Leibniz Universität Hannover

Antiferromagnetic Heisenberg spin chain of a few cold atoms in a 1D trap

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Outline

- Girardeau's Bose-Fermi mapping

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- Mapping for particles with spin

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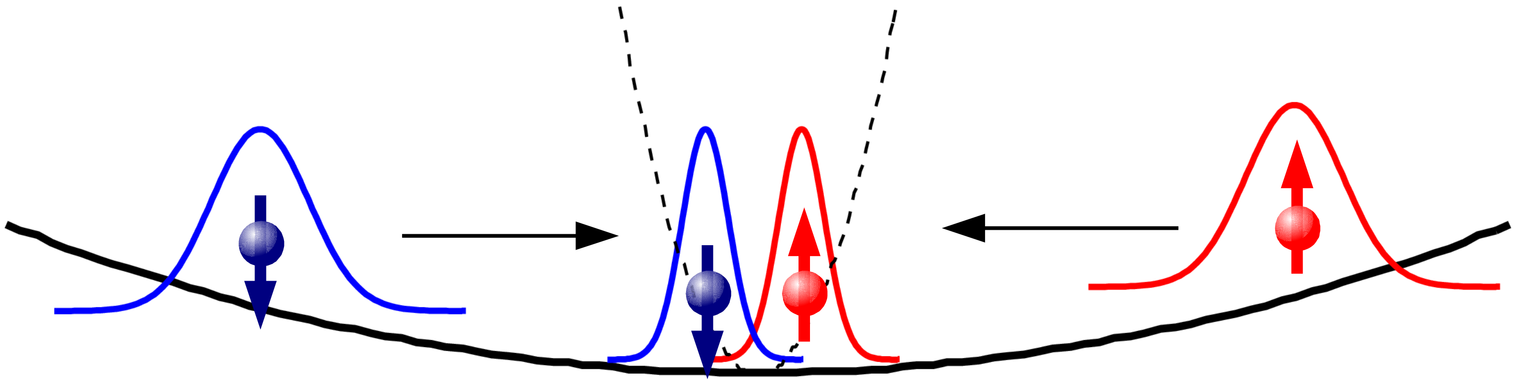
- Girardeau's Bose-Fermi mapping
- Mapping for particles with spin
- Spin chain without lattice
- Application to the experiment
- Numerical methods

System



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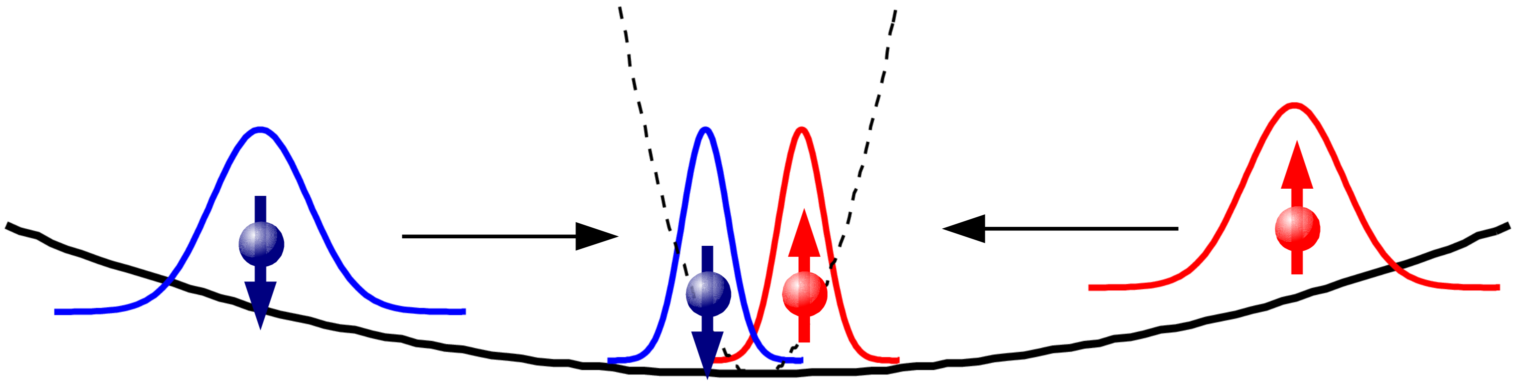

$$H = \sum_i \left[-\frac{\hbar^2}{2m} \frac{\partial^2}{\partial z_i^2} + V(z_i) \right] + g \sum_{i<j} \delta(z_i - z_j)$$

