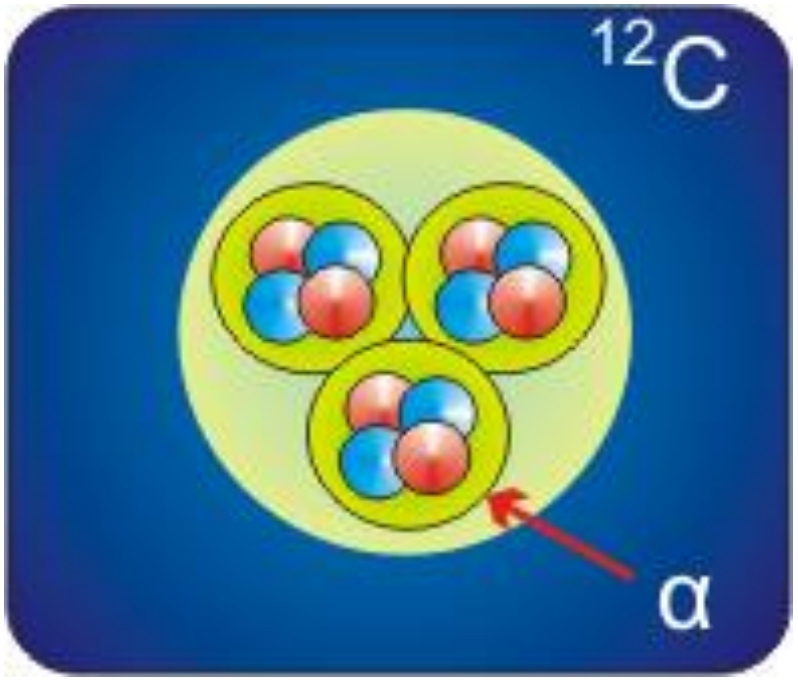
Electromagnetic selection rules for ^{12}C in a 3 α cluster model

Lorenzo Fortunato
Univ. Padova, Italy

In collaboration with G. Stellin and A.Vitturi



Work done within the **IN:Theory** project of the Univ. Padova

Presented at EFB23 Aarhus (DK) 8/8/16

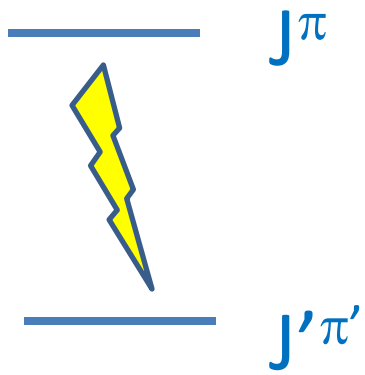
Dipartimento di Fisica e Astronomia Galileo Galilei

INFN Istituto Nazionale di Fisica Nucleare

Summary

In this talk I will touch the following topics:

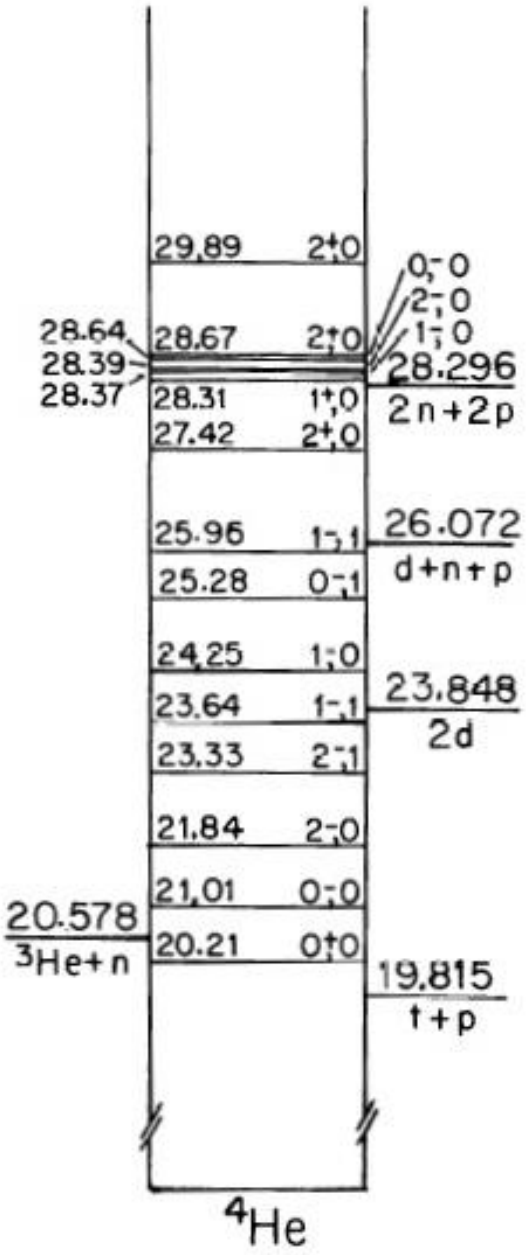
- α particle as a building block for cluster models
- Molecular α clusters model: *Wheeler Phys.Rev. 52 (1937)*
- Algebraic cluster model: *Bijker and Iachello PRC 61 (2000); ibid. Ann.Phys. (2002)*
- Experimental facts: *Freer at al PRC83(2011); Marín-Lámbarri et al. PRL 113 (2014)*
- A few details on the point-group theory
- **Electromagnetic selection rules ← this was missing!**
- Suggestions for crucial experiments



The content of this talk is mostly taken from:

