Recent progresses in *ab-initio* studies of low-energy few-nucleon reactions of astrophysical interest

Laura E. Marcucci

University of Pisa
INFN-Pisa

$^{\epsilon}FB^{23}$
Aarhus, Denmark, 8-12 August 2016

Results obtained in collaboration with:

- A. Kievsky & M. Viviani (INFN-Pisa - Italy)
- E. Tognelli, S. Degl'Innocenti, & P. Prada Moroni (Univ. of Pisa - Italy)
- L. Girlanda (Univ. of Salento - Italy)
- G. Mangano (INFN-Napoli - Italy)
- J. Golak (Jagiellonian University, Cracow - Poland)
- R. Schiavilla & A. Baroni (ODU & Jefferson Lab. - USA)
- S. Pastore (LANL - USA)
- M. Piarulli (ANL - USA)

Outline

- *Ab-initio* framework for few-nucleon systems
    - (Standard/Old fashion) phenomenological approach (PhenAp)
    - Chiral effective field theory (χEFT)
- Tests
    - Electromagnetic structure of $A = 2, 3, 4$ nuclei
    - Muon capture on light nuclei
- Results
    - $p + p \rightarrow d + e^+ + \nu_e$
    - $p + d \rightarrow {}^3\mathrm{He} + \gamma$
- Ongoing work
    - The weak sector in χEFT: $\Rightarrow$ triton β-decay
- Outlook

Theoretical framework: *ab-initio* studies

A-nucleon system $\leftrightarrow$ Cross section σ [or S-factor $S(E)$]

Ingredients

- Realistic nuclear Hamiltonian
  $H = T + \sum_{i<j} V_{ij} + \sum_{i<j<k} V_{ijk}$
- *Ab-initio* method to solve the A-body quantum problem (HH method for $A = 3, 4$ both bound and scattering)
  $\longrightarrow$ M. Viviani's talk
- Realistic nuclear electroweak currents j^{EW}

Nuclear Hamiltonian

- PhenAp $\rightarrow$ AV18/UIX
- χEFT $\rightarrow$ N3LO(Idaho)/N2LO(local form) – $\Lambda = 500, 600$ MeV

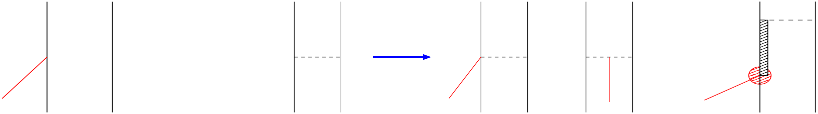
Electroweak currents in PhenAp

EW operators: $\rho^\gamma, \mathbf{j}^\gamma$; $\rho^{V/A}, \mathbf{j}^{V/A}$ but CVC $\Rightarrow \rho^V/\mathbf{j}^V \rightarrow \rho^\gamma/\mathbf{j}^\gamma$

- $\mathbf{j}^\gamma \rightarrow$ Current Conservation Relation (CCR) $\longrightarrow \mathbf{q}\cdot\mathbf{j}^\gamma \propto [\rho^\gamma, H]$

Realistic model

1b operators **2b operators**: Meson-exchange currents + $\mathbf{j}^{MD} \perp \mathbf{q}$

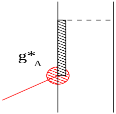


CCR with T at $\mathcal{O}(\frac{1}{m})$ | Interplay potential-current: MI | MD

CCR "exact" with AV18/UIX [L.E. Marcucci *et al.*, PRC **72**, 014001 (2005)]

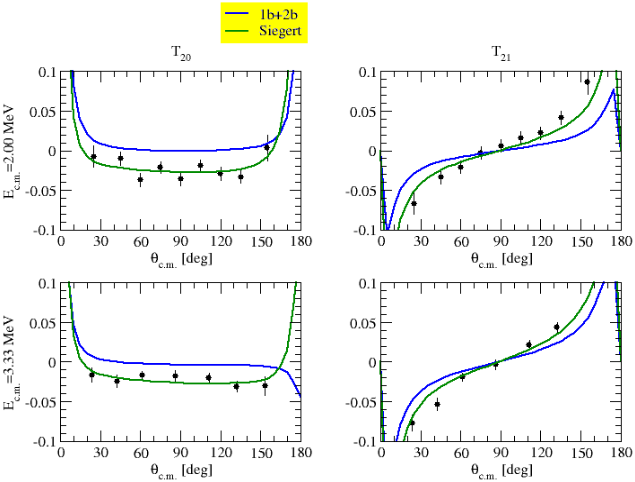
- $\mathbf{j}^A \rightarrow$ no CCR $\Rightarrow$ **MD**

