Break-up channels in Muon Capture on ^{3}He and ^{3}H

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Outline

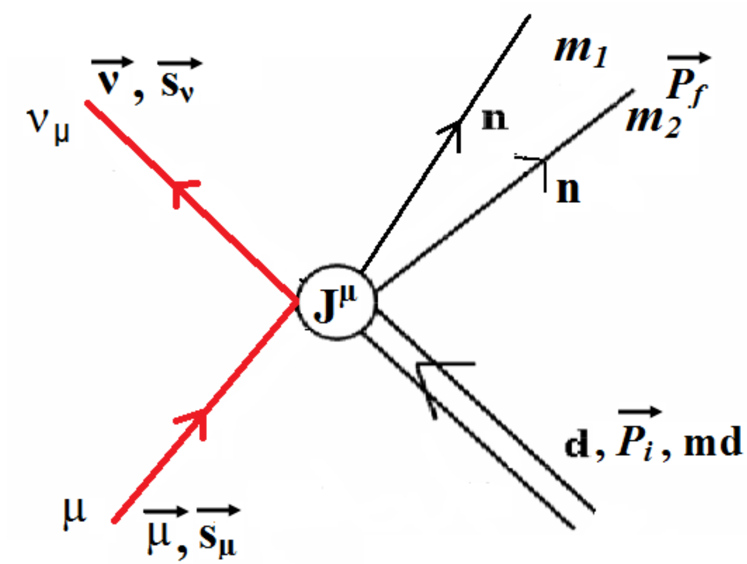
- Sketch of formalism
- Benchmark for $\mu^-+d \rightarrow \nu_\mu+n+n$ and $\mu^-+{}^3He \rightarrow \nu_\mu+{}^3H$
- Break-up channels: $\mu^-+{}^3He \rightarrow \nu_\mu+n+d$, $\mu^-+{}^3He \rightarrow \nu_\mu+n+n+p$

  and $\mu^-+{}^3H \rightarrow \nu_\mu+n+n+n$

  - predictions for total capture rates
  - comparison with recent experimental data
- Conclusions and outlook

Sketch of formalism

$$T_{fi} \propto L_\lambda N^\lambda$$



$$L_\lambda = \bar{u}\left(\vec{\nu}, s_\nu\right)\gamma_\lambda\left(1-\gamma_5\right)u\left(\vec{\mu}, s_\mu\right)$$

final nuclear state

nuclear weak current operator

initial nuclear state

$$N^\lambda = \left\langle \Psi_f \middle| j^\lambda \middle| \Psi_i \right\rangle$$

→ observables

Sketch of formalism (*cont.*)

Weak 3N current operator: single nucleon and many-nucleon parts

$$j^{\lambda} = \sum_{i} j^{\lambda}(i) + \sum_{i<j} j^{\lambda}(i,j) + j^{\lambda}(1,2,3) + \ldots$$

single nucleon current

$$j^{\lambda}(\vec{p}\,', s'; \vec{p}, s) = \bar{u}(\vec{p}\,', s') \begin{pmatrix} (g_1^V - 2m\, g_2^V)\gamma^{\lambda} \\ + g_2^V (p + p')^{\lambda} \\ + g_1^A \gamma^{\lambda}\gamma^5 \\ + g_2^A (p - p')^{\lambda} \gamma^5 \end{pmatrix} \tau_{-}\, u(\vec{p}, s)$$

$g_1^V, g_2^V, g_1^A, g_2^A$

weak nucleon formfactors

isospin lowering operator: proton „in", neutron „out"

Sketch of formalism (*cont.*)

Single nucleon current operator is used either in the nonrelativistic form or with $1/m^2$ corrections (RC) in the form borrowed from the Pisa group

Many-body current operators are discussed in many papers.
see for example:

L.E. Marcucci *et al.*, Phys.Rev.C **63**, 015801 (2001)
L.E. Marcucci *et al.*, Phys.Rev.C **83**, 014002 (2011)
L.E. Marcucci *et al.*, Phys. Rev. Lett. **108**, 052502 (2012)
G. Shen *et al.*, Phys. Rev. C **86**, 035503 (2012)
S. Ando *et al.*, Phys. Lett. B **533**, 25 (2002)
….

We take the 2N weak current operators from the meson-exchange model but neglect contributions from Delta excitations

Sketch of formalism (*cont.*)

Muon capture from the lowest K-shell of the muonic atom !

$$\psi_K(r) \equiv \psi_{100}(r) = \sqrt{\frac{(Z\,m'\alpha)^3}{\pi}}\, e^{-Z\,m'\alpha\, r}$$

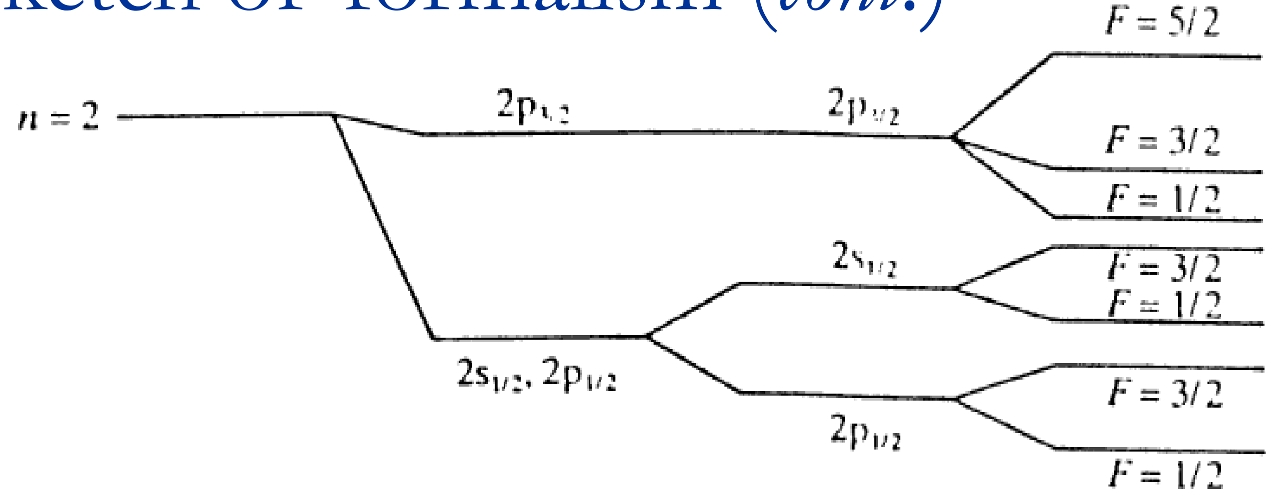
$$m' \equiv \frac{m_\mu m_Z}{m_\mu + m_Z}$$

