Three-body calculation of the *1s* level shift in kaonic deuterium with realistic multichannel $\bar{K}N$ interaction

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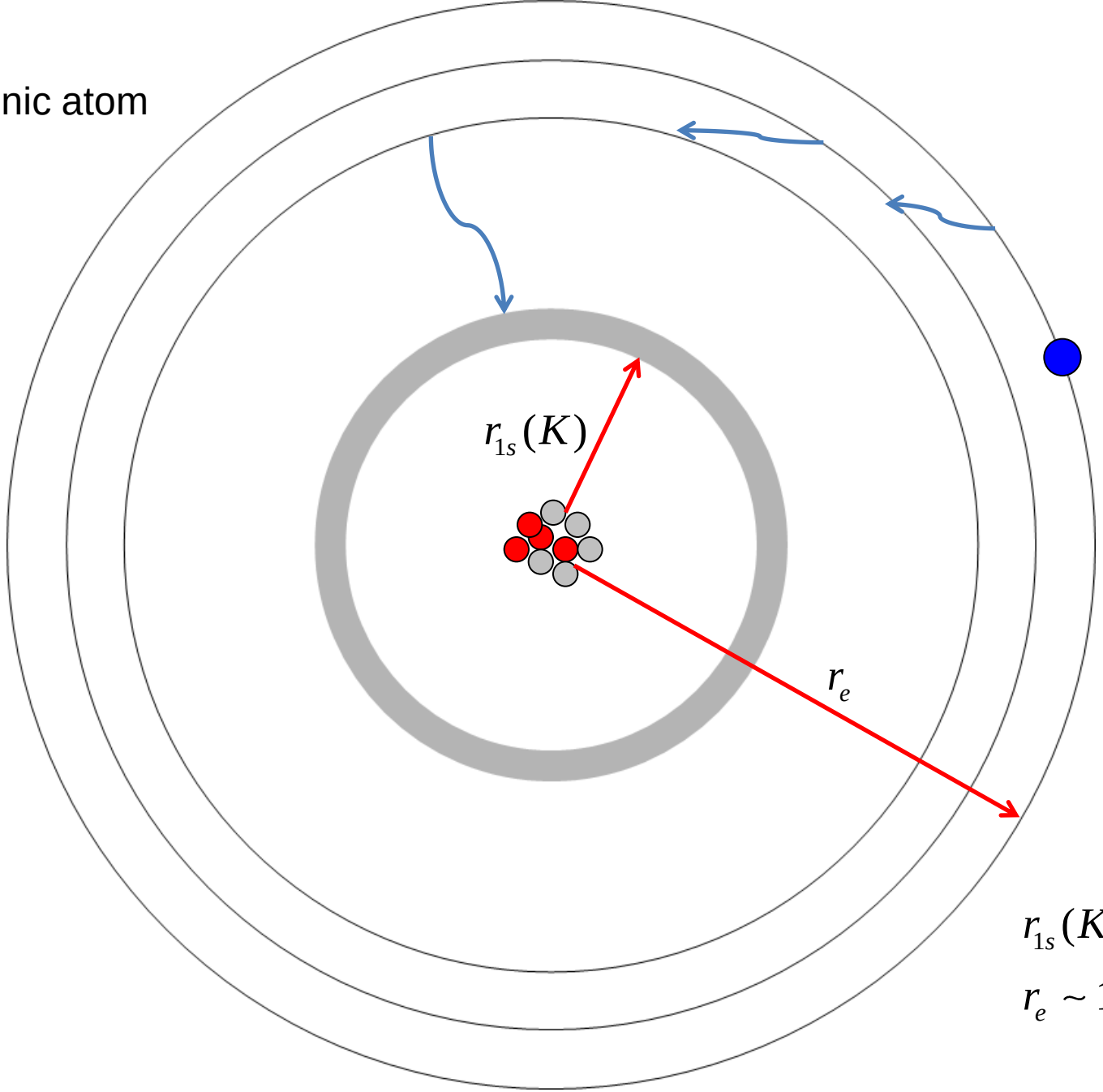
Hadronic atom



Negative hadron

$r_{1s}(Kd) \sim 70\ fm$

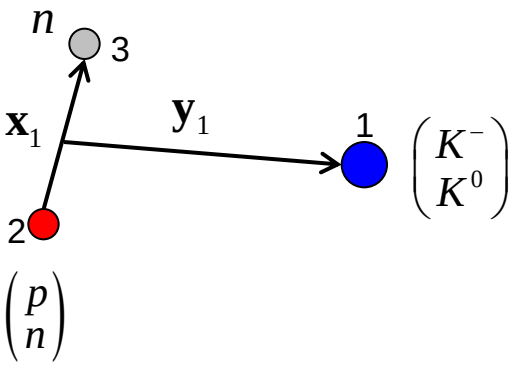
$r_e \sim 10^{-9}\ cm$

A.Deloff in „Fundamentals in hadronic atom theory”:

„.. the conventional picture of hadronic atoms (is) based on a two-body model Hamiltonian in which all strong interaction effects have been simulated by an absorptive potential representing the complicated interaction between the hadron and the nucleus...”

Well, it is not a two-body system

The simplest case to study deviation from two-body picture:
hadronic deuterium (kaonic in our case) :



3-body problem

Jacobi coordinates $(\mathbf{x}_i, \mathbf{y}_i),\ i = 1,2,3$

Hamiltonian $H = H_0 + v_1^s(x_1) + v_2^s(x_2) + v_3^s(x_3) - \frac{e^2}{x_3} P$

$$H_0 = -\frac{1}{2\mu_i}\Delta_{\mathbf{x}_i} - \frac{1}{2\mu_{i,jk}}\Delta_{\mathbf{y}_i} = h_0(x_i) + h_0(\mathrm{y}_i) = \ldots$$

with $v_3^s = \begin{pmatrix} v_{pK^-} & v_{pK^-,nK^0} \\ v_{nK^0,pK^-} & v_{nK^0} \end{pmatrix}$ and $P = \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix}$

Powerful methods to treat 3-body problem:

(a) - Faddeev equations
(b) - variational methods (w.f. expansion in coordinate space)

But:

(a) - everlasting problem with the long range Coulomb force (especially attractive)
(b) - two very different – and relevant - distance scales in hadronic atoms

Some years ago Z. Papp proposed a method for simultaneous treatment of short range and Coulomb forces in 3-body problem.

Basic idea: transform the Faddeev integral equations into matrix equations using a special discrete and complete set of the Coulomb Sturmian functions as a basis.

Successfully applied for short range + repulsive Coulomb forces (nuclear case) and purely Coulomb systems with attraction and repulsion.

Coulomb Sturmian (CS) functions

$$\langle \mathbf{r} \mid nlm \rangle = \langle \mathbf{r} \mid \mu \rangle = N_{nl} r^l e^{-br} L_n^{2l+1}(2br) Y_{lm}(\hat{\mathbf{r}})$$

b - range parameter

$$\langle \mu \mid \frac{1}{r} \mid \mu' \rangle = \delta_{\mu\mu'} \qquad \langle \mathbf{r} \mid \tilde{\mu} \rangle = \langle \mathbf{r} \mid \frac{1}{r} \mid \mu \rangle \qquad \langle \mu \mid \tilde{\mu}' \rangle = \langle \tilde{\mu} \mid \mu' \rangle = \delta_{\mu\mu'}$$

biorthogonal set

discrete and complete

$$\sum_{\mu=0}^{\infty} |\mu\rangle\langle\tilde{\mu}| = \sum_{i=0}^{\infty} |\tilde{\mu}\rangle\langle\mu| = \hat{1} \approx \sum_{i=0}^{N} |\mu\rangle\langle\tilde{\mu}| \qquad \mu = nlm$$

The most remarkable feature of the CS basis: the matrices of $(z - h_0)$ and $(z - h_c)$ are tridiagonal. This allows to set up an infinite tridiagonal set of equations for the matrix elements of $g_0(z) = (z - h_0)^{-1}$ and $g_c(z) = (z - h_c)^{-1}$, which can be solved exactly for any z.

Unfortunately, due to the lack of time, I have to rush through the formalism. For those, who are interested, the details can be found in arXiv 1608.01802

Appeared today !!

