Solvable Models for a Few Atoms in a Few One-Dimensional Wells

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Permanent: American University, Temporary: Aarhus University

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References:

- N.L. Harshman, "One-Dimensional Traps, Two-Body Interactions, Few-Body Symmetries: I. One, Two, and Three Particles," Few-Body Systems, 75, 11-43 (2016), arXiv: 1501.00215
- N.L. Harshman, "One-Dimensional Traps, Two-Body Interactions, Few-Body Symmetries: II. *N* Particles," Few-Body Systems, 75, 45-69 (2016), arXiv: 1505.00659
- N.L. Harshman, "Symmetries of Two Interacting Particles in One-Dimensional Double Wells," to appear.

The Model

$$H = \sum_{i=1}^{N} \left(-\frac{\hbar^2}{2m} \frac{\partial^2}{\partial x_i^{\,2}} + V(x_i) \right)$$

N non-interacting particles in trap

$$+\tau \sum_{i=1}^{N} \sum_{j=1}^{M-1} \delta(x_i - a_j)$$

(*M*-1) barriers, *M* wells

$$+\gamma \sum_{\langle i,j \rangle} \delta(x_i - x_j)$$

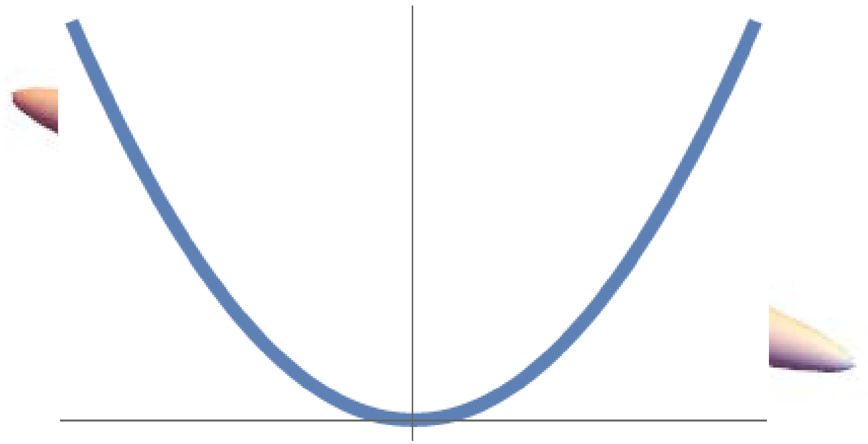
Contact interactions

Symmetry, integrability, and solvability depend on

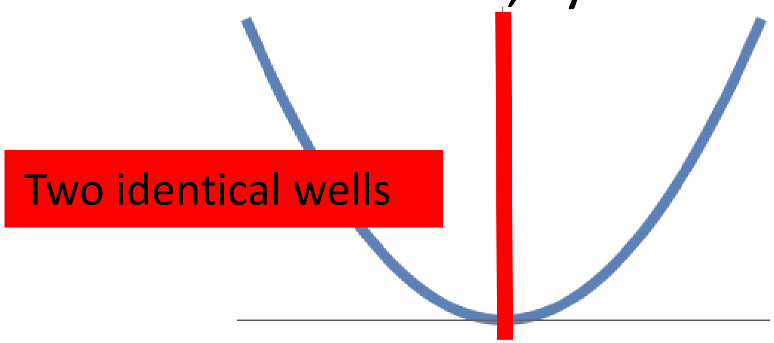
$$N, V(x), \tau, a_j, \gamma$$

Why study symmetries of one-dimensional few-body, few-well models?

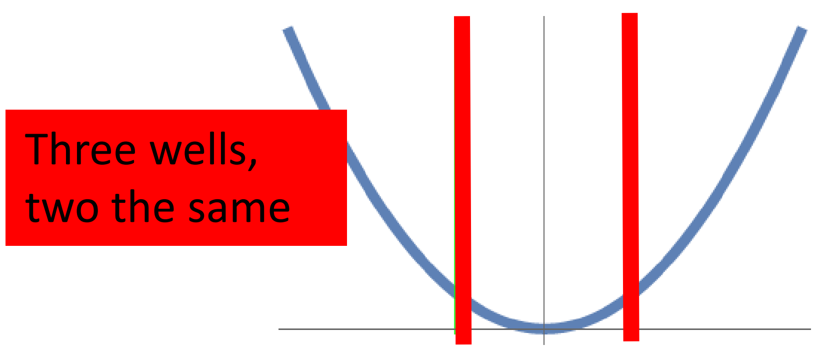
- Experimentally relevant: one-dimensional traps with tunable barriers and interactions