Motivation

- Experiments with cold atoms

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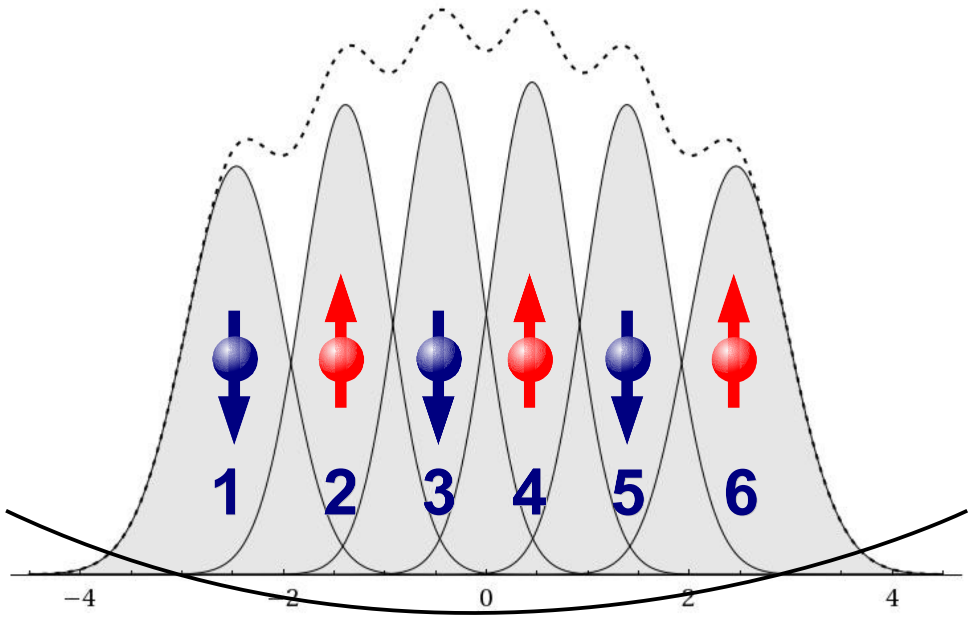
- Simple exact solution for infinitely strong interactions