Faddeev equations for our system

Assuming that particles 1,2 and 3 are distinguishable, we have 3 coupled particle channels: $(K^-, n_1, p_2), (K^-, p_1, n_2), (K^0, n_1, n_2)$ and correspondingly a column wave function Ψ ,which is then separated into the ususal Faddeev components:

$$\Psi = \begin{pmatrix} \Psi^{K^- n_1 p_2} \\ \Psi^{K^- p_1 n_2} \\ \Psi^{K^0 n_1 n_2} \end{pmatrix} = \begin{pmatrix} \Psi_1^{K^- n_1 p_2} \\ \Psi_1^{K^- p_1 n_2} \\ \Psi_1^{K^0 n_1 n_2} \end{pmatrix} + \begin{pmatrix} \Psi_2^{K^- n_1 p_2} \\ \Psi_2^{K^- p_1 n_2} \\ \Psi_2^{K^0 n_1 n_2} \end{pmatrix} + \begin{pmatrix} \Psi_3^{K^- n_1 p_2} \\ \Psi_3^{K^- p_1 n_2} \\ \Psi_3^{K^0 n_1 n_2} \end{pmatrix}$$

Using particle representation of the interactions we get coupled Faddeev equations for the 9 unknown functions. Symmetrization with respect to barion labels 1,2 simplifies the system: symmetric and antisymmetric combinations are decoupled. Since the deuteron is antisymmetric in these indices $(I=0)$, $\Psi_1^{K^0 n_1 n_2}$ disappears from the equations and we have to keep the 4 antisymmetric combinations (4, not 3, due to the dual nature of particle pair 3).

For them the Noble form of homogeneous Faddeev equations (Coulomb interaction is added to H_0) reads:

$$\Psi_{np}(x_1,y_1) = G_{np}(x_1,y_1;E)\,v_{np}(x_1)\left(\Psi_{K^-n}(x_2,y_2)+\Psi_{K^-p}(x_3,y_3)\right)$$

$$\Psi_{K^-n}(x_2,y_2) = G_{K^-n}(x_2,y_2;E)\,v_{K^-n}(x_2)\left(\Psi_{np}(x_1,y_1)+\Psi_{K^-p}(x_3,y_3)\right)$$

$$\begin{pmatrix}\Psi_{K^-p}(x_3,y_3)\\ \Psi_{K^0n}(x_3,y_3)\end{pmatrix} = G_3(x_3,y_3;E)\,v_3(x_3)\begin{pmatrix}\Psi_{np}(x_1,y_1)+\Psi_{K^-n}(x_2,y_2)\\ -\Psi_{K^0n}(x_2,y_2)\end{pmatrix}$$

with

$$G_{np}(x_1, y_1; E) = \left(E - h_0(x_1) - h_0(y_1) - v_{np}(x_1) + e^2 \left| -\frac{1}{2} x_1 + y_1 \right|^{-1} \right)^{-1}$$

$$G_{K^- n}(x_2, y_2; E) = \left(E - h_0(x_2) - h_0(y_2) - v_{K^- n}(x_2) + e^2 \left| \frac{m_N}{m_N + m_K} x_2 + y_2 \right|^{-1} \right)^{-1}$$

while $G_3(x_3, y_3; E)$ and $v_3(x_3)$ are 2x2 matrices:

$$v_3(x_3) = \begin{pmatrix} v_{pK^-}(x_3) & v_{pK^-, nK^0}(x_3) \\ v_{nK^0, pK^-}(x_3) & v_{nK^0}(x_3) \end{pmatrix}$$

$$G_3(x_3, y_3; E) = \left(\begin{pmatrix} E - h_0(x_3) - h_0(y_3) + e^2 / x_3 & 0 \\ 0 & E - h_0(x_3) - h_0(y_3) \end{pmatrix} - v_3(x_3) \right)^{-1}$$

The Coulomb interaction is the same in each Green's function, expressed in different Jacobi coordinates

Introducing a double CS basis for each set of Jacobi coordinates

$$\langle x_i y_i \mid \mu \rangle_i = \langle x_i \mid \mu_x \rangle \langle y_i \mid \mu_y \rangle; \;\; \mu = (\mu_x, \mu_y)$$

the unknown functions Ψ_i for $i = np, K^- n, K^- p, K^0 n$ can be expanded on this basis:

$$\Psi_i(x_i, y_i) = \sum_{\mu}^{N_i} \langle x_i y_i \mid \mu \rangle_i \, X^i_\mu; \;\; X^i_\mu = {}_i\langle \tilde{\mu} \mid \Psi_i(x_i, y_i) \rangle$$

Before writing down the matrix equations for the X^i_μ two intermediate steps are needed.

(1) When operators expressed in one set of Jacobi coordinates act on functions, depending on another set, a characteristic feature of Faddeev equations, we have to insert a transformation matrix:

$$\hat{O}(x_i, y_i)\Psi_j(x_j, y_j) \Rightarrow \sum_{\mu', \mu''} {}_i\langle \mu | \hat{O}(x_i, y_i) | \mu' \rangle_i \, M^{(ij)}_{\mu' \mu''} \, X^{j}_{\mu''}$$

where $M^{(ij)}$ is the overlap matrix of the two CS basis sets, depending on different Jacobi coordinates:

$$M^{(ij)}_{\mu' \mu''} = {}_i\langle \mu' | \mu'' \rangle_j$$

They can be calculated numerically.

(2) When calculating the matrix elements of the Green-operators, two cases has to be distinguished. In $G_3(x_3, y_3)$ the Coulomb interaction depends on its "native" relative coordinate x_3 thus it corresponds to a Green-operator of two non-interacting subsystems, sharing a common total energy. For this case, a calculation scheme exists. In the other two Green-operators the Coulomb interaction depends on both "native" coordinates and in order to reduce them to a calculable form, we have to split the Coulomb interaction into "channel" and "polarization" parts:

$$\frac{e^2}{|sx_i + y_i|} = V^{ch}(y_i) + U_i(x_i, y_i) \; ; \; (s \text{ is a mass-coefficient})$$

with

$$V^{ch}(y_i) = \frac{e^2}{y_i}; \; U_i(x_i, y_i) = \frac{e^2}{|sx_i + y_i|} - \frac{e^2}{y_i}$$

$V^{ch}(y_i)$ - Coulomb interaction of the spectator with the c.m. of the interacting pair (no influence on internal motion)

$U_i(x_i, y_i)$ - responsible for the distortion of the internal motion of the pair due to displacement of the Coulomb interaction from the charged particle to the c.m.

The Green-operators of the first two Faddeev equations satisfy the resolvent equations

$$G_{np} = G_{np}^{ch} + G_{np}^{ch} U_{np} G_{np} \quad \text{and} \quad G_{K^-n} = G_{K^-n}^{ch} + G_{K^-n}^{ch} U_{K^-n} G_{K^-n}$$

with the channel Green operators

$$G_{np}^{ch}(x_1, y_1; E) = \left(E - h_0(x_1) - h_0(y_1) - v_{np}(x_1) + \frac{e^2}{y_1} \right)^{-1}$$

$$G_{K^-n}^{ch}(x_2, y_2; E) = \left(E - h_0(x_2) - h_0(y_2) - v_{K^-n}(x_2) + \frac{e^2}{y_2} \right)^{-1}$$

and the polarization potentials U_{np} and U_{K^-n}.

With their help the first two Faddeev equations can be rewritten as

$$\Psi_{np}(x_1, y_1) =$$

$$G^{ch}_{np}(x_1, y_1; E)[U_{np}(x_1, y_1)\Psi_{np}(x_1, y_1) + v_{np}(x_1)(\Psi_{K^-n}(x_2, y_2) + \Psi_{K^-p}(x_3, y_3))]$$

$$\Psi_{K^-n}(x_2, y_2) =$$

$$G^{ch}_{K^-n}(x_2, y_2; E)[U_{K^-n}(x_2, y_2)\Psi_{K^-n}(x_2, y_2) + v_{K^-n}(x_2)(\Psi_{np}(x_1, y_1) + \Psi_{K^-p}(x_3, y_3))]$$

Now all matrix elements are in calculable form and our final matrix equation reads $\mathbf{X} = \mathbf{A}(E)\mathbf{X}$ with

$$\mathbf{X} = \begin{pmatrix} \mathbf{X}_{np} \\ \mathbf{X}_{K^-n} \\ \mathbf{X}_{K^-p} \\ \mathbf{X}_{K^0n} \end{pmatrix} \; ; \; \mathbf{A}(E) =$$

$$\begin{pmatrix} \mathbf{G}^{ch}_{np}\mathbf{U}_{np} & \mathbf{G}^{ch}_{np}\mathbf{v}_{np}\mathbf{M}^{(12)} & \mathbf{G}^{ch}_{np}\mathbf{v}_{np}\mathbf{M}^{(13)} & 0 \\ \mathbf{G}^{ch}_{K^-n}\mathbf{v}_{K^-n}\mathbf{M}^{(21)} & \mathbf{G}^{ch}_{K^-n}\mathbf{U}_{K^-n} & \mathbf{G}^{ch}_{K^-n}\mathbf{v}_{K^-n}\mathbf{M}^{(23)} & 0 \\ (\mathbf{G}_3\mathbf{v}_3)_{K^-p}\mathbf{M}^{(31)} & (\mathbf{G}_3\mathbf{v}_3)_{K^-p}\mathbf{M}^{(32)} & 0 & -(\mathbf{G}_3\mathbf{v}_3)_{K^-p,K^0n}\mathbf{M}^{(32)} \\ (\mathbf{G}_3\mathbf{v}_3)_{K^0n,K^-p}\mathbf{M}^{(31)} & (\mathbf{G}_3\mathbf{v}_3)_{K^0n,K^-p}\mathbf{M}^{(32)} & 0 & -(\mathbf{G}_3\mathbf{v}_3)_{K^0n}\mathbf{M}^{(32)} \end{pmatrix} .$$

Bold face letters stand for the vectors and matrices in the corresponding Coulomb Sturmian basis. Our task is to find the (complex) solution E of the equation $Det(\mathbf{I} - \mathbf{A}(E)) = 0$ close to the unperturbed value $E_0 = E_d + \varepsilon_{1s}(K^-d)$.