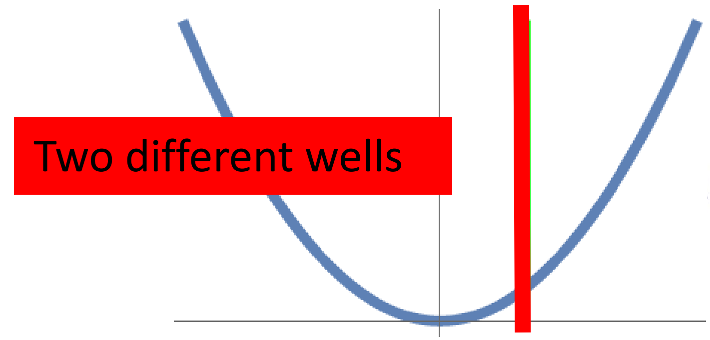
One barrier, symmetric



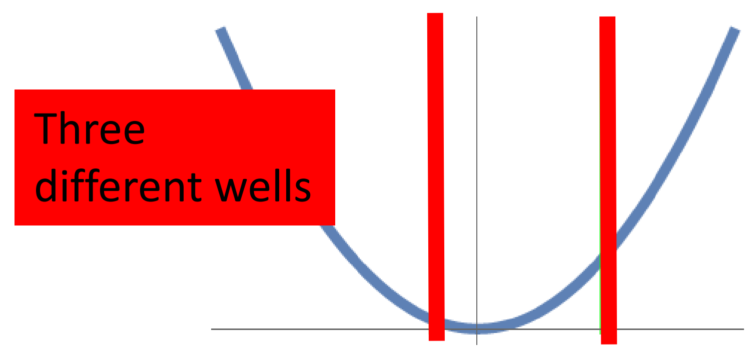
Two barriers, symmetric



One barrier, asymmetric



Two barriers, asymmetric



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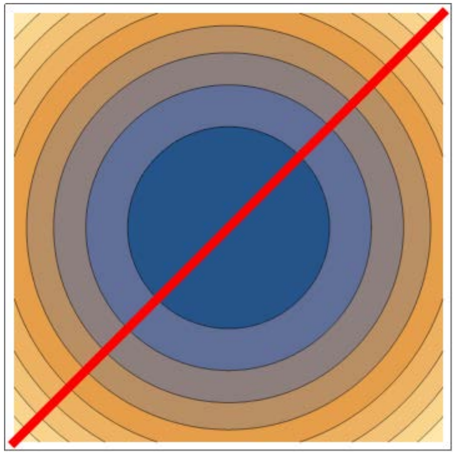
- Identify integrable and solvable models
  - Mathematical touchstones
  - Integrability and chaos

Symmetries of Limiting Cases

Besides parity and permutation symmetry

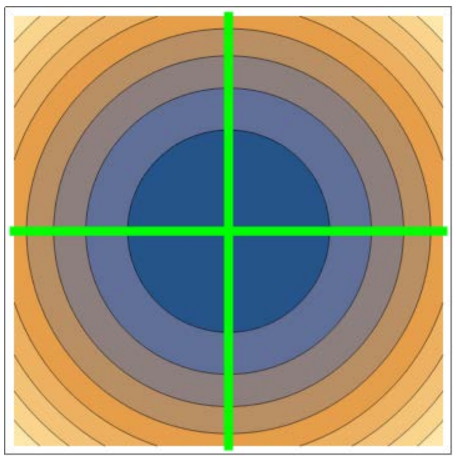
- Symmetry of separability
  - No interactions $\gamma = 0$
- Well permutation symmetry
  - No tunneling between wells $\tau \rightarrow \infty$
- Ordering permutation symmetry
  - Unitary limit $\gamma \rightarrow \infty$



$\tau = 0, \gamma = 0$ — Algebraically solvable

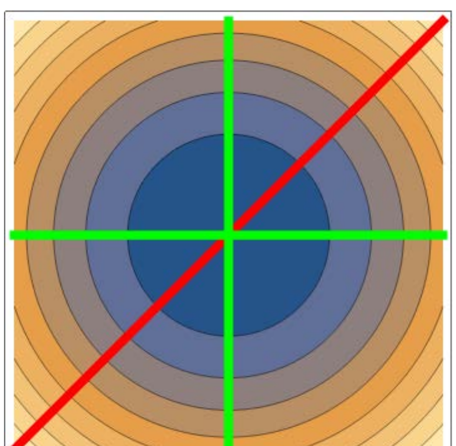
$\tau = 0, \gamma \neq 0$ — Solvable

$\tau = 0, \gamma \to \infty$ — Algebraically solvable



$\tau \neq 0, \gamma = 0$ — Solvable

$\tau \to \infty, \gamma = 0$ — Algebraically solvable



$\tau \to \infty, \gamma \neq 0$ — Solvable

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$\tau \to \infty, \gamma \to \infty$ — Algebraically solvable

