Few-body universality: from Efimov effect to super Efimov effect

Yusuke Nishida (Tokyo Tech)

The 23rd European Conference on Few-Body Problems in Physics

August 8-12 (2016) @ Aarhus

Plan of this talk

1. Universality of Efimov effect

⇒ Condensed matter physics

nature physics
ARTICLES
PUBLISHED ONLINE: 13 JANUARY 2013 | DOI: 10.1038/NPHYS2523

Efimov effect in quantum magnets

Yusuke Nishida*, Yasuyuki Kato and Cristian D. Batista



2. Novel few-body universality

⇒ Super Efimov effect

PRL **110,** 235301 (2013) PHYSICAL REVIEW LETTERS week ending 7 JUNE 2013

Super Efimov Effect of Resonantly Interacting Fermions in Two Dimensions

Yusuke Nishida,[1] Sergej Moroz,[2] and Dam Thanh Son[3]
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(Received 18 January 2013; published 4 June 2013)



Few-body universality

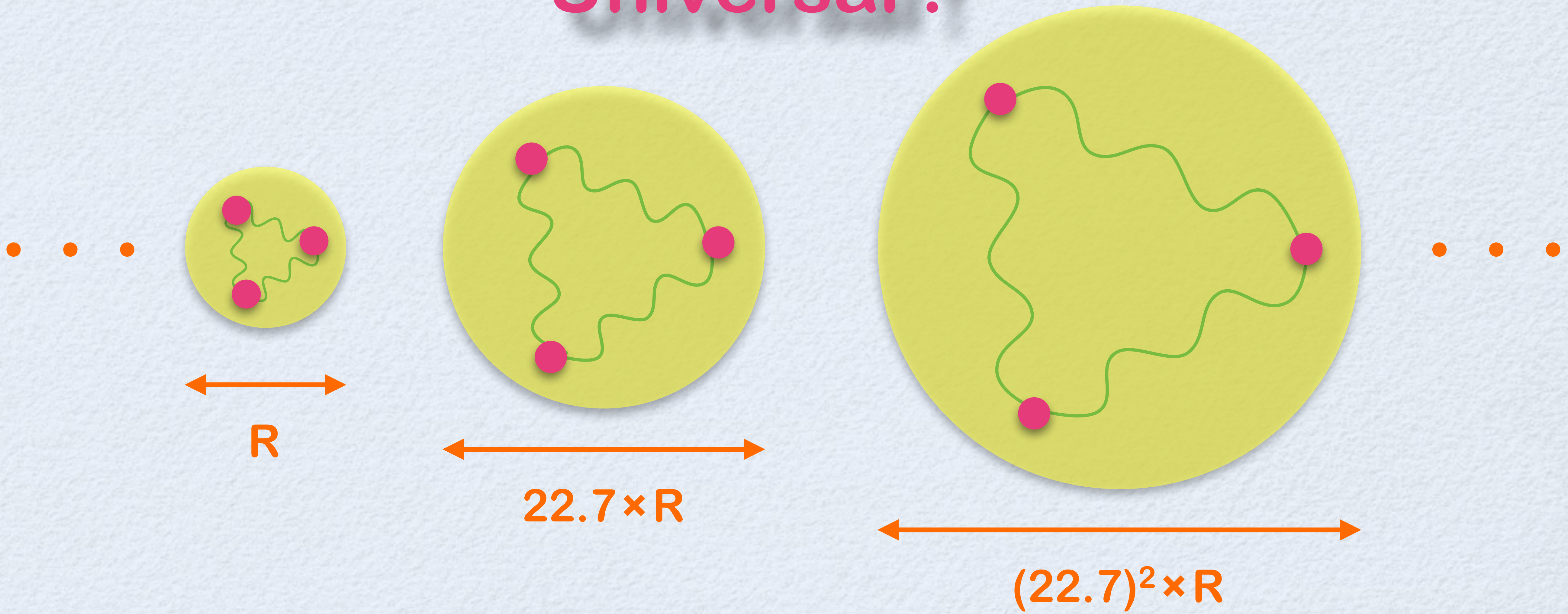
Efimov effect (1970)

- 3 bosons
- 3 dimensions
- s-wave resonance

→ Infinite bound states with exponential scaling

$$E_n \sim e^{-2\pi n}$$

Universal !



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$$E_n \sim e^{-2\pi n}$$

Universal !

nuclear physics

- nucleons
- halo nucleus
- ...

atomic physics

- helium atoms
- cold atoms
- ...

Few-body universality

Efimov effect (1970)

- 3 bosons
- 3 dimensions
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→ Infinite bound states with exponential scaling

$$E_n \sim e^{-2\pi n}$$

Universal !

nuclear physics

condensed matter

atomic physics

Efimov effect in solid states ?

× electrons (fermions with long-range repulsion)

- bosonic collective excitations !?