The crucial point of the method is the calculation of the matrix elements of the Green-operators, all of which correspond now to those of non-interacting subsystems. They can be written in the form

$$G(x,y;E) = (E - h_0(x) - h_0(y) - u_1(x) - u_2(y))^{-1}$$

and for them the following convolution integral representation exists:

$$G(x,y;E) = \oint_c g_1(x;\varepsilon) g_2(y;E-\varepsilon)\, d\varepsilon$$

with $g_1(x;z) = (z - h_0(x) - u_1(x))^{-1}$ and $g_2(\mathrm{y};z) = (z - h_0(y) - u_2(y))^{-1}$.

The contour c in original formulation "encircles the spectrum of g_1 without penetrating the spectrum of g_2". For practical purposes this can be reformulated as "the path c divides the complex plane into two non-intersecting parts the singularities of g_1 being on its left side, while those of g_2 on its right side.

Obviously, the double-Sturmian matrix elements of G can be expressed in the same way through the matrix elements of g_1 and g_2, each in its own basis.

For our operators

$$G_{np}^{ch} \Rightarrow \; u_1(x) = v_{np}(x) \; ; \; u_2(y) = -\frac{e^2}{y}$$

$$G_{K^- n}^{ch} \Rightarrow \; u_1(x) = v_{K^- n}(x) \; ; \; u_2(y) = -\frac{e^2}{y}$$

$$G_3 \Rightarrow u_1(x) = v_3(x) + \begin{pmatrix} -e^2/x & 0 \\ 0 & 0 \end{pmatrix} \quad \text{(a matrix)} \; ; \; u_2(y) = 0$$

It has to be noted, that G_{np}^{ch} plays a special role: its lowest pole coincides with our “unperturbed” energy eigenvalue $E_0 = E_d + \varepsilon_{1s}(K^- d)$.

The choice of c depends on the particular physical problem

- for “ordinary” 3-body problem (real energy, bound or scattering states) on the physical sheet
- for 3-body problem with absorptive potentials (complex energy eigen-values) – still on the physical sheet. Our previous test calculation

(P. Doleschall, J. Révai and N. V. Shevchenko PLB **744** (2015) 105

was of this type

- search for resonance poles in scattering – complex eigenvalue on the non-physical sheet. The path must be continued to the non-physical sheet (several papers of Z. Papp et al.)
- interaction with several thresholds and cuts – complicated Riemann surface of at least one of the g_i -s. Our present case: the sought eigenvalue, a quasi bound state is on one of the unphysical sheets.
- the Coulomb Sturmian matrix elements strongly oscillate along the real axis; it is desirable to move the integration path into the complex plane as much as possible

After a brief outline of the general formalism, some details of the present calculation

$\bar{K}N$ interactions

- test calculation: simple one-term separable potentials with (constant) complex coupling strength to account for the absorption;
- present: 3 versions of realistic, multichannel potentials

SIDD1 - $\bar{K}N \leftrightarrow \pi\Sigma$ coupled channels, one-pole structure of the $\Lambda(1405)$

SIDD2 - $\bar{K}N \leftrightarrow \pi\Sigma$ coupled channels, two-pole structure of the $\Lambda(1405)$

Chiral - $\bar{K}N \leftrightarrow \pi\Sigma \leftrightarrow \pi\Lambda$ coupled channels, energy dependent coupling strength, channel coupling according to chiral perturbation theory

Separable potentials with simple Yamaguchi form-factors.
Reproduce all known $\bar{K}N$ data, including the $1s$ level shift of kaonic hydrogen.

np interactions

- simple one-term attractive separable potential, reproduces the deuteron binding energy and size
- two-term attractive plus repulsive separable potential, reproduces the deuteron and the $^3S_1$ phase shifts untill 300 MeV

All potentials were constructed and fitted to experimental data earlier and already used in Coulomb-less Faddeev calculations for the $\bar{K}NN$ system.

N. V. Shevchenko, Nucl.Phys. **A890-891**,50(2012)

N. V. Shevchenko and J. Révai, Phys.Rev. **C90**,034003(2014)

J. Révai and N. V. Shevchenko, Phys.Rev. **C90**,034004(2014)

Reduction to the $\bar{K}NN$ particle channel

"Exact optical potential" for a given channel – reproduces the elastic t-matrix of the multichannel interaction in that channel both on- and off- energy shell. For separable potentials its construction is straightforward, e.g. for a two-channel interaction:

$$\hat{V} = \begin{pmatrix} |g_{\bar{K}N}\rangle \lambda_{\bar{K}N} \langle g_{\bar{K}N}| & |g_{\bar{K}N}\rangle \lambda_{\bar{K}N,\pi\Sigma} \langle g_{\pi\Sigma}| \\ |g_{\pi\Sigma}\rangle \lambda_{\pi\Sigma,\bar{K}N} \langle g_{\bar{K}N}| & |g_{\pi\Sigma}\rangle \lambda_{\pi\Sigma} \langle g_{\pi\Sigma}| \end{pmatrix}$$

we have

$$\hat{V}^{opt}_{\bar{K}N} = |g_{\bar{K}N}\rangle \lambda^{opt}_{\bar{K}N}(E) \langle g_{\bar{K}N}|$$

$$\lambda^{opt}_{\bar{K}N}(E) = \lambda_{\bar{K}N} + \frac{\lambda^2_{\bar{K}N,\pi\Sigma} \langle g_{\pi\Sigma}|G^0_{\pi\Sigma}(E)|g_{\pi\Sigma}\rangle}{1 - \lambda_{\pi\Sigma}\langle g_{\pi\Sigma}|G^0_{\pi\Sigma}(E)|g_{\pi\Sigma}\rangle}$$

where $G^0_{\pi\Sigma}(E)$ is the free Green-operator in the "excluded" channel.
3-channel case – similar, somewhat more complicated. Green-operators, t-matrices calculated from $\hat{V}^{opt}_{\bar{K}N}$ carry the full analytical structure (poles, branch points and cuts) of the original multichannel interaction.

Integration path

Depends on the properties of the two subsystem Green-operators and on the location of the sought eigenvalue on the Riemann surface of the system. Our shifted energy eigenvalue E is somewhat below the $d + K^-$ threshold, on the physical sheet for the closed $(K^- np)$ channel, and on the closest unphysical sheet for the open $(\pi\Sigma n)$ (or $(\pi\Lambda n)$ in the chiral case) channels.