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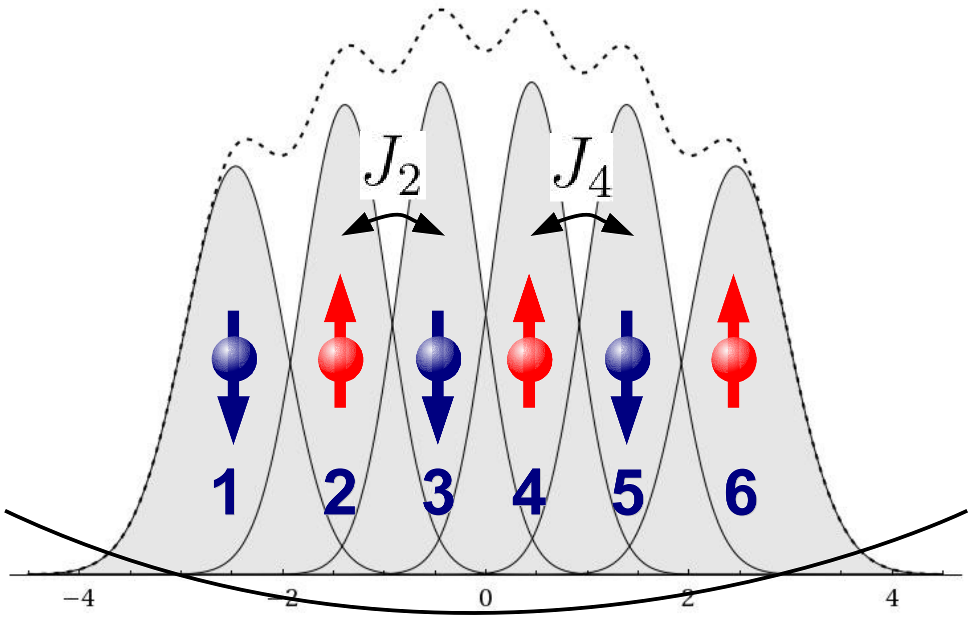
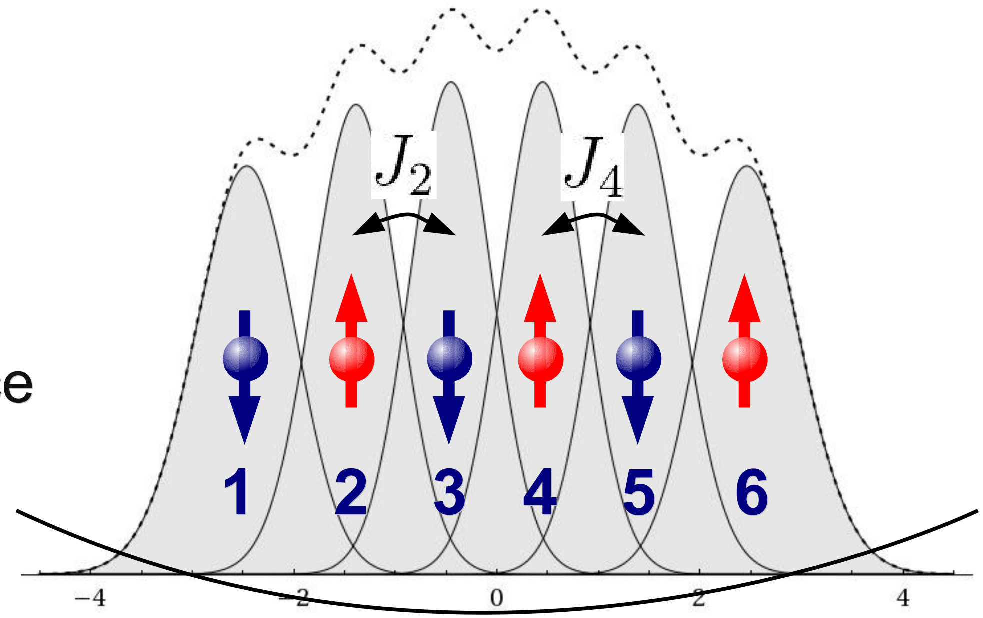


Heisenberg model

$$H_{\text{eff}} \propto \pm \sum_{i=1}^{N-1} J_i \, \vec{S}_i \cdot \vec{S}_{i+1}$$

Motivation

- Experiments with cold atoms
- Simple exact solution for infinitely strong interactions
- Quantum magnetism without lattice



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Antiferromagnetic Heisenberg spin chain of a few cold atoms in a 1D trap

Fermi-Bose mapping for spinless hard-core bosons

$$\psi_B = \left[\frac{1}{\sqrt{N!}}\det[\phi_i(z_j)]_{i,j=1,\ldots,N}\right]$$

Fermi-Bose mapping for spinless hard-core bosons

$$\psi_B = \left[\prod_{i<j} \mathrm{sgn}(z_i - z_j)\right]\left[\frac{1}{\sqrt{N!}}\det[\phi_i(z_j)]_{i,j=1,\dots,N}\right]$$

Fermi-Bose mapping for spinless hard-core bosons

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$$\psi_B^{(0)} = \left| \frac{1}{\sqrt{N!}} \det[\phi_i(z_j)]_{i,j=1,\ldots,N} \right|$$

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$$\left|\psi_B^{(0)}\right|^2 = \left|\psi_F^{(0)}\right|^2$$

Mapping for spinless distinguishable particles

$$\psi_{\mathrm{id}} = \left[\prod_{i=1}^{N-1}\theta(z_{i+1}-z_i)\right]\left|\psi_F^{(0)}\right|$$

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particle ordering $z_1 < z_2 < z_3 < \cdots < z_N$

Mapping for spinless distinguishable particles

$$\psi_{\rm id} = \sqrt{N!}\left[\prod_{i=1}^{N-1}\theta(z_{i+1}-z_i)\right]\left|\psi_F^{(0)}\right|$$