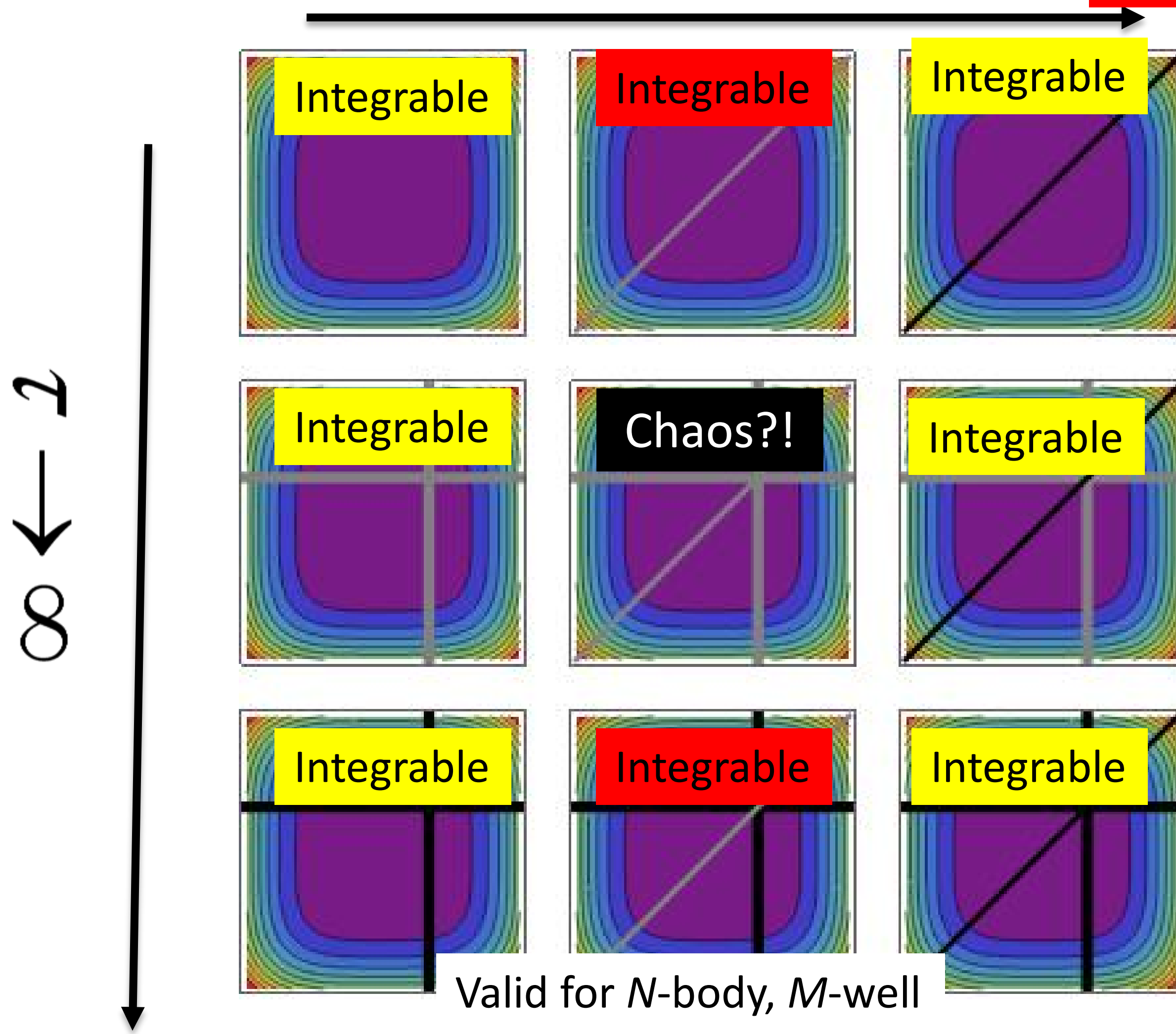
$\tau \neq 0, \gamma \neq 0$ — Nuffin!

Model space

Integrable for all 1-D traps

$\gamma \to \infty$

Integrable for some *N*, some 1-D traps



Why study symmetries of one-dimensional few-body, few-well models?

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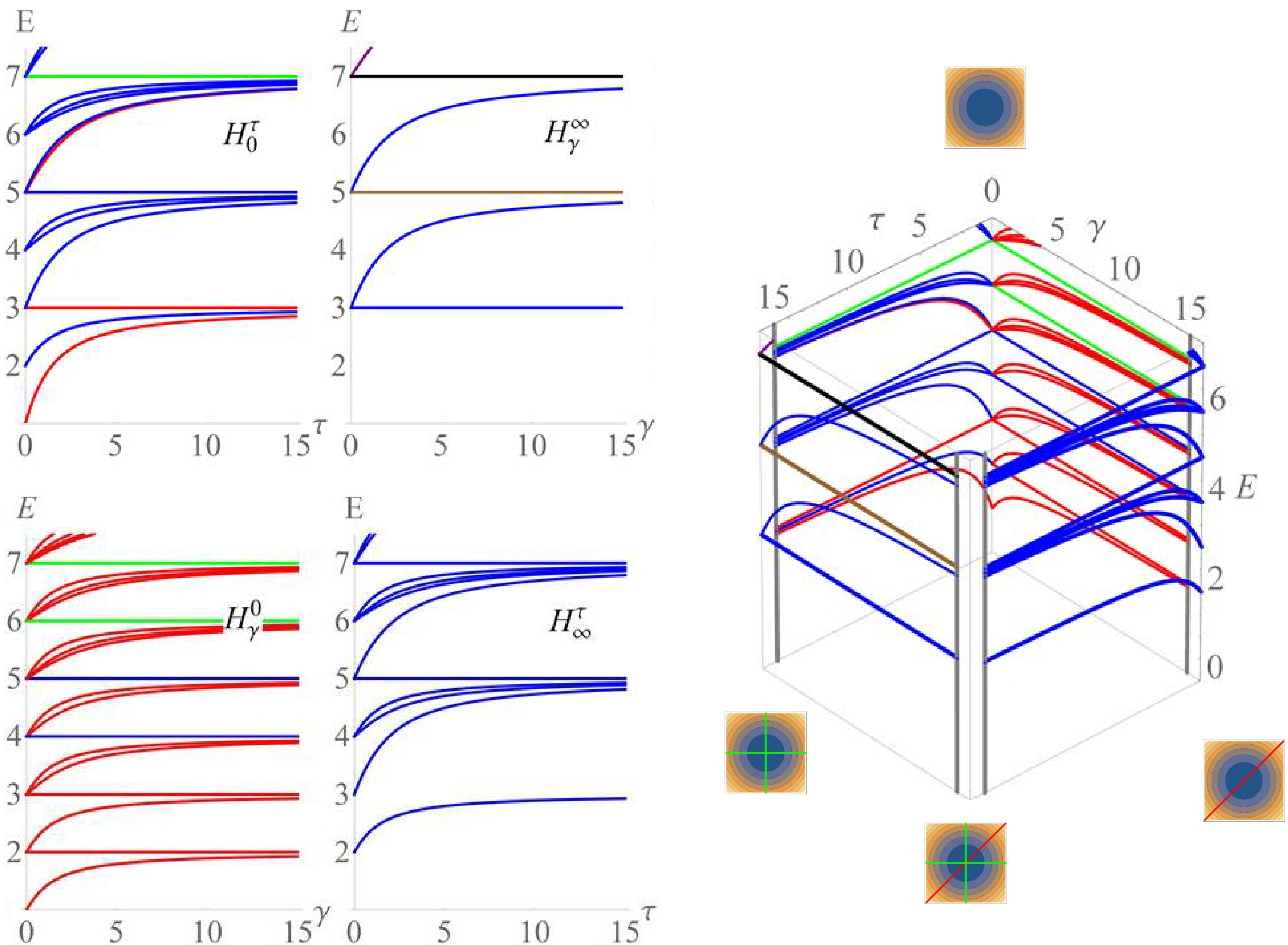
- Digitization and control of few-body quantum states for quantum information processing
  - Quantum abacus
  - Quantum combinatorics