Largest contribution to $\mathbf{j}^A$(MD) from



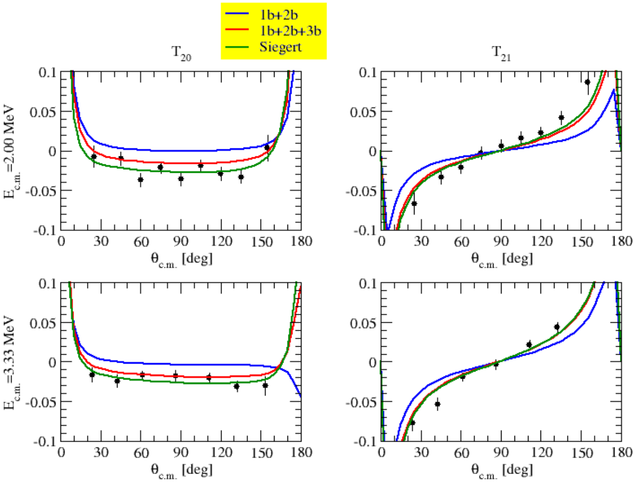
g^*_A fit to observable: GT_{Exp} of tritium β-decay

T_{20} & T_{21} for $p + \vec{d} \longrightarrow {}^3\mathrm{He} + \gamma$ with AV18/UIX



L.E. Marcucci *et al.*, PRC **72**, 014001 (2005)

T_{20} & T_{21} for $p + \vec{d} \to {}^3\mathrm{He} + \gamma$ with AV18/UIX



L.E. Marcucci *et al.*, PRC **72**, 014001 (2005)

Electroweak currents in χEFT: a short history

$\mathbf{j}^\gamma$

- Park *et al.* in heavy-baryon χPT (HBχPT)
  $\rightarrow$ since $\simeq$ 1995
- Pastore *et al.* in time-ordered perturbation theory (TOPT)
  $\rightarrow$ since 2009
- Kölling *et al.* with the unitary transform method
  $\rightarrow$ in parallel since 2009

$\mathbf{j}^A$

- Park *et al.* in HBχPT $\rightarrow$ since $\simeq$ 2000
- Baroni *et al.* in TOPT $\rightarrow$ PRC **93**, 015501 (2016)

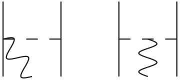
To be remarked:

- Park *et al.*: until now only available FULL set of operators

Electroweak currents in χEFT: power counting for $\mathbf{j}^\gamma$

$\mathcal{O}(Q^{-2})$ 

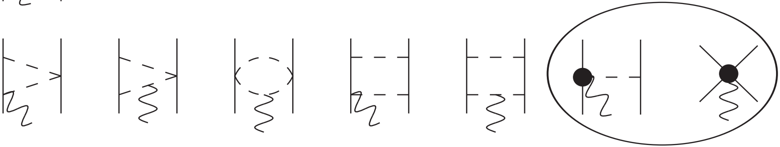
$$\mathbf{j}^{(-2)} \propto [e_N(1)(\mathbf{p}_1' + \mathbf{p}_1) + i\mu_N(1)\sigma_1 \times \mathbf{q}] \times \delta(\mathbf{p}_2' - \mathbf{p}_2) + 1 \leftrightarrow 2$$

$\mathcal{O}(Q^{-1})$ 

$\text{"standard" one} - \text{pion} - \text{exchange}$

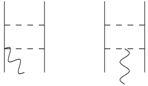
$\mathcal{O}(Q^{0})$ 

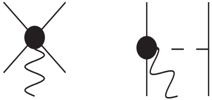
$\blacksquare$ $= \text{relativistic corrections}$

$\mathcal{O}(Q^{1})$ 

Comments

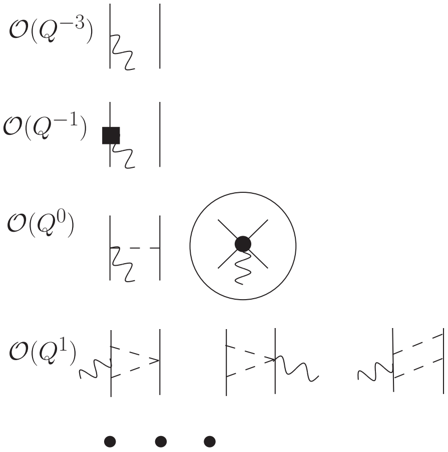
- Similar results between Pastore *et al.* and Kölling *et al.*
- Differences with Park *et al.*

  for the box-diagrams 

  for the terms 

- LECs fitted to selected $A = 2, 3$ EM observables
  $[(\sigma_{np})\ \mu_d, \mu^{S/V}(A = 3)]$

Electroweak currents in χEFT: power counting for $\mathbf{j}^A$



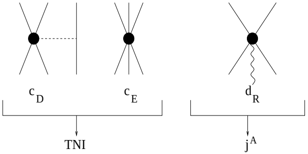
Note:

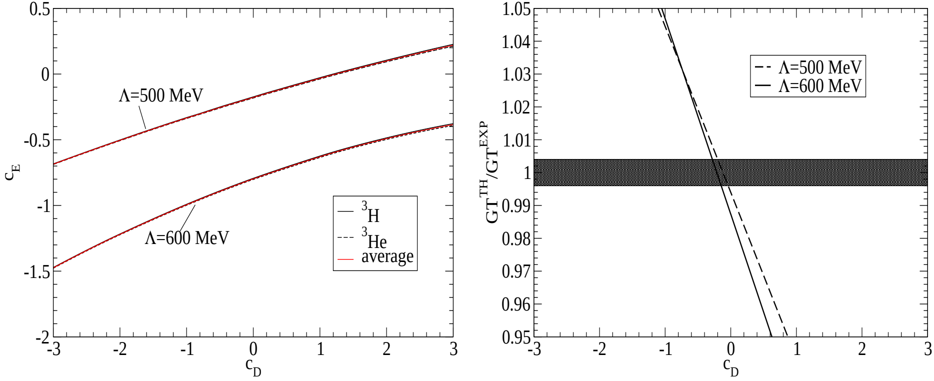
- $\mathcal{O}(Q^1)$: two-pion-exchange
  A. Baroni *et al.*, PRC **93**, 015501 (2016)
- Park *et al.* up to $\mathcal{O}(Q^0)$
  $\rightarrow$ one LEC - d_R

$$d_R = \frac{M_N}{\Lambda_\chi g_A} c_D + \frac{1}{3} M_N (c_3 + 2c_4) + \frac{1}{6}$$

A. Gardestig and D.R. Phillips, PRL **96**, 232301 (2006)

- fit c_D and c_E (in TNI at N2LO) to $B(A=3)$ and GT_{Exp}





$\Rightarrow \{c_D; c_E\}_{\rm MAX}$ and $\{c_D; c_E\}_{\rm MIN}$

Model	Λ [MeV]	c_D	c_E	B(^{4}He) [MeV]	$^2a_{nd}$ [fm]
N3LO/N2LO*	500	1.0	−0.029	28.36	0.675
N3LO/N2LO	500	−0.12	−0.196	28.49	0.666
N3LO/N2LO	600	−0.26	−0.846	28.64	0.696
Exp.				28.30	0.645(10)

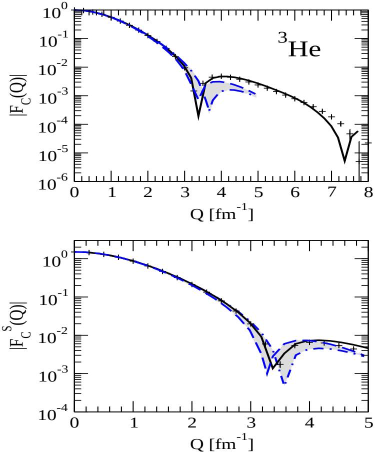
L.E. Marcucci *et al.*, PRL **108**, 052502 (2012); M. Viviani *et al.*, PRL **111**, 172302 (2013)

Electromagnetic structure of $A = 2, 3, 4$ nuclei

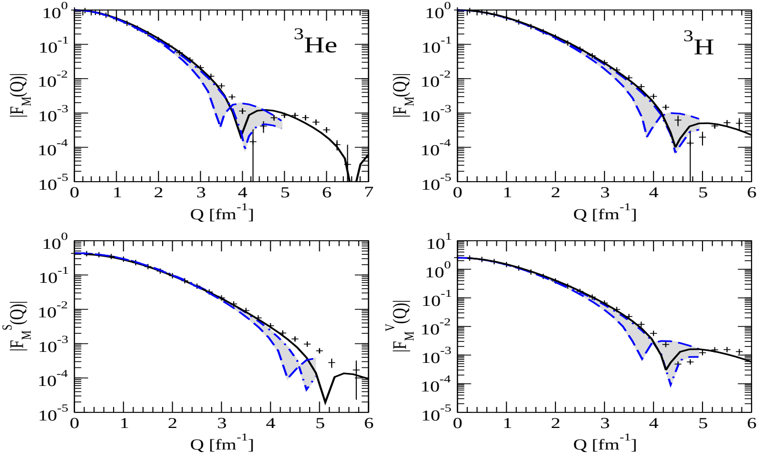
	PhenAp	χEFT	Exp.
$r_c(d)$ [fm]	**2.119**	**2.126 ± 0.004**	2.130 ± 0.010
$\mu(d)$ [n.m.]	0.847	0.8574	0.8574
$Q(d)$ [fm^2]	**0.280**	**0.2836 ± 0.0016**	0.2859 ± 0.0003
$r_c(^3\mathrm{He})$ [fm]	**1.928**	**1.962 ± 0.004**	1.973 ± 0.014
$r_m(^3\mathrm{He})$ [fm]	1.909	1.920 ±0.007	1.976 ±0.047
$\mu(^3\mathrm{H})$ [n.m.]	2.953	2.979	2.979
$\mu(^3\mathrm{He})$ [n.m.]	−2.125	−2.128	−2.128
$r_c(^4\mathrm{He})$ [fm]	**1.639**	**1.663 ± 0.011**	1.681 ± 0.004