G.Stellin, L.F. and A.Vitturi, **J.Phys. G: Nucl. Part. Phys. 43(2016) 085104**

α – particle



MAIN PROPERTIES:

- The alpha particle or $^{4}_{2}He_{2}$ is a very stable nucleus.
- It's total B.E. = 28.296 MeV
- No excited states up to the first threshold (t+p)
- Threshold for neutron and proton separation at about 20 MeV
- Boson with $J^{\pi} = 0^{+}$ -> Bose-Einstein Condensation (BEC)

Therefore:
It is a very stable system, that does not change state unless there are at least 20 MeV available. One could think it is the ideal building block for clusterized nuclei

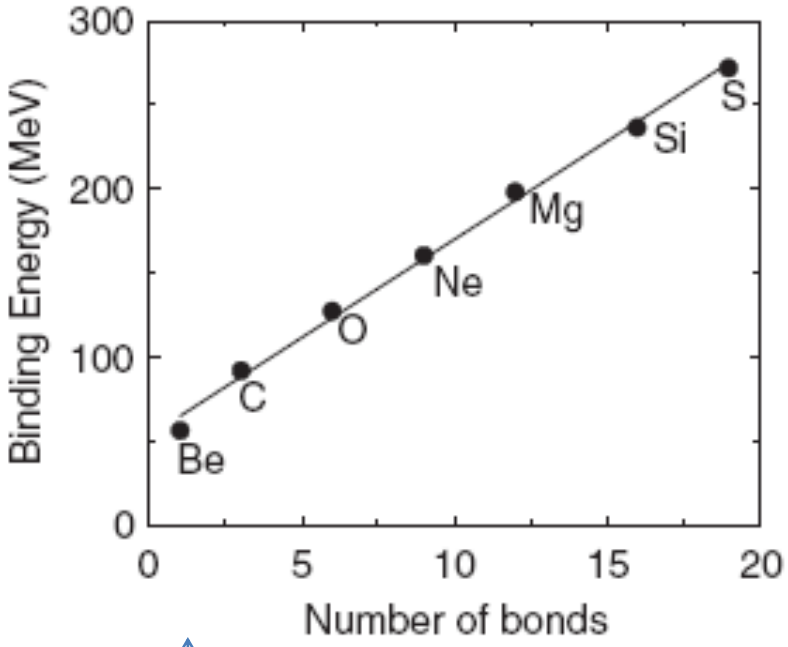
OTHER PROPERTIES :

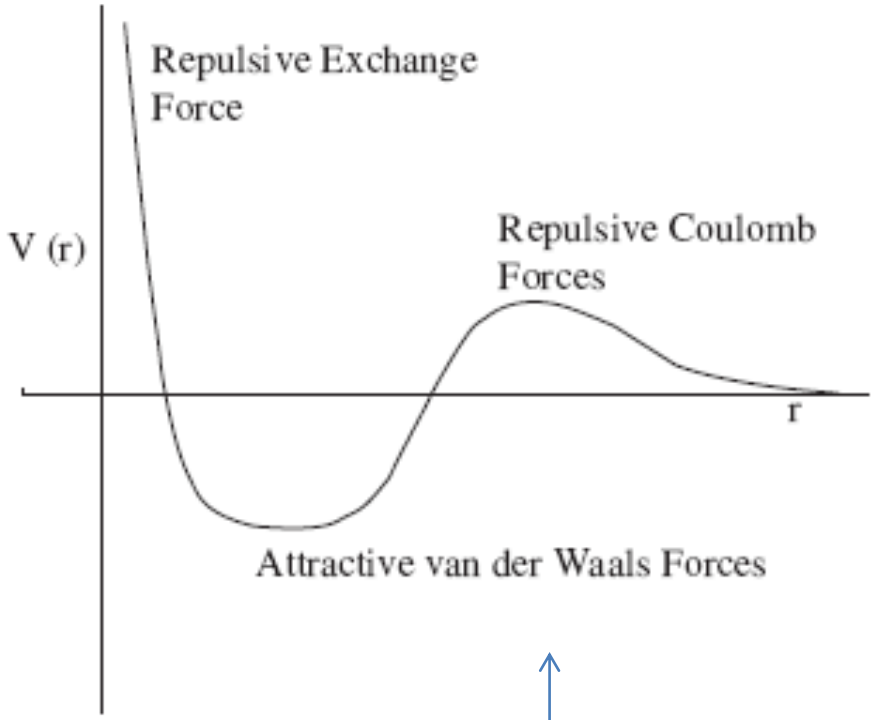
- It has N=Z=2, therefore it is the first even-even nucleus
- It is extremely important in nuclear physics because many nuclei decay by emission of an alpha particle
- It is also very important in reactions because (α, α') is a perfect isoscalar probe (for exciting Giant Resonances for example)