Efimov effect in quantum magnets

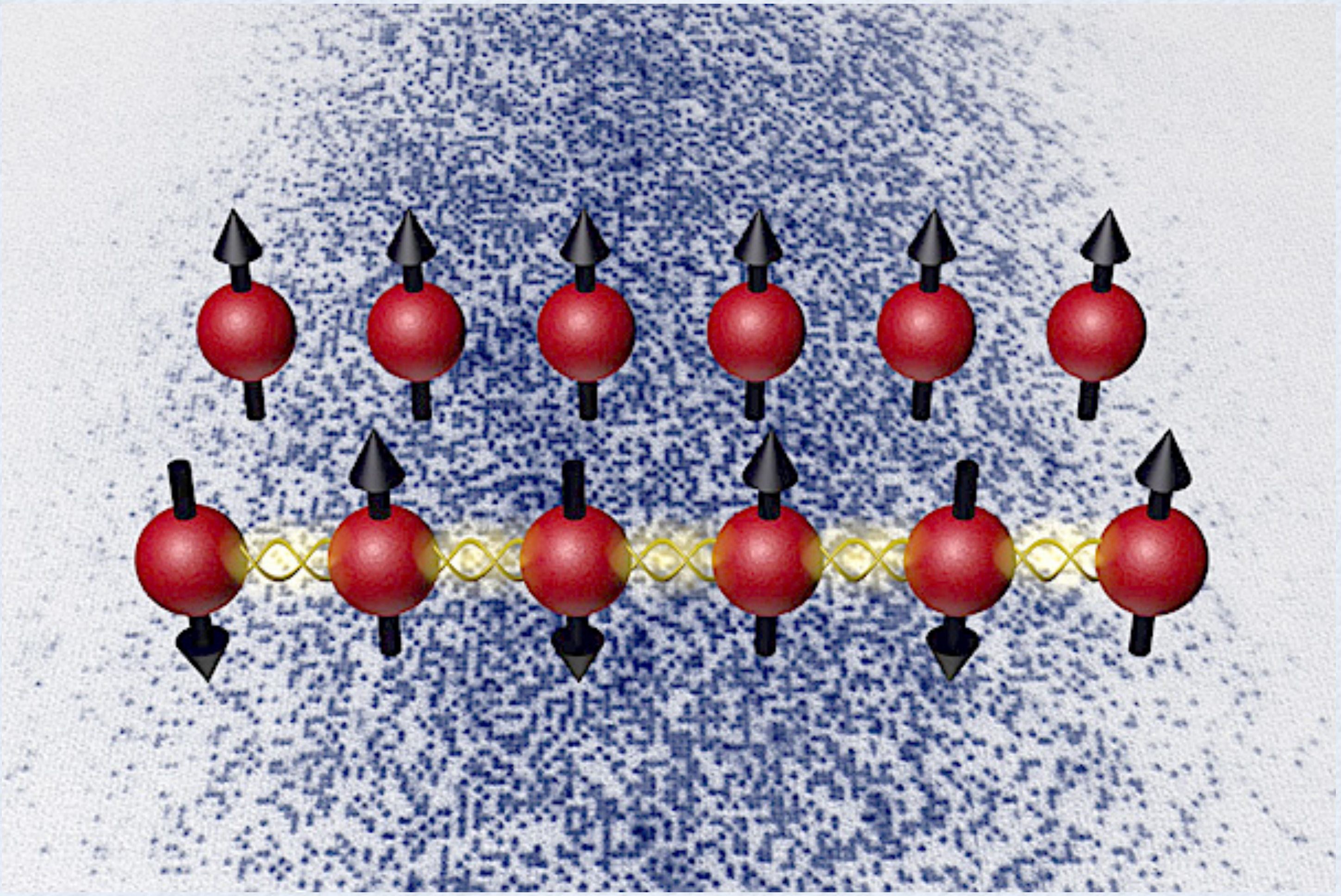
Quantum magnet

Anisotropic Heisenberg model on a 3D lattice

$$H = -\sum_{r}\left[\sum_{\hat{e}}(JS_r^+S_{r+\hat{e}}^- + J_zS_r^zS_{r+\hat{e}}^z) + D(S_r^z)^2 - BS_r^z\right]$$

exchange anisotropy

single-ion anisotropy



Quantum magnet

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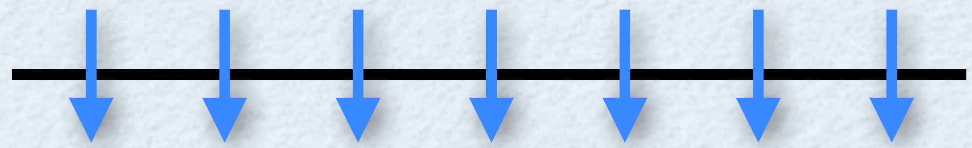
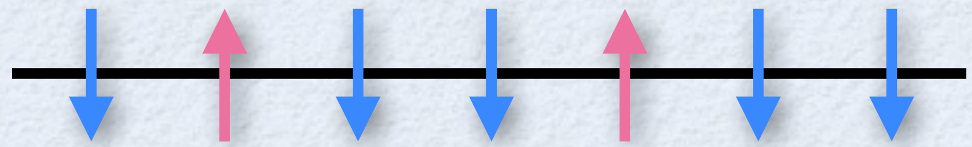
exchange anisotropy — single-ion anisotropy

Spin-boson correspondence

 ⇔

fully polarized state (B→∞) — No boson = vacuum

 ⇔ 

N spin-flips — N bosons = magnons

Quantum magnet

Anisotropic Heisenberg model on a 3D lattice

$$H = -\sum_{r}\left[\sum_{\hat{e}}(J S_r^+ S_{r+\hat{e}}^- + J_z S_r^z S_{r+\hat{e}}^z) + D(S_r^z)^2 - B S_r^z\right]$$

xy-exchange coupling

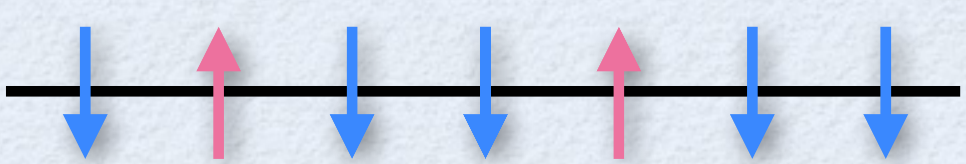
⇔ hopping

single-ion anisotropy

⇔ on-site attraction

z-exchange coupling

⇔ neighbor attraction



⇔

N spin-flips

N bosons = magnons

Quantum magnet

Anisotropic Heisenberg model on a 3D lattice

$$H = -\sum_{r}\left[\sum_{\hat{e}}\left(J S_r^{+} S_{r+\hat{e}}^{-} + J_z S_r^z S_{r+\hat{e}}^z\right) + D(S_r^z)^2 - B S_r^z\right]$$

xy-exchange coupling
⇔ hopping

single-ion anisotropy
⇔ on-site attraction

z-exchange coupling
⇔ neighbor attraction

Tune these couplings to induce
scattering resonance between two magnons
⇒ Three magnons show the Efimov effect

Two-magnon resonance

Schrödinger equation for two magnons

$$E\Psi(r_1, r_2) = \Big[SJ \sum_{\hat{e}} (2 - \nabla_{1\hat{e}} - \nabla_{2\hat{e}})$$

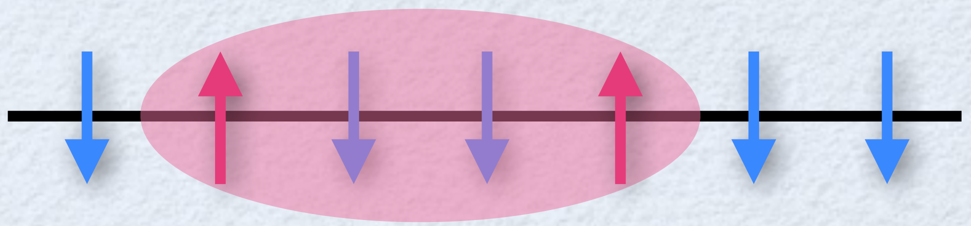
← hopping

$$+ J \sum_{\hat{e}} \delta_{r_1,r_2} \nabla_{2\hat{e}} - J_z \sum_{\hat{e}} \delta_{r_1,r_2+\hat{e}} - 2D\delta_{r_1,r_2} \Big] \Psi(r_1, r_2)$$

neighbor/on-site attraction

Scattering length between two magnons

$$\lim_{|r_1 - r_2| \to \infty} \Psi(r_1, r_2) \Big|_{E=0} \to \frac{1}{|r_1 - r_2|} - \frac{1}{a_s}$$



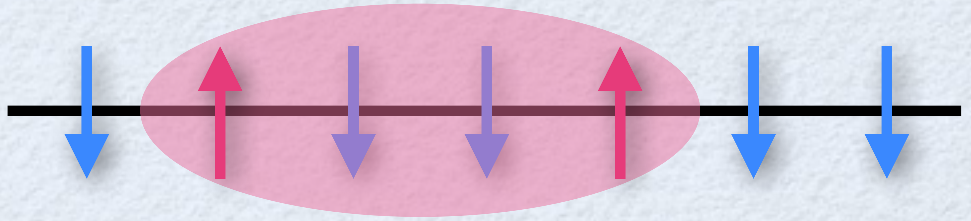
Two-magnon resonance

Scattering length between two magnons

$$\frac{a_s}{a} = \frac{\frac{3}{2\pi}\left[1 - \frac{D}{3J} - \frac{J_z}{J}\left(1 - \frac{D}{6SJ}\right)\right]}{2S - 1 + \frac{J_z}{J}\left(1 - \frac{D}{6SJ}\right) + 1.52\left[1 - \frac{D}{3J} - \frac{J_z}{J}\left(1 - \frac{D}{6SJ}\right)\right]}$$

