Fermi magnon in an optical lattice

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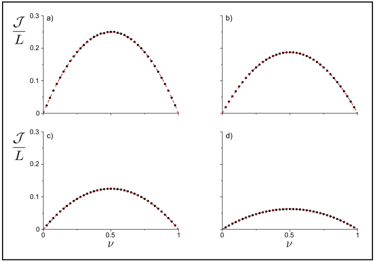
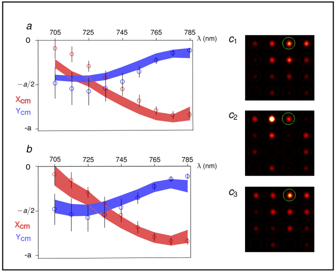
09/08/2016

Work In QOCA, Heriot-Watt

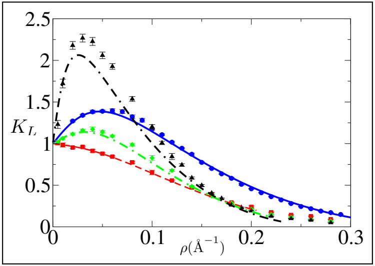


"Driven Topological Systems in the Classical Limit", **Duncan**, Öhberg and Valiente, arXiv:1607.05282

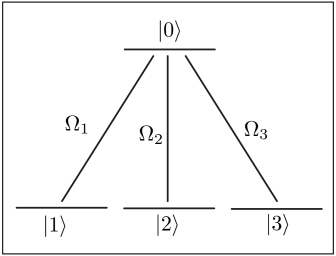


"Experimental observation of anomalous topological edge modes in a slowly-driven photonic lattice", Mukherjee, Spracklen *et al.*, arXiv:1604.05612

Work In QOCA, Heriot-Watt



"Few-Body Route to One-Dimensional Quantum Liquids", Valiente and Öhberg arXiv:1607.08604



"Light-induced gauge fields for ultracold atoms", Goldman, Juzĩunas, Öhberg, Spracklen and Spielman, Rep. Prog. Phys. 77, 12 (2014)

Contents

The System

Strongly-interacting spin-1/2 Fermi gas in 1D

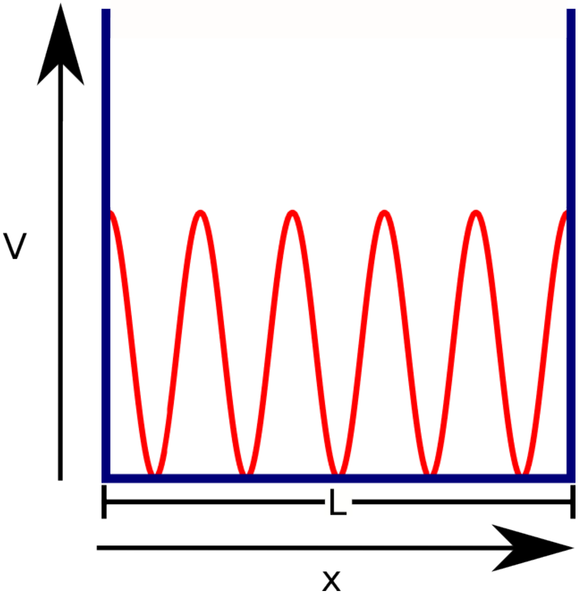
$$H = \sum_i \left(\frac{p_i^2}{2m} + V(x_i) \right) + g \sum_{i>j} \delta(x_i - x_j) \tag{1}$$

Have periodic potential within trap $V(x+d) = V(x)$, d is spacing between wells. We set our system to between $x = 0$ and $x = L = 1$

$$V(x) = V_0 cos(2\pi L_w x) \tag{2}$$

Define L_w - number of wells, with a number of particles N and length of the system $L = L_w d$. Setting $d = 1$.

Setting $\hbar = 1$, $m = 1$ and $V_0 = 5$. Will refer to filling, $\nu = N/L_w$.


V
L
x

Spin Chain Description

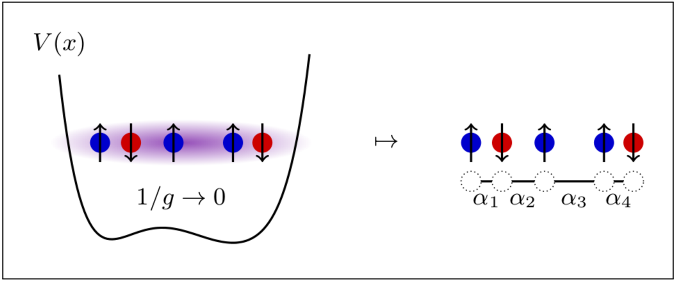


Figure taken from Loft *et al.*, arXiv:1603.02662 (2016)

Volosniev *et al.*, Nat. Comm. 5, 5300 (2014)
Volosniev *et al.*, PRA 91, 023620 (2015)
Deuretzbacher *et al.*, PRA 90, 013611 (2014)

Around the Tonks-Girardeau limit ($1/g \to 0$), charge degrees of freedom (DOF) are frozen, but the spin DOF are described to $O(1/g)$ by

$$H = E_0 - \frac{1}{2}\sum_{k=1}^{N-1} \frac{\alpha_k}{2g}\left(\boldsymbol{\sigma}^k \cdot \boldsymbol{\sigma}^{k+1} - \mathbb{I}\right) \tag{3}$$

E_0 is the non–interacting system energy, that is for all spins polarized in the same direction.

Volosniev *et al.*, PRA 91, 023620 (2015)
Deuretzbacher *et al.*, PRA 90, 013611 (2014)
Experiment: Murmann *et al.*, PRL 115, 215301 (2015)

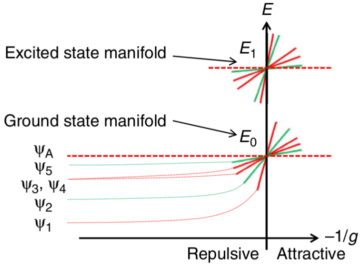
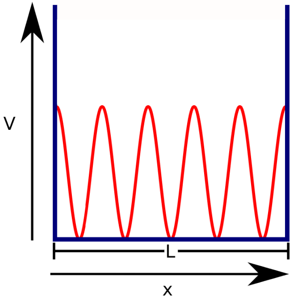


Figure taken from Volosneiv *et al.*, Nat. Comm. 5, 5300 (2014)

$$E = E_0 - \frac{K}{g}\ , \ (K > 0) \qquad (4)$$

K is the energy spectrum of the spin chain.

Volosniev *et al.*, Nat. Comm. 5, 5300 (2014)
Volosniev *et al.*, PRA 91, 023620 (2015)
Deuretzbacher *et al.*, PRA 90, 013611 (2014)
Experiment: Murmann *et al.*, PRL 115, 215301 (2015)



We need to obtain the exchange coefficients (α_k) for the spin chain with our potential $V(x) = V_0 cos\left(\frac{2\pi}{d}x\right)$.

Can do this with an open source code, CONAN, develeloped here in Aarhus by Nikolaj Zinner's group.

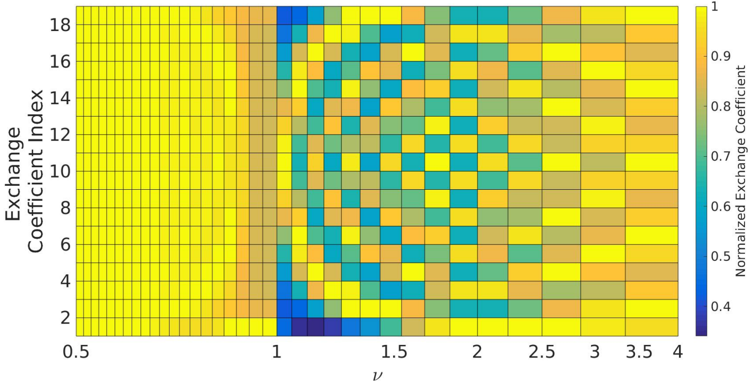
The CONAN code can be summarised,

1. Takes a one dimensional potential in a mathematical form that is defined in a hard wall trap from 0 to L, for a number of particles N.
2. Using the single particle solutions for the trap and potential, CONAN obtains the (N-1) exchange coefficients for the spin chain.