E
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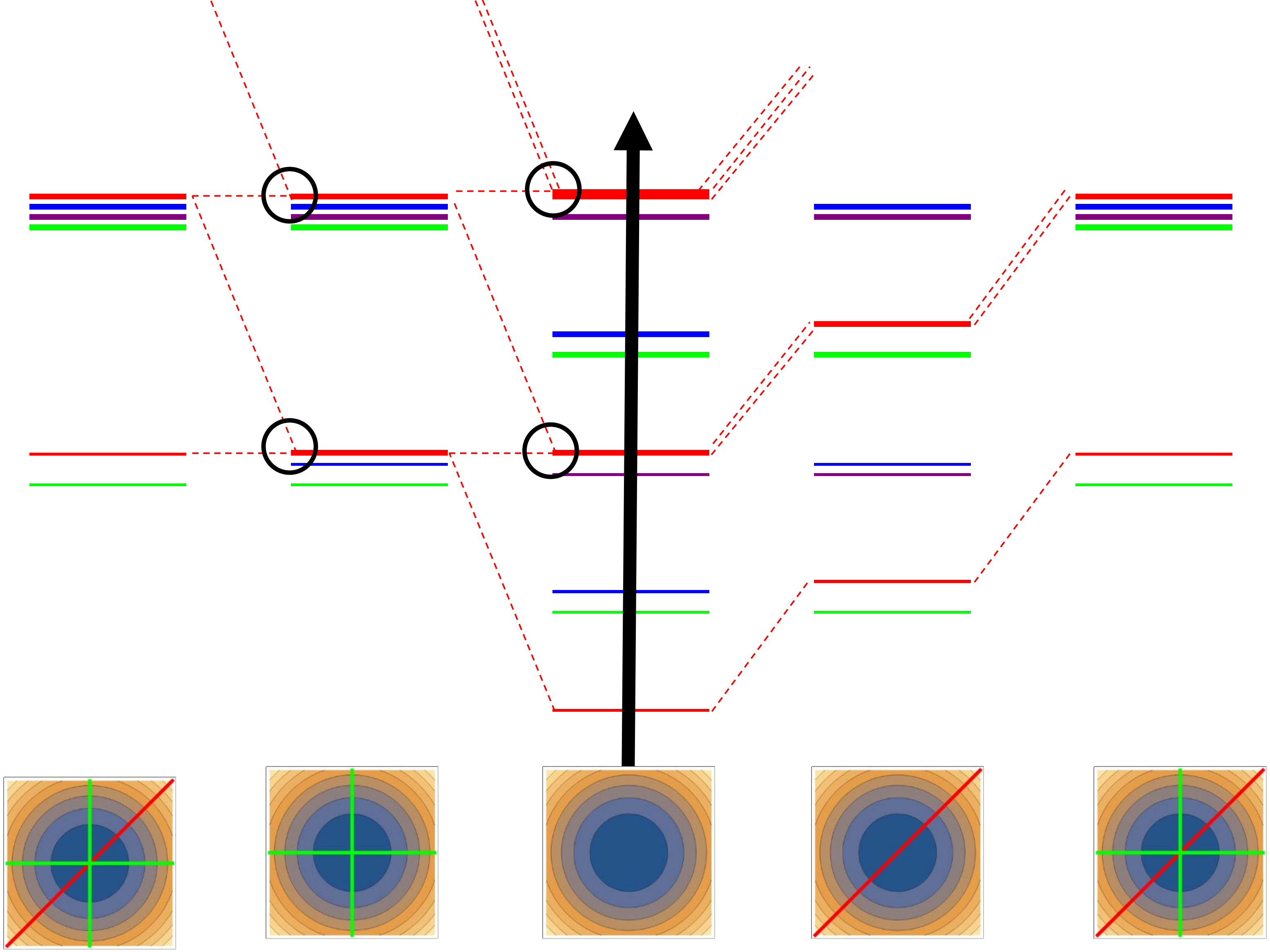

Positive parity bosons   Negative parity bosons

Positive parity fermions   Negative parity fermions

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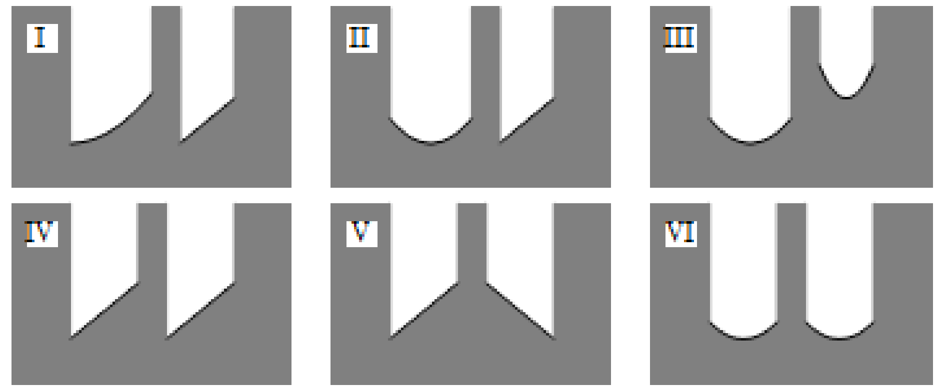
- Bridge to many-body, many-well models