L. Fortunato

Saturation of binding energy

W. von Oertzen et al. / Physics Reports 432 (2006) 43–113



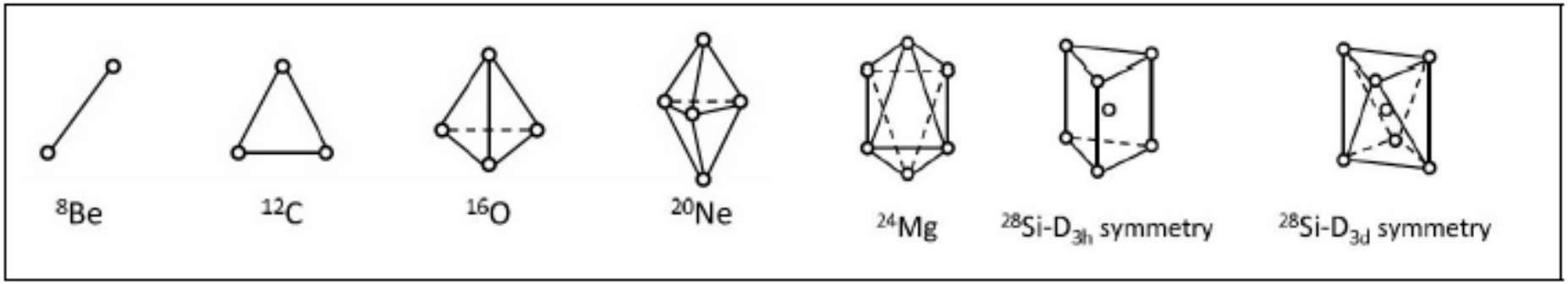


Heisenberg (1935) noticed that the B.E. is proportional to the number of saturated bonds between alpha constituents.

Hafstad and Teller (1938) developed a model based on a α-α potential based on similar well-known phenomena in molecular physics (van der Waals).

L. Fortunato

Molecular models of the nucleus



The model of Wheeler portrays the even-even alpha-conjugate nuclei as **molecular structures** made up of point-like objects that can perform **vibrations and rotations** around geometrical **equilibrium configurations**. These equilibrium shapes can be found minimizing the total PES built from alpha-alpha potentials and should be **understood in a dynamical sense**: unlike ions in ordinary rigid molecules that stay close to the equilibrium configurations, the alpha particles inside the nucleus have **large fluctuations** (translational and rotational motions). This is an intrinsic feature due to dimensions and energies involved in the nuclear domain.

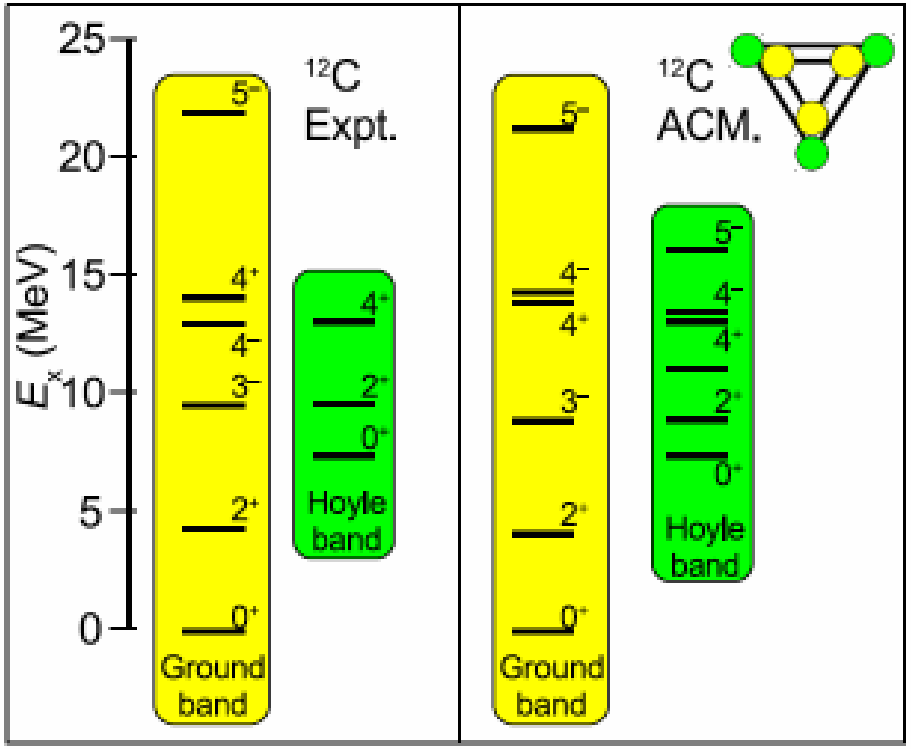
The point-group symmetry that is relevant in the case of 3 alpha particles at the vertexes of an equilater triangle is:

$$D_{3h}$$

L. Fortunato

Algebraic cluster model

A modern version of the alpha cluster models is the **Algebraic Cluster Model (ACM) of Bijker and Iachello** that treats the hamiltonian for vibrations and rotations using the appropriate U(7) Lie algebra and imposing the constraints due to D_{3h} point-group symmetry. This model is very successfull in reproducing part of the spectrum, that is attributed to cluster states.