L.E. Marcucci *et al.*, JPG **43**, 023002 (2016)

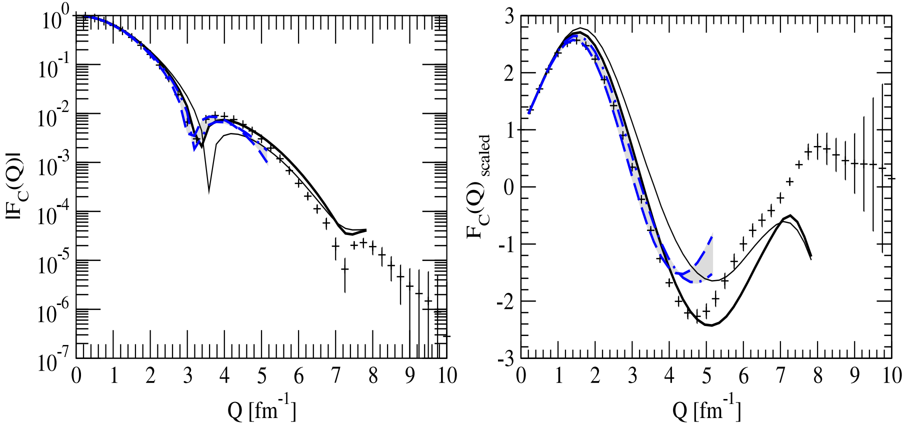
$A = 3$ charge form factors



$A = 3$ magnetic form factors



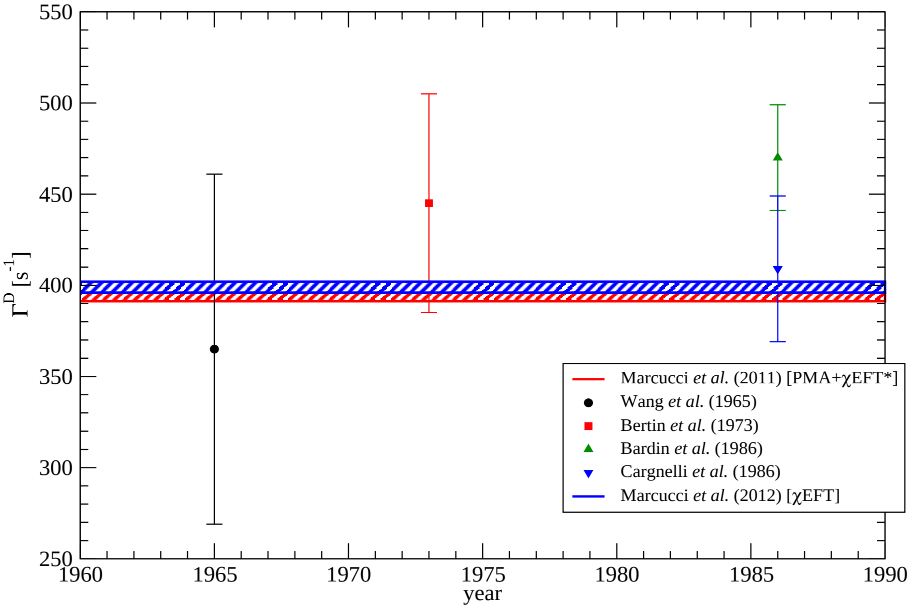
$A = 4$ form factor



Scaling function: $F_C(Q)_{\rm scaled} = F_c(Q)/[F_0 \exp(-Q/k)]$, with $F_0 = 1$ & $k = 0.760488$ fm^{-1}

Muon capture on $A = 2, 3$ nuclei

- $\mu^- + d \rightarrow n + n + \nu_\mu \longrightarrow$ capture rate in the doublet hyperfine state Γ^D
- $\mu^- + {}^3\mathrm{He} \rightarrow {}^3\mathrm{H} + \nu_\mu$ Γ_0
- $\mu^- + {}^3\mathrm{He} \rightarrow n + d + \nu_\mu \longrightarrow$ total capture rate Γ_{nd}
- $\mu^- + {}^3\mathrm{He} \rightarrow n + n + p + \nu_\mu$ Γ_{nnp}



550
500
450
400
350
300
250
1960
1965
1970
1975
1980
1985
1990
year
Γ^D [s^{-1}]
Marcucci *et al.* (2011) [PMA+χEFT*]
Wang *et al.* (1965)
Bertin *et al.* (1973)
Bardin *et al.* (1986)
Cargnelli *et al.* (1986)
Marcucci *et al.* (2012) [χEFT]