reduced mass of the muonic atom

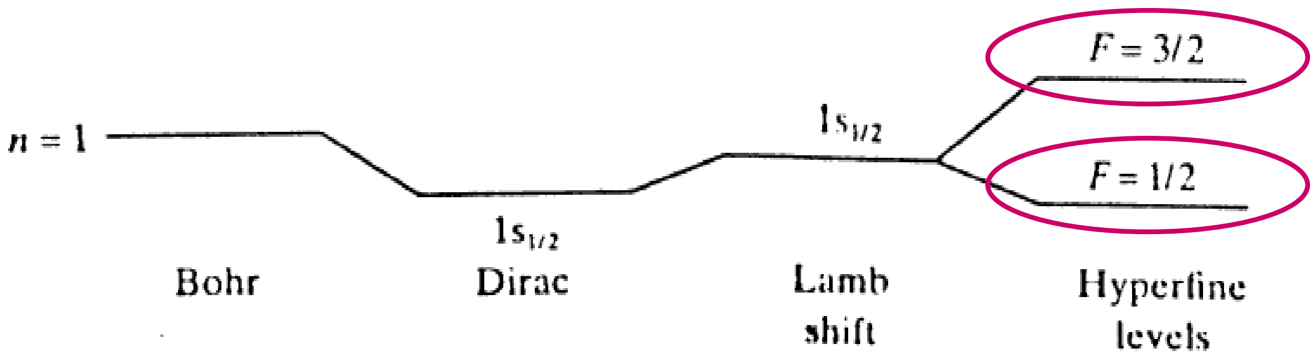
$$E_1 = -\frac{Z^2 \alpha^2 m'}{2}$$

muon binding energy negligible for *Z=1,2*

Sketch of formalism (*cont.*)



Hyperfine structure in deuteron



quadruplet states

doublet states

Sketch of formalism (*cont.*)

Momentum space calculations employing partial wave decomposition

Methods developed for electromagnetic reactions
(see for example: Phys. Rept. **415**, 89 (2005); Eur. Phys. J. A **24**, 31 (2005))
extended to describe following reactions

$$\mu^- + d \rightarrow \nu_\mu + n + n$$

$$\mu^- + {}^3\mathrm{He} \rightarrow \nu_\mu + {}^3\mathrm{H}$$

$$\mu^- + {}^3\mathrm{He} \rightarrow \nu_\mu + d + n$$

$$\mu^- + {}^3\mathrm{He} \rightarrow \nu_\mu + p + n + n, \;\; \mu^- + {}^3\mathrm{H} \rightarrow \nu_\mu + n + n + n$$

$${}^3\mathrm{H} \rightarrow \bar{\nu}_e + e + {}^3\mathrm{He}$$

$$\ldots$$

Benchmark for $\mu^- + d \rightarrow \nu_\mu + n + n$ *(cont.)*

Doublet (F=1/2) and quadruplet (F=3/2) capture rates in s^{-1}
calculated with the AV18 NN potential (neutron mass is used)

	F=1/2 PW	F=1/2 full	F=3/2 PW	F=3/2 full
SNC	351.8	382.3	9.8	11.4
SNC+MEC	356.9	391.0	10.3	12.1

↑

in agreement with results of the Pisa group:
L.E. Marcucci *et al.*, Phys.Rev. C **83**, 014002 (2011)

Benchmark for $\mu^- + {}^3He \rightarrow \nu_\mu + {}^3H$

$$N^\lambda = \langle \Psi^{3H} | j^\lambda | \Psi^{3He} \rangle$$

Three-nucleon Hamiltonian	Capture rate Γ in s^{-1}	
Bonn B	1360	
chiral NNLO version 1	1379	
chiral NNLO version 2	1312	
chiral NNLO version 3	1350	
chiral NNLO version 4	1394	
chiral NNLO version 5	1332	
AV18	1295	
AV18 with MEC	1353	
AV18 + Urbana IX	1324	1316
AV18 + Urbana IX with MEC	1386	1385

in agreement with results of the Pisa group:
L.E. Marcucci *et al.*, Phys.Rev.C **83**, 014002 (2011)