From this have an eigenvalue problem for the spin chain, that can be solved numerically.

Loft *et al.*, arXiv:1603.02662 (2016)

Spin Chain Exchange Coefficients, $N = 20$



Two clear regimes, 1) $\nu < 1$, and 2) $\nu > 1$

Single Magnon

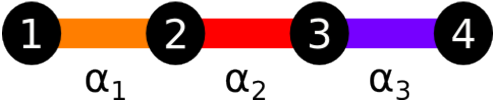
Have a single magnon, e.g. for $N = 4$

$$|\downarrow\uparrow\uparrow\uparrow\rangle, \quad |\uparrow\downarrow\uparrow\uparrow\rangle, \quad |\uparrow\uparrow\downarrow\uparrow\rangle, \quad |\uparrow\uparrow\uparrow\downarrow\rangle$$

We write this for general N as

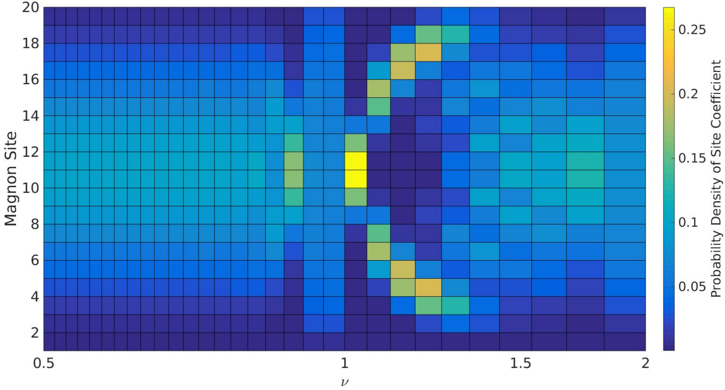
$$| \Psi\rangle = \sum_{j=1}^{N} \psi(j) \, |\uparrow \cdots \uparrow (\downarrow)_j \uparrow \cdots \uparrow\rangle \tag{5}$$

with the mangnon on the jth site. We will write our wavefunctions in this section as a function of $\psi(j)$, the coefficient at the jth site.



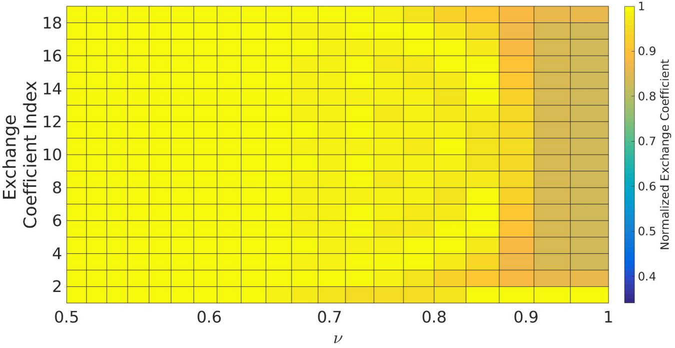
$$\hat{K} = \begin{pmatrix} \alpha_1 & -\alpha_1 & 0 & 0 \\ -\alpha_1 & \alpha_1 + \alpha_2 & -\alpha_2 & 0 \\ 0 & -\alpha_2 & \alpha_2 + \alpha_3 & -\alpha_3 \\ 0 & 0 & -\alpha_3 & \alpha_3 \end{pmatrix} \quad (6)$$

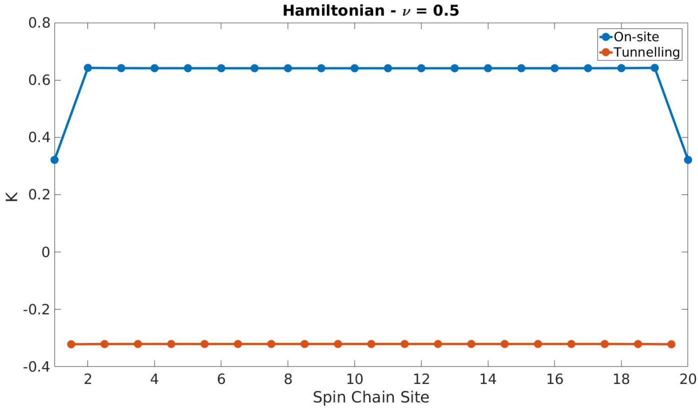
Groundstates $N = 20$



$\nu < 1$


α1
α2
α3
α4


Exchange
Coefficient Index
18
16
14
12
10
8
6
4
2
0.5
0.6
0.7
0.8
0.9
1
ν
Normalized Exchange Coefficient
1
0.9
0.8
0.7
0.6
0.5
0.4



$$-J\left[\psi(j+1)+\psi(j-1)\right]+V\psi(j)=E\psi(j), \quad j\neq 1,N \qquad (7)$$

Can analytically obtain the spectra

$$E = V - 2Jcos\left(k\right) \tag{8}$$

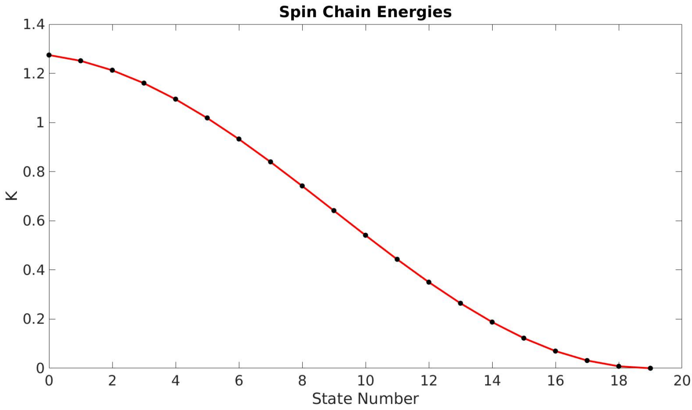
and the wavefunctions,

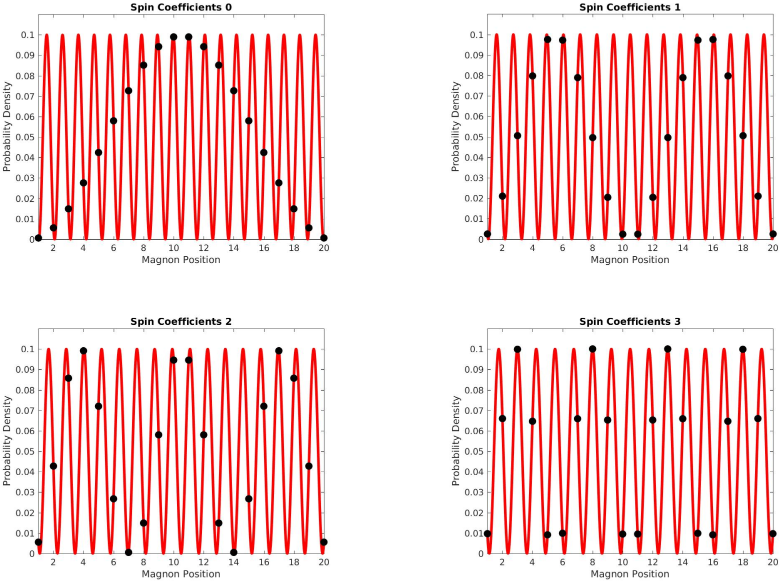
$$\psi_m = \frac{1}{\sqrt{2N}} \sum_{j=1}^{N-1} \left(e^{ik_m j} + e^{-ik_m (j-1)} \right) \tag{9}$$

where

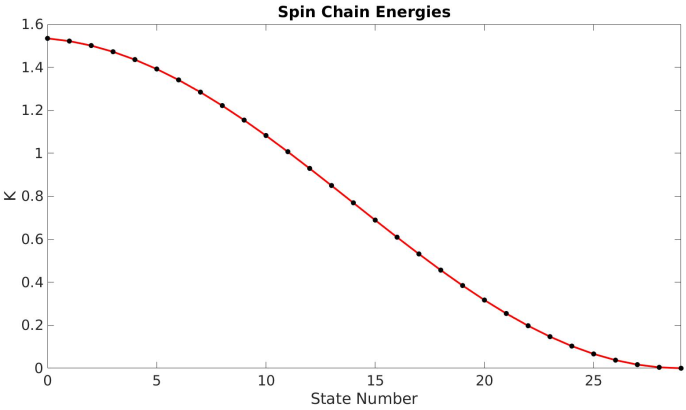
$$k_m = \frac{\pi m}{N} \qquad m = 0, 1, \cdots, N-1 \tag{10}$$