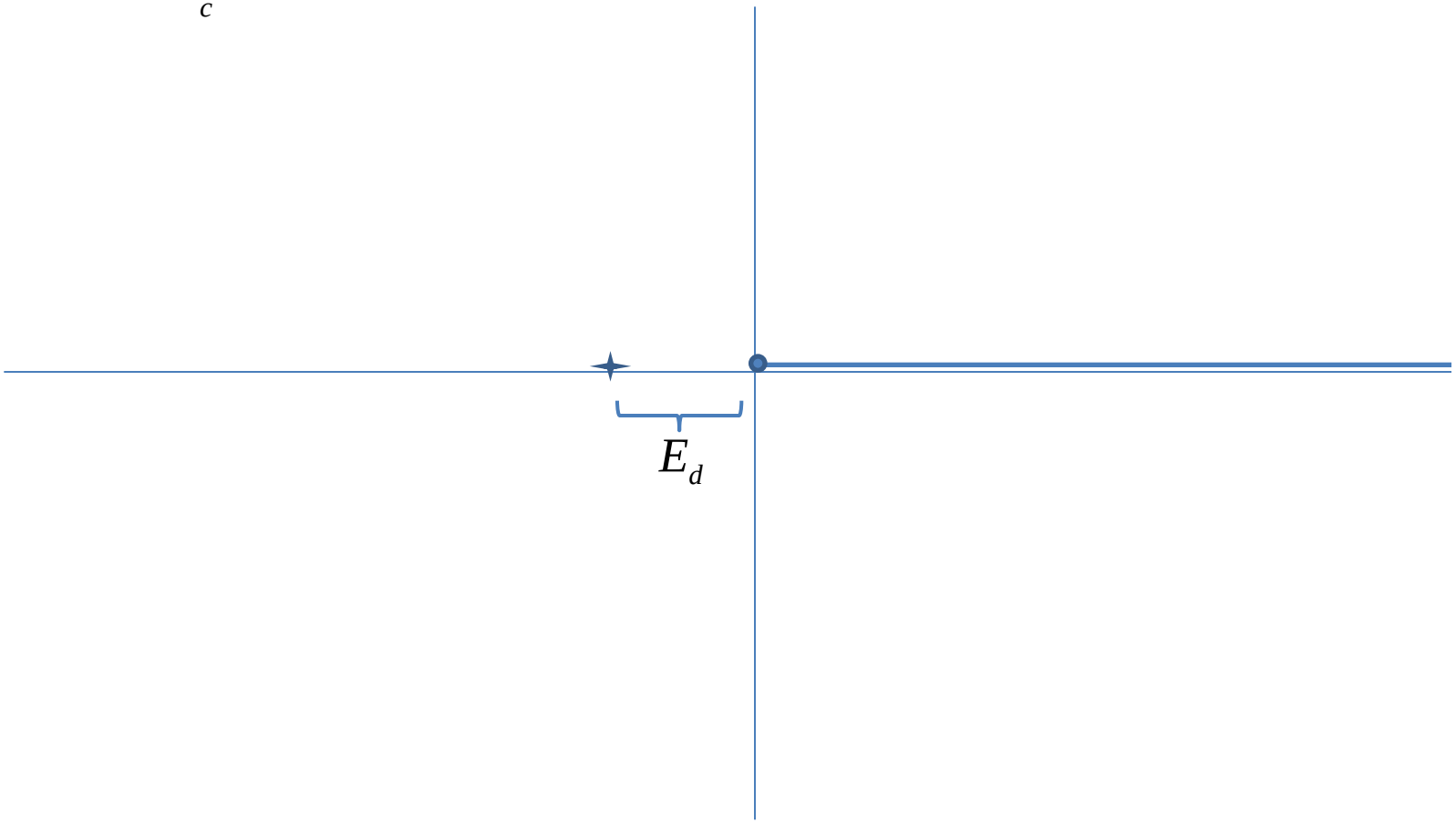
Two examples:

(a) channel $(K^- np)$, interacting pair (np) - physical sheet

(b) channel $(K^- np)$, interacting pair $(K^- n)$; in this case the $(K^- n)$ interaction – due to the "exact optical" construction – "remembers" the $(\pi\Sigma)$ cut, and with respect to it, the eigenvalue E is on the unphysical sheet.

$$G_{np}^{ch}(x_1, y_1; E) = \oint_c g_{np}(x_1; \varepsilon) g_c(y_1; E - \varepsilon)\, d\varepsilon$$

ε



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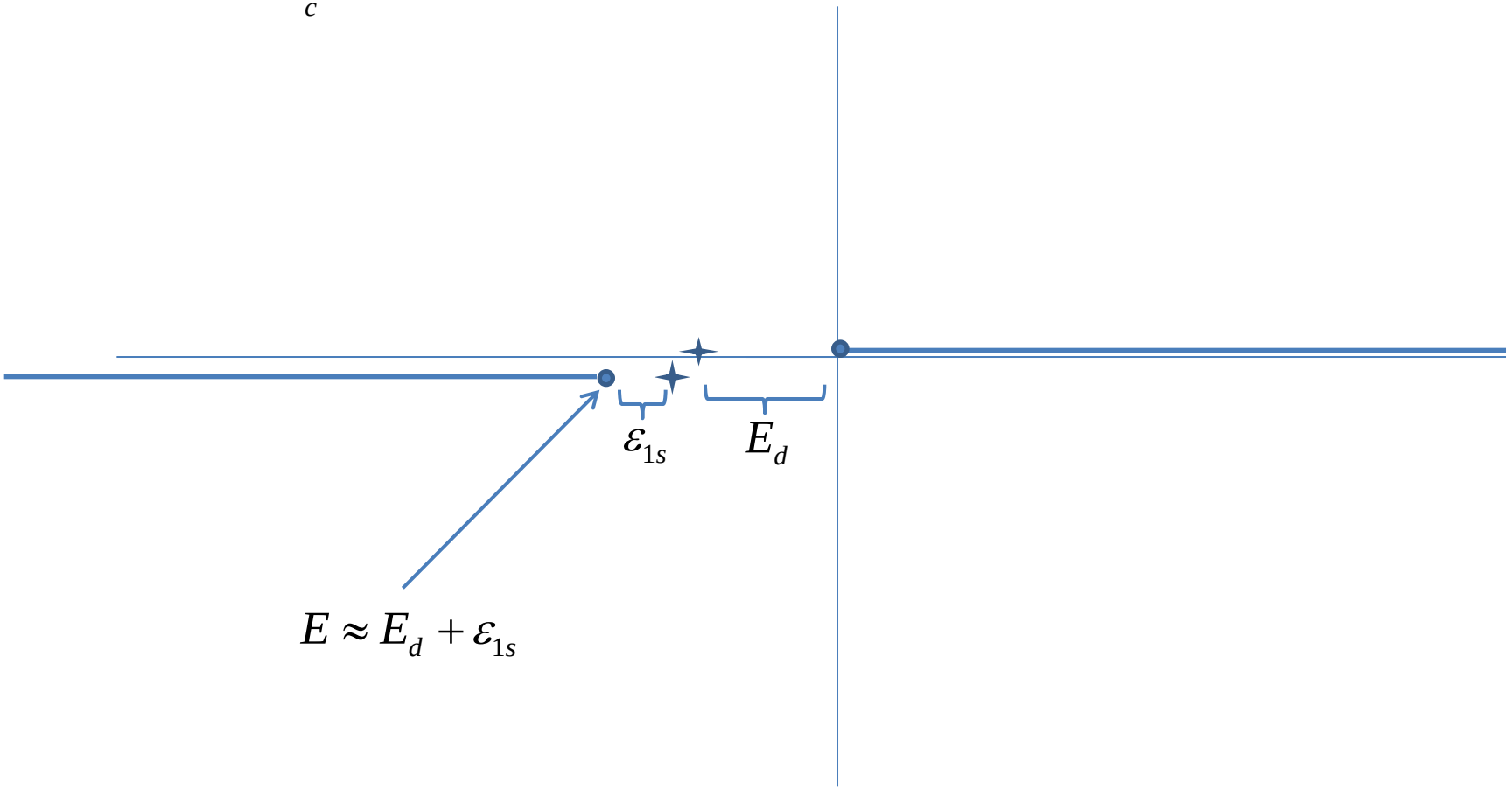

