good performance, stable behaviour and easy parameter choice
- Combination of **attractive** 2-body and **repulsive** 3-body potential

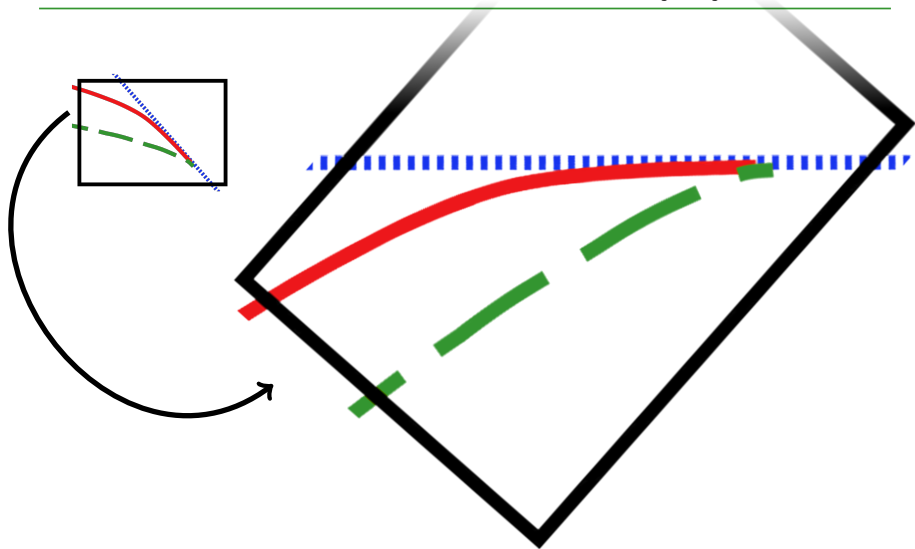
$$V_{iN} = V_0 \exp\left(-\frac{r_{iN}^2}{2r_0^2}\right), \quad W_{ijN} = W_0 \exp\left(-\frac{r_{ij}^2 + r_{jN}^2 + r_{iN}^2}{16r_0^2}\right)$$

- $N - 1$  atoms of mass  $M$ ,  $N$ th atom of mass  $m$
- Natural energy scale of the problem:  $E_s = \frac{1}{2r_0^2} \frac{m+M}{Mm}$
- Approximation valid, if binding Energies  $|E| \ll E_s$

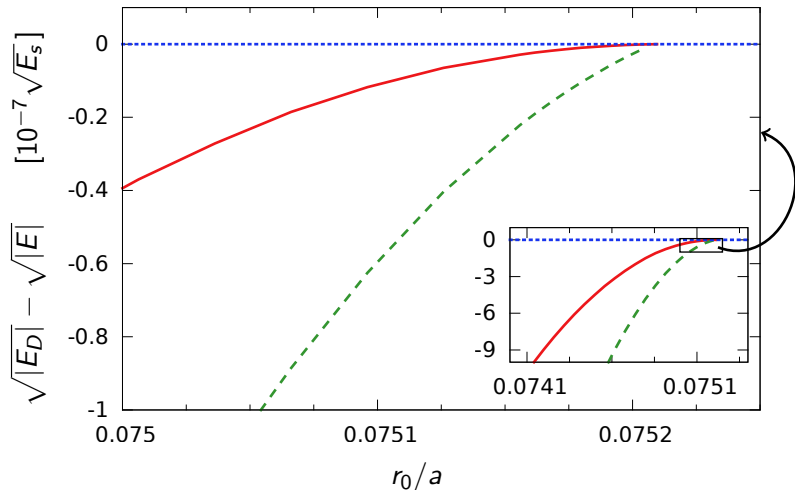
# Efimov Plot for Heteronuclear Four-body System