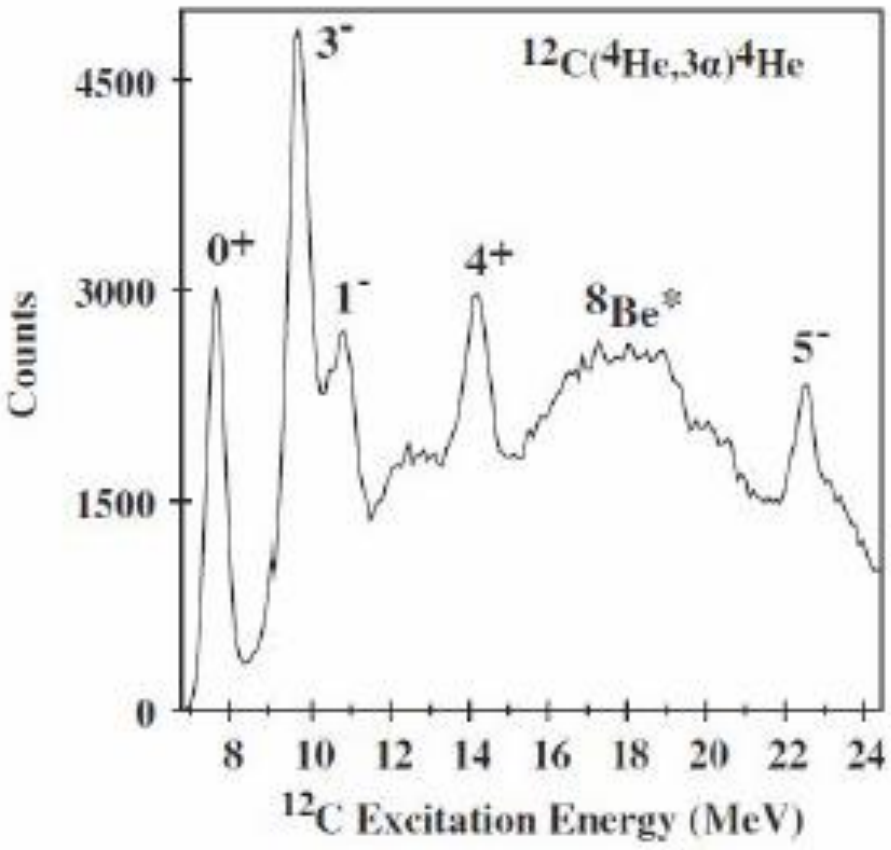


- The ground state band has a characteristic pattern of rotational levels that DO NOT OBEY the commonly known rules for rotational bands. In particular they have a specific spin-parity scheme that is unique to the D3h symmetry! Only a few values of K are allowed.
- The Hoyle band is intepreted as a first vibrational breathing excitation of the triangular configuration.

Bijker and Iachello PRC 61 (2000); ibid. Ann.Phys. (2002)

L. Fortunato

Experiments



Measuments on the spectrum of 12C have been done, mainly from the group in Birmingham.
In particular the 5- states predicted from theory has been found at the right energy.

It seems that a very **relevant question to be asked at this moment** is:

What kind of electromagnetic selection rules should we expect in such a highly symmetric geometric arrangement of alpha particles?

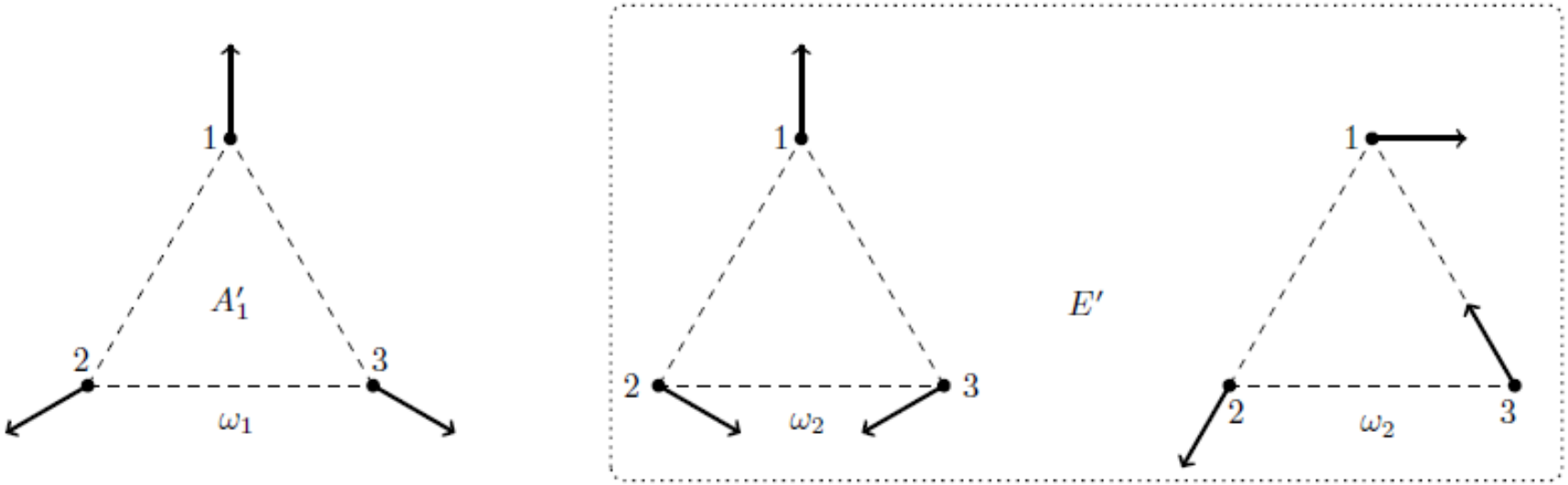
- *Freer at al PRC83(2011);*
- *Marín-Lámbarri et al. PRL 113 (2014)*

A brief outline of our working programme …

1. Identify the geometrical symmetry (equilateral triangle)
2. Find the normal modes of vibrations (3 in the present case)
3. Find the moments of inertia (oblate top in this case)
4. Construct the tables of allowed rotational-vibrational states that obey that symmetry (i.e. reproduce the results of Wheeler in a modern fashion)
5. Find the character under transformation of the point-group for the E.M. operators
6. Check the allowed and forbidden transitions with the Vanishing integral rule (from group theory)

Normal modes of vibrations

We have calculated the normal modes of vibrations and the respective coordinates:



Breathing mode (unique)

Bending mode (doubly degenerate)

Normal modes are labeled by the irreducible representations of the D3h group. It turns out that the 3N-6 = 3 normal modes of motion are of two kinds, a breathing mode of A-type and a bending mode of E-type.