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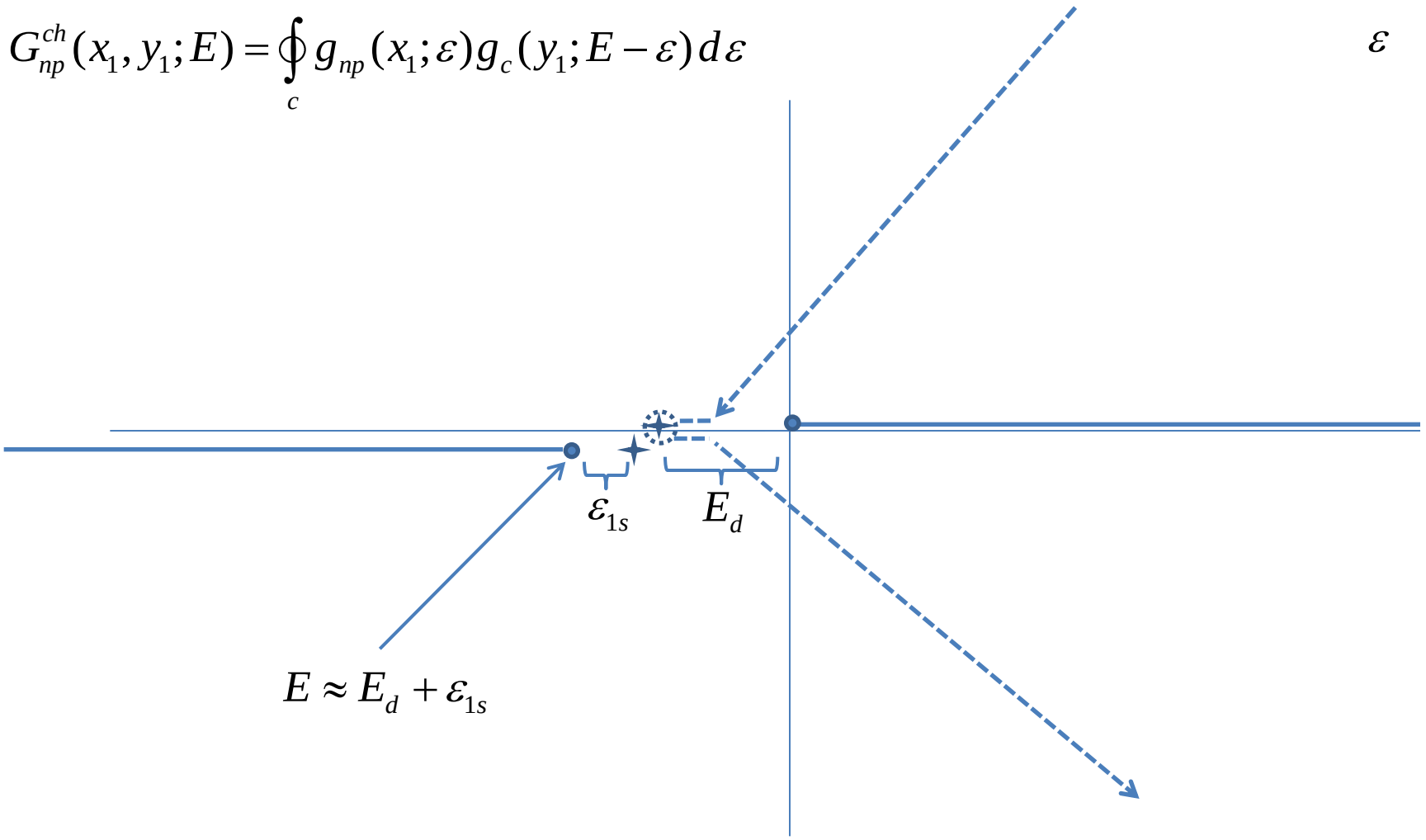
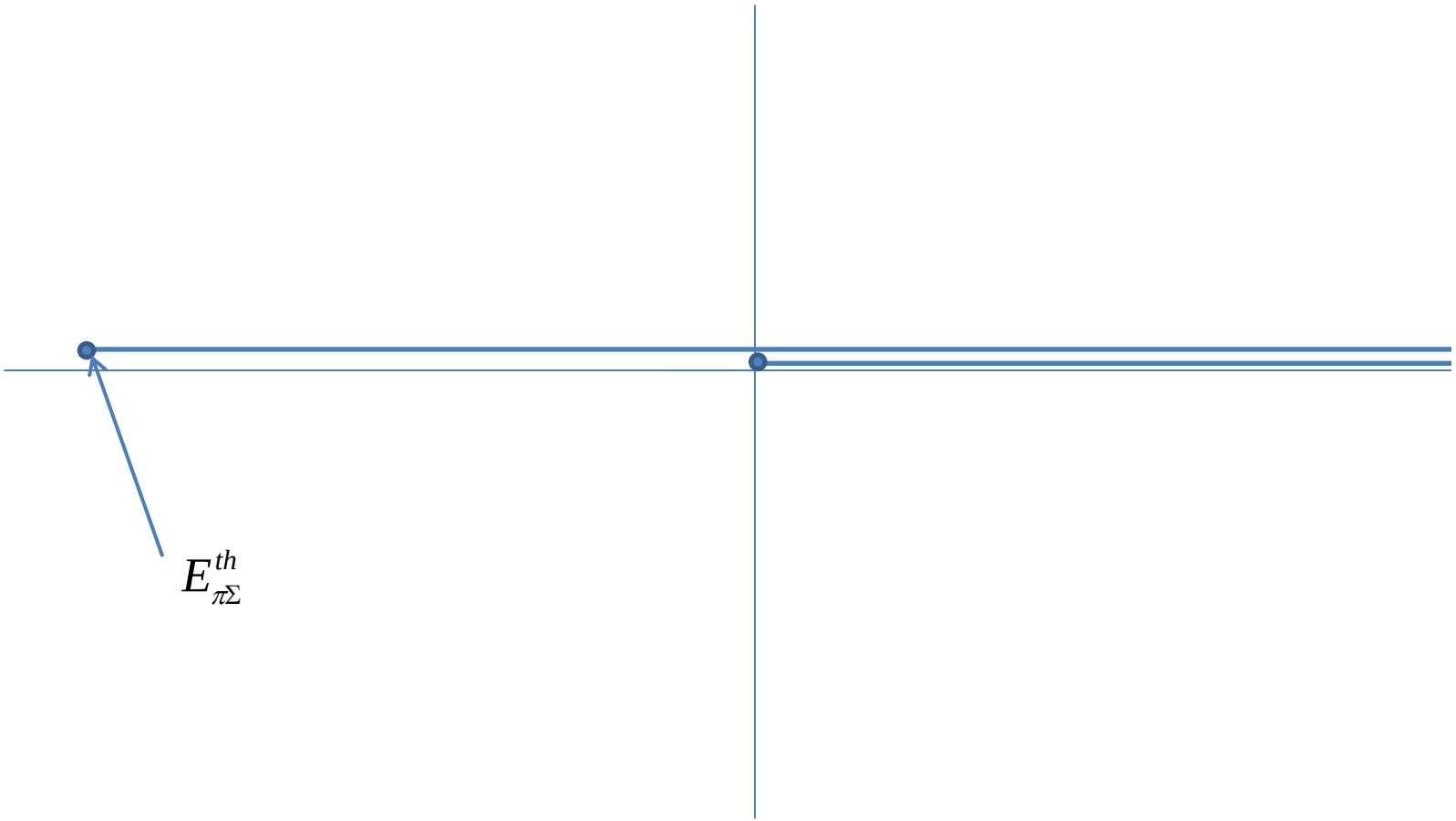
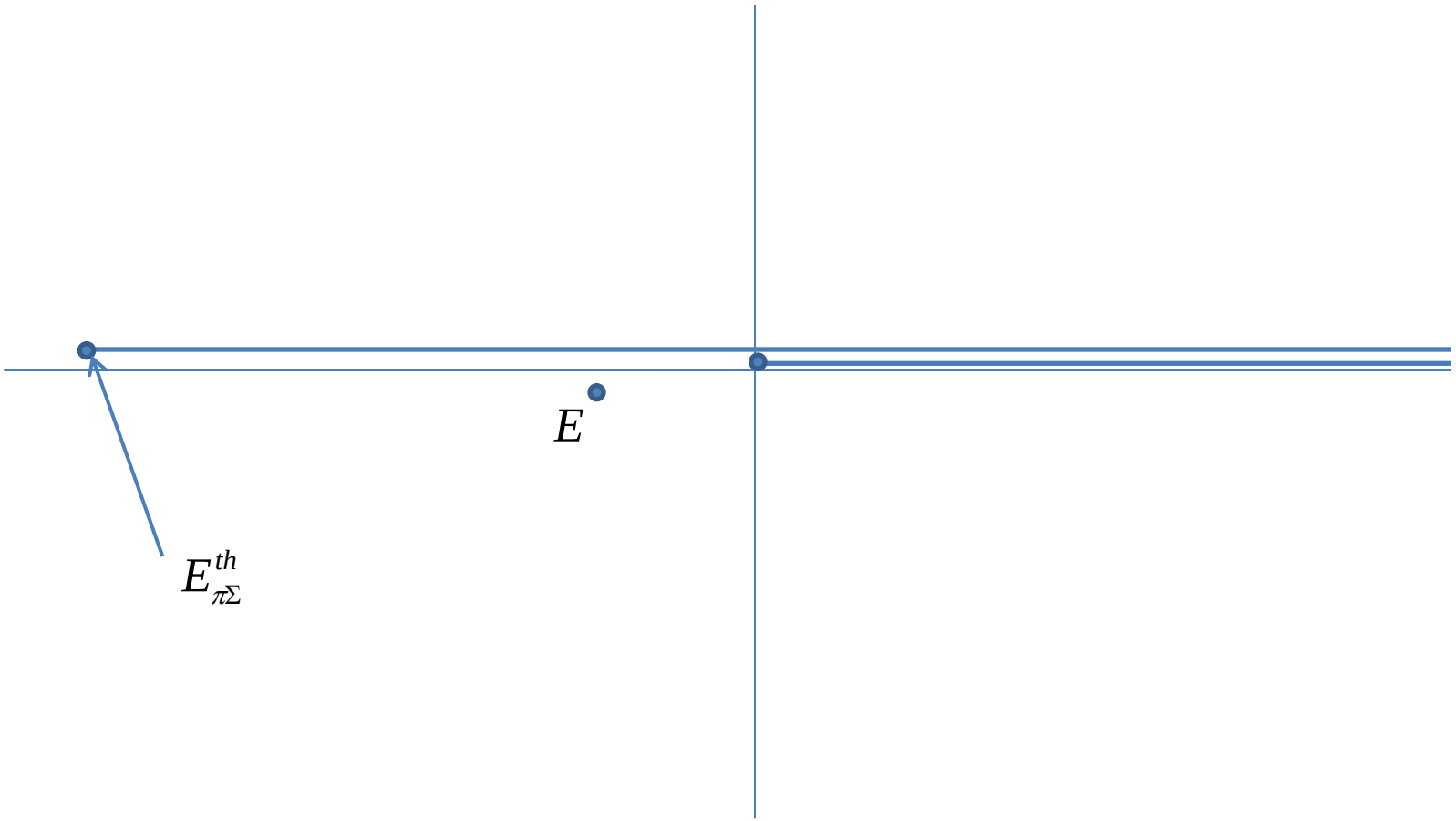