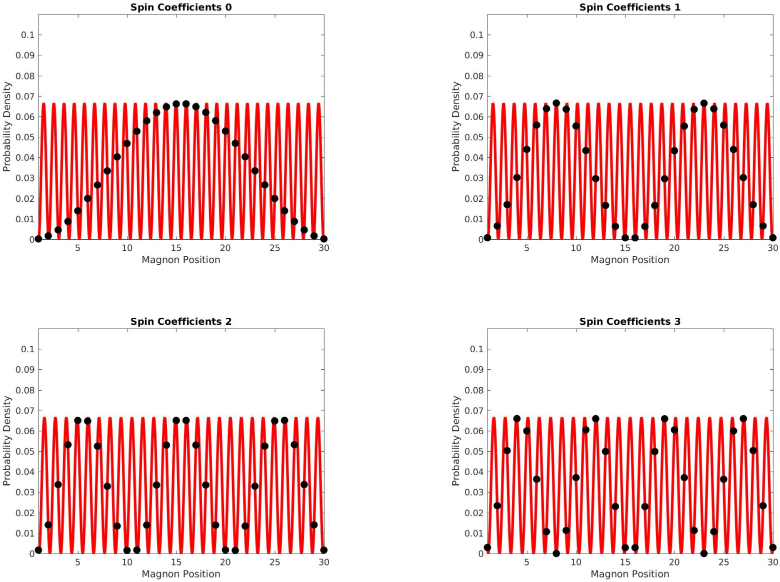
$N = 20$


Spin Chain Energies
K
State Number
1.4
1.2
1
0.8
0.6
0.4
0.2
0
0
2
4
6
8
10
12
14
16
18
20

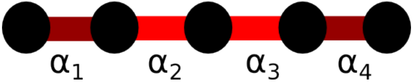

Spin Coefficients 0
Spin Coefficients 1
Spin Coefficients 2
Spin Coefficients 3
Probability Density
Magnon Position
0
0.01
0.02
0.03
0.04
0.05
0.06
0.07
0.08
0.09
0.1
2
4
6
8
10
12
14
16
18
20

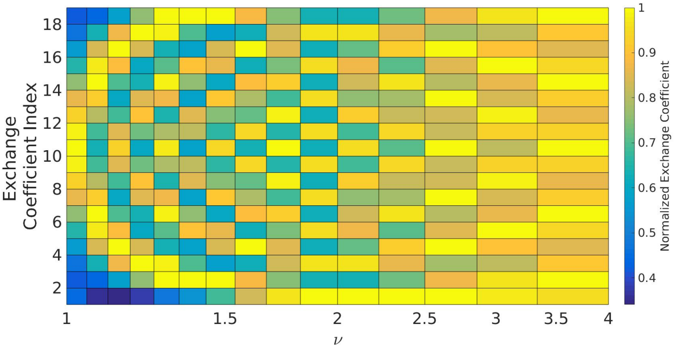
$N = 30$

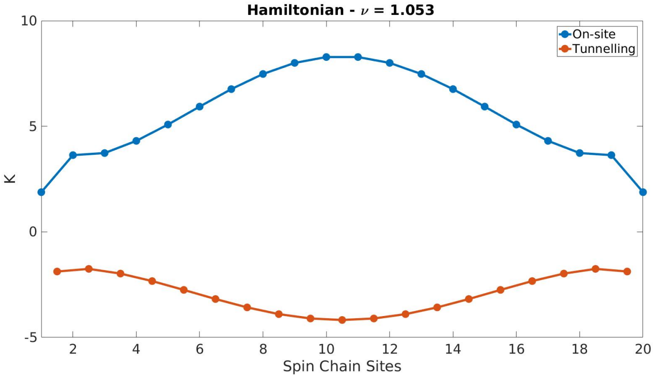

Spin Chain Energies
K
1.6
1.4
1.2
1
0.8
0.6
0.4
0.2
0
0
5
10
15
20
25
State Number


Spin Coefficients 0
Probability Density
Magnon Position
Spin Coefficients 1
Probability Density
Magnon Position
Spin Coefficients 2
Probability Density
Magnon Position
Spin Coefficients 3
Probability Density
Magnon Position

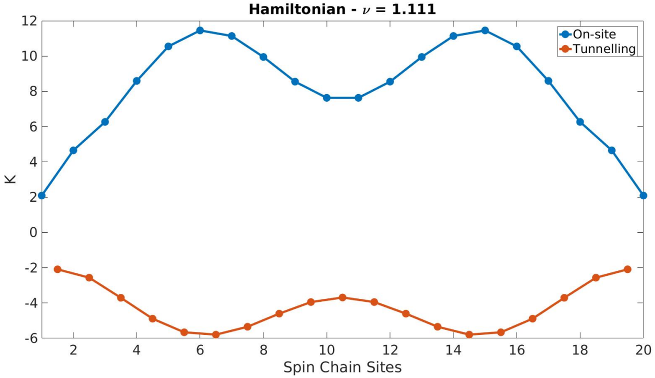
$\nu > 1$


α1
α2
α3
α4


Exchange Coefficient Index
18
16
14
12
10
8
6
4
2
1
1.5
2
2.5
3
3.5
4
ν
Normalized Exchange Coefficient
1
0.9
0.8
0.7
0.6
0.5
0.4



Hamiltonian - ν = 1.053
On-site
Tunnelling
K
Spin Chain Sites
10
5
0
-5
2
4
6
8
10
12
14
16
18
20

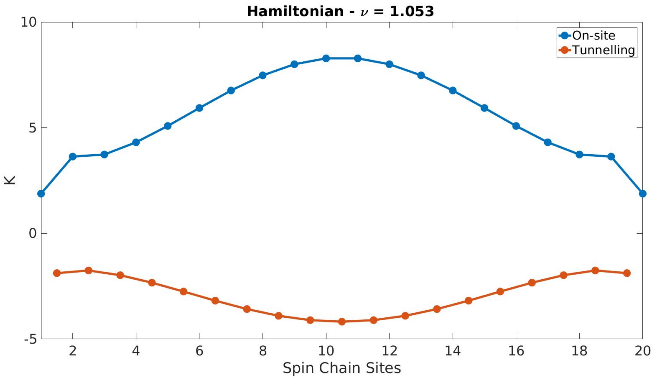

Hamiltonian - ν = 1.111
On-site
Tunnelling
K
Spin Chain Sites
12
10
8
6
4
2
0
-2
-4
-6
2
4
6
8
10
12
14
16
18
20

Get well-like structure for the spin chain potential. Which scales as

$$\nu = 1 + P/N \tag{11}$$

P matches the number of 'wells' in the spin chain potential.

The P extra particles over the lattice interact with the rest of the atoms in each lattice site, and the hard wall trap at the edges. Results in well-like exchange coefficients.



Can find analytical solution by approximating that

$$V_j = V^0 + V^1 cos\left(\alpha j\right)$$
$$J_{j,j+1} = J^0 + J^1 cos\left(\alpha(j + 1/2)\right). \quad (12)$$

We then expand around centre, to give a constant and a inverted harmonic well

$$J_{j,j+1} \approx J^0 + J^1 \tag{13}$$

$$V_j \approx V^0 + V^1 - \frac{1}{2} V^1 \alpha^2 j^2 \tag{14}$$

The spin chain Hamiltonian has the same form as for a single particle on a lattice

$$H = \sum_{j=-(N-1)/2}^{(N-1)/2} \left[V_j \hat{n}_j - J_{j,j+1} \left(\hat{a}_j^\dagger \hat{a}_{j+1} + \hat{a}_{j+1}^\dagger \hat{a}_j \right) \right]. \tag{15}$$

With the approximation to the single well as a harmonic oscillator, get expected solutions with modified mass and frequency,

$$\psi_n(x) \approx \mathcal{N}(2^n n!)^{-\frac{1}{2}} H_n\left(\sqrt[4]{\frac{\mathcal{V}}{\mathcal{J}}}x\right) e^{-\sqrt[4]{\frac{\mathcal{V}}{\mathcal{J}}}\frac{x^2}{2}} cos(\pi x) \tag{16}$$

$$E_n \approx E_{off} - 2\sqrt{\mathcal{J}\mathcal{V}}\left(n + \frac{1}{2}\right). \tag{17}$$