...a very lengthy calculation made extremely short...

Details to be found in the **master thesis of G.Stellin**, Univ. Padova (Italy), 2015 and in a recent presentation at the QPTN worhshop in Prague (may 2016)

rigid rotor hamiltonian

$$\hat{H}_{RR} = \frac{1}{2}\left(\hat{P}_1^2 + \omega_1^2\hat{Q}_1^2\right) + \frac{1}{2}\sum_{i=2}^{3}\left(\hat{P}_i^2 + \omega_2^2\hat{Q}_i^2\right) + \frac{\hat{J}^2}{2I_{xx}^e} - \frac{\hat{J}_z^2}{2}\left(\frac{1}{I_{xx}^e} - \frac{1}{I_{zz}^e}\right).$$

- two wavefunctions of harmonic oscillator

$$\psi_V = \langle Q_1|\frac{(a^\dagger)^{n_1}}{\sqrt{n_1!}}|0\rangle\langle Q_2,Q_3|\frac{(a_R^\dagger)^{\frac{n_2+m}{2}}}{\sqrt{\frac{n_2+m}{2}!}}\frac{(a_L^\dagger)^{\frac{n_2-m}{2}}}{\sqrt{\frac{n_2-m}{2}!}}|0\rangle = \Phi_{n_1}(Q_1)\Phi_{n_2,m}(Q_2,Q_3)$$

- Wigner D matrices

$$\Psi_{Rs}(\varphi,\theta,\chi) \equiv \Psi_{J,K,M}(\varphi,\theta,\chi) = \sqrt{\frac{2J+1}{8\pi^2}}D^{J*}_{MK}(\varphi,\theta,\chi)$$

Vibrations and rotations are decoupled in the present model. Additional couplings (higher order) might be considered, that might lift some degeneration.