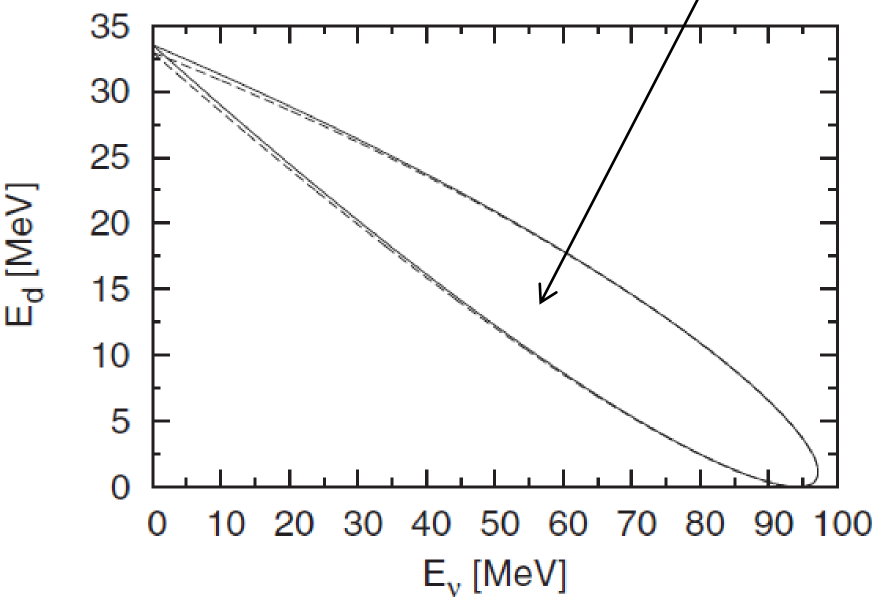
Exp: P. Ackerbauer *et al.* 1496 ± 4

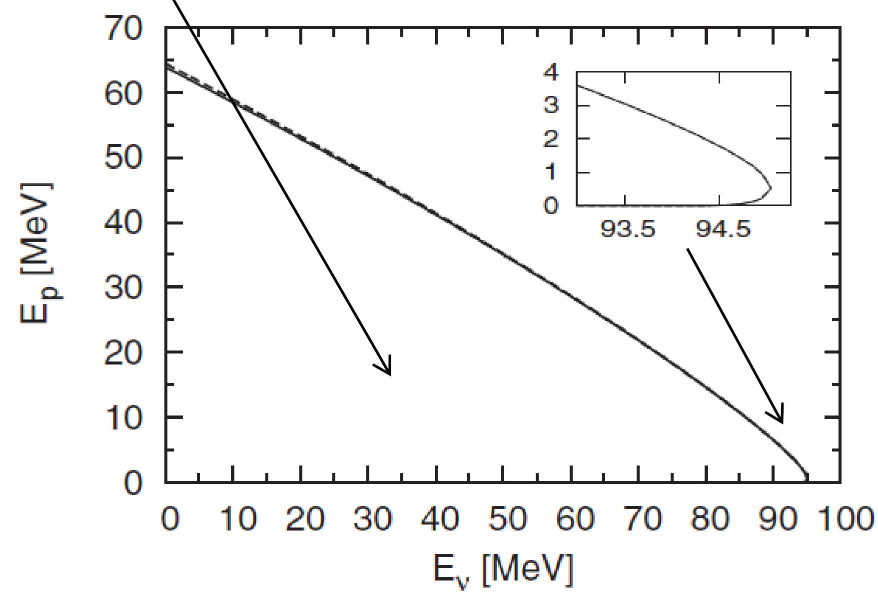
Break-up channels

$$\mu^- + {}^3\mathrm{He} \rightarrow \nu_\mu + d + n$$

$$\mu^- + {}^3\mathrm{He} \rightarrow \nu_\mu + p + n + n$$

kinematically allowed regions





Relativistic and nonrelativistic curves overlap

no relativity in the kinematics !

Break-up channels *(cont.)*

$$N^{\lambda} = \left\langle \Psi_f^{(-)} \middle| j^{\lambda} \middle| \Psi^{3He} \right\rangle$$

two-body or three-body scattering state

channel state

$$\left| \Psi_f^{(-)} \right\rangle = \lim_{\varepsilon \to 0^+} \frac{-i\varepsilon}{E - i\varepsilon - H} \left| \phi_f \right\rangle$$

formal definition

$$H = H_0 + V_{23} + V_{13} + V_{12} + V_{123}$$
$$\equiv H_0 + V_1 + V_2 + V_3 + V_4$$

full 3N Hamiltonian

Break-up channels *(cont.)*

Operators in 3N space:

(1) 3N force decomposed as

$$V_4 = V_4^{(1)} + V_4^{(2)} + V_4^{(3)}$$

$V_4^{(i)}$ is symmetric under the exchange of nucleons j and k, i≠j≠k≠i

(2) free 3N propagator introduced

$$G_0 \equiv \lim_{\varepsilon \to 0^+} \frac{1}{E + i\varepsilon - H_0}$$

(3) 2N off-shell t-matrix generated via LSE:

$$t = V + V G_0 t$$

(4) identical nucleons → permutation operator:

$$P = P_{12}P_{23} + P_{13}P_{23}$$

Break-up channels *(cont.)*

Auxiliary equation and quadratures

$$\left|U^{\lambda}\right\rangle = \left\{ t\,G_0 + \frac{1}{2}(1+P)V_4^{(1)}\,G_0(1+t\,G_0) \right\}(1+P)\,j^{\lambda}\left|\Psi^{3He}\right\rangle$$

$$+\left\{ t\,G_0 P + \frac{1}{2}(1+P)V_4^{(1)}\,G_0(1+t\,G_0)P \right\}\left|U^{\lambda}\right\rangle ,$$

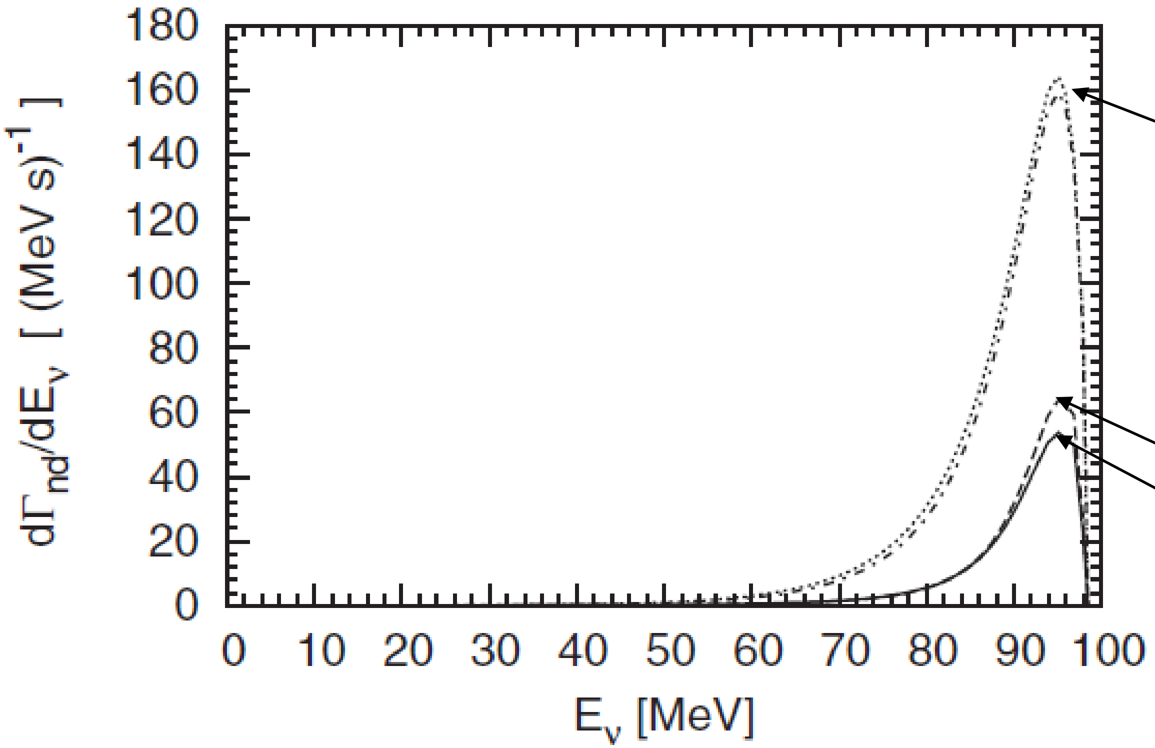
$$N^{\lambda,nd} = \left\langle \phi_{nd}\right|(1+P)\,j^{\lambda}\left|\Psi^{3He}\right\rangle + \left\langle \phi_{nd}\right|P\left|U^{\lambda}\right\rangle,$$