Types of Symmetries

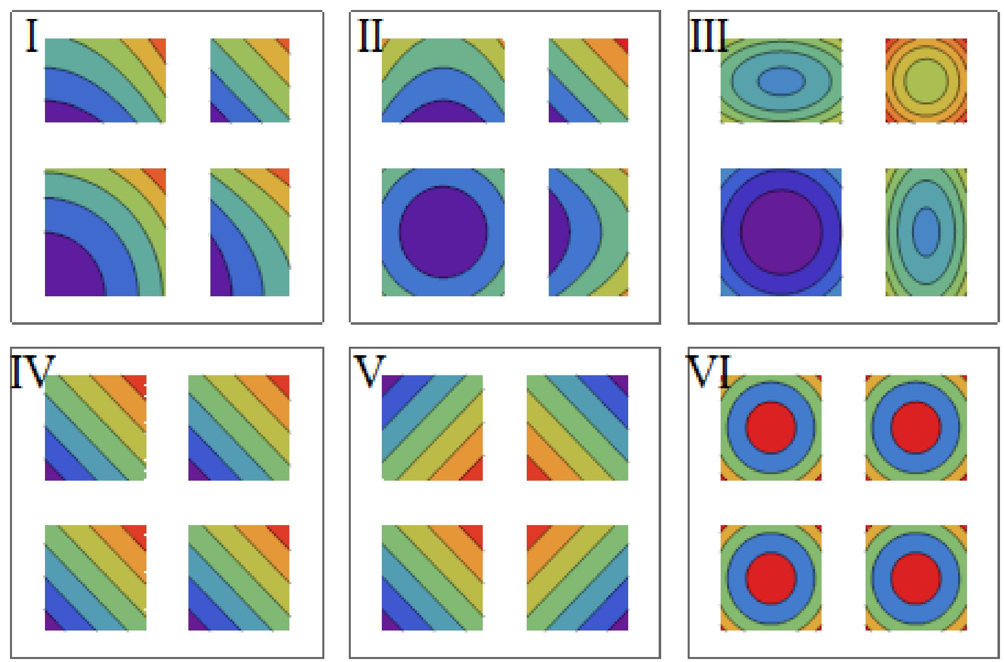
- Configuration space:
  - Parity, particle permutations, well permutations, ordering permutations
  - Linear vs. non-linear; global vs. local
  - Single-particle generated vs. emergent