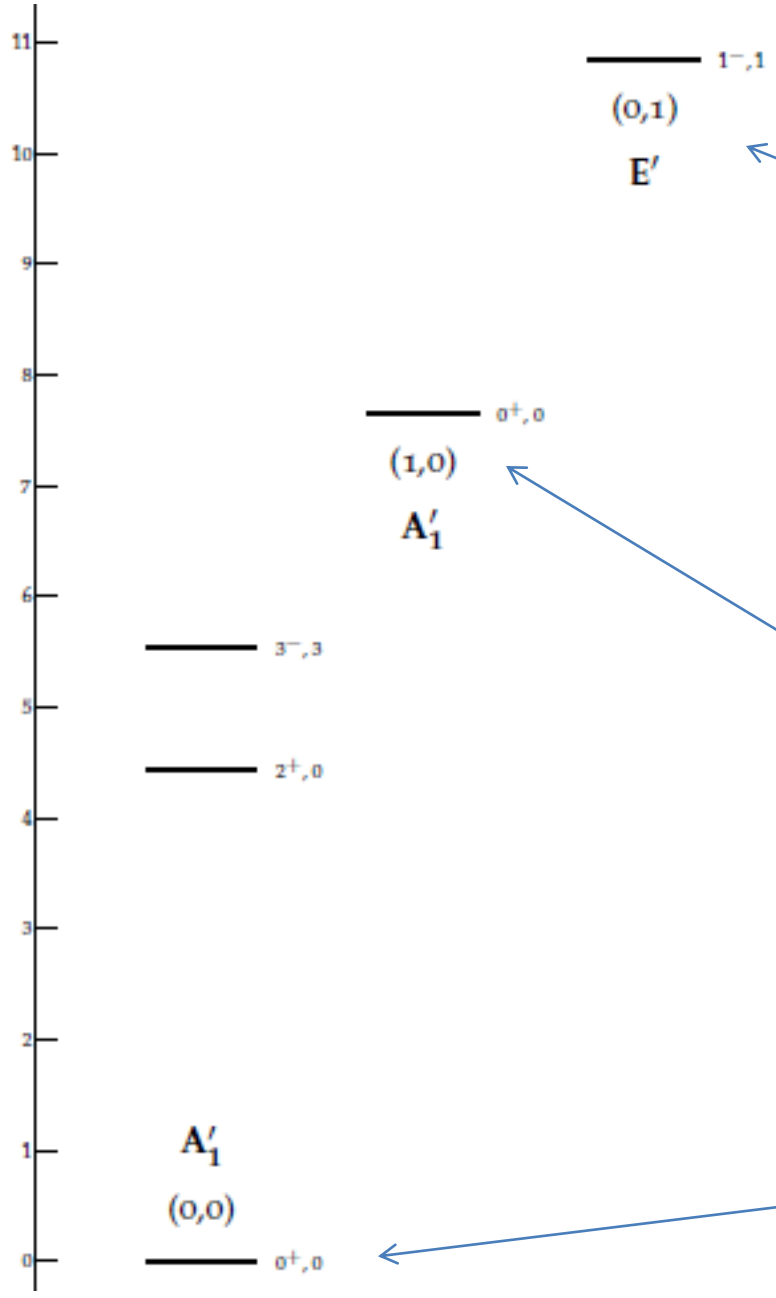
L. Fortunato

Rotational-Vibrational states

$n_1 = 0, 1, 2, 3, \ldots; n_2 =$			0	1	2	3	4	5
J	$\lvert K\rvert$	$D_{3h}(N)$	A_1'	E'	$A_1' \oplus E'$	$A_1' \oplus A_2' \oplus E'$	$A_1' \oplus E' \oplus E'$	$A_1' \oplus A_2' \oplus E' \oplus E'$
0	0	A_1'	1	0	1	1	1	1
1	0	A_2'	0	0	0	1	0	1
	1	E''	0	1	1	1	2	2
2	0	A_1'	1	0	1	1	1	1
	1	E''	0	1	1	1	2	1
	2	E'	0	1	1	1	2	2
3	0	A_2'	0	0	0	1	0	1
	1	E''	0	1	1	1	2	2
	2	E'	0	1	1	1	2	2
	3	$A_1'' \oplus A_2''$	1	0	1	2	1	2
4	0	A_1'	1	0	1	1	1	1
	1	E''	0	1	1	1	2	2
	2	E'	0	1	1	1	2	2
	3	$A_1'' \oplus A_2''$	1	0	1	2	1	2
	4	E'	0	1	1	1	2	2
5	0	A_2'	0	0	0	1	0	1
	1	E''	0	1	1	1	2	2
	2	E'	0	1	1	1	2	2
	3	$A_1'' \oplus A_2''$	1	0	1	2	1	2
	4	E'	0	1	1	1	2	2
	5	E''	0	1	1	1	2	2

L. Fortunato

Schematic Rotational-Vibrational spectrum



Bending mode $n_2=1$

Symmetric representation band (Hoyle band) $n_1=1$

Ground state band

L. Fortunato

Electromagnetic operators and matrix elements

Reduced transition probabilities between any two states are defined as:

$$\mathrm{B}(R\lambda) = \sum_{M_i, M_f} \sum_{\mu} \left| \langle J_f, M_f, K_f, n_{1f}, n_{2f} | \Omega(R)_{\lambda,\mu} | J_i, M_i, K_i, n_{1i}, n_{2i} \rangle \right|^2$$

Where the electric and magnetic multipole operators in the lab frame are related through rotations to the intrinsic operators, ω(R):

$$\Omega_{\lambda\mu}(R) = \sum_{\nu=-\lambda}^{\lambda} D^{\lambda *}_{\mu\nu}(\varphi, \theta, \chi) \omega_{\lambda\nu} \ (R)$$

$$\omega_{\lambda\mu}(E) = \int \mathrm{d}^3 r \ \rho(\vec{r}) r^{\lambda} Y_{\lambda\mu}(\theta, \varphi)$$

$$\omega_{\lambda\mu}(M) = \frac{1}{ec} \frac{1}{\lambda + 1} \int \mathrm{d}^3 r \ \nabla \left[r^{\lambda} Y_{\lambda\mu}(\theta, \varphi) \right] \cdot [\vec{r} \times \vec{j}(\vec{r})].$$

where ρ and **j** are the charge density and the current density operators.

L. Fortunato

Transformation properties of E.M. operators

The characters of multipole oprators can be found recursively taking symmetric powers of the 3-dim representation, according to the following formula

$$\chi^{\Gamma^{(\lambda)}}[R] = \frac{1}{3}\left[2\chi^{\Gamma}[R]\chi^{\Gamma^{(\lambda-1)}}[R] + \frac{1}{2}\left(\chi^{\Gamma}[R^2] - \chi^{\Gamma}[R]^2\right)\chi^{\Gamma^{(\lambda-2)}}[R] + \chi^{\Gamma}[R^{\lambda}]\right]$$

$\mathcal{D}_{3h}$	$\mathbb{I}$	$2C_3$	$3C_2$	σ_h	$2S_3$	$3\sigma_V$
$\Gamma[\omega_{1\mu}(E)] = A_2'' + E'$	3	0	-1	1	-2	1
$\Gamma[\omega_{2\mu}(E)] = A_1' + E' + E''$	5	-1	1	1	1	1
$\Gamma[\omega_{3\mu}(E)] = A_1' + A_2' + A_2'' + E' + E''$	7	1	-1	1	1	1
$\Gamma[\omega_{4\mu}(E)] = A_1' + A_1'' + A_2'' + 2E' + E''$	9	0	1	1	-2	1
$\Gamma[\omega_{5\mu}(E)] = A_1' + A_2' + A_2'' + 2E' + 2E''$	11	-1	-1	1	1	1
$\Gamma[\omega_{6\mu}(E)] = 2A_1' + A_2' + A_1'' + A_2'' + 2E' + 2E''$	13	1	1	1	1	1

$\mathcal{D}_{3h}$	$\mathbb{I}$	$2C_3$	$3C_2$	σ_h	$2S_3$	$3\sigma_V$
$\Gamma[\omega_{1\mu}(M)] = A_2' + E''$	3	0	-1	-1	2	-1
$\Gamma[\omega_{2\mu}(M)] = A_1'' + E' + E''$	5	-1	1	-1	-1	-1
$\Gamma[\omega_{3\mu}(M)] = A_1'' + A_2' + A_2'' + E' + E''$	7	1	-1	-1	-1	-1
$\Gamma[\omega_{4\mu}(M)] = A_1' + A_1'' + A_2' + 2E' + E''$	9	0	1	-1	2	-1
$\Gamma[\omega_{5\mu}(M)] = A_1'' + A_2' + A_2'' + 2E' + 2E''$	11	-1	-1	-1	-1	-1
$\Gamma[\omega_{6\mu}(M)] = A_1' + 2A_1'' + A_2' + A_2'' + 2E' + 2E''$	13	1	1	-1	-1	-1