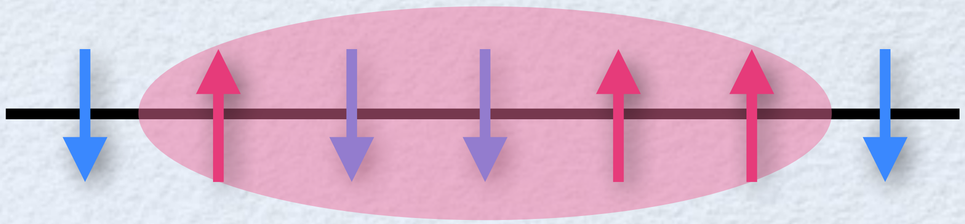
→ **Two-magnon resonance** ($a_s \to \infty$)

- $J_z/J = 2.94$ (spin-1/2)
- $J_z/J = 4.87$ (spin-1, D=0)
- $D/J = 4.77$ (spin-1, ferro $J_z=J>0$)
- $D/J = 5.13$ (spin-1, antiferro $J_z=J<0$)
- …



Three-magnon spectrum

At the resonance, three magnons form bound states with binding energies E_n



- **Spin-1/2**

n	E_n/J	$\sqrt{E_{n-1}/E_n}$
0	-2.09×10^{-1}	—
1	-4.15×10^{-4}	22.4
2	-8.08×10^{-7}	22.7

- **Spin-1, D=0**

n	E_n/J	$\sqrt{E_{n-1}/E_n}$
0	-5.16×10^{-1}	—
1	-1.02×10^{-3}	22.4
2	-2.00×10^{-6}	22.7

- **Spin-1, J_z=J>0**

n	E_n/J	$\sqrt{E_{n-1}/E_n}$
0	-5.50×10^{-2}	—
1	-1.16×10^{-4}	21.8

- **Spin-1, J_z=J<0**

n	E_n/J	$\sqrt{E_{n-1}/E_n}$
0	-4.36×10^{-3}	—
1	-8.88×10^{-6}	22.2

Three-magnon spectrum

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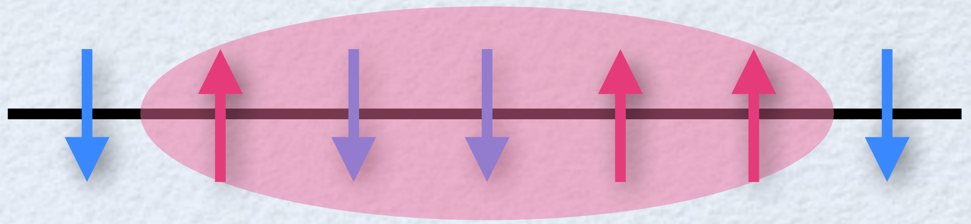

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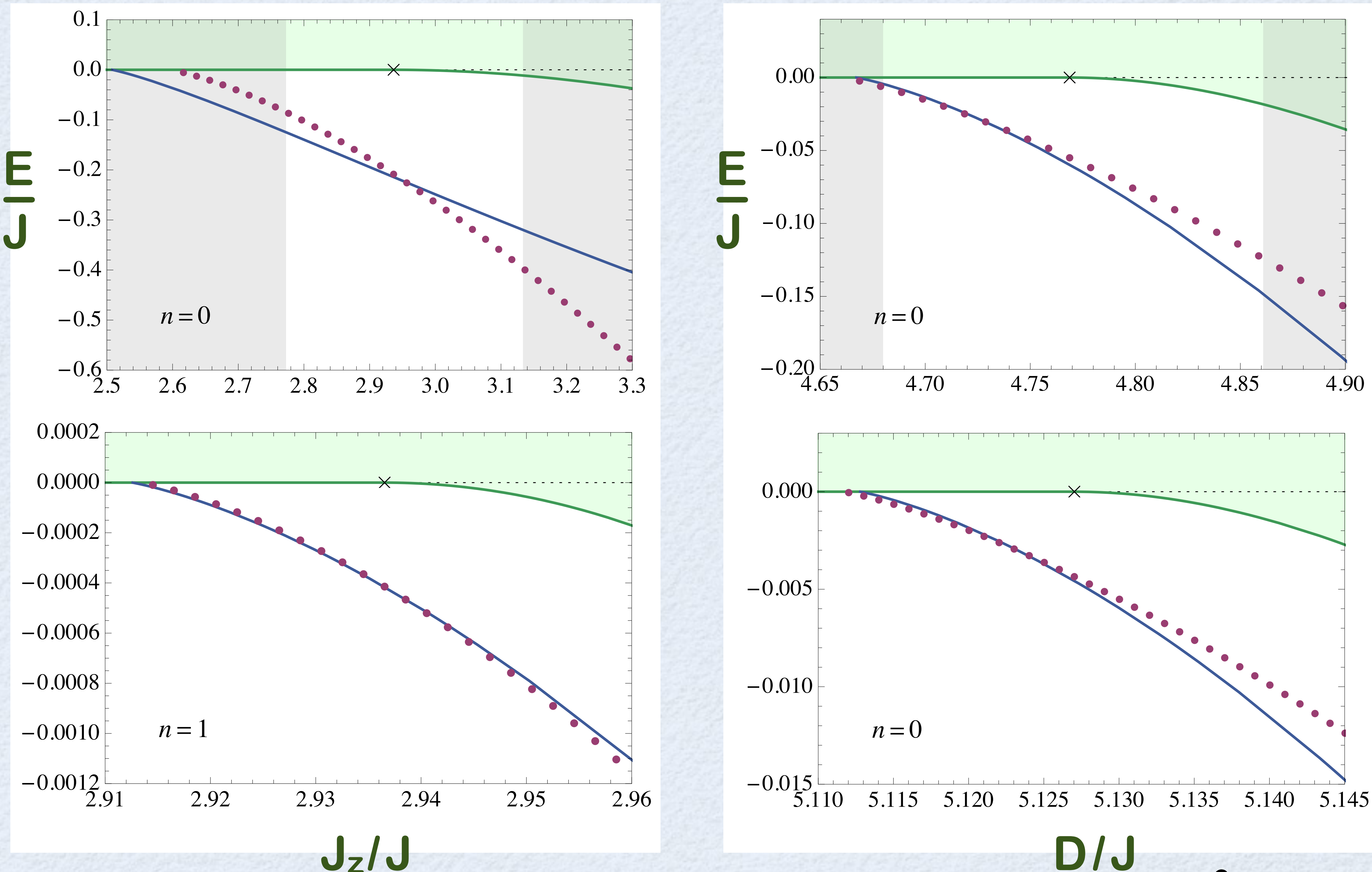
Universal scaling law by ~ 22.7 confirms they are Efimov states !

