IN THE NAME OF GOD

A New Method for Calculating the Baryons Mass under the Phenomenological Interaction Potential

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Outline of the peresentation

- The hypercentral Constituent Quark Model (hCQM).

- Exact Analytical Solution of the Schrödinger Equation for Killingbeck and Isotonic Oscillator Potential

- The Gürsey Radicati Mass Formula and Generalized GR Mass Formula

- Calculating the Masses of Nonstrange Baryons Resonances

The hypercentral Constituent Quark Model

❖ Nonstrange baryon resonances with u and d quarks can be classified using the non-relativistic quark model. The Constituent Quark Models (CQMs) have been recently widely applied to the description of baryon properties and most attention has been devoted to the spectrum [1-7].

❖ We consider baryons as bound states of three quarks. After removing the center of mass coordinate R, the internal quark motion is described by the Jacobi coordinates :

$$\vec{\rho} = \frac{1}{\sqrt{2}}(\vec{r}_1 - \vec{r}_2) \qquad \vec{\lambda} = \frac{1}{\sqrt{6}}(\vec{r}_1 + \vec{r}_2 - 2\vec{r}_3) \qquad (1)$$

Such that:

$$m_\rho = \frac{2m_1 m_2}{m_1 + m_2} \qquad m_\lambda = \frac{3m_3(m_1 + m_2)}{2(m_1 + m_2 + m_3)} \qquad (2)$$

Here m_1, m_2 and m_3 are the constituent quark masses.

The hypercentral Constituent Quark Model

❖ In order to describe three - quark dynamics, it is convenient to introduce the hyperspherical coordinates, which are obtained by replacing the absolute values ρ and λ by:

$$r = \sqrt{\rho^2 + \lambda^2} \qquad \xi = \arctan\left(\frac{\rho}{\lambda}\right) \tag{3}$$

where r is the hyperradius and ξ is the hyperangle.

In our model the interaction potential is assumed as:

$$V(r) = \underbrace{ar^2 + br + \frac{c}{r}}_{\text{Killingbeck potential}} + \underbrace{\frac{d}{r^2} + \frac{hr}{r^2+1} + \frac{kr^2}{(r^2+1)^2}}_{\text{Isotonic oscillator potentials}}, \tag{4}$$

Solution of the Schrödinger Equation for the Phenomenological Interaction Potential

- First we will solve the Schrödinger equation with this potential exactly by means of the ansatz method, and give the closed-form expressions for the energies then by using the generalized GR mass formula we can try to find the baryons mass. For hypercentral potentials, the Schrödinger equation, in the hyperspherical coordinates, is simply reduced to a single hyperradial equation, while the angular and hyperangular parts of the 3q-states are the known hyperspherical harmonics [8].
- Therefore the Hamiltonian will be:

$$H = \frac{-\hbar^2}{2\mu}\nabla^2 + V(r), \qquad (5)$$

The Schrödinger equation for a system containing three quarks with a potential $V(r)$ and by considering of $\psi_{\nu\gamma} = u_{\nu\gamma}(r)r^{-5/2}$ can be written as:

$$\frac{d^2u_{\nu\gamma}(r)}{dr^2} + 2\mu\left[E - V(r) - \frac{(2\gamma+5)(2\gamma+3)}{8\mu r^2}\right]u_{\nu\gamma}(r) = 0, \qquad (6)$$

where γ is the grand angular quantum number, μ is the reduced mass and $u_{\nu\gamma}(r)$ is the hyperradial wave function.