$$G_{K^-n}^{ch}(x_2, y_2; E) = \oint_c g_{K^-n}(x_2; \varepsilon) g_c(y_2; E - \varepsilon)\, d\varepsilon$$



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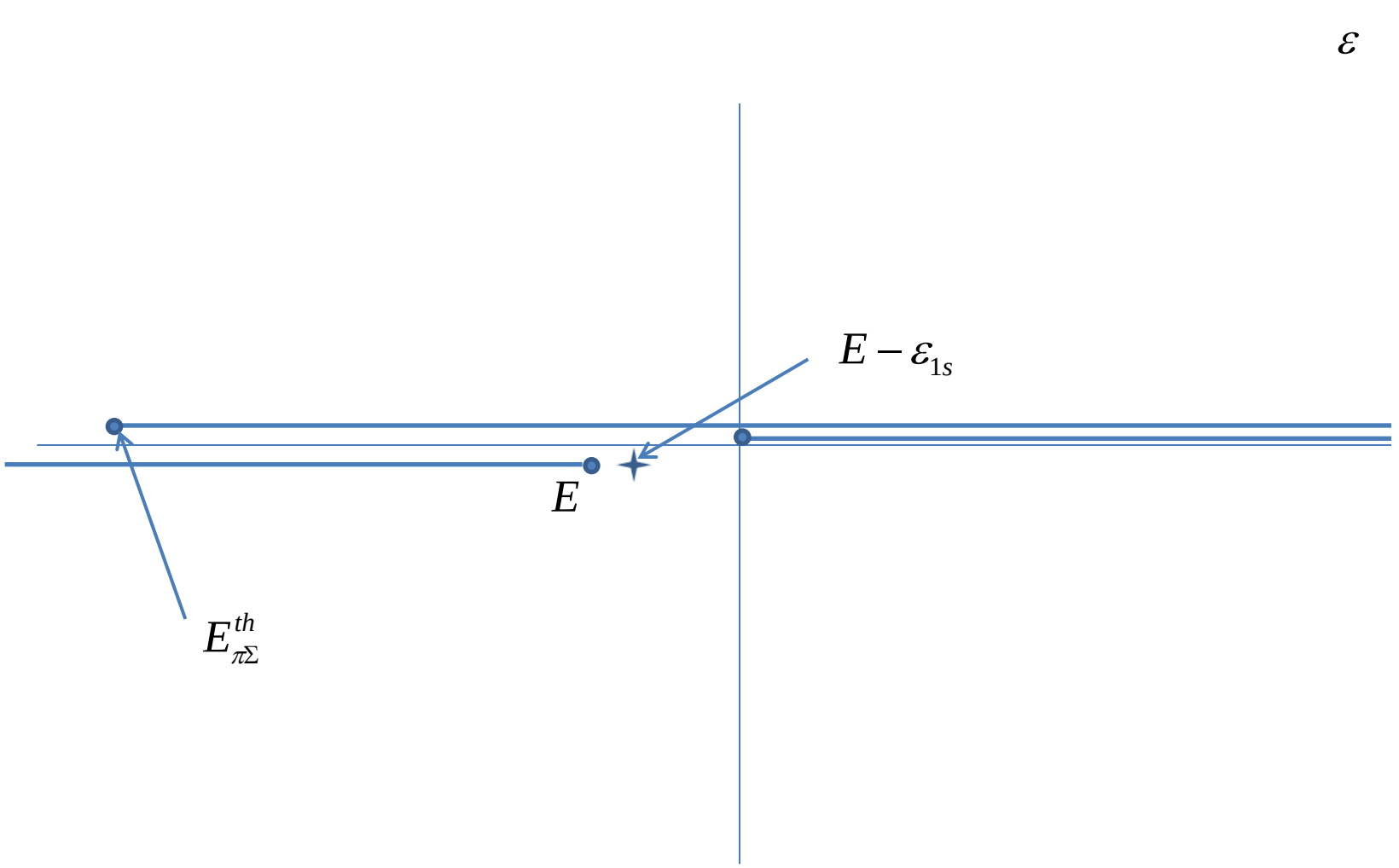
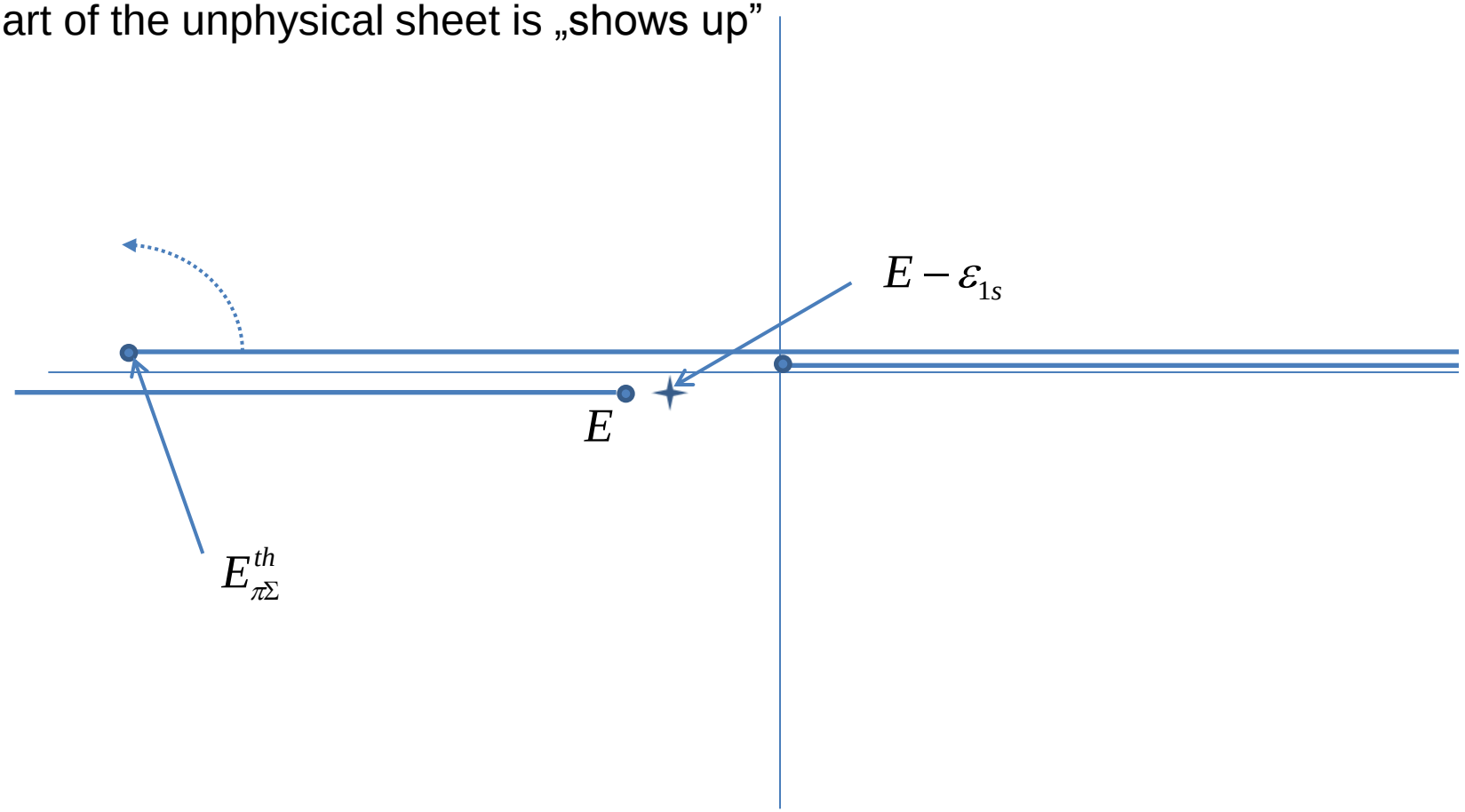


$$G^{ch}_{K^-n}(x_2, y_2; E) = \oint_c g_{K^-n}(x_2; \varepsilon) g_c(y_2; E - \varepsilon)\, d\varepsilon$$