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$$\psi_P = \sqrt{N!}\left[\prod_{i=1}^{N-1}\theta\big(z_{P(i+1)}-z_{P(i)}\big)\right]\left|\psi_F^{(0)}\right|$$

particle ordering $z_{P(1)} < z_{P(2)} < z_{P(3)} < \cdots < z_{P(N)}$

Mapping for spinless distinguishable particles

$$\boxed{\psi_{\mathrm{id}} = \sqrt{N!}\left[\prod_{i=1}^{N-1} \theta(z_{i+1} - z_i)\right] \left|\psi_F^{(0)}\right|}$$

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particle ordering $z_{P(1)} < z_{P(2)} < z_{P(3)} < \cdots < z_{P(N)}$

useful properties of
sector wave functions $\langle P|P'\rangle = \delta_{P,P'} \qquad \hat{P}|P'\rangle = |P \circ P'\rangle$

Mapping for particles with spin

$$|\psi_{m_1,\ldots,m_N}\rangle = \qquad |\mathrm{id}\rangle|m_1, m_2, \ldots, m_N\rangle$$

Mapping for particles with spin

$$|\psi_{m_1,\ldots,m_N}\rangle = \left(\frac{1}{\sqrt{N!}}\sum_P (\pm 1)^P \hat{P}\right)\Big(|\mathrm{id}\rangle|m_1, m_2, \ldots, m_N\rangle\Big)$$

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$$|\psi_\chi\rangle = \left(\frac{1}{\sqrt{N!}}\sum_P (\pm 1)^P \hat{P}\right)\left[|\mathrm{id}\rangle\underbrace{\left(\sum_{m_1,\ldots,m_N} c_{m_1,\ldots,m_N}|m_1,\ldots,m_N\rangle\right)}_{|\chi\rangle}\right]$$

Mapping for particles with spin

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fermionization + spin chain

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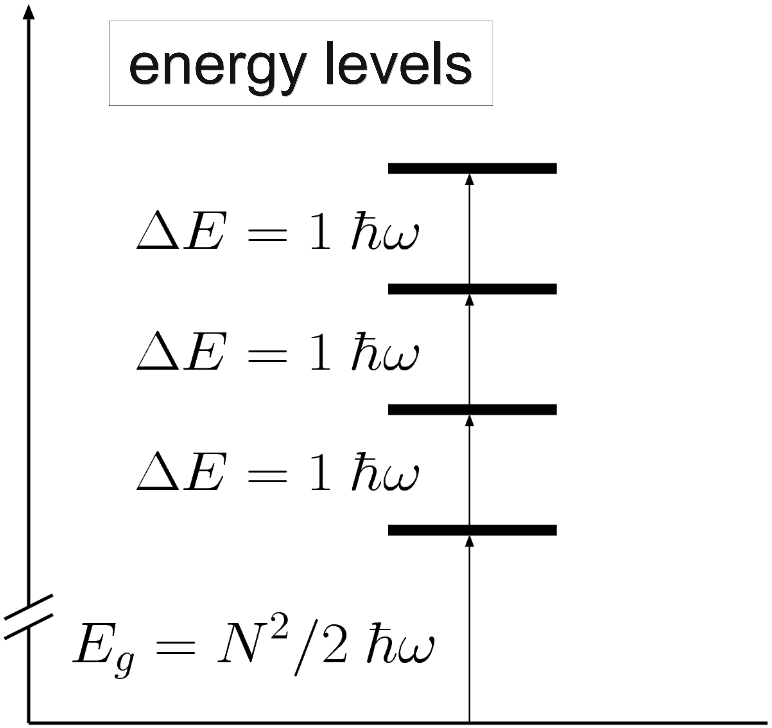
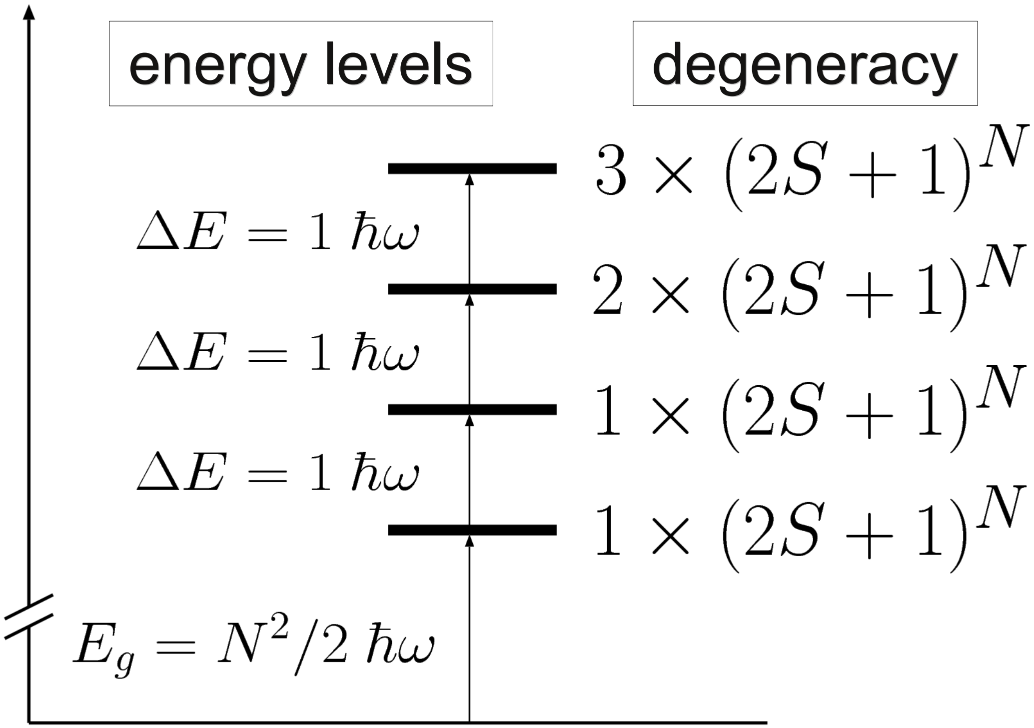