Muon capture on $A=3$

Γ_0(PhenAp)=1495(11) s^{-1}

$\Gamma_0(\chi$EFT)=1494(21) s^{-1}

vs. Γ_0(Exp)=1496(4) s^{-1}

L.E. Marcucci *et al.*, PRC **83**, 014002 (2011); PRL **108**, 052502 (2012)

PhenAp:1b+FE	capture rate Γ in s^{-1}			
	Γ_{nd}		Γ_{nnp}	$\Gamma_{nd}+\Gamma_{nnp}$
	SPW	Full	Full	Full
AV18	2046	604	169	773
AV18+Urbana IX	1956	544	154	698
Experimental results:				
O.A. Zaĭmidoroga *et al.*, PLB **6**, 100 (1963)				660 ± 160
L.B. Auerbach *et al.*, PRB **138**, 127 (1965)				$665\ ^{+170}_{-430}$
E.M. Maev *et al.*, Hyp. Interact. **101/102**, 423 (1996)				720 ± 70
V.M. Bystritsky *et al.*, PRA **69**, 012712 (2004)				
method I	491 ± 125		187 ± 11	678 ± 126
method II	497 ± 57		190 ± 7	687 ± 60

J. Golak *et al.*, PRC **90**, 024001 (2014) $\longrightarrow$ J. Golak's talk

The proton-proton weak capture reaction

$S(E)$ in χEFT and PhenAp

- Energy range 2 keV – 100 keV
- PhenAp or χEFT + FULL EM interaction
- *pp* $L \leq 1$ partial waves: ${}^1S_0$ + all *P*-waves

$S(0)$ cumulative contributions (in 10^{-23} MeV fm^2)

	${}^1S_0$	$\cdots + {}^3P_0$	$\cdots + {}^3P_1$	$\cdots + {}^3P_2$
PhenAp	4.000(3)	4.003(3)	4.015(3)	4.033(3)
χEFT	4.008(6)	4.011(6)	4.020(6)	4.030(6)

$$S(0) = \mathbf{(4.030 \pm 0.006)} \times 10^{-23} \ \textbf{MeV fm}^2$$

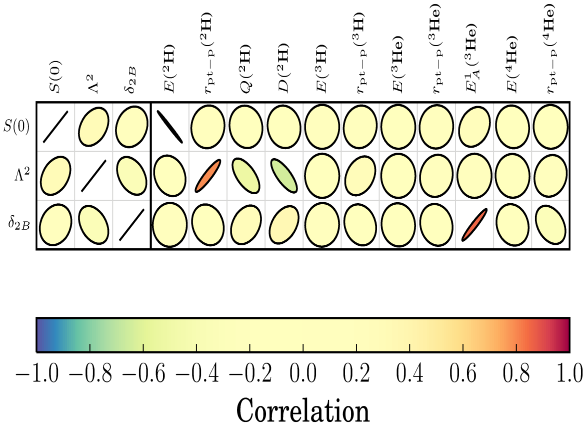
vs.

$$S(0)^{\rm SFII} = (4.01 \pm 0.04) \times 10^{-23} \ \text{MeV fm}^2$$

SFII: E.G. Adelberger *et al.*, RMP **83**, 195 (2011)

L.E. Marcucci *et al.*, PRL **110**, 192503 (2013)

Latest calculation in χEFT consistent up to N2LO

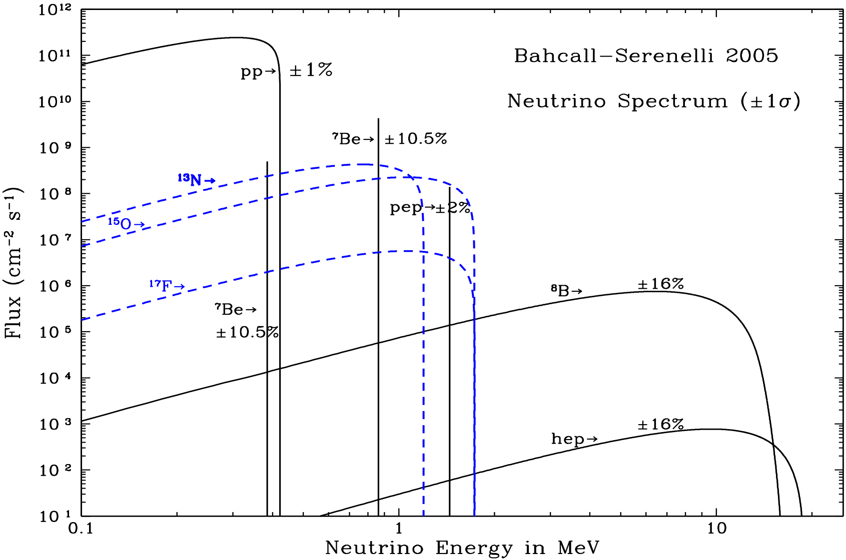


$$S_{\rm cor}(0) = (4.047^{+0.024}_{-0.032}) \times 10^{-23}\ \mathrm{MeV\ fm}^2$$