Re-defining, how the square root is taken for the $\pi\Sigma$ channel, the cut can be turned and the needed part of the unphysical sheet is „shows up”

ε



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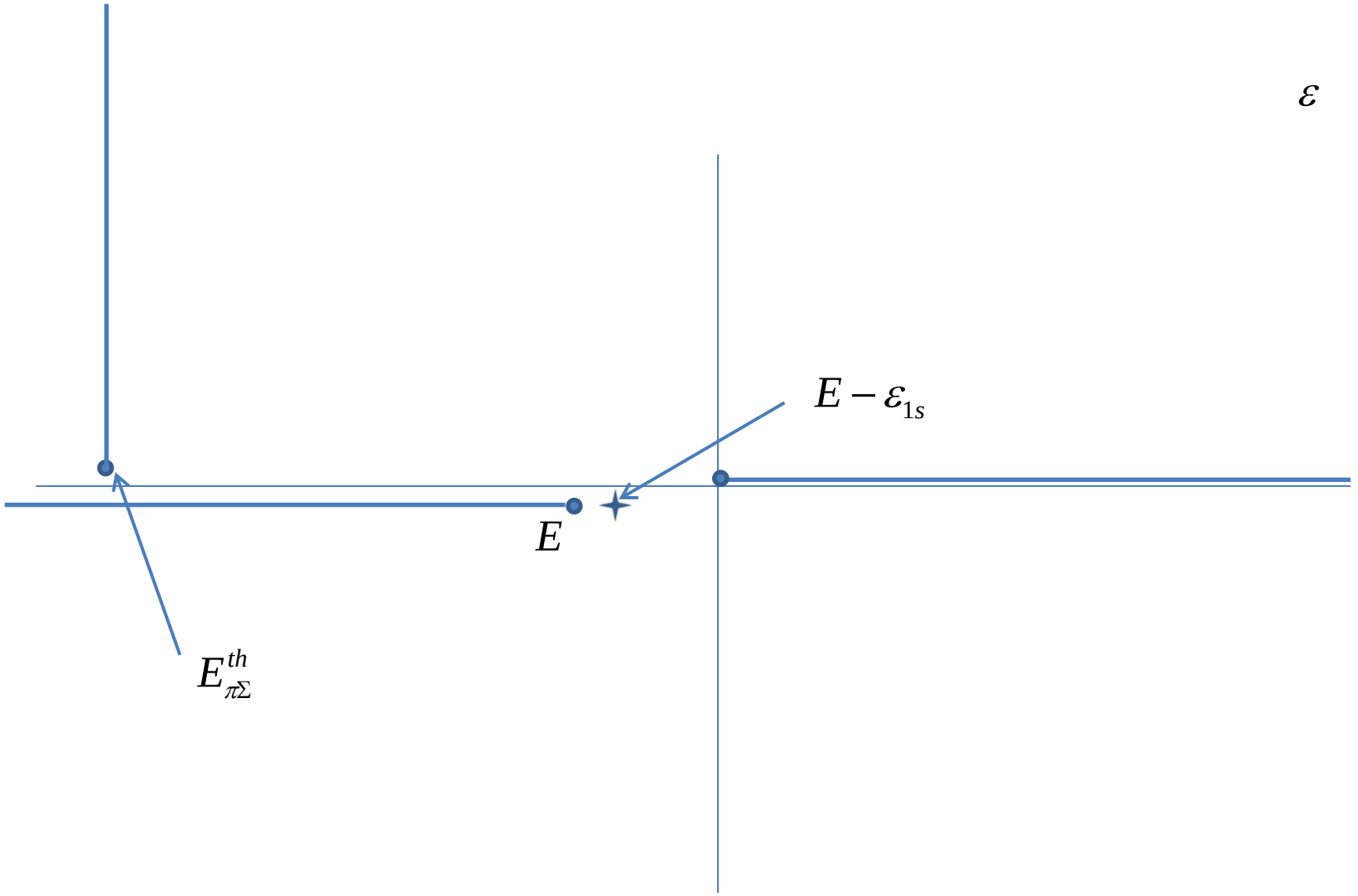

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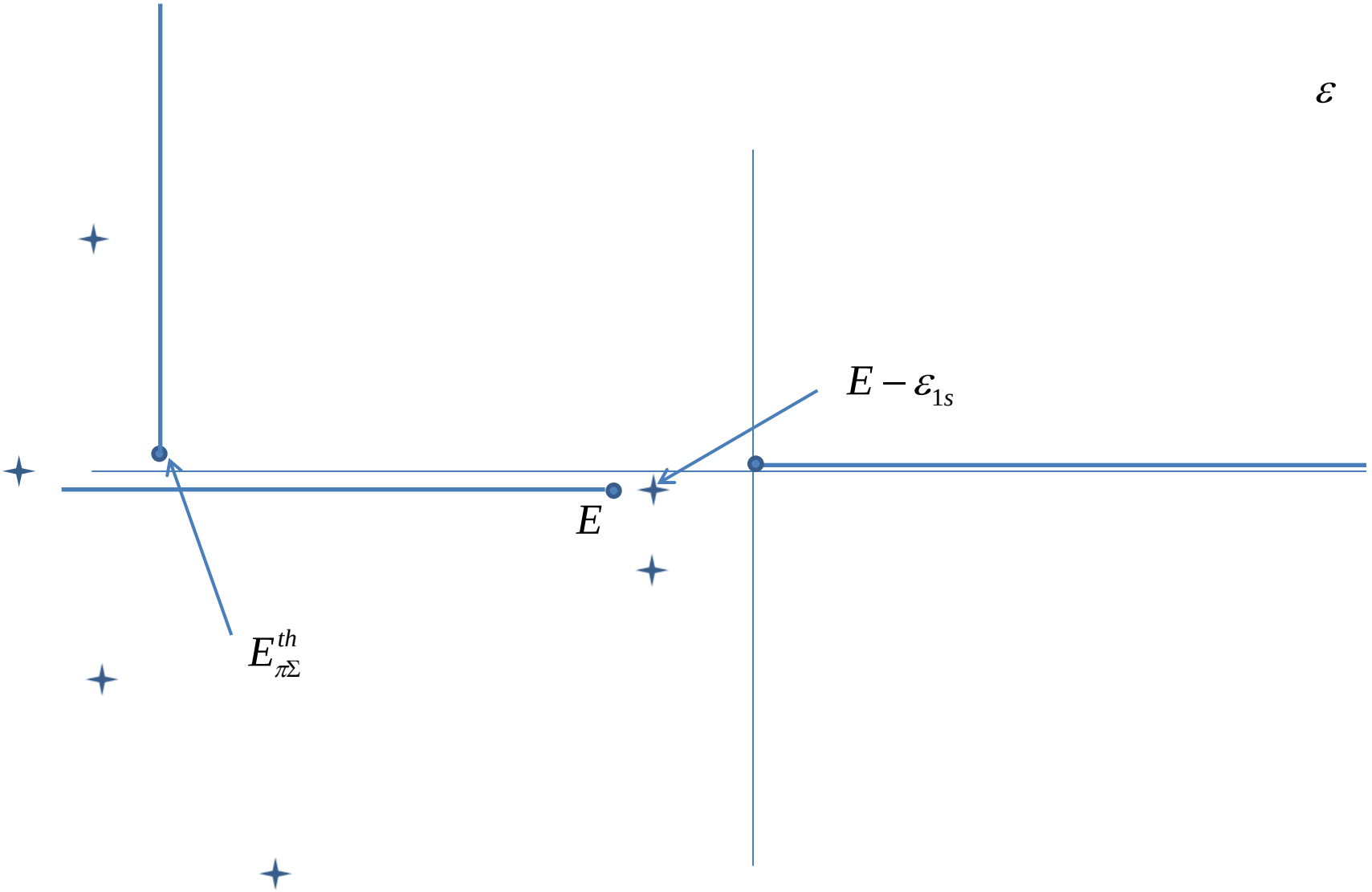