Fermionization + spin chain



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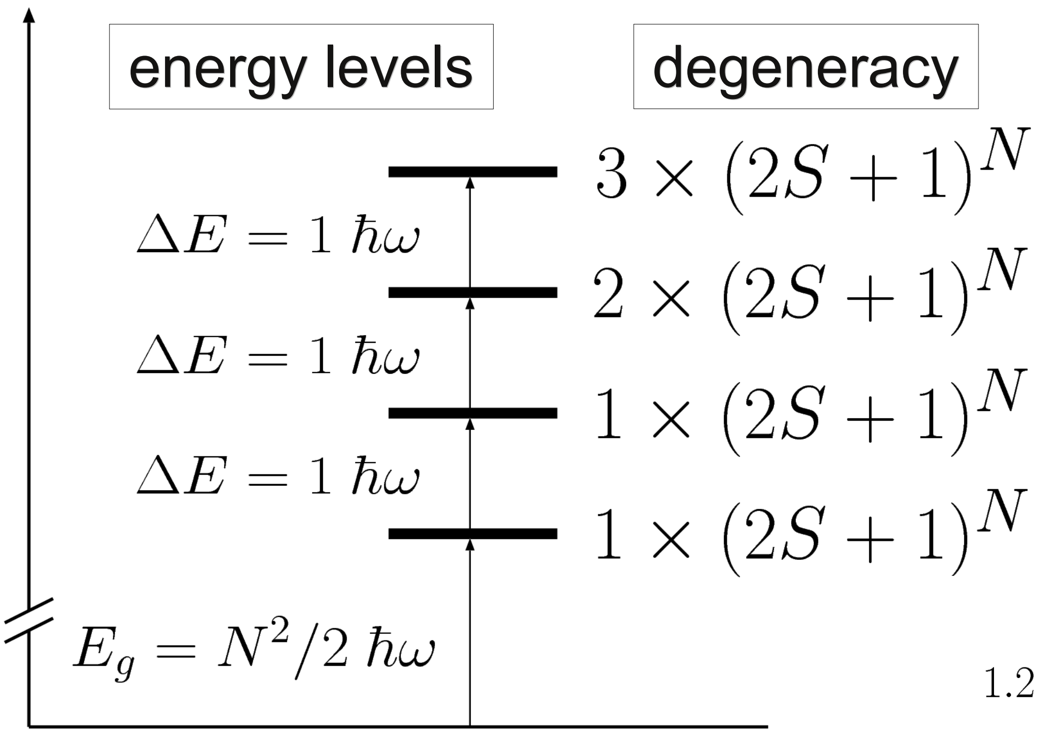