By putting Eq. (4) in Eq. (6) we obtain the following equation:

$$\frac{d^2u_{\nu\gamma}(r)}{dr^2}=-2\mu\left[E-ar^2-br-\frac{c}{r}-\frac{d}{r^2}-\frac{hr}{r^2+1}-\frac{kr^2}{(r^2+1)^2}-\frac{(2\gamma+5)(2\gamma+3)}{8\mu r^2}\right]u_{\nu\gamma}(r), \quad (7)$$

by regarding $\frac{r^2}{(r^2+1)^2}=\frac{1}{r^2+1}-\frac{1}{(r^2+1)^2}$ we have

$$\frac{d^2u_{\nu\gamma}(r)}{dr^2}=[-2\mu E+2\mu ar^2+2\mu br+2\mu\frac{c}{r}+2\mu\frac{d}{r^2}+2\mu\frac{hr}{r^2+1}+2\mu\frac{k}{r^2+1}$$
$$-2\mu\frac{k}{(r^2+1)^2}+\frac{(2\gamma+5)(2\gamma+3)}{4r^2}]u_{\nu\gamma}(r), \quad (8)$$

We suppose the following form for the wave function:

$$u_{\nu\gamma}(r)=g(r)\exp(f(r)). \quad (9)$$

Solution of the Schrödinger Equation for the Phenomenological Interaction Potential

Now for the functions $f(r)$ and $g(r)$ we make use of the ansatz [9,10]:

$$g(r)=\begin{cases}1 & \nu=0\\ \prod_i^{\nu}\left(r-\alpha_i^{\nu}\right) & \nu\geq 1\end{cases} \qquad (10)$$

$$f(r)=\alpha r^2+\beta r+\lambda \ln r+\eta \ln(r^2+1) \quad , \quad \alpha>0,$$

From Eq. (9) we obtain:

$$u''_{\nu,\gamma}(r)=\left[f''(r)+f'^2(r)+\frac{2f(r)g'(r)+g''(r)}{g(r)}\right]u_{\nu,\gamma}(r), \qquad (11)$$

and from Eq. (10) we have:

$$f'^2(r)=4\alpha^2r^2+\beta^2+4\alpha\beta r+\frac{\lambda^2}{r^2}+\frac{4\eta^2}{r^2+1}-\frac{4\eta^2}{(r^2+1)^2}+\frac{4\lambda\eta}{r^2+1}+4\alpha\lambda+8\alpha\eta-\frac{8\alpha\eta}{r^2+1}+\frac{2\beta\lambda}{r}+4\beta\eta\frac{r}{r^2+1},$$

$$f''(r)=2\alpha-\frac{\lambda}{r^2}-\frac{2\eta}{r^2+1}+\frac{4\eta}{(r^2+1)^2}, \qquad (12)$$

By putting Eq. (12) in Eq. (11) we will have:

$$u''_{0,\gamma}(r)=[4\alpha^2r^2+\beta^2+4\alpha\lambda+8\alpha\eta+2\alpha+4\alpha\beta r+\frac{\lambda^2}{r^2}-\frac{\lambda}{r^2}+\frac{2\beta\lambda}{r}$$
$$+\frac{4\eta^2}{r^2+1}+\frac{4\lambda\eta}{r^2+1}-\frac{2\eta}{r^2+1}-\frac{8\alpha\eta}{r^2+1}-\frac{4\eta^2}{(r^2+1)^2}+\frac{4\eta}{(r^2+1)^2}+4\beta\eta\frac{r}{r^2+1}]u_{0,\gamma}(r), \qquad (13)$$

By Comparing Eqs. (8) and (13), we can obtain:

$$\lambda^2-\lambda-2\mu d-\frac{(2\gamma+5)(2\gamma+3)}{4}=0,\quad 4\alpha^2=2\mu a,\quad -4\eta^2+4\eta=-2\mu k,$$
$$4\beta\eta=2\mu h,\quad 4\eta^2+4\lambda\eta-8\alpha\eta-2\eta=2\mu k,\quad 4\alpha\beta=2\mu b,$$
$$2\beta\lambda=2\mu c,\quad \beta^2+4\alpha\lambda+8\alpha\eta+2\alpha=-2\mu E, \tag{14}$$