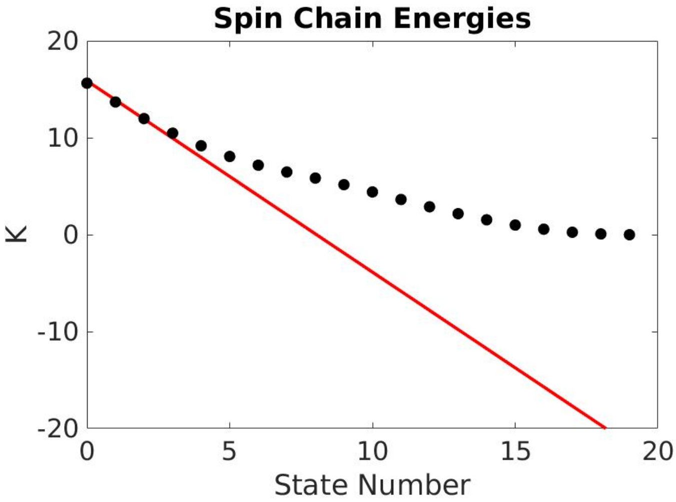
with

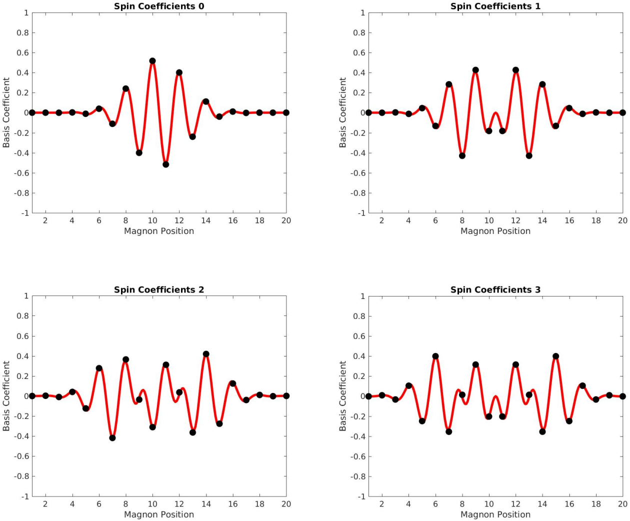
$$\mathcal{V} = \frac{1}{2} V^1 \alpha^2 \tag{18}$$

$$\mathcal{J} = (J^0 + J^1) \tag{19}$$

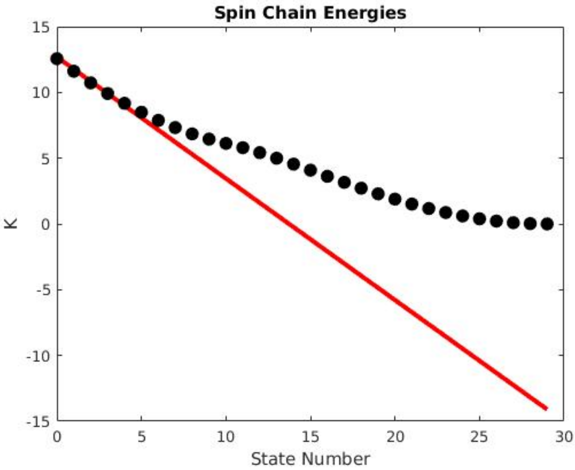
Valiente and Petrosyan, EPL, 83 (2008) 30007

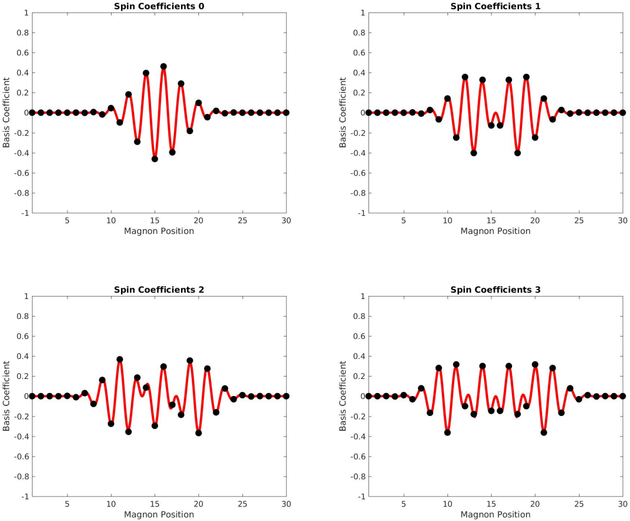
$N = 20$

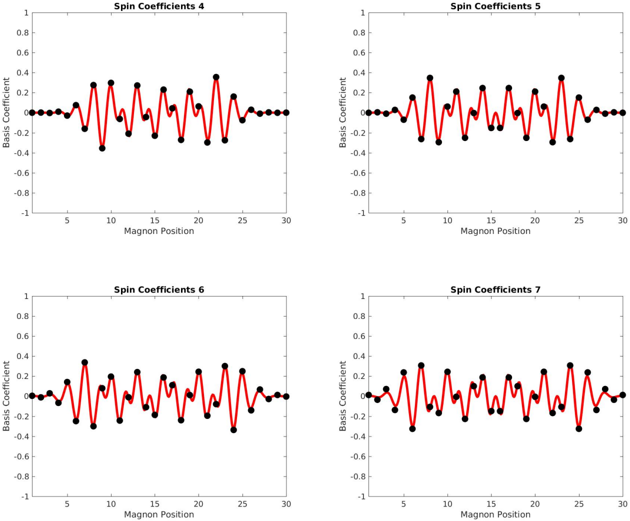



Spin Coefficients 0
Spin Coefficients 1
Spin Coefficients 2
Spin Coefficients 3
Basis Coefficient
Magnon Position
1
0.8
0.6
0.4
0.2
0
-0.2
-0.4
-0.6
-0.8
-1
2 4 6 8 10 12 14 16 18 20

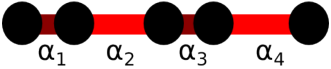
$N = 30$

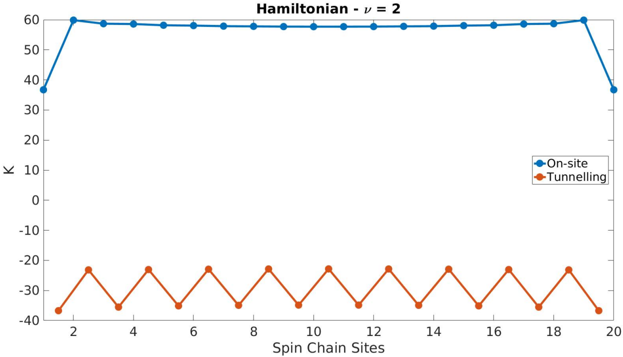

Spin Chain Energies
K
State Number
15
10
5
0
-5
-10
-15
0
5
10
15
20
25
30


Spin Coefficients 0
Spin Coefficients 1
Spin Coefficients 2
Spin Coefficients 3
Basis Coefficient
Magnon Position


Spin Coefficients 4
Spin Coefficients 5
Spin Coefficients 6
Spin Coefficients 7
Basis Coefficient
Magnon Position

$\nu = 2$


α1
α2
α3
α4


Hamiltonian - ν = 2
60
50
40
30
20
10
0
-10
-20
-30
-40
K
On-site
Tunnelling
2
4
6
8
10
12
14
16
18
20
Spin Chain Sites

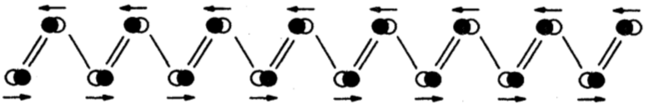


Figure from Heeger, *et al.* Mod. Phys. 60, 3 (1988)

This is approximately the, Su-Schriefer-Heeger Model (polyacetylene), have alternating values of the tunnelling t_1 and t_2