$$N^{\lambda,nnp} = \left\langle \phi_{nnp}\right|(1+P)\,j^{\lambda}\left|\Psi^{3He}\right\rangle + \left\langle \phi_{nnp}\right|t\,G_0(1+P)\,j^{\lambda}\left|\Psi^{3He}\right\rangle$$

$$+\left\langle \phi_{nnp}\right|P\left|U^{\lambda}\right\rangle + \left\langle \phi_{nnp}\right|t\,G_0 P\left|U^{\lambda}\right\rangle$$

Break-up channels: $\mu^- + {}^3He \rightarrow \nu_\mu + n + d$

J. Golak *et al.*, Phys. Rev. C **90**, 024001 (2014)

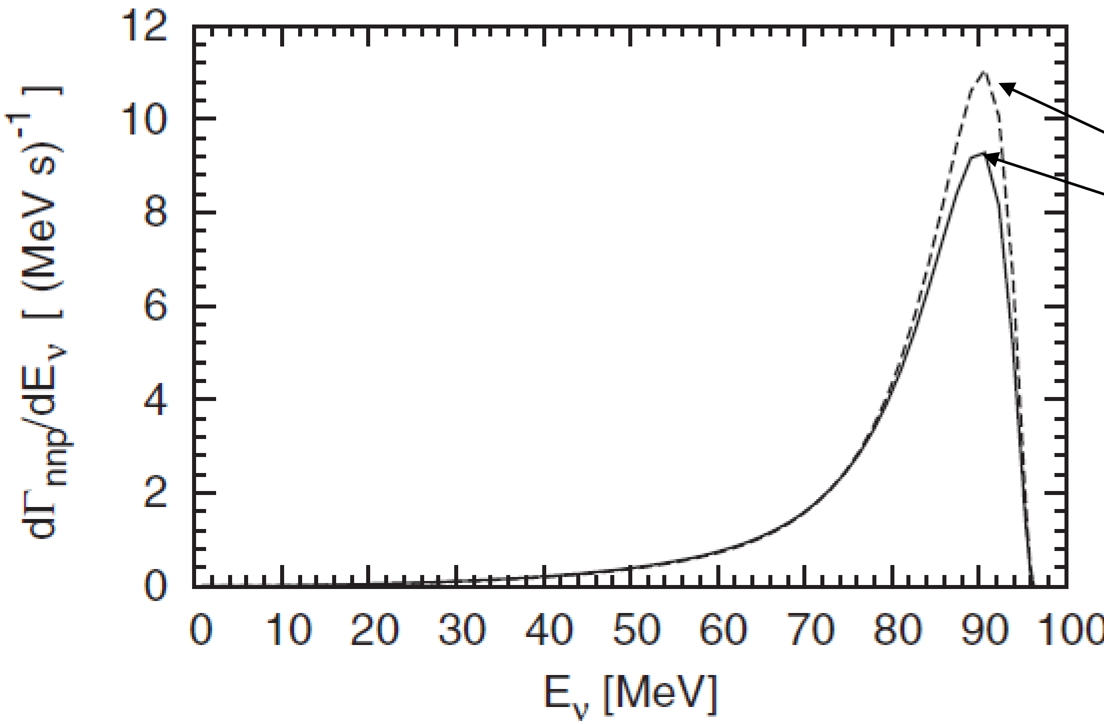


Plane wave predictions with AV18

Full predictions:
with AV18
with AV18 + Urbana IX

Break-up channels: $\mu^- + {}^3He \rightarrow \nu_\mu + n + n + p$

J. Golak *et al.*, Phys. Rev. C **90**, 024001 (2014)



Full predictions:
AV18
AV18 + Urbana IX

Break-up channels for ^{3}He: summary

	Capture rate Γ in s^{-1}				
	Γ_{nd}			Γ_{nnp}	$\Gamma_{nd} + \Gamma_{nnp}$
	PW	SPW	Full	Full	Full
AV18 ($j_{\max} = 3$)	1917	2046	604	169	773
AV18 ($j_{\max} = 4$)	1917	2046	606	170	776
AV18 + Urbana IX ($j_{\max} = 3$)	1853	1956	544	154	698
Earlier theoretical predictions:					
Yano [34]		510		160	670
Philips *et al.* [35]		414		209	623
Congleton [36]					650
Experimental results:					
Zaĭmidoroga *et al.* [37]					660 ± 160
Auerbach *et al.* [38]					665^{+170}_{-430}
Maev *et al.* [39]					720 ± 70
Bystritsky *et al.* [15]					
Method I		491 ± 125		187 ± 11	678 ± 126
Method II		497 ± 57		190 ± 7	687 ± 60

← best predictions !

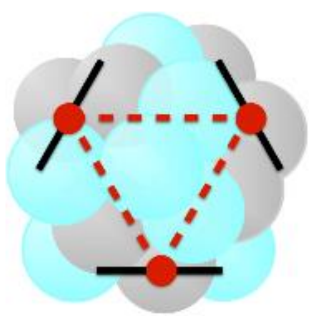
Improved chiral forces

2N forces published recently:
E. Epelbaum, H. Krebs, U.-G.Meiβner, Eur. Phys. J. A **5**, 53 (2015);
and Phys. Rev. Lett. **115**, 122301 (2015),
(later also consistent 3N forces and currents)

will be applied within LENPIC „to understand nuclear structure and reactions with chiral forces"

http://www.lenpic.org

Low Energy Nuclear Physics
International Collaboration



LENPIC

Evgeny Epelbaum, Hermann Krebs

Sven Binder, Kai Hebeler,
Joachim Langhammer, Robert Roth

Richard J. Furnstahl,

Angelo Calci

Pieter Maris, Hugh Potter, James Vary

Andreas Nogga

Veronique Bernard

Jacek Golak, Roman Skibiński,
Kacper Topolnicki, Henryk Witała

Hiroyuki Kamada

Ulf-G.Meißner

Results with the improved chiral potential for $\mu^- + {}^3He \rightarrow \nu_\mu + n + d$