Three-magnon spectrum

- Spin-1/2



- S=1, J_z=J>0
- S=1, J_z=J<0

Agree with universal prediction : $E_n = -\lambda^{-2n}\,\frac{\kappa_*^2}{m}\,F\!\left(\frac{\lambda^n}{\kappa_* a_s}\right)$

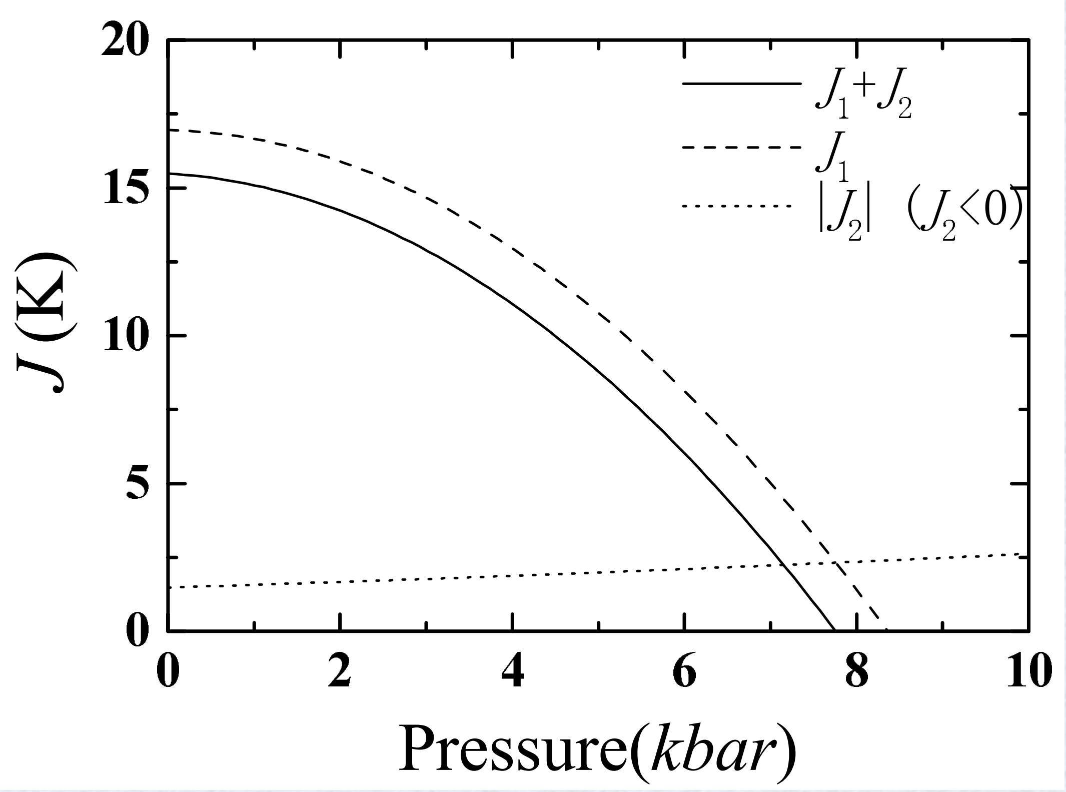
Toward experimental realization

1. Find a good compound
   whose anisotropy is close to the critical value

   E.g. Ni-based organic ferromagnet with D/J~3 (critical 4.8)

   R. Koch et al., Phys. Rev. B 67, 094407 (2003)

C.f. TDAE-C_{60}



T. Kawamoto et al, JPSJ (2001)

2. Tune the exchange coupling
   with pressure to induce
   the two-magnon resonance

3. Observe the Efimov states
   of three magnons with

- absorption spectroscopy
- inelastic neutron scattering
- electron spin resonance

  [see Y.N., PRB88, 224402 (2013)]

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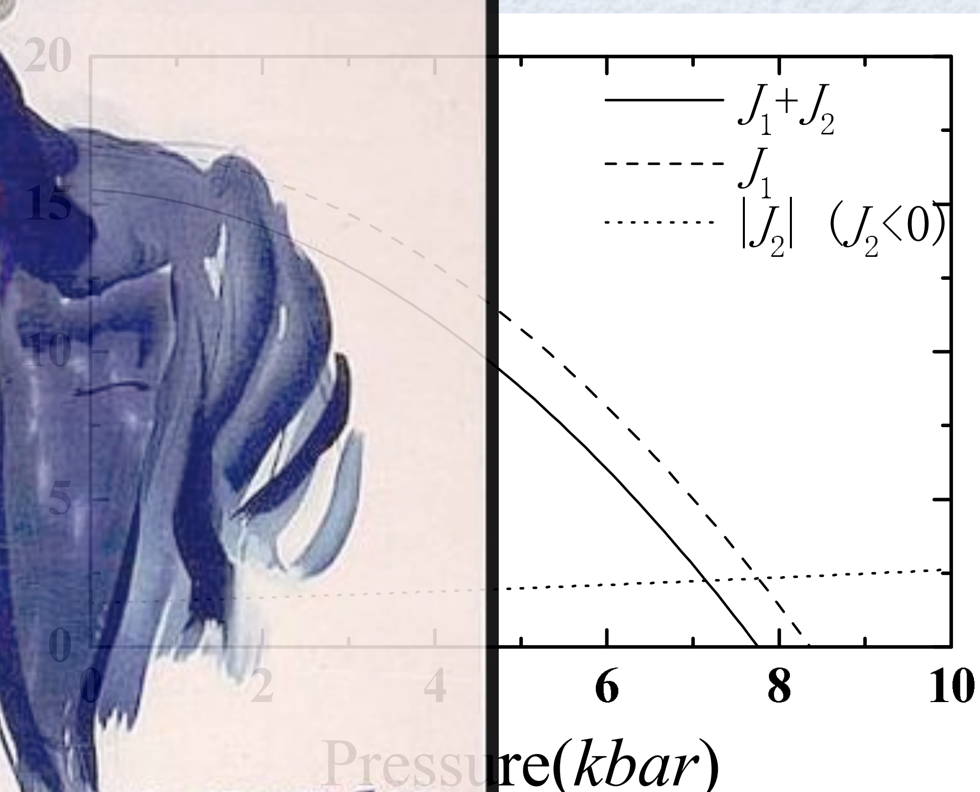
   R. Koch et al., Phys. Rev. B 6