# Efimov Plot for Heteronuclear Four-body System

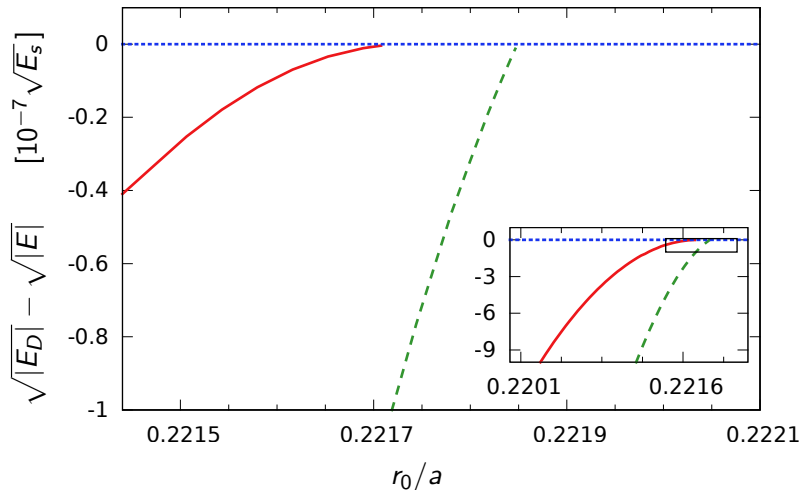


# ${}^7\text{Li}$ - ${}^6\text{Li}$ Mixture

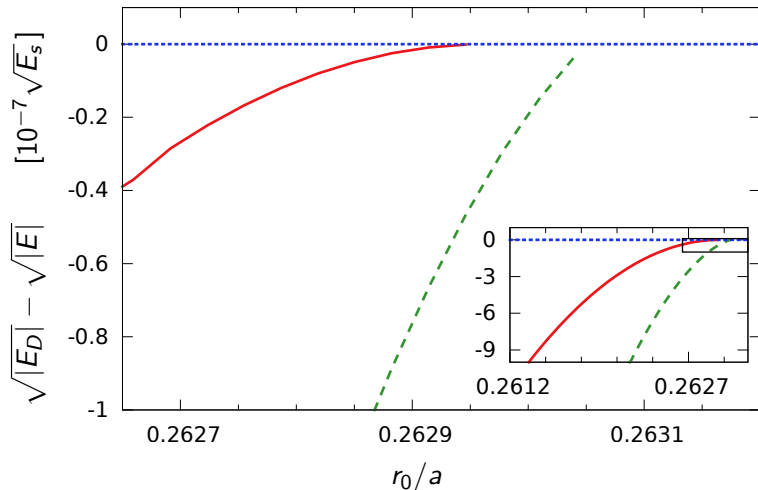




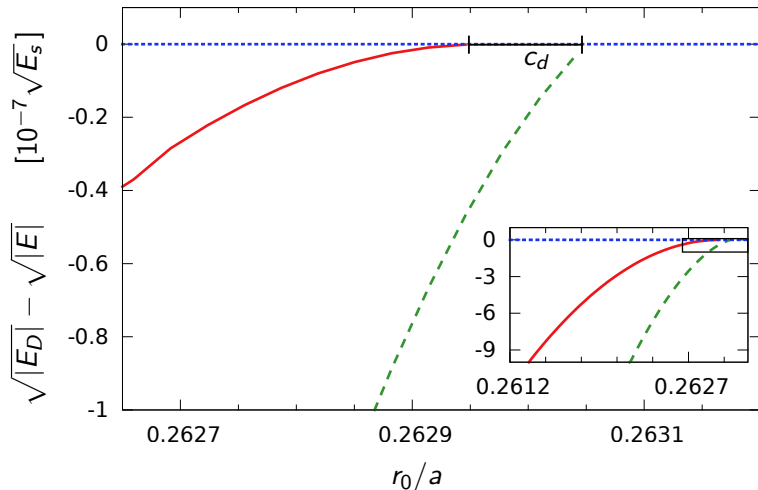
# $^{87}\text{Rb}$ - $^7\text{Li}$ Mixture