(single nucleon current with RC)

R. Skibński *et al.*, Phys. Rev. C **93**, 064002 (2016)

Chiral order	R=0.8 fm	R=0.9 fm	R=1 fm	R=1.1fm	R=1.2 fm	$\Gamma_{max} - \Gamma_{min}$
LO	262	282	312	350	392	130
NLO	535	525	515	504	492	44
N2LO	547	539	529	518	507	40
N3LO	584	586	592	596	603	19
N4LO	590	584	583	587	595	12

AV18 604

very weak dependence on the regulator parameter R

Results with the improved chiral potential for $\mu^- + {}^3He \rightarrow \nu_\mu + n+n+p$ (single nucleon current with RC)

R. Skibński *et al.*, Phys. Rev. C **93**, 064002 (2016)

Chiral order	R=0.8 fm	R=0.9 fm	R=1 fm	R=1.1fm	R=1.2 fm	Γ_{max} - Γ_{min}
LO	95	99	105	113	120	26
NLO	159	157	154	151	148	11
N2LO	161	159	157	154	151	10
N3LO	169	169	171	172	175	6
N4LO	170	169	169	170	173	4

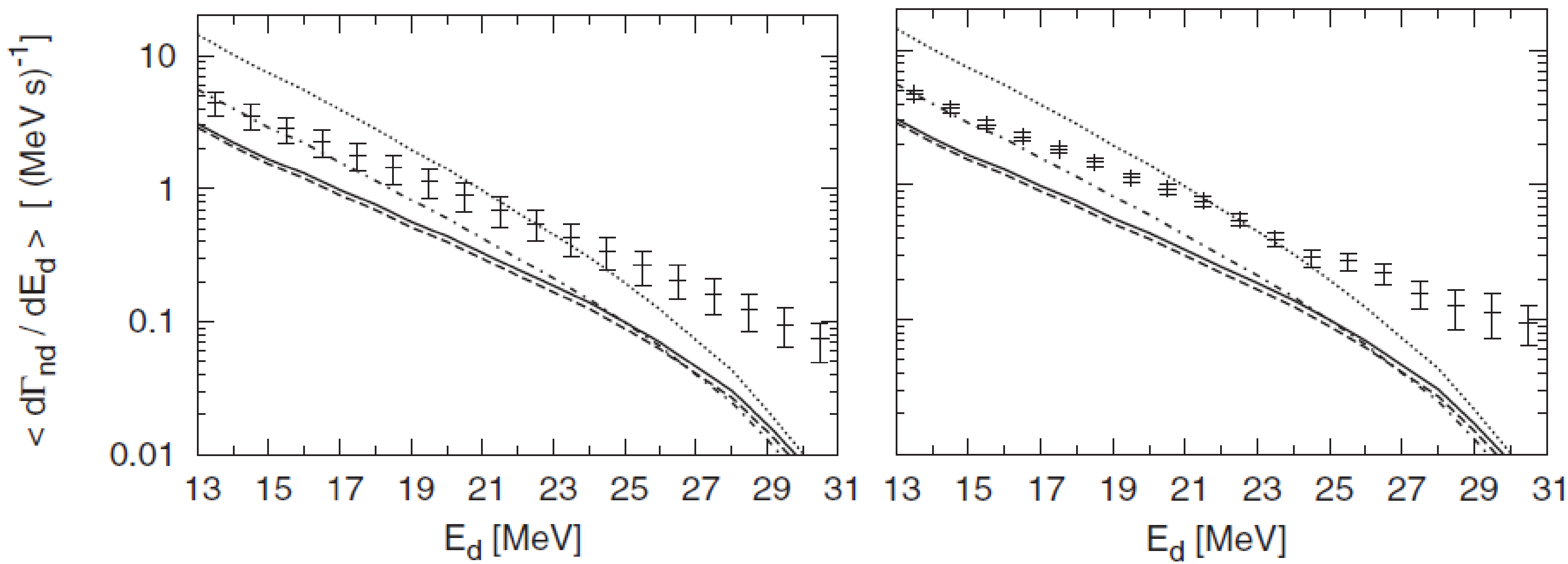
AV18 169

very weak dependence on the regulator parameter R

Break-up channels for ^{3}He: theory vs. experiment

Two sets of data from Bystritsky *et al.*, Phys. Rev. A **69**, 012712 (2004)