Well Permutation Symmetry

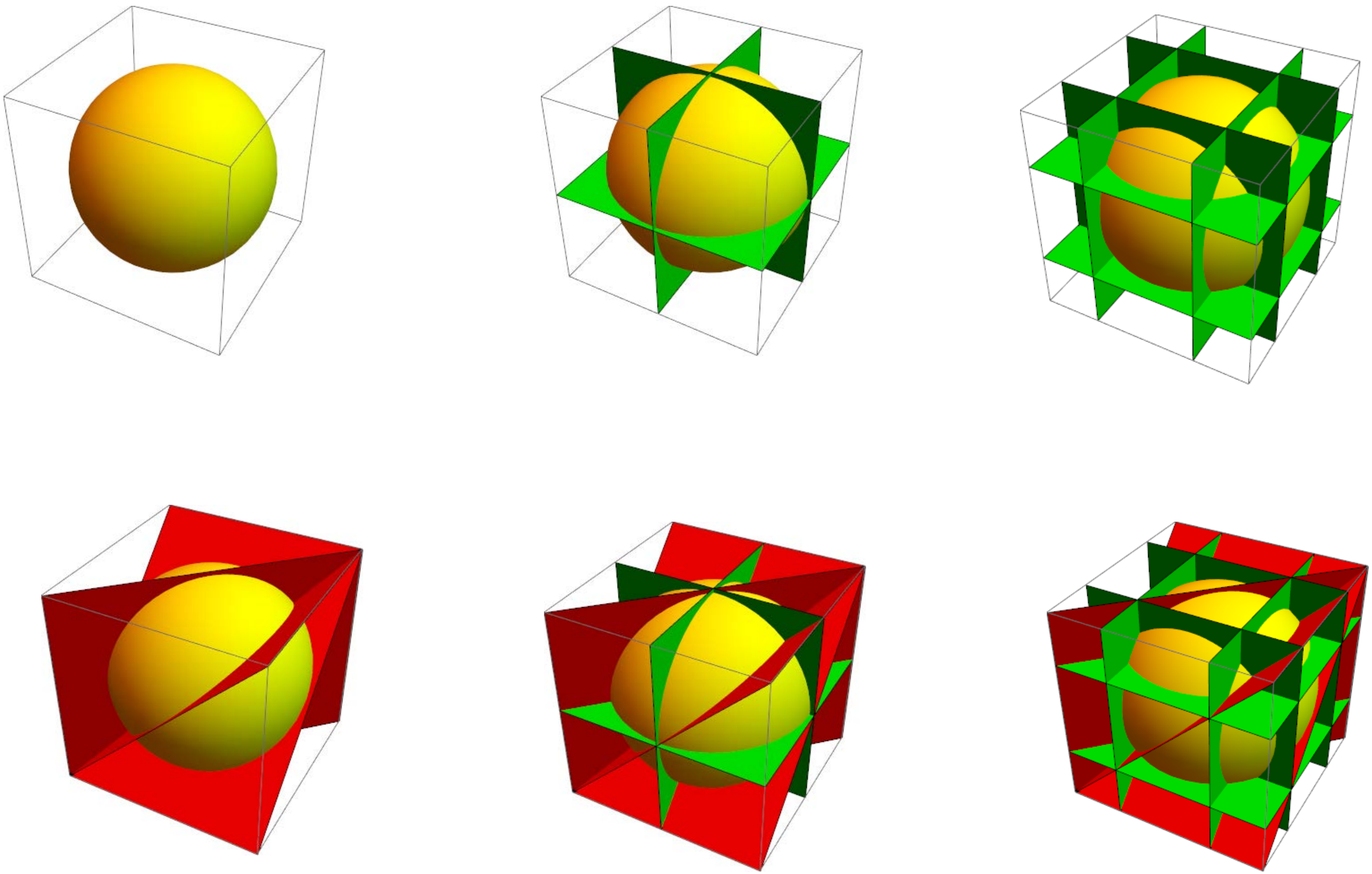
- One particle, two wells



- Two particles, two wells



Ordering Permutation Symmetry



Types of Symmetries

- Configuration space:
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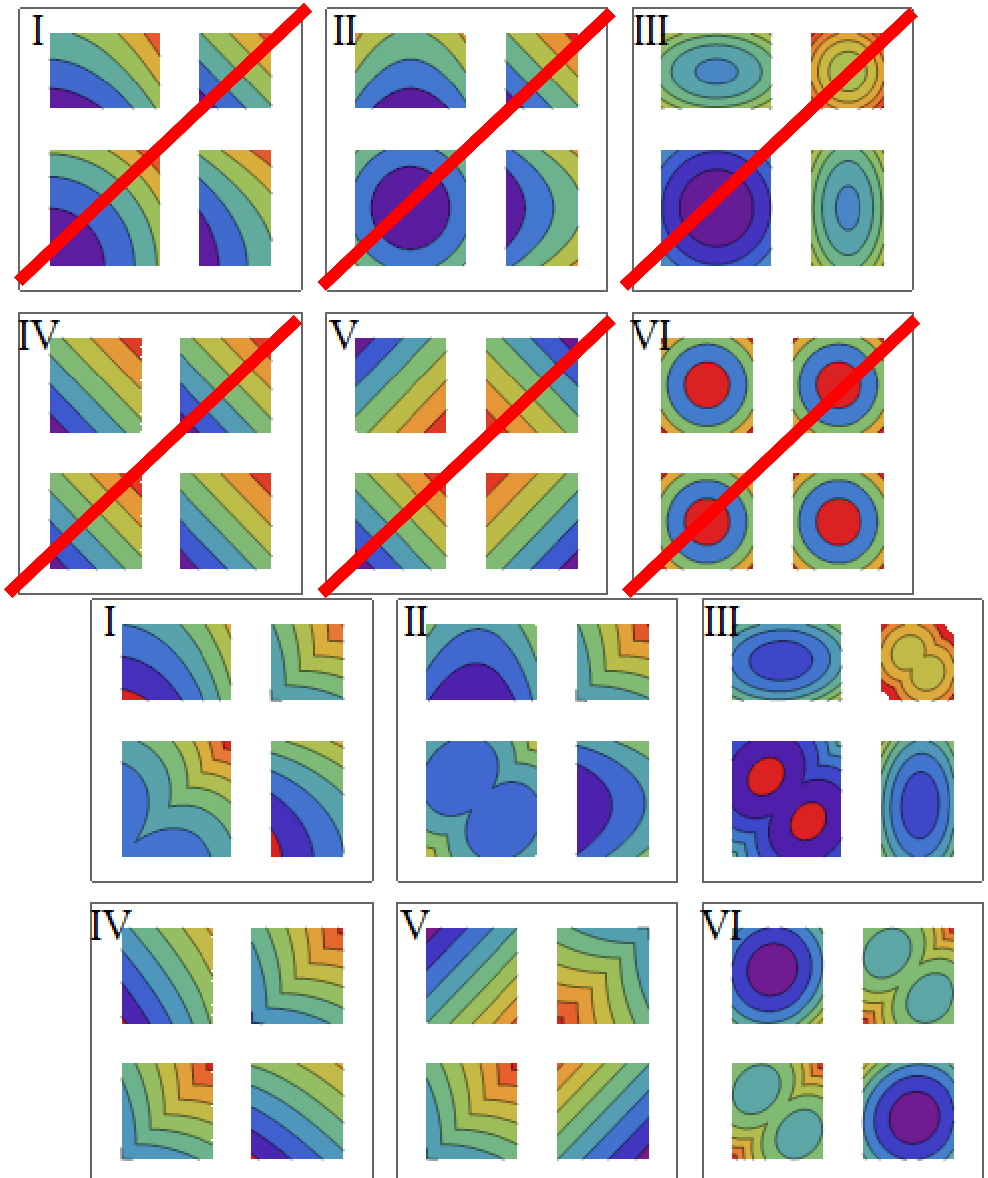
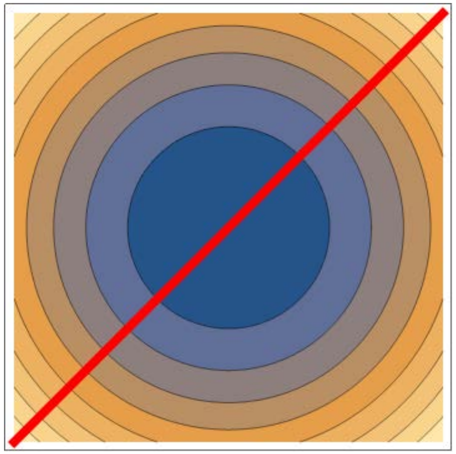
- Phase space:
  - Symmetry of separability
  - Harmonic oscillators: superintegrability and exact solvability