C.f. TDAE-C_{60}



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3. Observe the Efimov states
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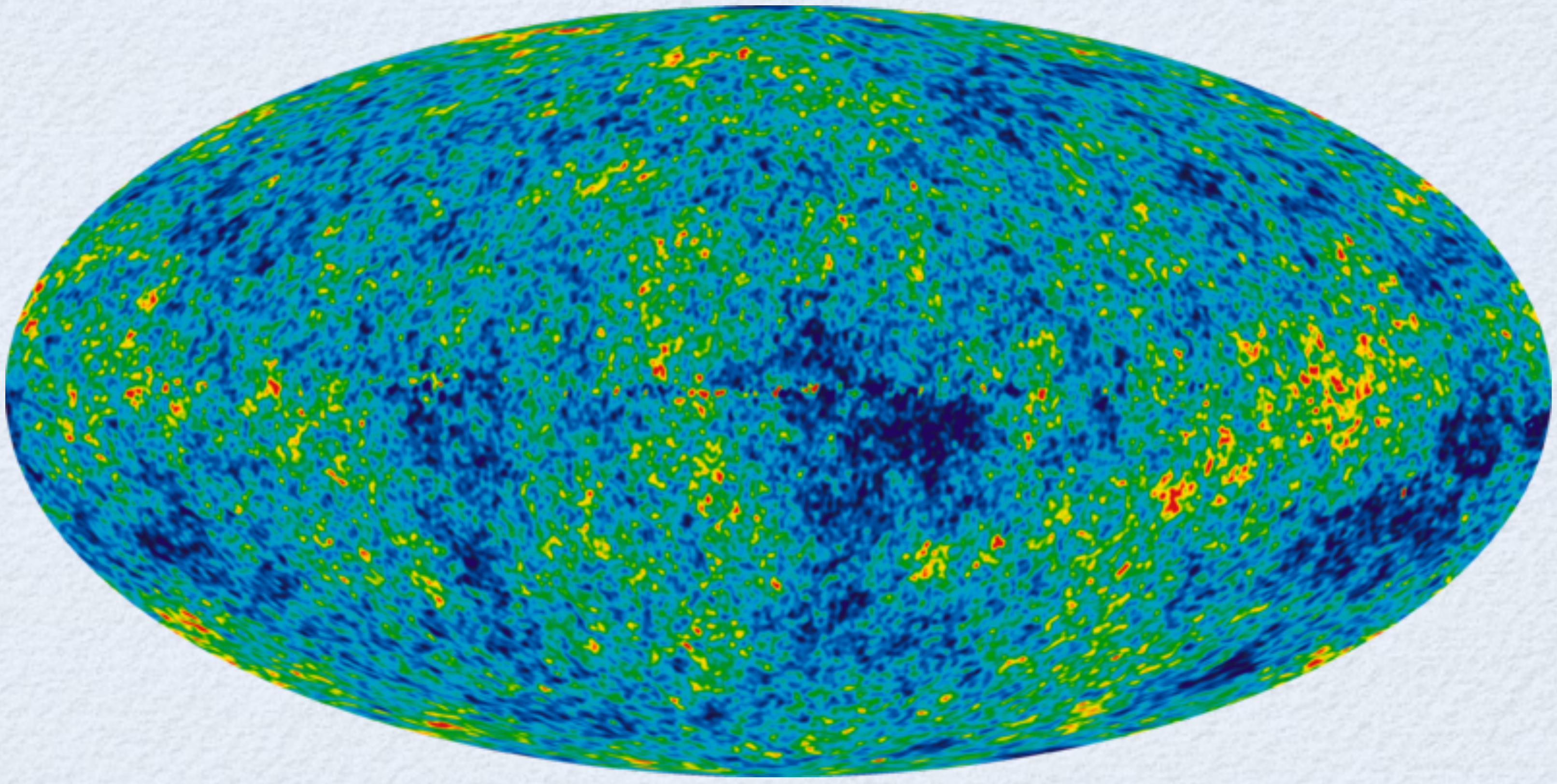
- absorption spectroscopy ... electron spin resonance
- inelastic neutron scattering [see Y.N. PRB 88, 224402 (2013)]

Find interested experimentalists !

Novel universality: Super Efimov effect



Few-body universality

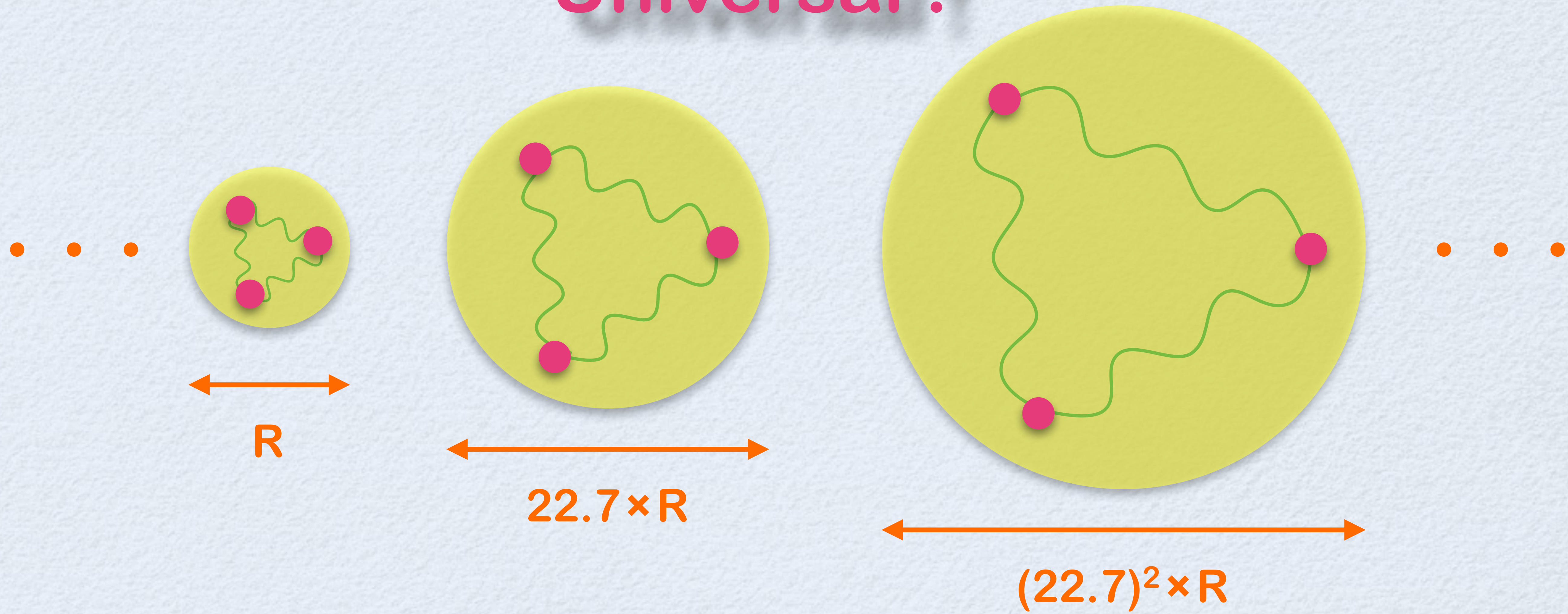
Efimov effect (1970)

- 3 bosons
- 3 dimensions
- s-wave resonance

→ Infinite bound states with exponential scaling

$$E_n \sim e^{-2\pi n}$$

Universal !



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Efimov effect in other systems ?

No, only in 3D with s-wave resonance

	s-wave	p-wave	d-wave
3D	O	x	x
2D	x	x	x
1D	x	x	

Y.N. & S.Tan, Few-Body Syst 51, 191 (2011)

Y.N, Phys Rev A 86, 012710 (2012)

Few-body universality

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Different universality in other systems ?

Yes, super Efimov effect in 2D with p-wave !

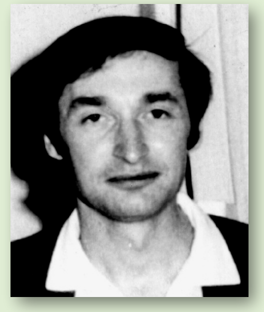
	s-wave	p-wave	d-wave
3D	O	x	x
2D	x	!x!	x
1D	x	x	

Y.N. & S.Tan, Few-Body Syst 51, 191 (2011)

Y.N, Phys Rev A 86, 012710 (2012)

Few-body universality

Efimov effect



- 3 bosons
- 3 dimensions
- s-wave resonance

exponential scaling

$$E_n \sim e^{-2\pi n}$$

Super Efimov effect

New !