$$\mu^- + {}^3\mathrm{He} \rightarrow \nu_\mu + d + n$$

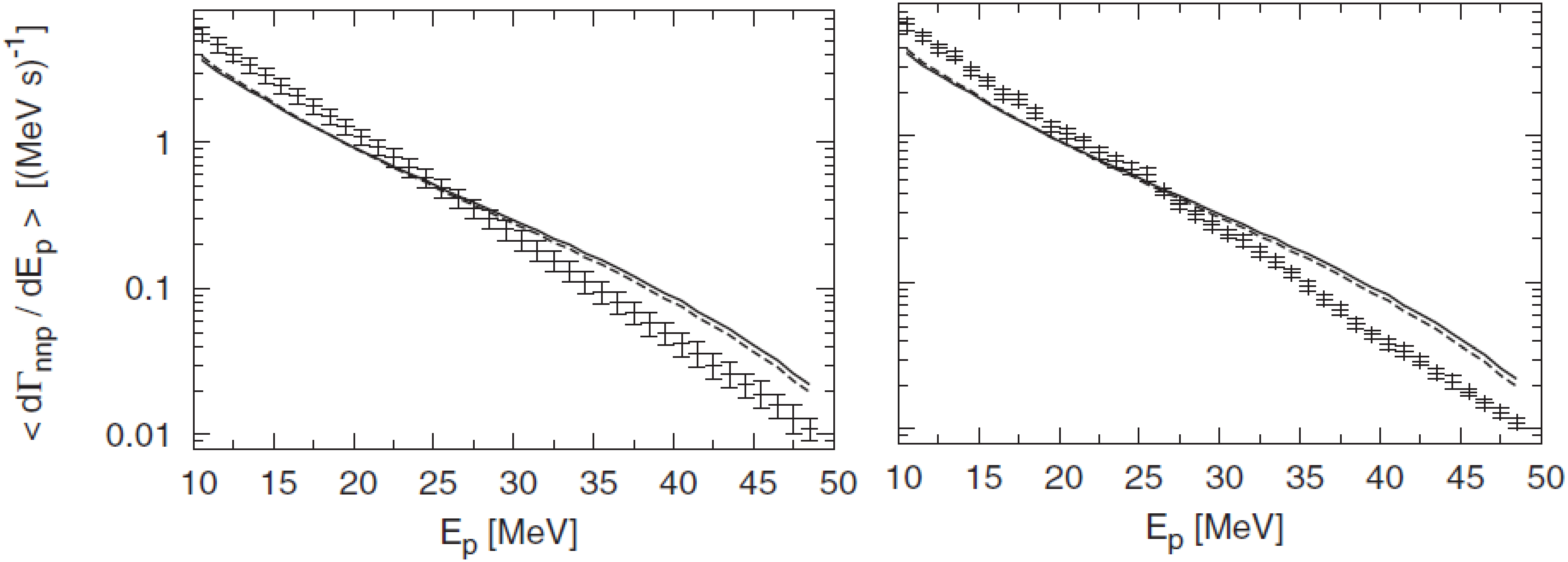


Our calculations with AV18 + Urbana IX underestimate the data !

Break-up channels *(cont.)*

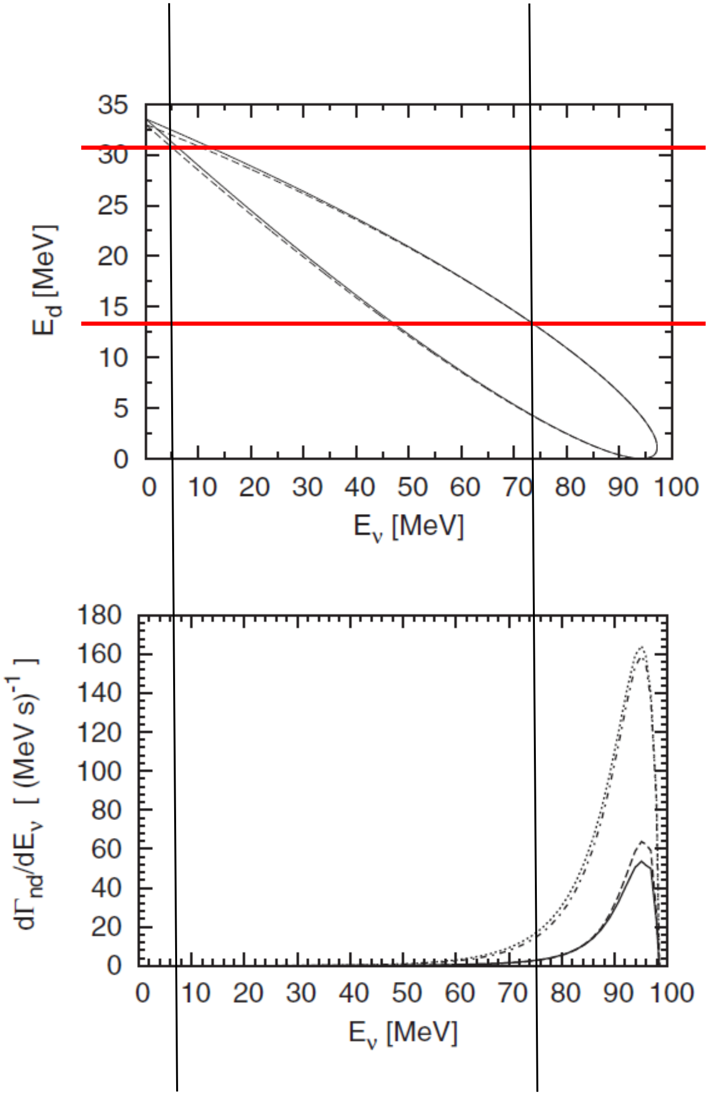
Data from Bystritsky *et al.*, Phys. Rev. A **69**, 012712 (2004)

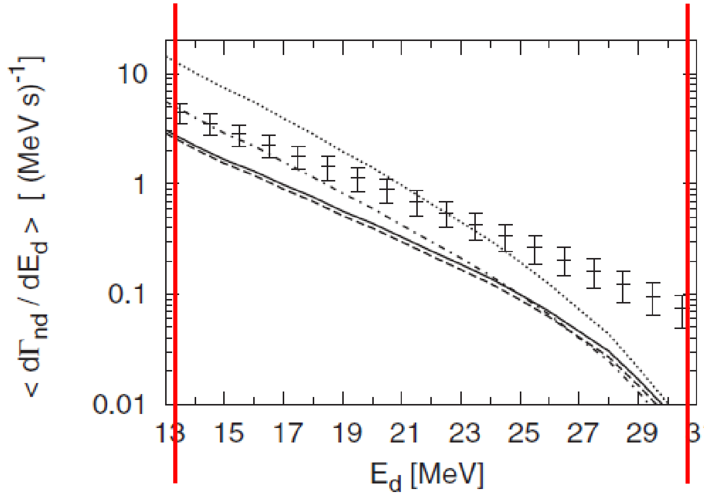
$$\mu^- + {}^3\text{He} \rightarrow \nu_\mu + p + n + n$$