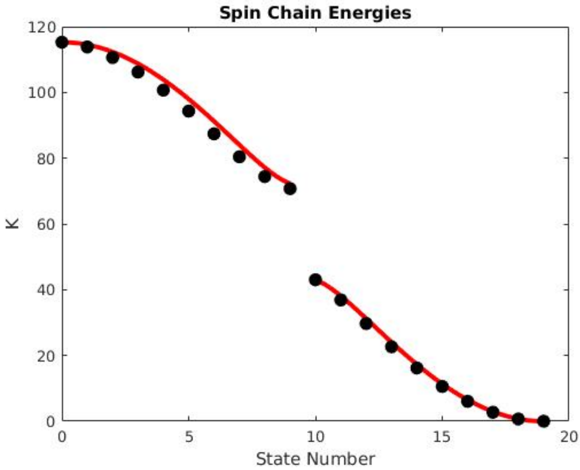
$$H = -\sum_{j=1}^{N/2} \left[t_1 \left(\hat{a}_j^\dagger \hat{b}_j + h.c. \right) + t_2 \left(\hat{b}_{j+1}^\dagger \hat{a}_j + h.c. \right) \right], \qquad (20)$$

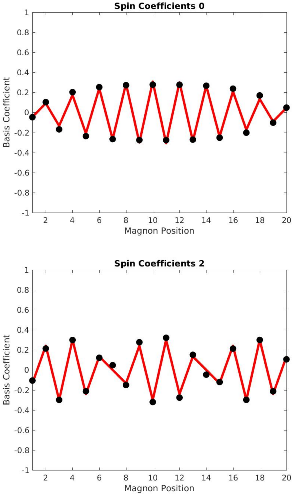
with energies

$$\epsilon(n) = \pm \left(t_1^2 + t_2^2 + 2 t_1 t_2 cos(kn) \right)^{1/2}. \qquad (21)$$

Su, Schriefer, Heeger, PRL 42, 25 (1979)
Heeger, Kivelson, Schrieffer, Su, Rev. Mod. Phys. 60, 3 (1988)

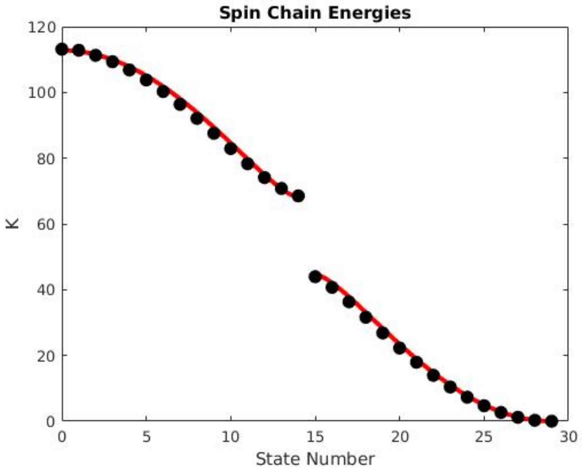
$N = 20$

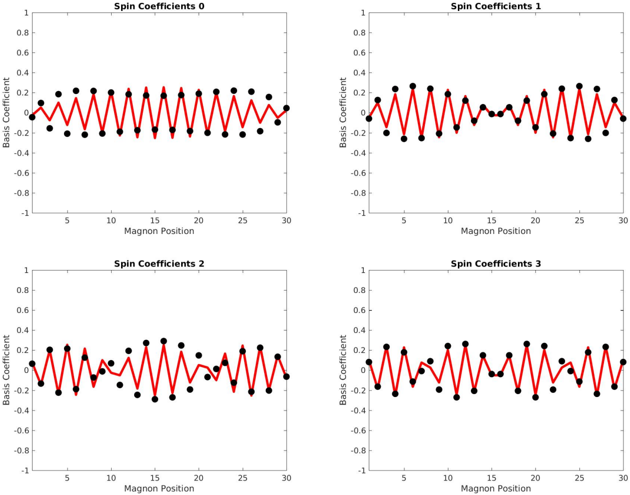

Spin Chain Energies
K
State Number
120
100
80
60
40
20
0
0
5
10
15
20



SSH wavefunctions found in Shen, “Topological Insulators: Dirac Equations in Condensed Matters”, Springer (2012), pp.74–79

$N = 30$


Spin Chain Energies
120
100
80
60
40
20
0
K
0
5
10
15
20
25
30
State Number



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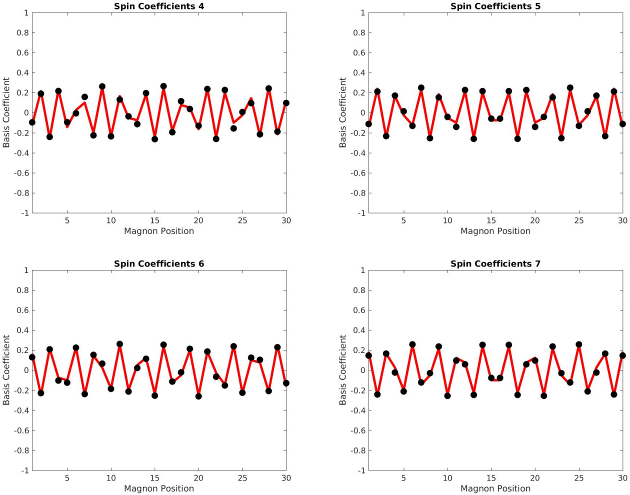
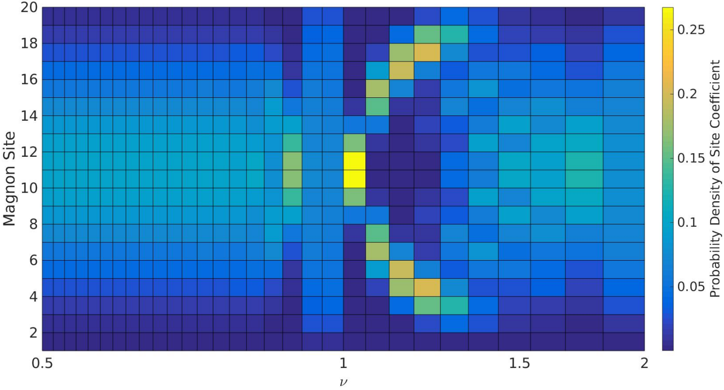


SSH wavefunctions found in Shen, "Topological Insulators: Dirac Equations in Condensed Matters", Springer (2012), pp.74–79

The Groundstates $N = 20$



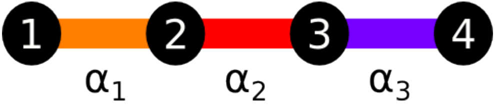
Two Magnons

We want to investigate interactions, magnon–magnon effects.
Require 2 magnons, for $N = 4$ (balanced case) would have basis of,

$$|\downarrow\downarrow\uparrow\uparrow\rangle, \ |\downarrow\uparrow\downarrow\uparrow\rangle, \ |\uparrow\downarrow\downarrow\uparrow\rangle, \ |\downarrow\uparrow\uparrow\downarrow\rangle, \ |\uparrow\downarrow\uparrow\downarrow\rangle, \ |\uparrow\uparrow\downarrow\downarrow\rangle$$

For N particles we have the general basis

$$| \Psi\rangle = \sum_{j_1<j_2}^{N} \psi(j_1, j_2) \, |\uparrow \cdots \uparrow (\downarrow)_{j_1} \uparrow \cdots \uparrow (\downarrow)_{j_2} \uparrow \cdots \uparrow\rangle \tag{22}$$



$$H = \begin{pmatrix} \alpha_2 & -\alpha_2 & 0 & 0 & 0 & 0 \\ -\alpha_2 & \alpha_1+\alpha_2+\alpha_3 & -\alpha_1 & -\alpha_3 & 0 & 0 \\ 0 & -\alpha_1 & \alpha_1+\alpha_3 & 0 & -\alpha_3 & 0 \\ 0 & -\alpha_3 & 0 & \alpha_1+\alpha_3 & -\alpha_1 & 0 \\ 0 & 0 & -\alpha_3 & -\alpha_1 & \alpha_1+\alpha_2+\alpha_3 & -\alpha_2 \\ 0 & 0 & 0 & 0 & -\alpha_2 & \alpha_2 \end{pmatrix}$$

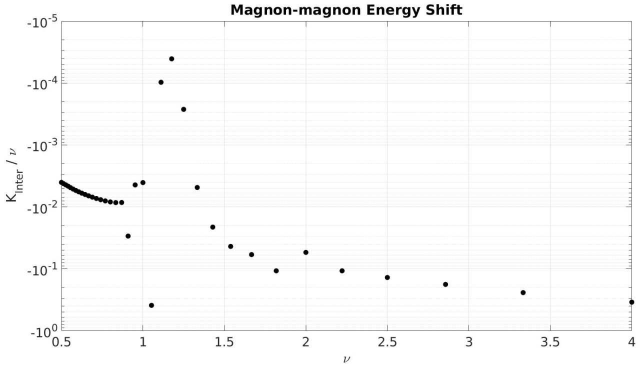
Quantify the interactions via the magnon–magnon interaction energy shift,

$$K_{inter} = K_0^I - K_0^{NI} \tag{23}$$

where K_0^I and K_0^{NI} are the interacting and non-interacting two magnon groundstates.