Equation (14) immediately yields:

$$\lambda=\frac{1+\sqrt{1+8\mu d+(2\gamma+5)(2\gamma+3)}}{2},\quad \alpha=-\sqrt{\frac{\mu a}{2}},\quad \eta=\frac{1+\sqrt{1+2\mu k}}{2},$$
$$\beta=\frac{\mu h}{2\eta},\quad b=\frac{2\alpha\beta}{\mu},\quad c=\frac{\beta\lambda}{\mu}, \tag{15}$$

- The energy eigenvalues are given as follows:

$$E_{\nu\gamma}=-\frac{1}{2\mu}(\beta^2+4\alpha\lambda+8\alpha\eta+2\alpha), \tag{16}$$

Gürsey Radicati Mass Formula

- The description of the nonstrange baryons spectrum obtained by the hypercentral Constituent Quark Model (hCQM) is fairly good and comparable to the results of other approaches, but in some cases the splitting within the various $SU(6)$ multiplets are too low. The preceding results [11, 12, 13] show that both spin and isospin dependent terms in the quark Hamiltonian are important. Description of the splitting within the $SU(6)$ baryon multiplets is presented by the **Gürsey Radicati mass formula** [14]:

$$M = M_0 + CC_2[SU_S(2)] + DC_1[U_Y(1)] + E[C_2[SU_I(2)] - \frac{1}{4}(C_1[U_Y(1)])^2] \qquad (17)$$

- where M_0 is the average energy value of the $SU(6)$ multiplet, $C_2[SU_S(2)]$ and $C_2[SU_I(2)]$ are the $SU(2)$ (quadratic) Casimir operators for spin and isospin, respectively, and $C_1[UY(1)]$ is the Casimir for the $U(1)$ subgroup generated by the hypercharge Y.

Generalized Gürsey Radicati Mass Formula

- This mass formula has tested to be successful in the description of the ground state baryon masses, however, as stated by Gürsey and Radicati , it is not the most general mass formula that can be written on the basis of a broken $SU(6)$ symmetry. In order to generalize Eq. (17), Giannini and his collegues considered a dynamical spin- flavor symmetry $SU_{SF}(6)$ [15] and described the $SU_{SF}(6)$ symmetry breaking mechanism by generalizing Eq. (17) as:

$$M = M_0 + AC_2[SU_{SF}(6)] + BC_2[SU_F(3)] + CC_2[SU_S(2)] + DC_1[U_Y(1)] + E\left[C_2[SU_I(2)] - \frac{1}{4}(C_1[U_Y(1)])^2\right] \quad (18)$$

- In Eq. (18) the spin term represents spin-spin interactions, the flavor term denotes the flavor dependence of the interactions, and the $SU_{SF}(6)$ term depends on the permutation symmetry of the wave functions.

Generalized Gürsey Radicati Mass Formula

- The generalized Gürsey Radicati mass formula Eq. (18) can be used to describe the light baryons spectrum, provided that two conditions are fulfilled. The first condition is the feasibility of using the same splitting coefficients for different SU (6) multiplets. This seems actually to be the case, as shown by the algebraic approach to the baryon spectrum [1]. The second condition is given by the feasibility of getting reliable values for the unperturbed mass values M_0 [15].

- Therefore, the light baryons masses are obtained by three quark masses and the eigenenergies of the radial Schrödinger equation with the expectation values of H_{GR} as follows:

$$M = 3m + E_{\nu\gamma} + \langle H_{GR} \rangle \qquad (19)$$

- In order to simplify the solving procedure, the constituent quarks masses are assumed to be the same for up and down quark flavors . In previous section we determined eigenenergies by exact solution of the radial Schrödinger equation for the hypercentral Potential. The expectation values of H_{GR} , is completely identified by the expectation values of the Casimir operators [16]:

Calculating the Masses of Nonstrange Baryons Resonances

$$\langle C_2[SU_{SF}(6)]\rangle = \begin{cases} \frac{45}{4}\, for\, [56] \\ \frac{33}{4}\, for [70] \\ \frac{21}{4}\, for [20] \end{cases}$$

$$\langle C_2[SU_I(2)]\rangle = I(I+1)$$

$$\langle C_1[U_Y(1)]\rangle = Y \tag{20}$$

$$\langle C_2[SU_S(2)]\rangle = S(S+1)$$

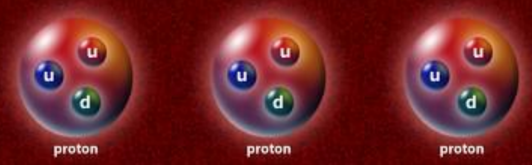
$$\langle C_2[\mathrm{SU}_F(3)]\rangle = \begin{cases} 3\ for\, [8] \\ 6\ for\, [10] \\ 0\ \ for\, [1] \end{cases}$$

For calculating the light baryons mass according to Eq. (19), we need to find the unknown parameters. For this purpose we choose a limited number of well-known light resonances and express their mass differences using H_{GR} and the Casimir operator expectation values:

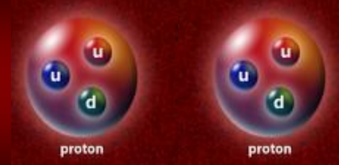
$$N(1650)S11 - N(1535)S11 = 3C$$

$$\Delta(1232)P33 - N(938)P11 = 9B + 3C + 3E \tag{21}$$

$$N(1535)S11 - N(1440)S11 = \left(E10 - E01\right) + 12A$$



Calculating the Masses of Nonstrange Baryons Resonances



- We found the C parameter from Eq. (21) and determined m, α, β, d and η and the three coefficients A, B and E of Eq. (20) in a simultaneous fit to the 3 and 4 star resonances of Table 2 which have been assigned as octet and decuplet states. The fitted parameters are reported in Table 1. The corresponding numerical values are given in Table 2. Comparison between our results and the experimental masses show that the light baryon spectra are, in general, fairly well reproduced.

Table 1

The fitted values of the parameters of the Eq. (19), obtained with resonances mass differences and global fit to the experimental resonance masses [17].

Parameter	A	B	C	E	m	α	β	d	η
Value	-18.23 MeV	-3.13 MeV	38.3	59.1 MeV	265 MeV	-0.21 MeV^2	0.573 MeV	0.401	0.51

Calculating the Masses of Nonstrange Baryons Resonances

Table 2

Mass spectrum of baryons resonances (in MeV) calculated with the mass formula Eq. (19). The column $M_{Our\ Calc}$ contains our calculations with the parameters of table 1.

- Comparison between our results and the experimental masses [17] show that the light baryon spectra are, in general, fairly well reproduced.