B. Acharya *et al.*, PLB **760**, 584 (2016) ⟶ B.D. Carlsson's talk

Effects on

- age of mid and old stellar clusters (1-12 Gyr)
- standard solar model predictions



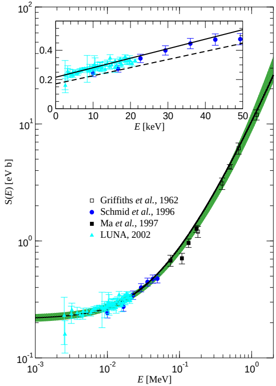
Neutrino fluxes relative differences

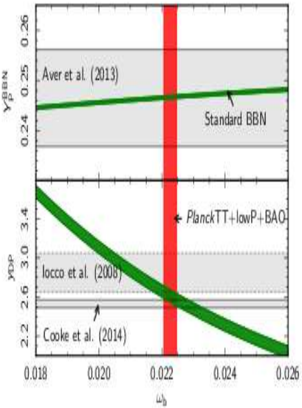
	MSV13(S+P) reference	MSV13(S)	NACRE99	SFII	JINA
		relative differences			
T_c [10^7 K]	1.54794	-1‰	-3‰	-2‰	-1‰
$\Phi^\nu_{\rm pp}$ [10^{10}]	6.020	1‰	2‰	2‰	1‰
$\Phi^\nu_{\rm pep}$ [10^8]	1.446	-2‰	-6‰	-2‰	-1‰
$\Phi^\nu_{\rm hep}$ [10^3]	8.584	-1‰	-3‰	<1‰	2‰
$\Phi^\nu_{\rm Be-7}$ [10^9]	4.503	**−1%**	**−3%**	**−1%**	-9‰
$\Phi^\nu_{\rm B-8}$ [10^6]	3.694	**−3%**	**−7%**	**−4%**	**−2%**
$\Phi^\nu_{\rm N-13}$ [10^8]	2.417	**−2%**	**−6%**	**−3%**	**−1%**
$\Phi^\nu_{\rm O-15}$ [10^8]	1.811	**−3%**	**−8%**	**−4%**	**−2%**
$\Phi^\nu_{\rm F-17}$ [10^6]	3.373	**−3%**	**−8%**	**−4%**	**−2%**

$$r_{12} \quad \rightarrow \quad \int_0^\infty S(E) \exp\left(-2\pi\eta - \frac{E}{kT_c}\right) dE \qquad \eta = \frac{Z_1 Z_2 \alpha}{v_{rel}}$$

E. Tognelli *et al.*, PLB **742**, 189 (2015)

New ongoing measurement by the **LUNA Collab.** at LNGS in the energy range of interest for BBN





SFII: E.G. Adelberger *et al.*, RMP **83**, 195 (2011)

Planck Collab., arXiv:1501.01589 (2015)

"New" calculation (PhenAp)

- $V_{NN} + V_{NNN} \rightarrow$ AV18+UIX $\Rightarrow A = 3$ systems well reproduced
- test of the nuclear scattering wave functions by calculating $\langle H \rangle = \langle \Psi^{LSJ} | H | \Psi^{LSJ} \rangle = E_{cm} - B_d$ in a box with $R_{box} = 70$ fm
- EM current $\rightarrow$ state-of-the-art (1b + 1b RC* + MEC)

* 1b $\mathcal{O}(1/m^3)$ term$\rightarrow$ RC "borrowed" from χEFT

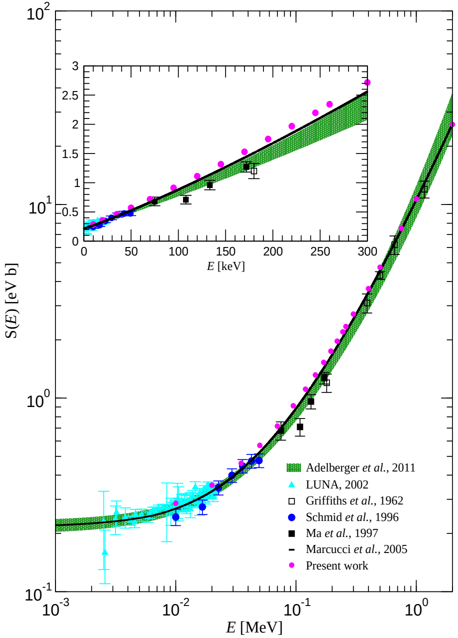
L. Girlanda *et al.*, PRL **105**, 232502 (2010): few % contribution to $n + d \rightarrow {}^3$H+γ

$\Rightarrow$ included in the study of $p + d \rightarrow {}^3$He+γ

PhenAp results

E [keV]	ΔS_{WF} [%]	$\Delta \mathbf{j}^{(RC)}$ [%]
35	1.1	+1.3
70	0.4	+2.1
95	0.3	+2.3
120	0.8	+2.4
145	0.4	+2.5
170	0.4	+2.6
195	0.4	+2.6
220	0.5	+2.8
245	0.4	+2.7