Break-up channels *(cont.)*

$$\mu^- + {}^3\mathrm{He} \rightarrow \nu_\mu + d + n$$

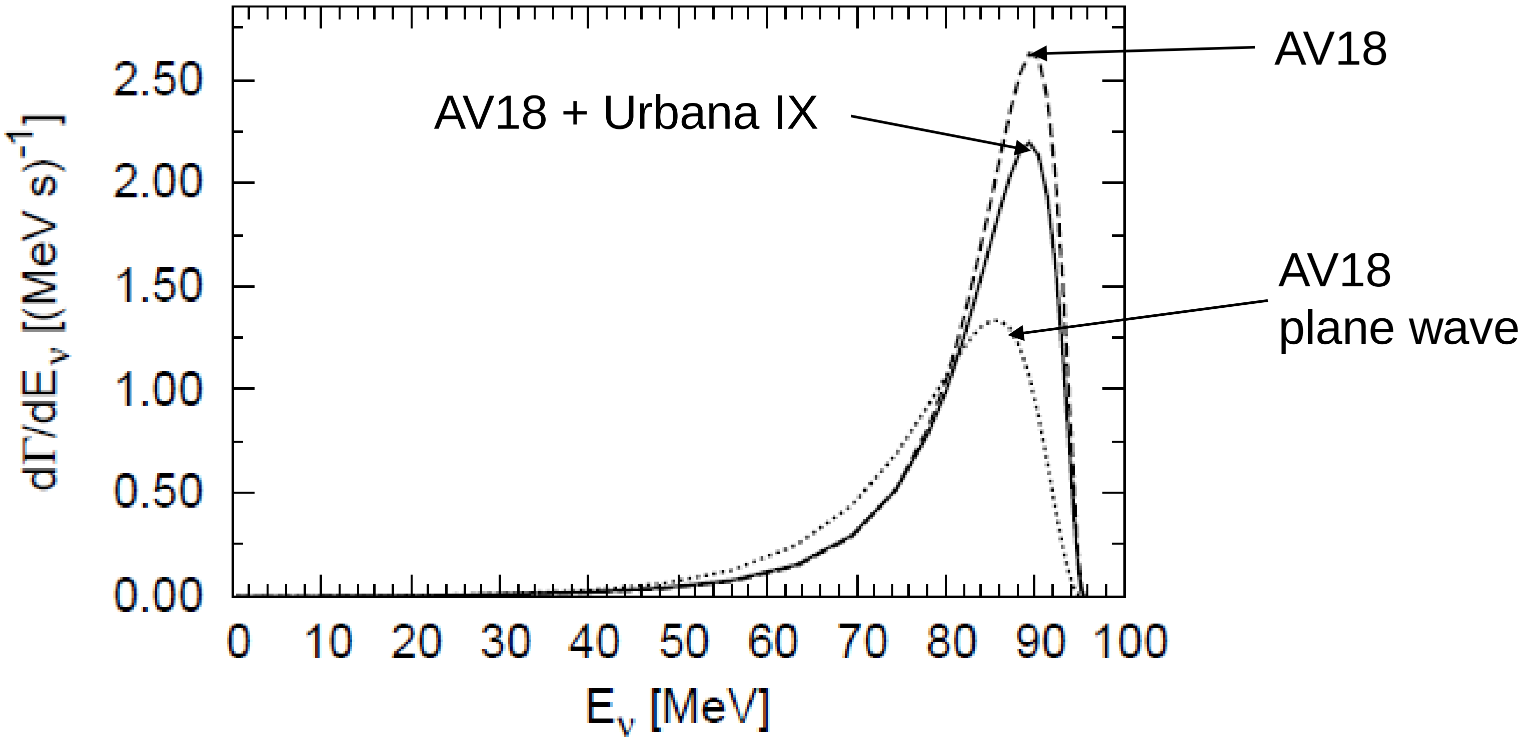




Is extrapolation possible at all ?
How to measure total capture rates ??

Break-up channels: $\mu^- + {}^3H \rightarrow \nu_\mu + n + n + n$

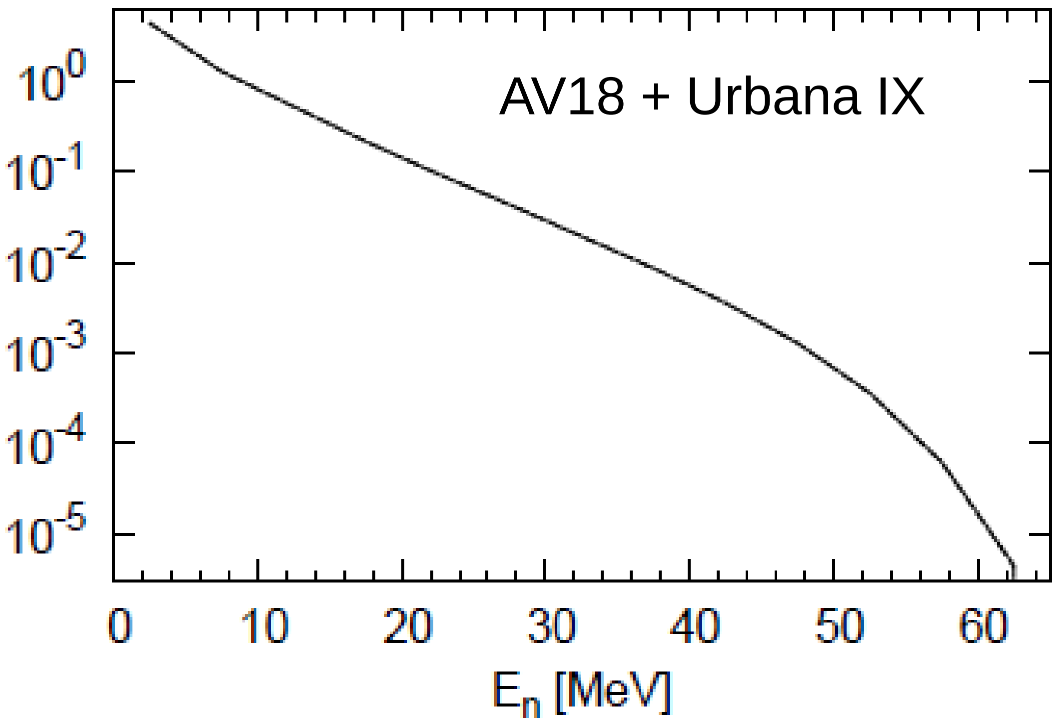
J. Golak *et al.*, arXiv:1605.05668 [nucl-th]



Break-up channels: $\mu^- + {}^3H \rightarrow \nu_\mu + n + n + n$

J. Golak *et al.*, arXiv:1605.05668 [nucl-th]

differential capture rate $d\Gamma/dE_n$
averaged over 5 MeV energy bins



low-energy neutrons decide about the total rate !