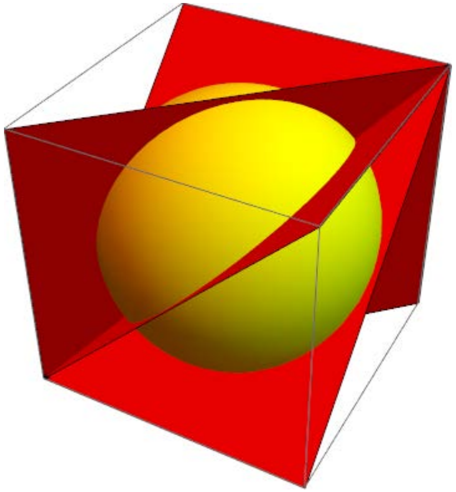
Symmetry of Separability

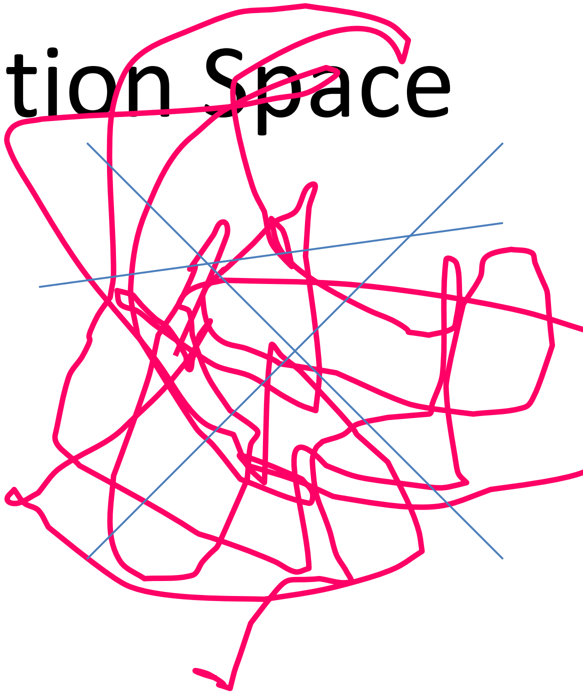
- No interactions
- Zero range interactions
- Finite range interactions

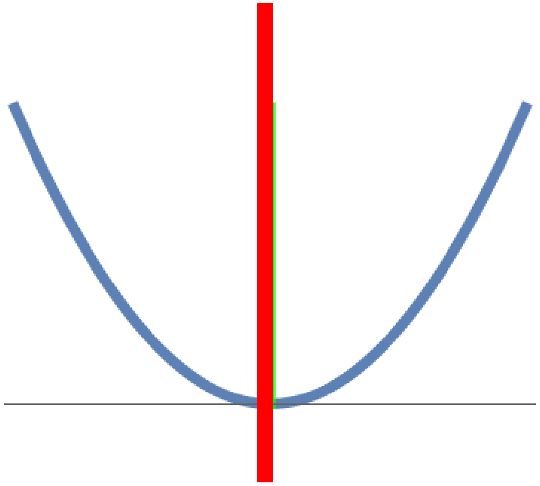


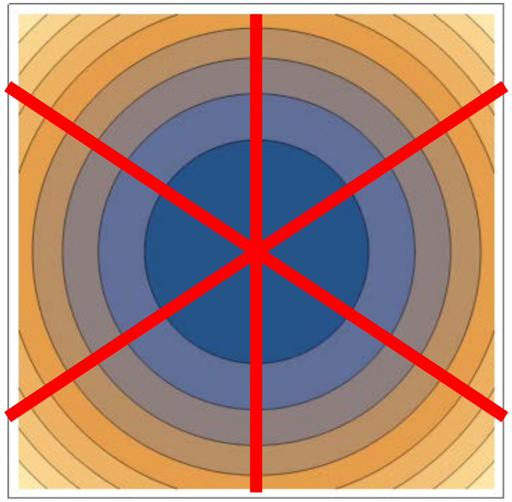
HO Separability in Configuration Space

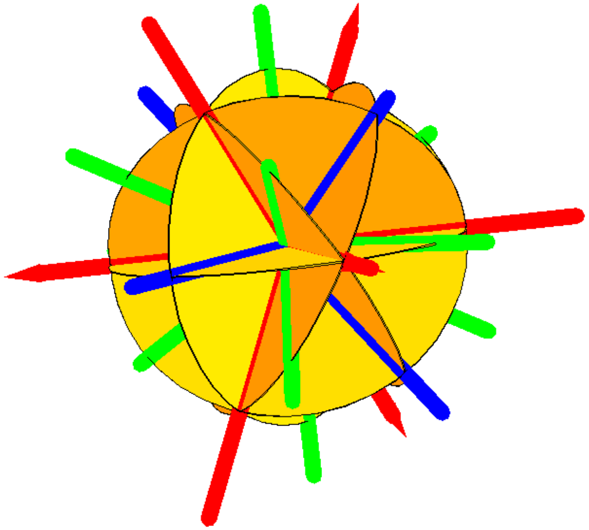












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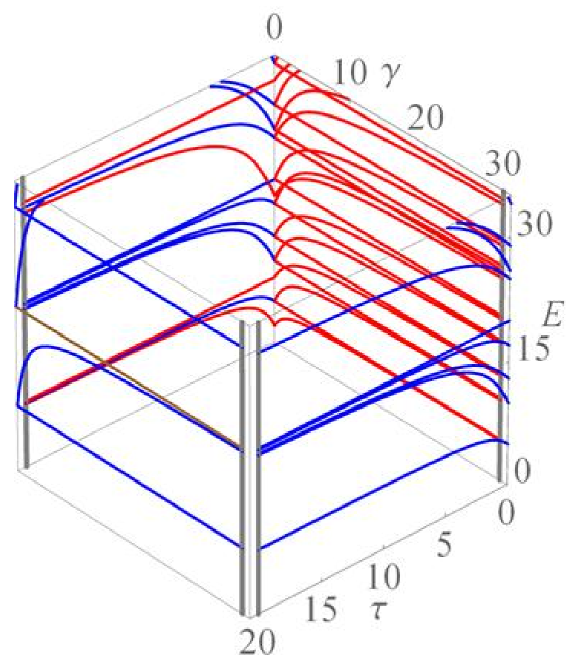
- Hilbert space:
  - Kinematic vs. dynamic
  - Total vs. partial: state permutation symmetry