L.E. Marcucci *et al.*, PRL **116**, 102501 (2016)



Implications for Big Bang Nucleosynthesis

- $S(E) \to$ primordial ^{2}H abundance
- ^{2}H/H abundance as function of $\Omega_b h^2$ and N_{eff}
- Likelihood function to get the best fit on $\Omega_b h^2$ and N_{eff}

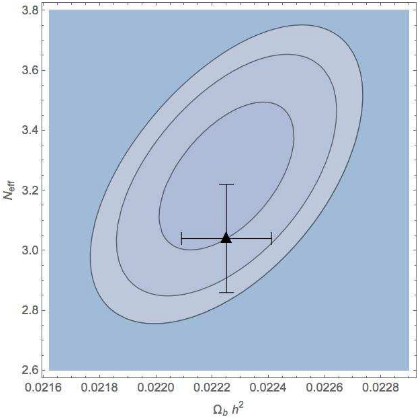
$^2\mathrm{H/H}|_{TH} = (2.46 \pm 0.03 \pm 0.03) \times 10^{-5}$
$\Omega_b h^2 \to$Planck 2015 & standard N_{eff}

vs.

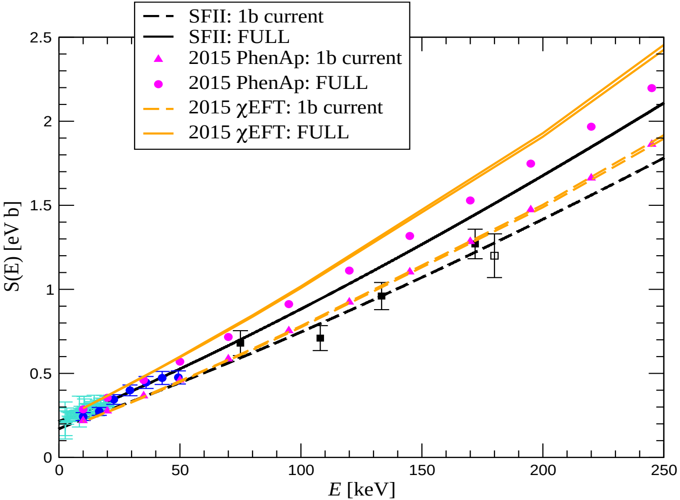
$^2\mathrm{H/H}|_{Exp}=(2.53 \pm 0.04) \times 10^{-5}$

L.E. Marcucci *et al.*, PRL **116**, 102501 (2016)

Erratum: PRL **117**, 049901 (2016)