Get effective attractive interactions in the spin chain ($K_{inter} < 0$).

Preliminary results

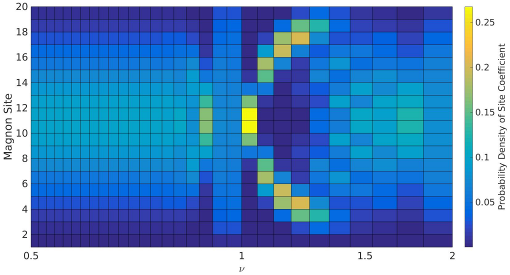


Possibly a transition between the two regimes, 1) Homogeneous $\nu < 1$, dilute 2-body interactions 2) Single well, localized close together, high energy shift. 3) Multiple wells can get 2 magnons further apart.

Summary

- For the magnon confined in a trap with an optical lattice potential have two main regimes dependent on reasonable filling.
- Approximately homogeneous system for $\nu < 1$, with plane wave solutions and standard spectra.
- Well-like potentials for the spin chain for $\nu > 1$, with low energy states approximated by harmonic well wavefunctions.
- Interesting Hamiltonian at $\nu = 2$, SSH model and can approximate low energy wavefunctions with particle in a box solutions.
- With 2 magnons, get attractive effective interactions in the system that are dependent on the regime.

Thank you for listening



HERIOT WATT UNIVERSITY

The Scottish Doctoral Training Centre in Condensed Matter Physics

CM-CDT

An EPSRC Centre for Doctoral Training in Condensed Matter Physics

SUPA

EPSRC Engineering and Physical Sciences Research Council

$$\alpha_k = 2\sum_{i=1}^{N}\sum_{j=1}^{N}\sum_{l=0}^{N-1-k}\frac{(-1)^{i+j+N-k}}{l!}\begin{pmatrix} N-l-2 \\ k-1 \end{pmatrix}$$
$$\int_a^b dx \frac{2m}{\hbar^2}\left(V(x)-E_i\right)\frac{d\psi_j}{dx} \tag{24}$$
$$\times\left[\frac{\partial^l}{\partial\lambda^l}\det\left[\left(B(x)-\lambda\mathbf{I}\right)^{(ij)}\right]\right]_{\lambda=0}+\sum_{i=1}^{N}\left[\frac{d\psi_i}{dx}\right].$$

Where E_i and ψ_i give the single particle energies and wavefunctions, $B(x)$ is the partial single particle wavefunction overlap matrix,

$$\left[B(x)\right]_{m,n} = \int_a^x dy \psi_m(y)\psi_n(y), \tag{25}$$

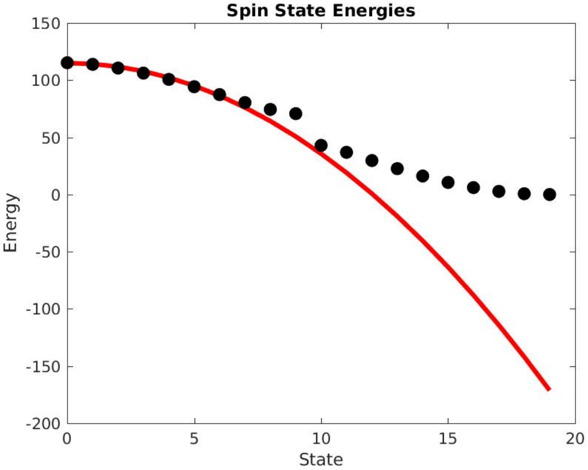
Loft *et al.*, arXiv:1603.02662 (2016)

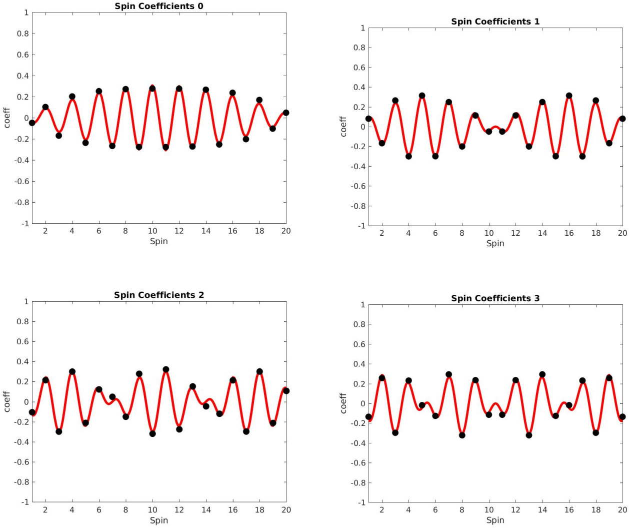
$\nu = 2$, approximative method

We can use similar method as to single well, to approximate this problem as that of a particle in a box with a augmented mass, giving

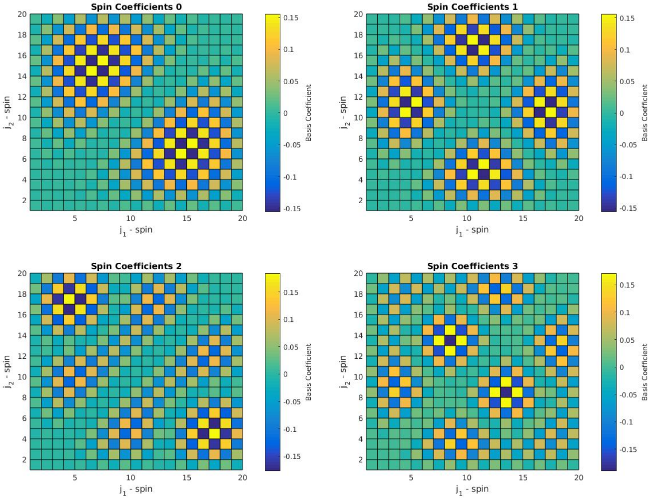
$$\psi_n(j) = \mathcal{N}(-1)^j sin\left(\frac{\pi n}{N+1} j\right), \tag{26}$$

$$E_n \approx V + 2\left(J + \delta J\right) - \left(J + \delta J\right)\frac{n^2\pi^2}{(N+1)^2}. \tag{27}$$

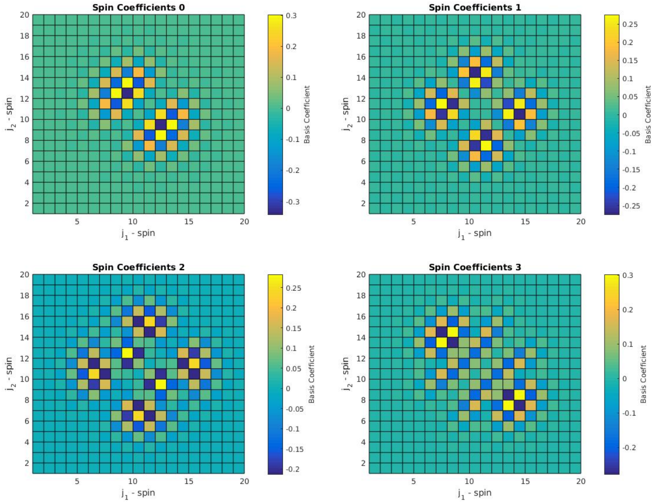

Spin State Energies
150
100
50
0
-50
-100
-150
-200
Energy
0
5
10
15
20
State


Spin Coefficients 0
Spin Coefficients 1
Spin Coefficients 2
Spin Coefficients 3
coeff
Spin
1
0.8
0.6
0.4
0.2
0
-0.2
-0.4
-0.6
-0.8
-1
2
4
6
8
10
12
14
16
18
20