What symmetry analysis gives

- Degeneracy of energy levels and spectroscopic labels
- Conserved quantities and efficient approximation schemes
- Adiabatic maps between energy level of different models
- Symmetry protected sectors of the Hilbert space
- Symmetry breaking analysis for more realistic models
- Diagnosis of integrability and solvability
- Different perspective on universality as emergent symmetry

Parameters	$a \neq 0$	$a = 0$
$\tau \neq 0, \gamma = 0$	$1, 2$	$1, 2$
$\tau \neq 0, \gamma \neq 0$	1	1
$\tau \neq 0, \gamma \to \infty$	2	2
$\tau \to \infty, \gamma = 0$	$1, 2$	$4, 8$
$\tau \to \infty, \gamma \neq 0$	$1, 2$	$2, 6$
$\tau \to \infty, \gamma \to \infty$	2	$2, 8$

γ	K
$\gamma = 0$	$(T_a \times P_2) \int W_{ABCD}$
$\gamma \neq 0$	$P_2 \int W_{AC} \times (T_a \times P_2) \int W_{BD}$
$\gamma \to \infty$	$T_a \int O_{2A} \int W_{AC} \times (T_a \times P_2) \int W_{BD}$



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References:

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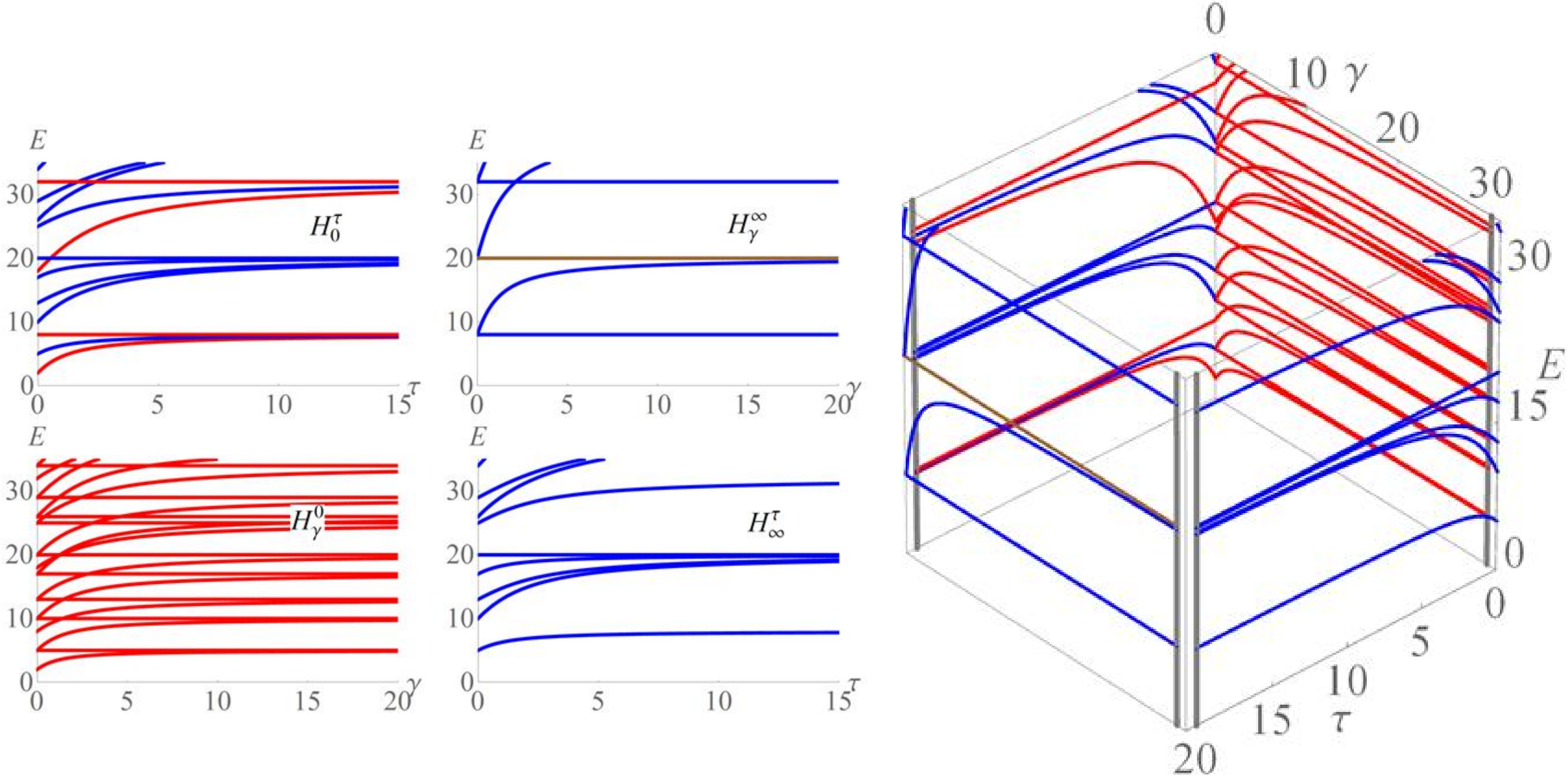

H_0^τ
H_γ^∞
H_γ^0
H_∞^τ
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