Baryon	Status	Mass(exp)[17]	State	$M_{Our\ Calc}$
N(938) P11	****	938	$^{2}8_{1/2}[56, 0^{+}]$	938.1
N(1440) P11	****	1420-1470	$^{2}8_{1/2}[56, 0^{+}]$	1447.9
N(1520) D13	****	1515-1525	$^{2}8_{3/2}[70, 1^{-}]$	1517.42
N(1535) S11	****	1525-1545	$^{2}8_{1/2}[70, 1^{-}]$	1517.42
N(1650) S11	****	1645-1670	$^{4}8_{1/2}[70, 1^{-}]$	1667.26
N(1675) D15	****	1670-1680	$^{4}8_{5/2}[70, 1^{-}]$	1667.26
N(1680) F15	***	1680-1690	$^{2}8_{5/2}[56, 2^{+}]$	1722.24
N(1700) D13	***	1650-1750	$^{4}8_{3/2}[70, 1^{-}]$	1667.26
N(1710) P11	***	1680-1740	$^{2}8_{1/2}[70, 0^{+}]$	1680.85
N(1720) P13	****	1700-1750	$^{2}8_{3/2}[56, 2^{+}]$	1722.24
N(2190) G17	****	2100-2200	$^{2}8_{7/2}[70, 3^{-}]$	2130.44
N(2220) H19	****	2200-2300	$^{2}8_{9/2}[56, 4^{+}]$	2258.21
N(2250) G19	****	2200-2350	$^{4}8_{9/2}[70, 3^{-}]$	2246.3
N(2600) I1,11	***	2550-2750	$^{2}8_{11/2}[70, 5^{-}]$	2574.34
Δ (1232) P33	****	1231-1233	$^{4}10_{3/2}[56, 0^{+}]$	1232.02
Δ (1600) P33	***	1550-1700	$^{4}10_{3/2}[56, 0^{+}]$	1590.13
Δ (1700) D33	****	1670-1750	$^{2}10_{3/2}[70, 1^{-}]$	1700.32
Δ (1905) F35	****	1865-1915	$^{4}10_{5/2}[56, 2^{+}]$	1892.51
Δ (1910) P31	****	1870-1920	$^{4}10_{1/2}[56, 2^{+}]$	1892.51
Δ (1920) P33	***	1900-1970	$^{4}10_{3/2}[56, 0^{+}]$	1942.26
Δ (1950) F37	****	1915-1950	$^{4}10_{7/2}[56, 2^{+}]$	1892.51
Δ (2420) H3, 11	****	2300-2500	$^{4}10_{11/2}[56, 4+]$	2464.45

Conclusion

- The overall good description of the spectrum which we obtain by this combination of potentials shows that our model can also be used to give a fair description of the energies of the excited multiplets up to three GeV and not only for the ground state octets and decuplets.

- Our model can also be used to give a fair description of the negative-parity resonance.

- Our model reproduces the position of the Roper resonances of the nucleon.

References

1. R. Bijker, F. Iachello and A. Leviatan, *Ann. Phys. (N.Y.)* **236**, 69 (1994).
2. M. Aiello, M. Ferraris, M.M. Giannini, M. Pizzo and E. Santopinto, *Phys. Lett.* **B387**, 215 (1996).
3. Z. Dziembowski, M. Fabre de la Ripelle and Gerald A. Miller, *Phys. Rev.* C53, **R2038** (1996).
4. M. Benmerrouche, N. C. Mukhopadhyay and J.-F. Zhang, *Phys. Rev. Lett.* **77**, 4716 (1996).
5. B. Chakrabarti, A. Bhattacharya, S. Mani, A. Sagari, *Acta Physica Polonica B*, Vol. 41, **No. 1** , 95-101(2010).
6. N. Isgur and G. Karl, *Phys. Rev.* **D20**, 1191 (1979).
7. M. M. Giannini, *Rep. Prog. Phys.* **54**, 453 (1991).
8. Ballot J and Fabre de la Ripelle M, *Ann. Phys., NY* **127** 62 (1980).

References

9. M. Znojil, *J. Math Phys.* **31**, (1990).

10. A. A. Rajabi and N. Salehi , *Iranian Journal of Physics Research*, 8(3), 169-175 (2008).

11. M. M. Giannini, E. Santopinto and A, Vassallo, *Eur. Phys. J.* **A12**, 447 (2001).

12. N. Salehi, A. A. Rajabi, *Modern Physics Letters A*, Vol. 24, No. **32**, 2631-2637 (2009).

13. H. Hassanabadi, A. A. Rajabi, Modern Physics Letters A, Vol. 24, Nos. 11-13, 1043-1046 (2009).

14. F. Gürsey and L.A. Radicati , *Phys. Rev. Lett.* **13**, 173 (1964).

15. M. M. Giannini, E. Santopinto, and A.Vassallo, *Eur.Phys.J.* **A25,** 241-247(2005).

16. R. Bijker, M.M.Giannini and E.Santopinto, Eur.Phys.J. A22, 319 (2004).

17. K.A. Olive et al. (Particle Data Group), Chin. Phys. C38, 090001 (2014).

Thanks

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