- 3 fermions
- 2 dimensions
- p-wave resonance

“doubly” exponential

$$E_n \sim e^{-2e^{3\pi n/4}}$$

PRL **110,** 235301 (2013) PHYSICAL REVIEW LETTERS week ending 7 JUNE 2013

Super Efimov Effect of Resonantly Interacting Fermions in Two Dimensions

Yusuke Nishida,[1] Sergej Moroz,[2] and Dam Thanh Son[3]
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[2]*Department of Physics, University of Washington, Seattle, Washington 98195, USA*
[3]*Enrico Fermi Institute, University of Chicago, Chicago, Illinois 60637, USA*
(Received 18 January 2013; published 4 June 2013)



Super Efimov effect

Super Efimov effect

- 3 fermions
- 2 dimensions
- p-wave resonance

⇒ Infinite bound states with doubly exponential scaling $E_n \sim e^{-2e^{3\pi n/4}}$

- Low-energy EFT for 2D p-wave scattering
- RG analysis for 3-body & 4-body couplings

⇛ Exact spectrum in the low-energy limit !

Two tetramers attached to every trimer with resonance energy $E_n \sim e^{-2e^{3\pi n/4-0.188}}$

Super Efimov effect

Super Efimov effect

- 3 fermions
- 2 dimensions
- p-wave resonance

⇒ Infinite bound states with doubly exponential scaling $E_n \sim e^{-2e^{3\pi n/4}}$

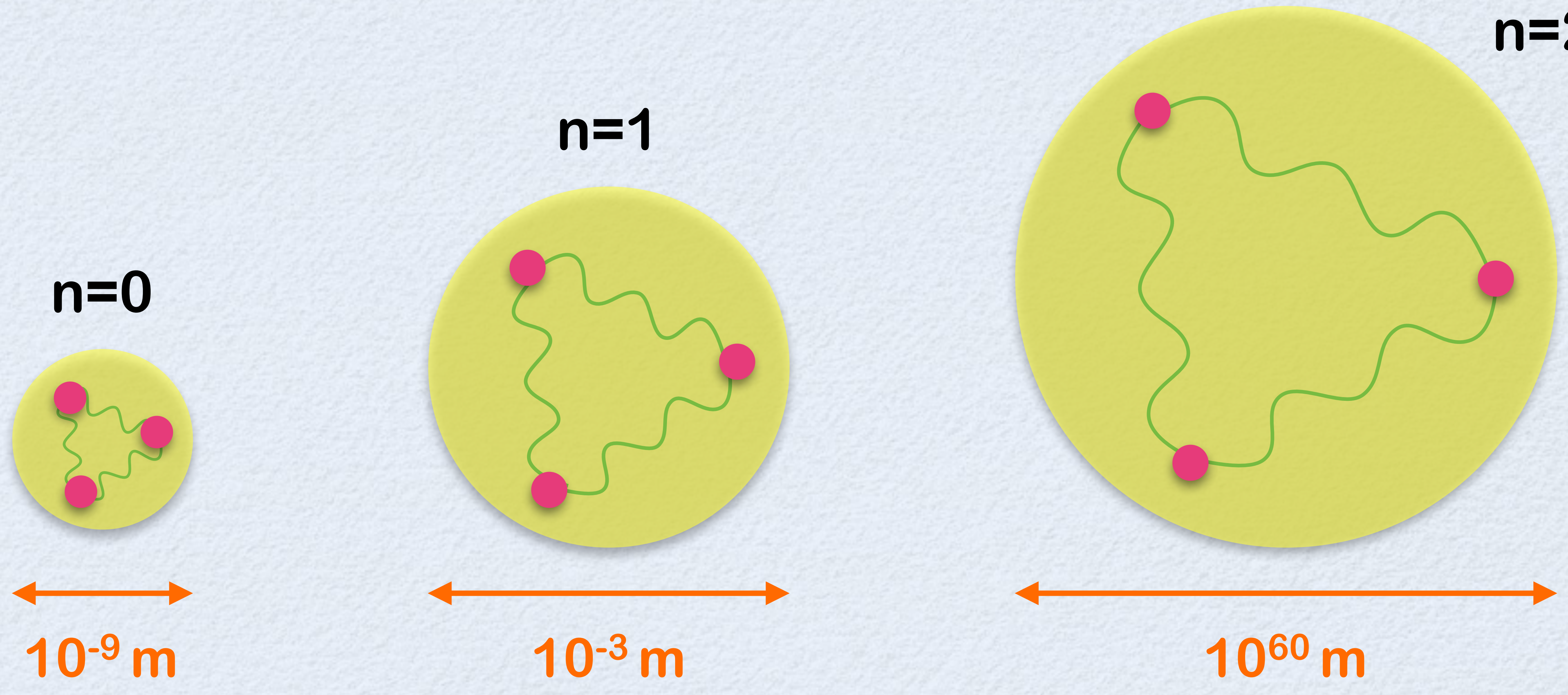
- **Low-energy EFT** and **RG analysis**
  (Nishida-Moroz-Son 2013)
- **STM equation** for model interaction
  (Nishida-Moroz-Son 2013, Levinsen-Cooper-Gurarie 2008)
- **Hyperspherical** ⇒ $V_{eff} \sim 1/[R \log(R)]^2$
  (Volosniev-Fedorov-Jensen-Zinner 2014, Gao-Wang-Yu 2015)
- **Mathematical proof** (Gridnev 2014)

Super Efimov effect

Super Efimov effect

- 3 fermions
- 2 dimensions
- p-wave resonance

Infinite bound states with doubly exponential scaling $E_n \sim e^{-2e^{3\pi n/4}}$



Super Efimov effect

Super Efimov effect

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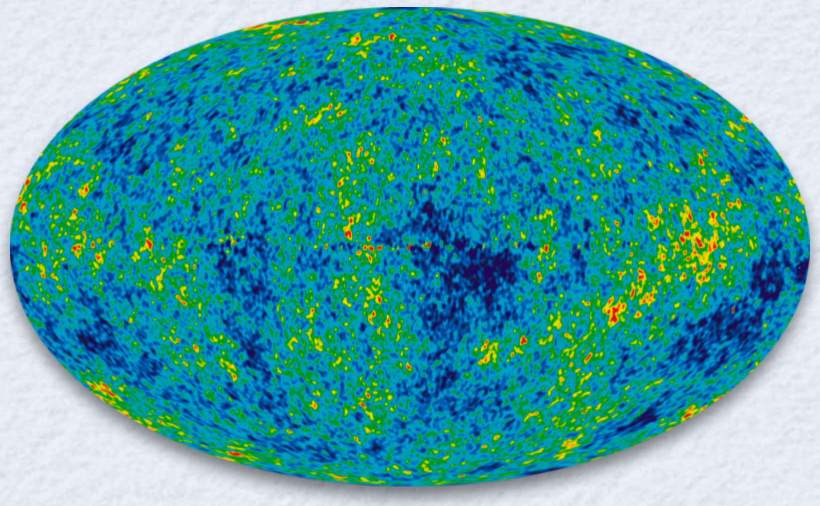
difficult to observe ?