$\nu = 0.5$


Spin Coefficients 0
Spin Coefficients 1
Spin Coefficients 2
Spin Coefficients 3
j_1 - spin
j_2 - spin
Basis Coefficient

$\nu = 1 + 1/20$


Spin Coefficients 0
Spin Coefficients 1
Spin Coefficients 2
Spin Coefficients 3
j1 - spin
j2 - spin
Basis Coefficient

MBL at $\nu = 0.5$

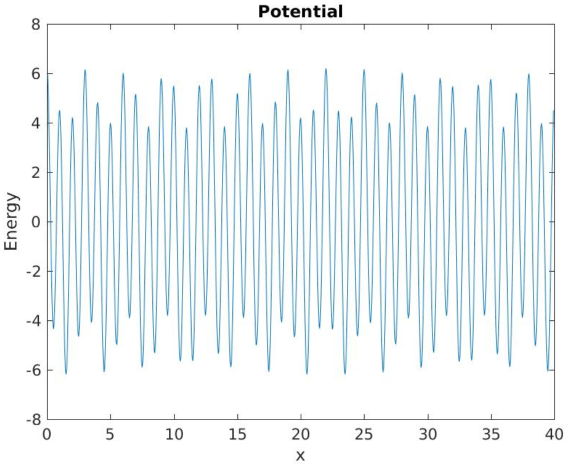
Now have a quasi-random potential

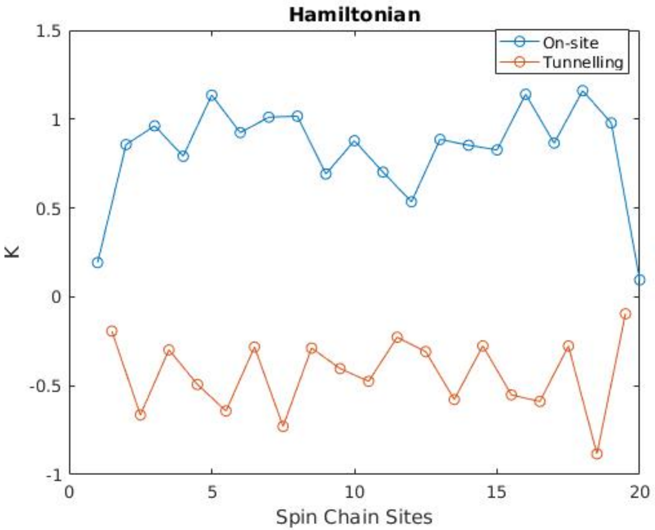
$$V(x) = V_1 cos(\tau_1 x) + V_2 cos(\tau_2 x) \quad (28)$$

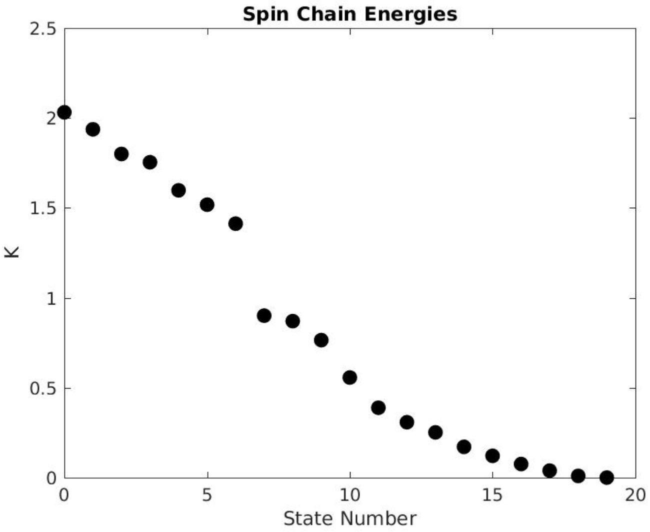
with τ_1 and τ_2 incommensurate.

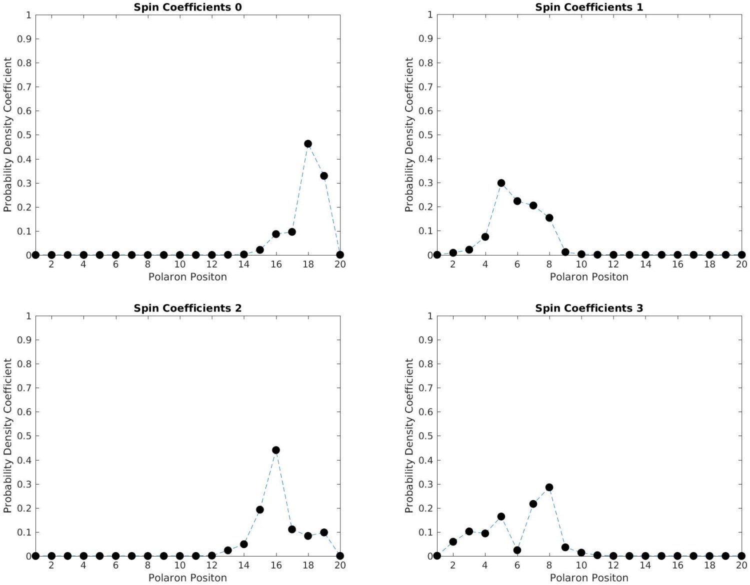
We set V_1 and scale through disorder for strength of V_2. Have 4 regimes dependent on V_2.
1) The clean case ($V_2 = 0$) already discussed.

$V_2/V_1 = 0.24$

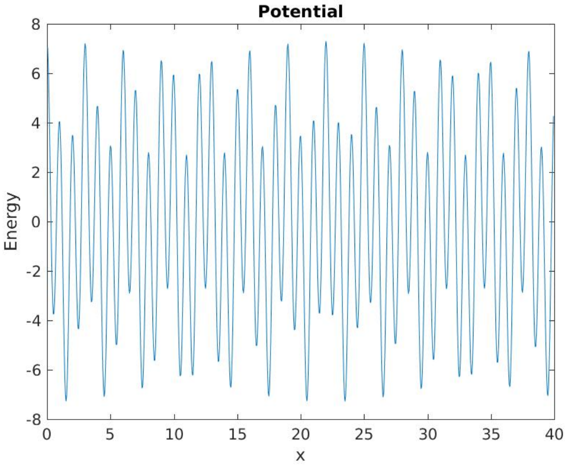


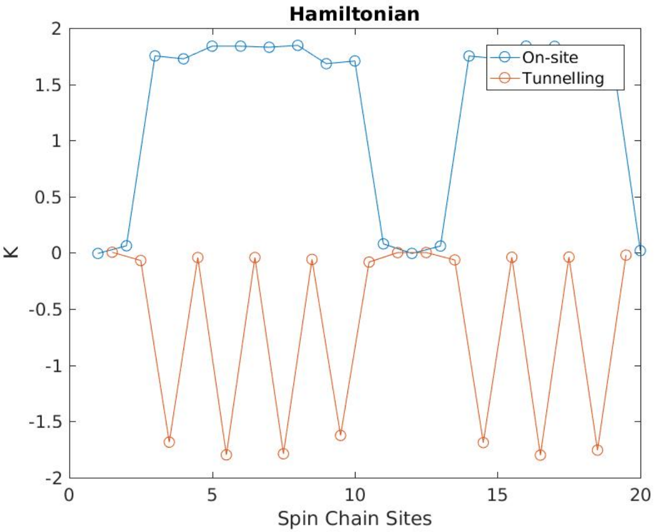

Hamiltonian
On-site
Tunnelling
K
Spin Chain Sites
1.5
1
0.5
0
-0.5
-1
0
5
10
15
20