# $^{133}\text{Cs}$ - $^6\text{Li}$ Mixture

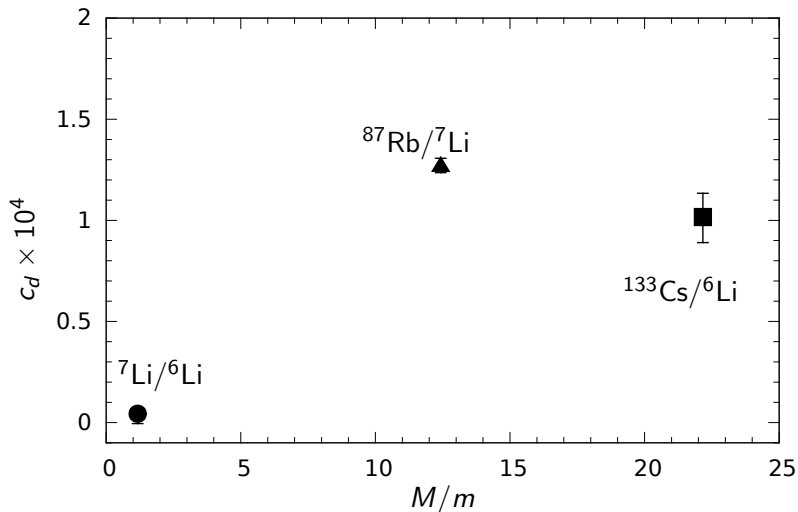


# $^{133}\text{Cs}$ - $^6\text{Li}$ Mixture



## Relative Crossing Point Positions

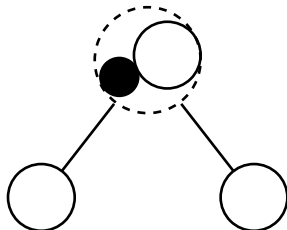
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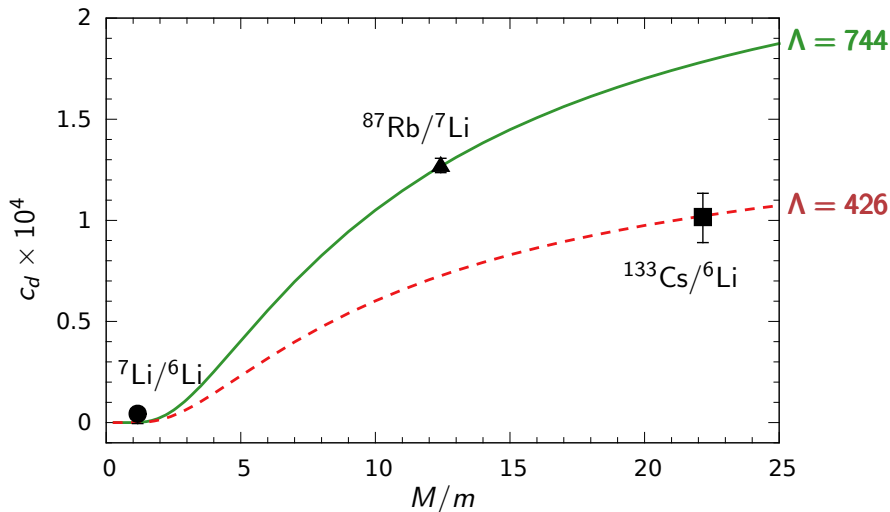
## Cross-checking with Effective STM Treatment

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- Before vanishing through the dimer threshold, the trimer becomes very weakly bound
- Scattering length can be approximated by inverse binding energy
- Close to threshold, the dimer-atom **scattering length** should be **large** ( $a_{Da}/r_0 \approx 10^7$ )
- Solve STM-equation for effective 3-body system (dimer-atom-atom)
- Tune 3-body parameter  $\Lambda$  to reproduce results



## Relative Crossing Point Positions II



## Summary and Outlook

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- First investigation into behaviour of heteronuclear trimer and tetramer at dimer threshold
- Trimer and tetramer cross into the dimer at almost the same point
- Results for crossing difference seem inconsistent  
→ more data points are needed
- Next steps: investigate dependence on potential shape and 3-body potential strength