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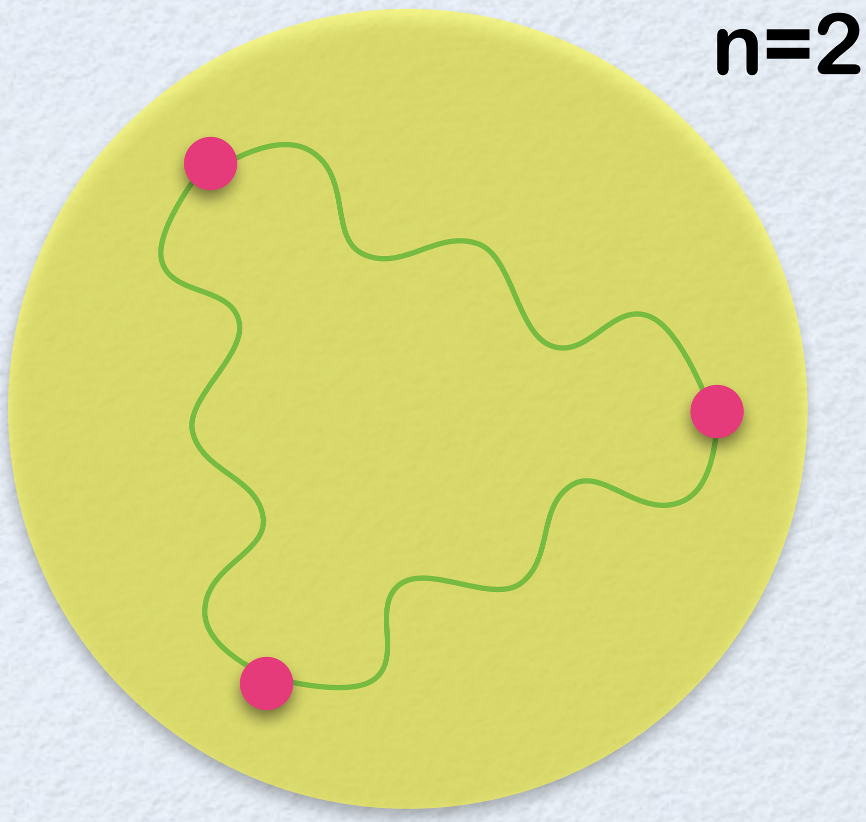


10^{26} m

10^{60} m

Efimov vs super Efimov

Efimov effect

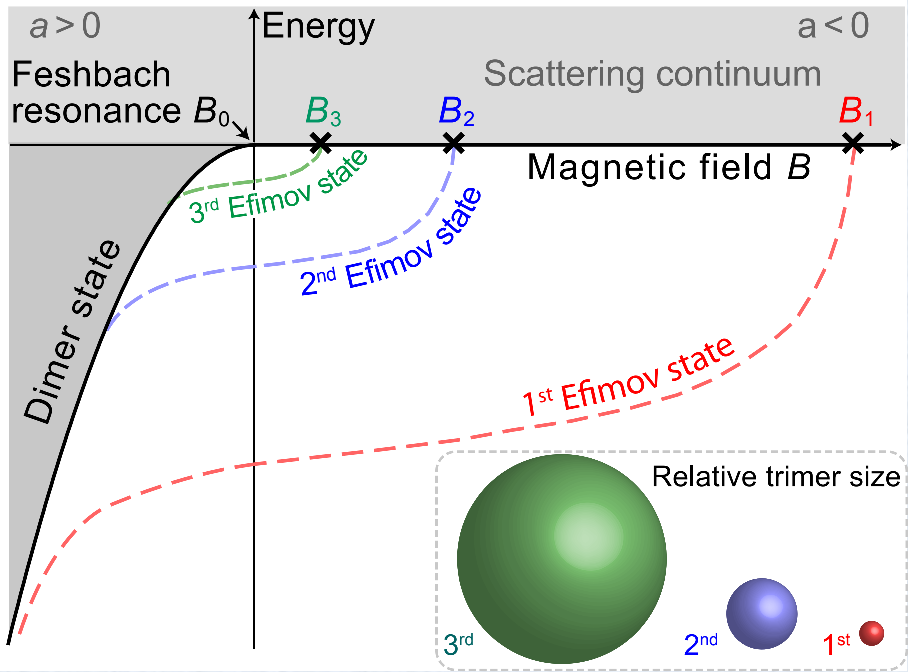
- 3 identical bosons
- 3 dimensions
- s-wave resonance

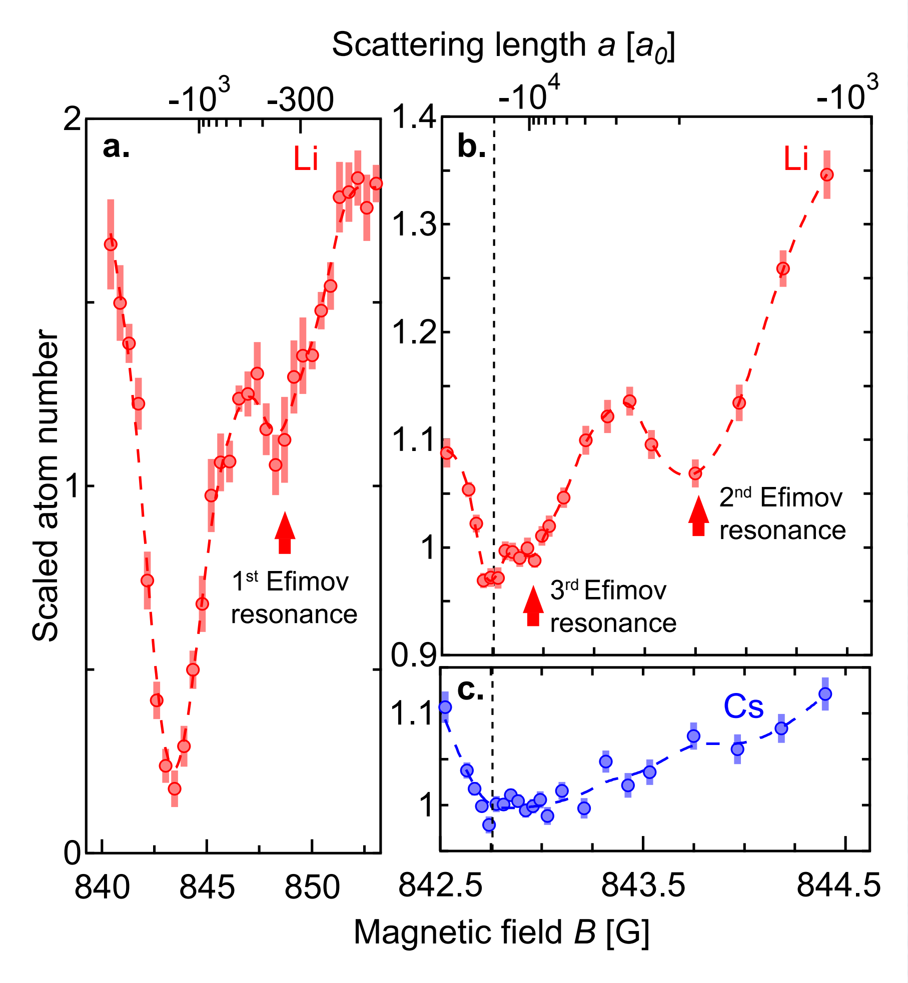
exponential scaling

$$\frac{E_{n+1}}{E_n} \to e^{-2\pi} \approx (22.7)^{-2}$$

$$(4.88)^{-2}$$

for ^{6}Li-^{133}Cs mixture





S.-K. Tung et al, PRL113 (2014)