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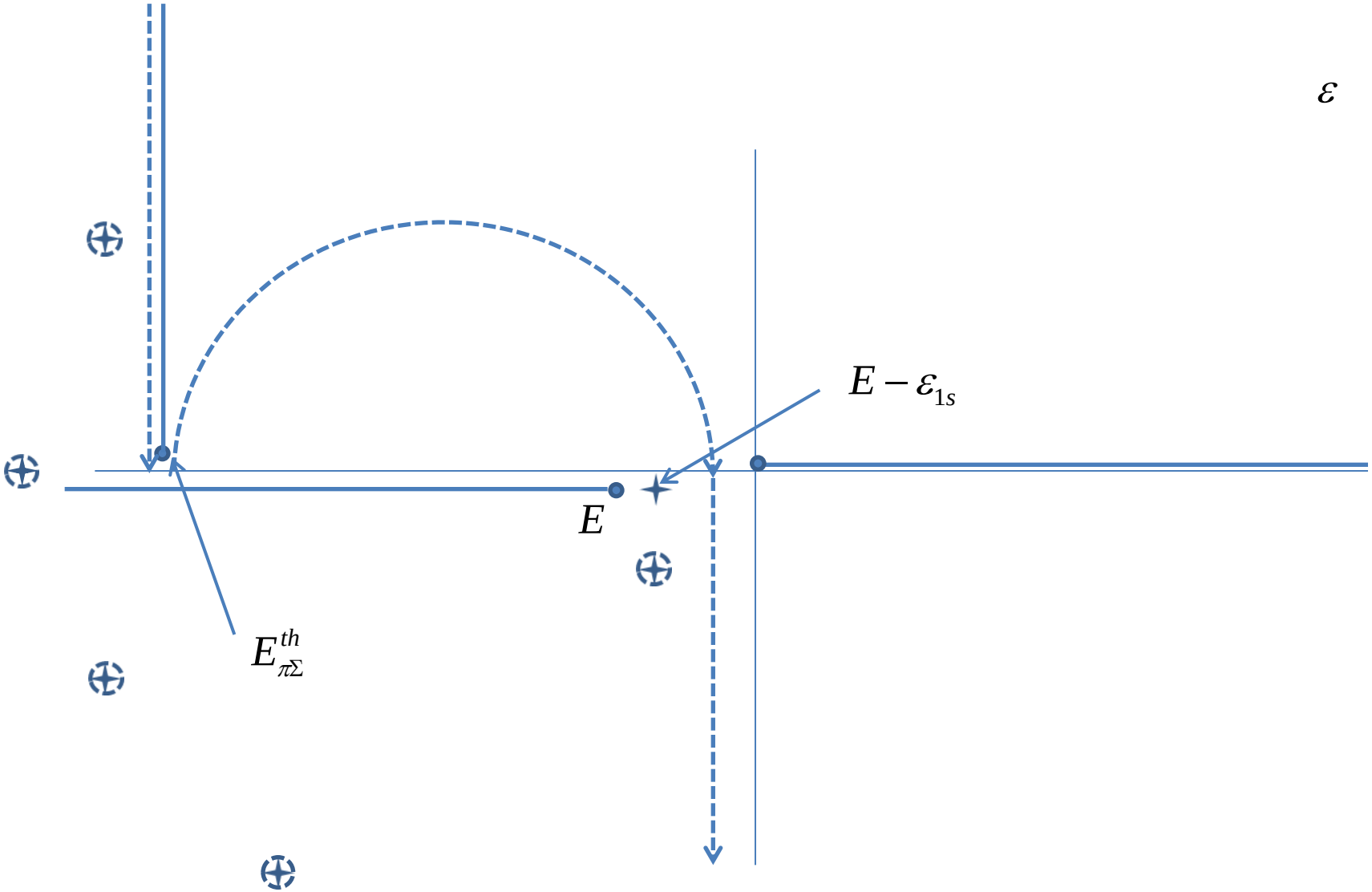

$$G^{ch}_{K^-n}(x_2, y_2; E) = \oint_c g_{K^-n}(x_2; \varepsilon) g_c(y_2; E - \varepsilon)\, d\varepsilon$$

Similar cosiderations for the G_3 (matrix) case and for the 3-channel chiral $\bar{K}N$ interaction – two cuts to be turned.

Energy dependent interactions and the convolution integral

Strong interactions enter the Faddeev equations in the form:

$G^{ch}_{K^-n}(x_2, y_2; E) v_{K^-n}(x_2)$ and $G_3(x_3, y_3; E) v_3(x_3)$. How to proceed, if the potentials depend on energy : $v_{K^-n}(x_2; z)$ and $v_3(x_3; z)$?

$$G^{ch}_{K^-n}(x_2, y_2; E) = \oint_c g_{K^-n}(x_2; \varepsilon) \, \mathrm{g}_c(y_2; E - \varepsilon) \, d\varepsilon$$

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$$G^{ch}_{K^-n}(x_2, y_2; E)\, v_{K^-n}(x_2) = \oint_c g_{K^-n}(x_2; \varepsilon) \qquad \mathrm{g}_c(y_2; E-\varepsilon)\, d\varepsilon$$

Similar cosiderations for the G_3 (matrix) case and for the 3-channel chiral $\bar{K}N$ interaction – two cuts to be turned.

Energy dependent interactions and the convolution integral

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And in the same way for $G_3(x_3, y_3) v_3(x_3)$.

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Fortunately, for separable potentials gv is even simpler, than g itself. Its Coulomb Sturmian matrix elements for Yamaguchi form-Factors can be calculated analytically.

The main results of the calculation can be summarized in the table

Basis		ΔE in eV					
n_{max} in each channel	Total basis size	$v_{\bar{K}N}$(SIDD1)		$v_{\bar{K}N}$(SIDD2)		$v_{\bar{K}N}$(Chiral)	
		v_{np}^{s}	v_{np}^{a+r}	v_{np}^{s}	v_{np}^{a+r}	v_{np}^{s}	v_{np}^{a+r}
20	1764	$692-439i$	$714-452i$	$711-448i$	$728-448i$	$762-461i$	$766-460i$
24	2500	$699-442i$	$739-456i$	$738-451i$	$753-455i$	$792-472i$	$802-477i$
28	3364	$706-442i$	$753-459i$	$755-455i$	$769-461i$	$809-480i$	$823-490i$
32	4356	$711-442i$	$761-461i$	$765-458i$	$776-466i$	$816-486i$	$832-497i$
36	5476	$713-442i$	$764-463i$	$770-461i$	$780-468i$	$819-489i$	$835-500i$
40	6724	$715-442i$	$766-464i$	$774-461i$	$781-469i$	$819-490i$	$836-502i$
44	8100	$716-442i$	$767-464i$	$776-461i$	$782-469i$	$820-491i$	$835-502i$

We can compare our (converged) 3-body results with those of commonly used approximations for the same level shifts :

$\bar{K}N$ potential	Corrected Deser from a_{Kd}	$\bar{K}d$ optical potential	3-body
SIDD1	$831 - 367i$	$785 - 509i$	$767 - 464i$
SIDD2	$840 - 364i$	$797 - 512i$	$782 - 469i$
Chiral	$881 - 363i$	$828 - 527i$	$835 - 502i$

ΔE in eV

It is evident, that the most popular and unconditionally trusted “corrected Deser formula” has little to do with the exact result, especially for the imaginary part of the level shift.

Conclusions

- The present calculations suggest, that the level shift ΔE should be in the in the range $\Delta E \sim (800 \pm 30) - (480 \pm 20)\ eV$, Now it is the turn of experimentalists.

- This is the first exact calculation of the level shift in a hadronic atom, which uses realistic, multichannel hadron-nucleon interaction and goes beyond the conventional two-body picture.

- For the strangeness nuclear physics the main significance of the results is not as much in the obtained numbers, as in the first possibility to relate an important an hopefully measurable observable of the $\bar{K}NN$ system to the input $\bar{K}N$ interactions without relying upon uncontrollable approximations.

- The proposed method can serve as an important tool in fixing the yet uncertain properties of the basic $\bar{K}N$ interactions.