PRELIMINARY RESULTS in χEFT: zoom for $E = 0 - 250$ keV

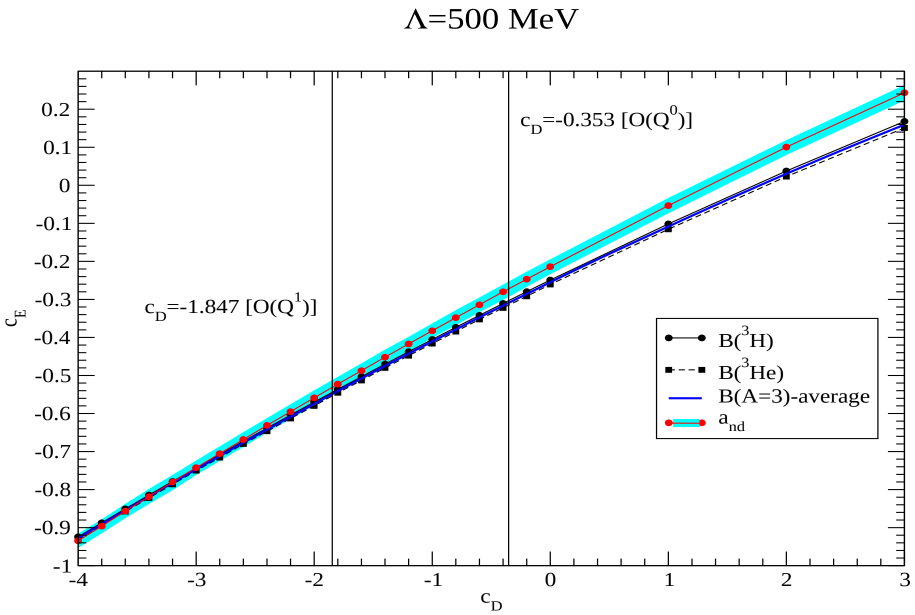


Ongoing work: the weak sector in χEFT

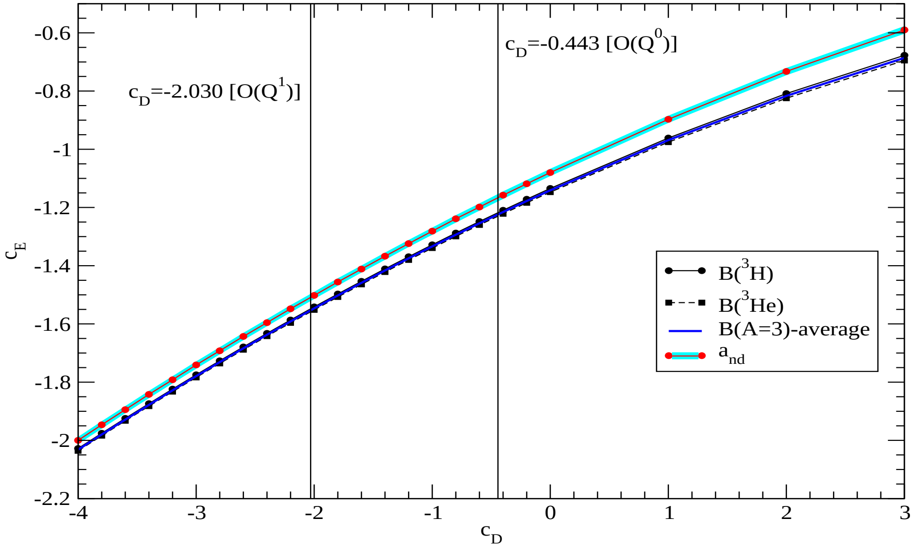
- Weak current up to $\mathcal{O}(Q^1)$ + N3LO(Idaho)/N2LO(local form)
- Fit c_D and c_E using $B(A=3)$ and GT_{Exp}

Λ [MeV]		c_D	c_E	a_{nd} [fm]
500	$\mathcal{O}(Q^0)$	-0.353	-0.305	0.665
	$\mathcal{O}(Q^1)$	-1.847	-0.548	0.654
600	$\mathcal{O}(Q^0)$	0.443	1.224	0.699
	$\mathcal{O}(Q^1)$	2.030	1.553	0.688
Exp.				0.645(10)

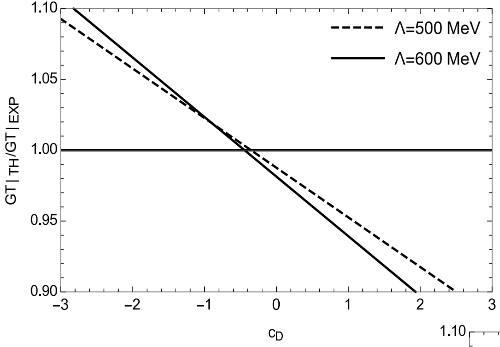
A. Baroni, arXiv:1605.01620, PRC in press


Λ=500 MeV
c_D=-0.353 [O(Q^0)]
c_D=-1.847 [O(Q^1)]
B(^3H)
B(^3He)
B(A=3)-average
a_nd
c_E
c_D
0.2
0.1
0
-0.1
-0.2
-0.3
-0.4
-0.5
-0.6
-0.7
-0.8
-0.9
-1
-4
-3
-2
-1
0
1
2
3

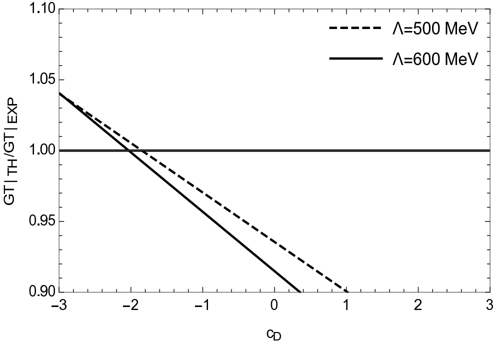
Λ=600 MeV



c_D=-2.030 [O(Q^1)]
c_D=-0.443 [O(Q^0)]
-0.6
-0.8
-1
-1.2
-1.4
-1.6
-1.8
-2
-2.2
c_E
-4
-3
-2
-1
0
1
2
3
c_D
B(^{3}H)
B(^{3}He)
B(A=3)-average
a_{nd}



$\mathcal{O}(Q^0)$



$\mathcal{O}(Q^1)$

Outlook

Using the χEFT weak current up to $\mathcal{O}(Q^1)$

- μ-capture on deuteron and ^{3}He
- $p + p \rightarrow d + e^+ + \nu_e$
- $p + {}^3\mathrm{He} \rightarrow {}^4\mathrm{He} + e^+ + \nu_e$ – the *hep* reaction

Within the PhenAp

- μ-capture on ^{3}He - breakup channels: beyond 1b term
- μ-capture on ^{3}H $[\mu^- + {}^3\mathrm{H} \rightarrow n + n + n + \nu_\mu] \Rightarrow T = 3/2$ channel $\longrightarrow$ J. Golak's talk