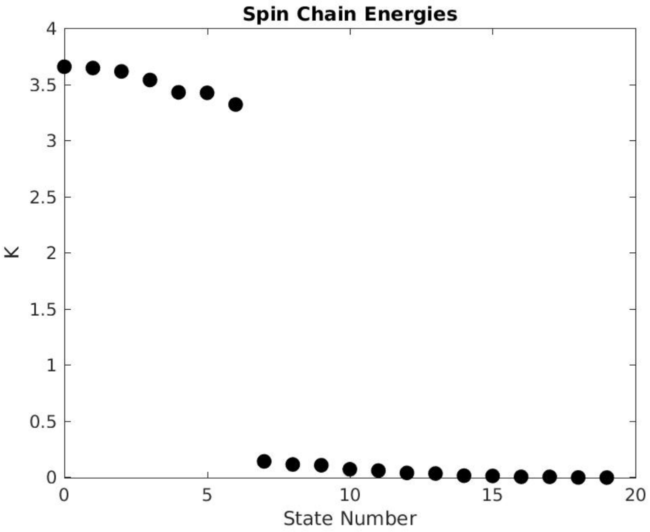

Spin Chain Energies
2.5
2
1.5
1
0.5
0
K
0
5
10
15
20
State Number

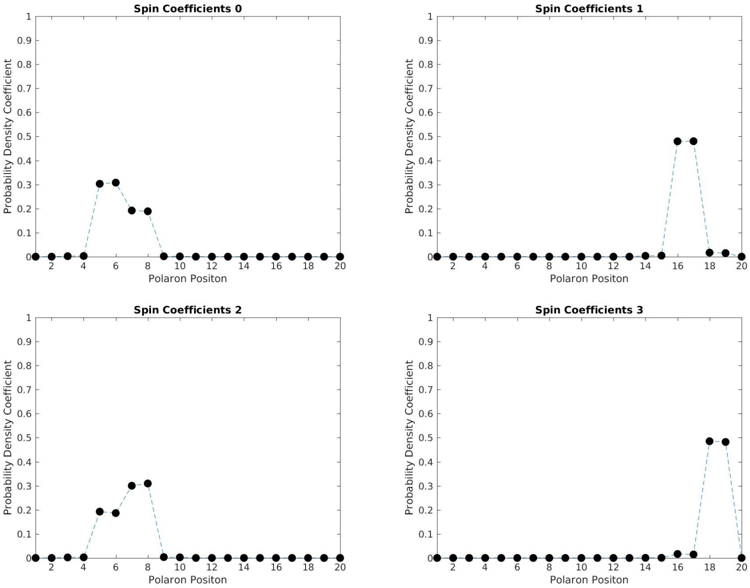

Spin Coefficients 0
Probability Density Coefficient
Polaron Positon
Spin Coefficients 1
Probability Density Coefficient
Polaron Positon
Spin Coefficients 2
Probability Density Coefficient
Polaron Positon
Spin Coefficients 3
Probability Density Coefficient
Polaron Positon

$V_2/V_1 = 0.46$

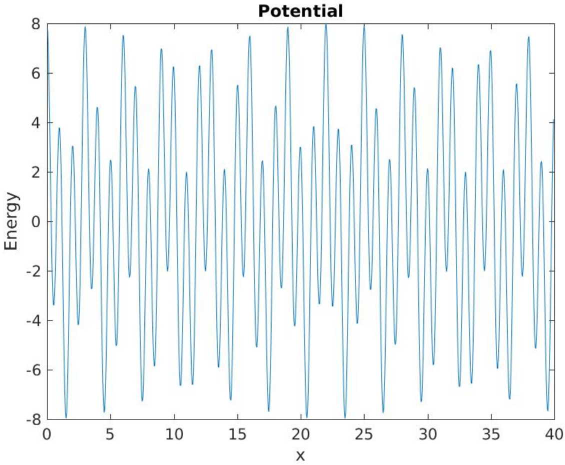


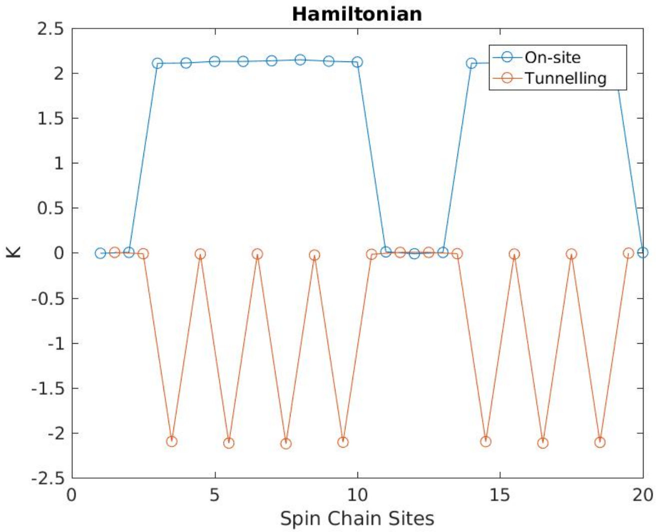

Hamiltonian
On-site
Tunnelling
K
Spin Chain Sites
2
1.5
1
0.5
0
-0.5
-1
-1.5
-2
0
5
10
15
20

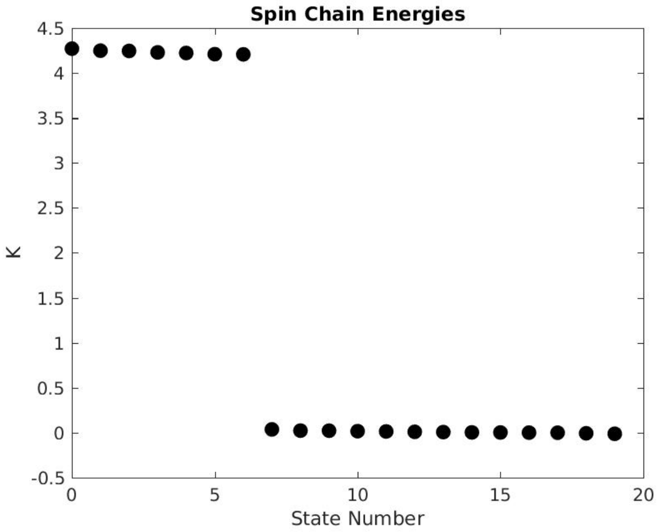

Spin Chain Energies
K
4
3.5
3
2.5
2
1.5
1
0.5
0
0
5
10
15
20
State Number

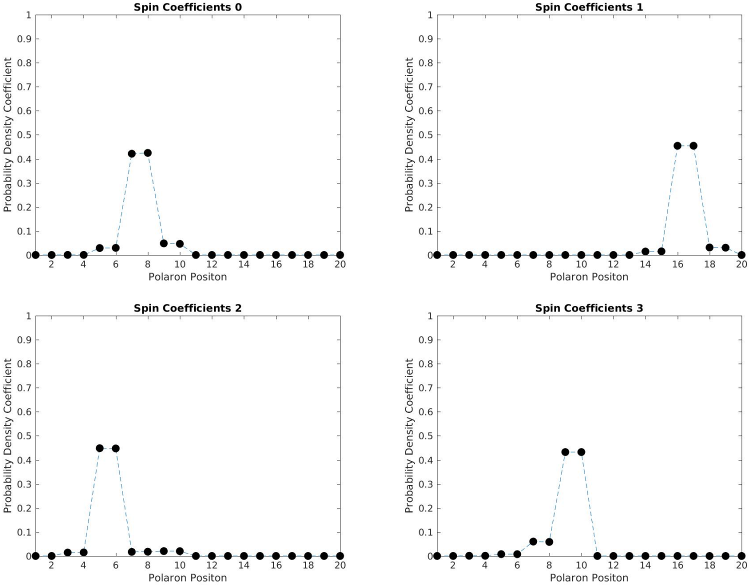

Spin Coefficients 0
Spin Coefficients 1
Spin Coefficients 2
Spin Coefficients 3
Probability Density Coefficient
Polaron Positon

$\frac{V_2}{V_1} = 0.6$




Hamiltonian
On-site
Tunnelling
K
Spin Chain Sites
2.5
2
1.5
1
0.5
0
-0.5
-1
-1.5
-2
-2.5
0
5
10
15
20


Spin Chain Energies
4.5
4
3.5
3
2.5
2
1.5
1
0.5
0
-0.5
K
0
5
10
15
20
State Number


Spin Coefficients 0
Spin Coefficients 1
Spin Coefficients 2
Spin Coefficients 3
Probability Density Coefficient
Polaron Positon
0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
1
2
4
6
8
10
12
14
16
18
20