Vanishing integral rules → Selection rules

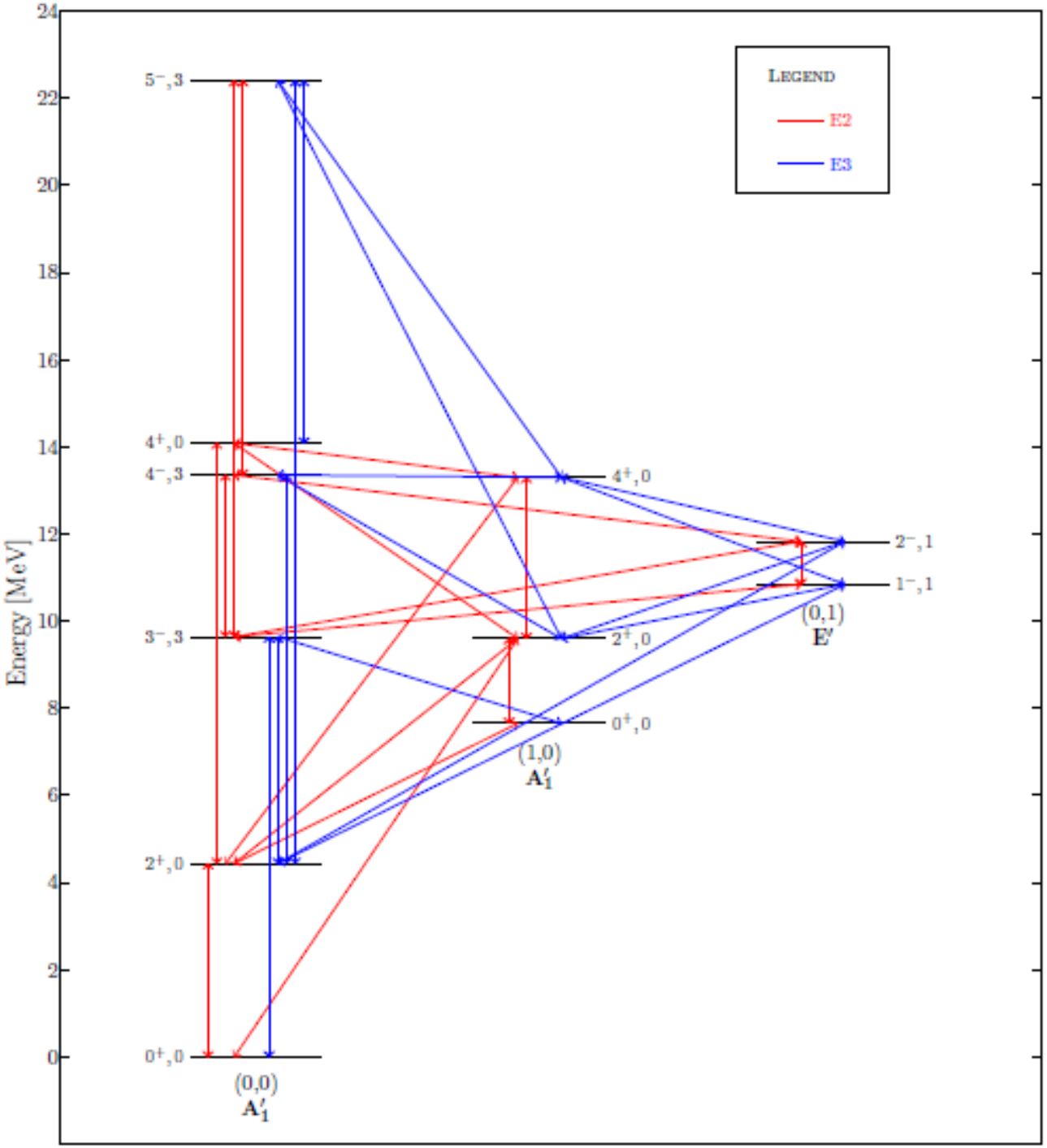
From group theory, the total character of the integrand (operators, initial and final states) should transform under the operations of the group as the totally symmetric representation (A_1' in this case) for the integral to be non-null.

$$\Gamma(\Psi_f) \otimes \Gamma[\Omega_{\lambda\mu}(R)] \otimes \Gamma(\Psi_i) \supset A_1' \qquad R = E, M.$$

$$\langle n_{1f} n_{2f} m_f | \omega_{\lambda\mu}(R) | n_{1i} n_{2i} m_i \rangle \neq 0 \leftrightarrow \Gamma(\Phi^E_{n_{2f}}) \otimes \Gamma[\omega_{\lambda\mu}(R)] \otimes \Gamma(\Phi^E_{n_{2i}}) \supset A_1'$$

The wavefunctions of the breathing vibration have been omitted because they do not contributed being totally symmetric.