Break-up channels: $\mu^- + {}^3H \rightarrow \nu_\mu + n + n + n$

		capture rate Γ in s^{-1}		
		$F=0$	$F=1$	total
AV18, without RC		138.1 (100.0)	3.97 (2.97)	37.5 (27.2)
AV18		133.6 (97.0)	4.21 (3.12)	36.5 (26.6)
AV18 + Urbana IX		118.7	3.92	32.6
earlier theoretical predictions:				
A.C. Phillips *et al.* (1975)	YAM			9.5 (6.1)
J. Torre *et al.* (1978)	RSC			(23.6)
	RSC RC			(28.2)
	SSC			(23.0)
	SSC RC			(27.6)
J. Torre *et al.* (1979)	SSC	122.8 (90.6)	3.58 (2.69)	33.4 (24.7)
	SSC RC	137.5 (102.0)	3.66 (2.78)	37.1 (27.6)
R.I. Dzhibuti *et al.* (1984)	V			35.7 (22.3)
	EH			29.9 (19.7)
	S1			33.1 (20.8)
	S2			35.5 (21.9)

best predictions !

Conclusions and outlook

- We have constructed a very robust momentum space framework to deal with several weak processes

$$\mu^- + d \rightarrow \nu_\mu + n + n$$

$$\mu^- + {}^3\mathrm{H}e \rightarrow \nu_\mu + {}^3\mathrm{H}$$

$$\mu^- + {}^3\mathrm{H}e \rightarrow \nu_\mu + d + n$$

$$\mu^- + {}^3\mathrm{H}e \rightarrow \nu_\mu + p + n + n, \quad \mu^- + {}^3\mathrm{H} \rightarrow \nu_\mu + n + n + n$$

$$^3\mathrm{H} \rightarrow \bar{\nu}_e + e^- + {}^3\mathrm{H}e$$

$$\ldots$$

$$\nu_\mu + {}^3\mathrm{H} \rightarrow \mu^- + {}^3\mathrm{H}e$$

Conclusions and outlook *(cont.)*

- We have tested it by comparing our numbers with the results obtained by the Pisa group for $\mu^- + d \rightarrow \nu_\mu + n + n$, $\mu^- + {}^3\mathrm{He} \rightarrow \nu_\mu + {}^3\mathrm{H}$ ($\mu^- + {}^3\mathrm{He} \rightarrow \nu_\mu + n + d$ in progress !)

- We have obtained first realistic estimates for the total capture rates in three 3N break-up channels:

$$\mu^- + {}^3\mathrm{He} \rightarrow \nu_\mu + d + n \quad (\Gamma_{\mathrm{nd}} = 544\, s^{-1}),$$

$$\mu^- + {}^3\mathrm{He} \rightarrow \nu_\mu + p + n + n \quad (\Gamma_{\mathrm{nnp}} = 154\, s^{-1}),$$

$$\mu^- + {}^3\mathrm{H} \rightarrow \nu_\mu + n + n + n \quad (\Gamma_{\mathrm{nnn}} = 32.6\, s^{-1}).$$

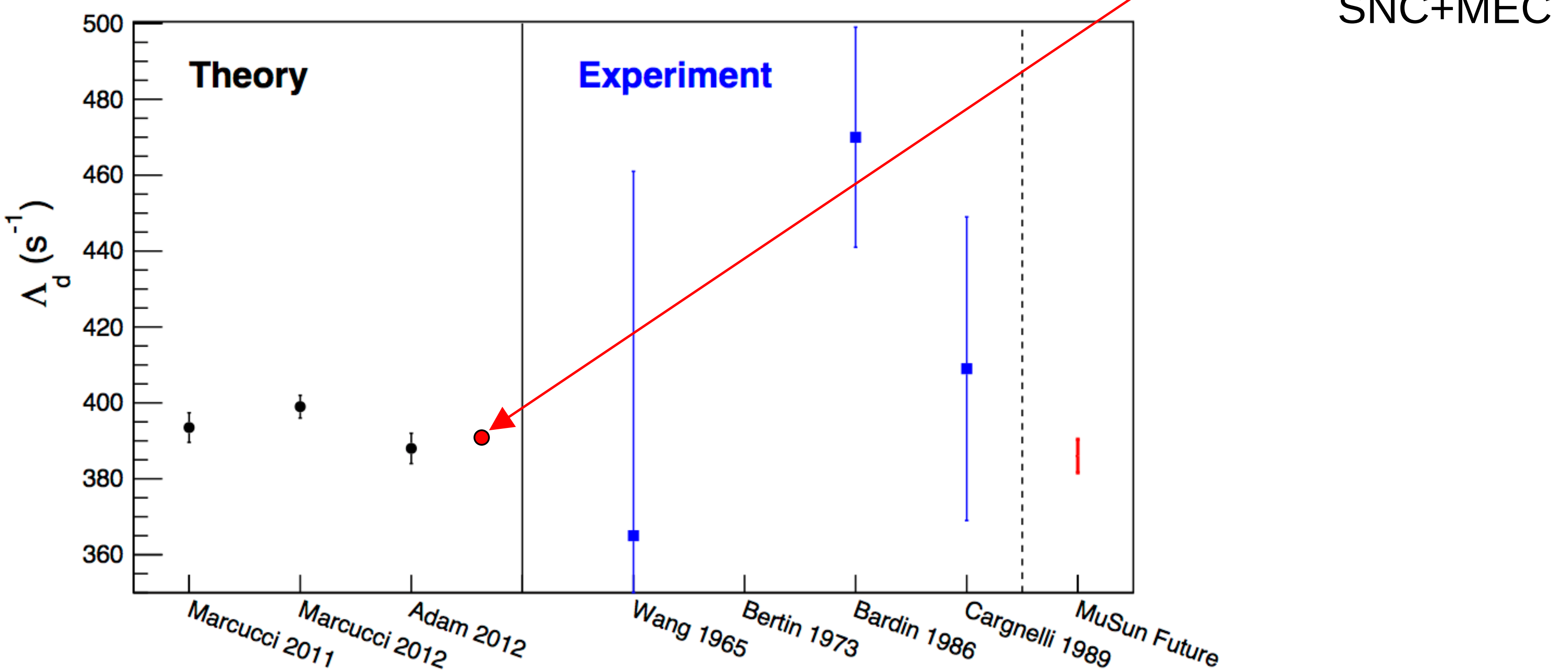
- We do not agree well with the results of the recent experiment by Bystritsky *et al.,* Phys. Rev. A **69**, 012712 (2004).

- We look forward to consistent chiral potentials and weak current operators within the LENPIC project

Conclusions and outlook *(cont.)*

- We are waiting for the results from the MuSun experiment

  http://muon.npl.washington.edu/exp/MuSun/



Conclusions and outlook *(cont.)*

- … and hoping for the precision measurements of the capture rates in the break-up channels

Thank you for your attention !