R. Pires et al, PRL112 (2014)

Efimov vs super Efimov

Efimov effect

- 3 identical bosons
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exponential scaling

$$\frac{E_{n+1}}{E_n} \to e^{-2\pi} \approx (22.7)^{-2}$$

$$(4.88)^{-2}$$

for ^{6}Li-^{133}Cs mixture

Super Efimov effect

- 3 identical fermions
- 2 dimensions
- p-wave resonance

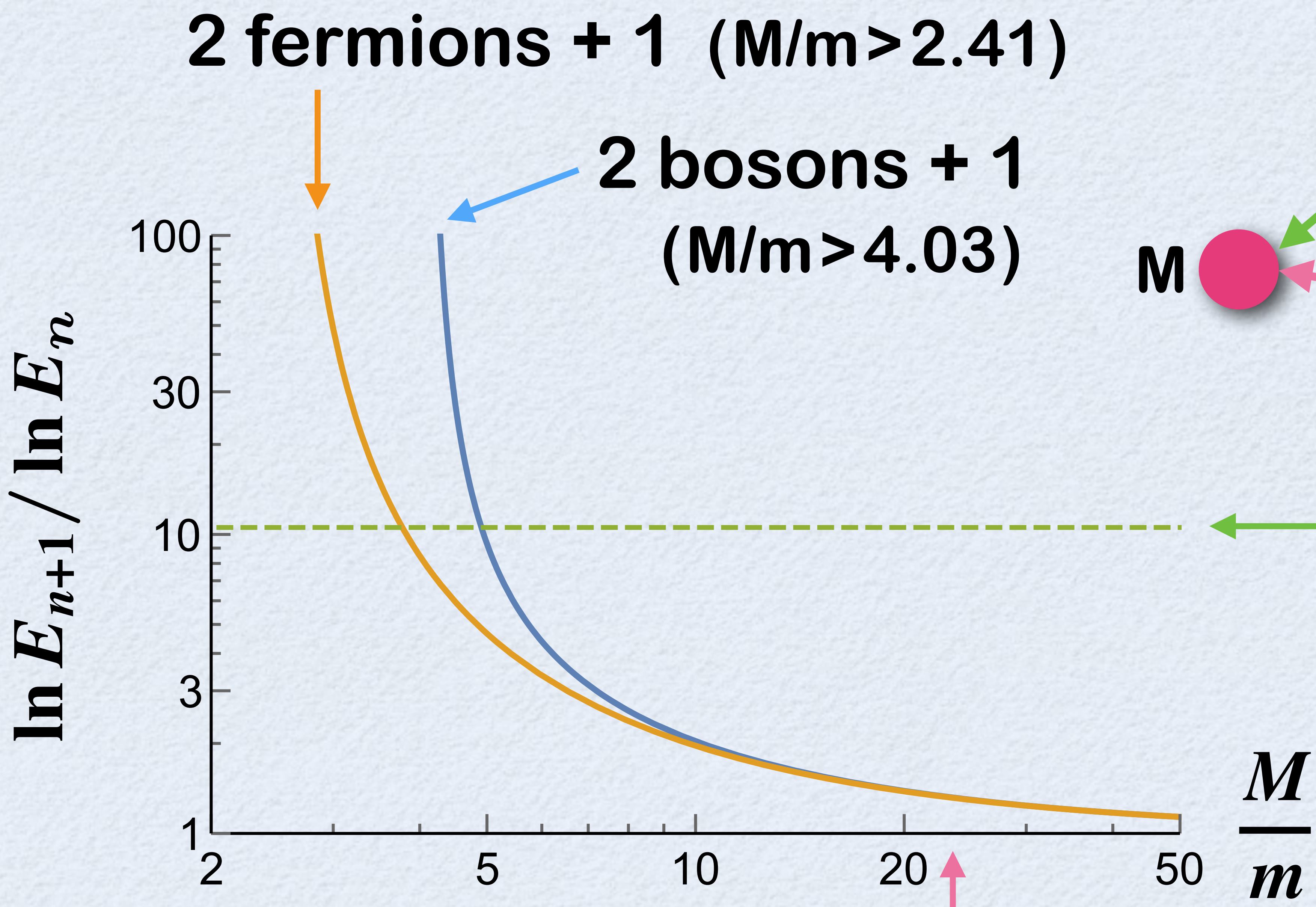
"doubly" exponential

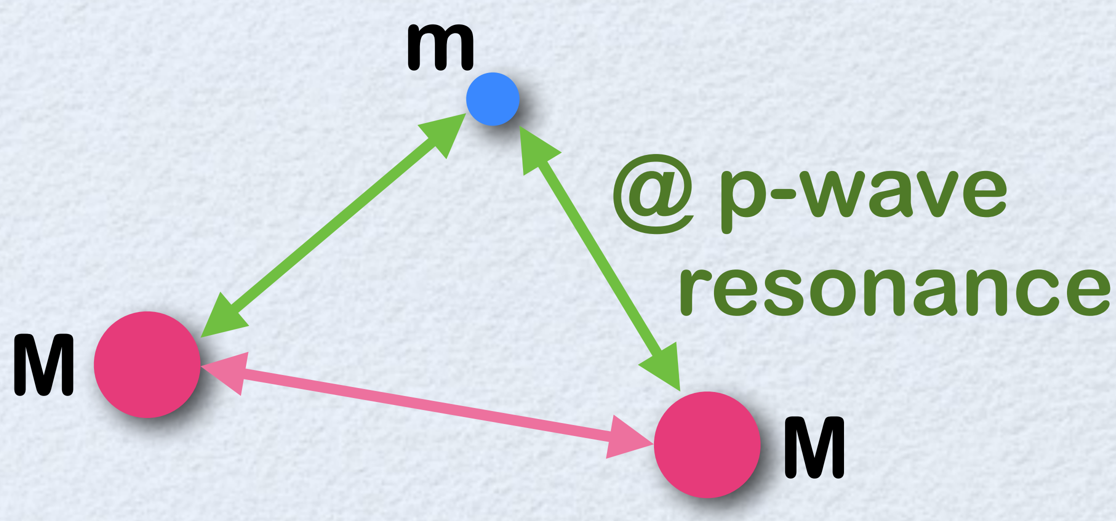
$$\frac{\ln E_{n+1}}{\ln E_n} \to e^{3\pi/4} \approx 10.55$$

???

for ^{6}Li-^{133}Cs mixture

Mass imbalance mixtures





$e^{3\pi/4} \approx 10.55$

for 3 identical fermions

$$\frac{\ln E_{n+1}}{\ln E_n} \approx 1.34 \text{ ! }$$ for ^{6}Li-^{133}Cs mixture

- p-wave resonance observed but 2D confinement necessary

M. Repp et al, Phys. Rev. A 87, 010701(R) (2013)

Summary

Few-body universality: Efimov effect

nuclear physics	condensed matter	atomic physics
prediction (1970)	proposal (2013)	realization (2006)

✓ **Novel playground ⇒ Quantum magnets**

Y.N, Y.K, C.D.B, Nature Physics 9, 93-97 (2013)

✓ **Novel universality ⇒ Super Efimov effect**

Y.N, S.M, D.T.S, Phys Rev Lett 110, 235301 (2013)

S.M, Y.N, Phys Rev A 90, 063631 (2014)

(mass imbalance may help to observe)