D_{3h}	$\mathbb{I}$	$2C_3$	$3C_2$	σ_h	$2S_3$	$3\sigma_V$
A_1'	1	1	1	1	1	1
A_2'	1	1	-1	1	1	-1
E'	2	-1	0	2	-1	0
A_1''	1	1	1	-1	-1	-1
A_2''	1	1	-1	-1	-1	1
E''	2	-1	0	-2	1	0



Pattern of Electric-type transitions:

- Dipole is forbidden
- E2 (red)
- E3 (blue)

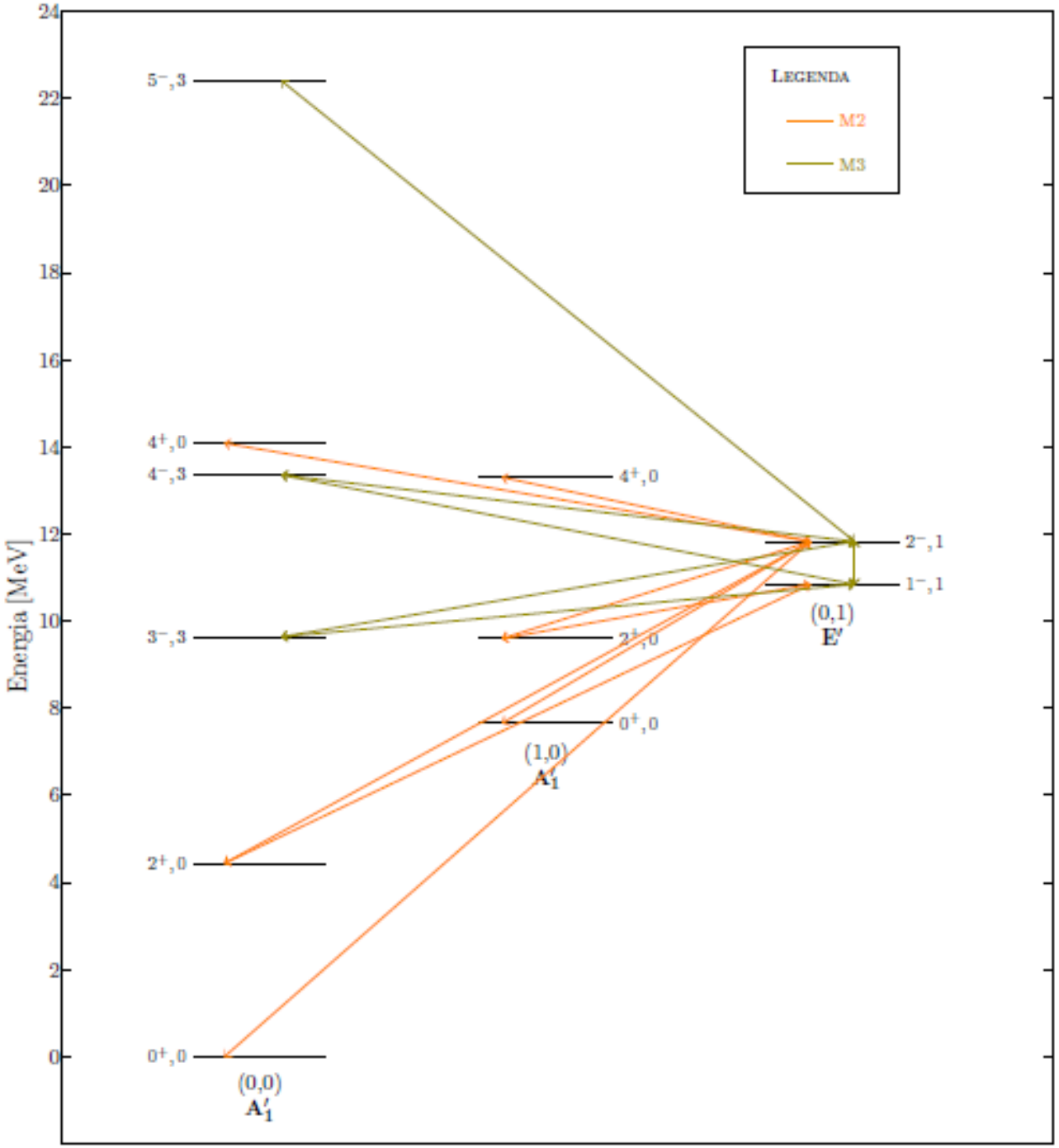
Notation:

J^{π} , K ----------

(n_1, n_2)

E

L. Fortunato



Pattern of Magnetic-type transitions:

- Dipole is forbidden
- M2 (orange)
- M3 (green)

Notation:

J^{π} , K ----------

(n_1, n_2)

M

L. Fortunato

Suggestions and summary

After reminding why the alpha cluster model for 12C is justified (both in the old fashioned model of Wheeler and in the new algebraic model of Bijker-Iachello), we have briefly summarized the procedure to obtain the allowed states and bands in the equilateral triangle configuration of three alpha particles.

We have obtained **electromagnetic selection rules** for this system (summarized in the table below) that **represent a very stringent test** that can be used to prove the validity of the alpha cluster model beyond any doubt!
These rules will of course break down in energy domains where the alpha particle itself can break or get excited.

$\Gamma(in.) \leftrightarrow \Gamma(fin.)$	Electric	Magnetic
$A_1' \leftrightarrow A_1'$	E2,3,$\cdots$	M4,6,7,$\cdots$
$E' \leftrightarrow E'$	E2,3,$\cdots$	M2,3,$\cdots$
$A_1' \leftrightarrow E'$	E2,3,$\cdots$	M2,3,$\cdots$

Therefore we suggest that decisive experiments should be conducted in order to try and measure the B(Eλ) and B(Mλ) values between excited states of 12C.

Table Summary of allowed electric and magnetic multipole transitions between bands of a system with $\mathcal{D}_{3h}$ symmetry.

L. Fortunato