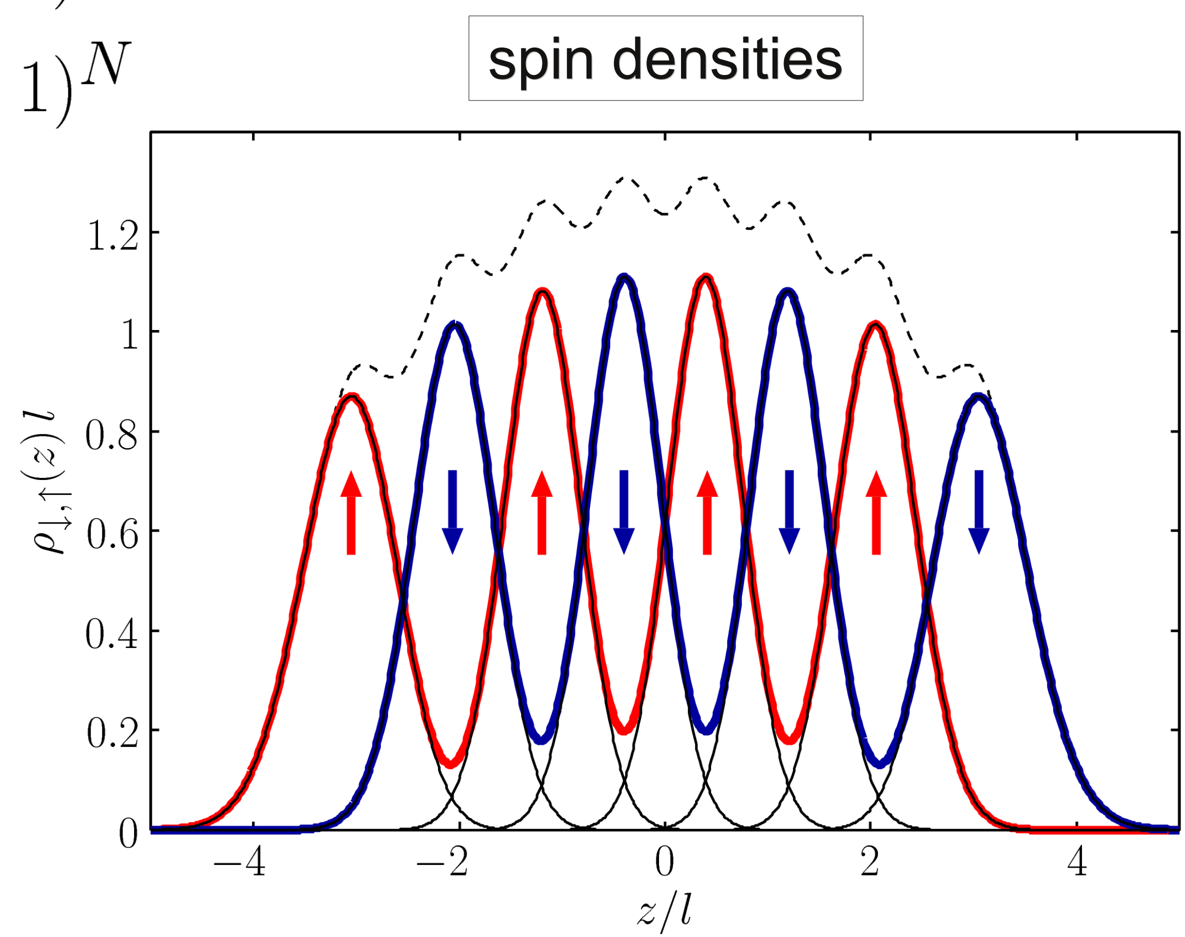

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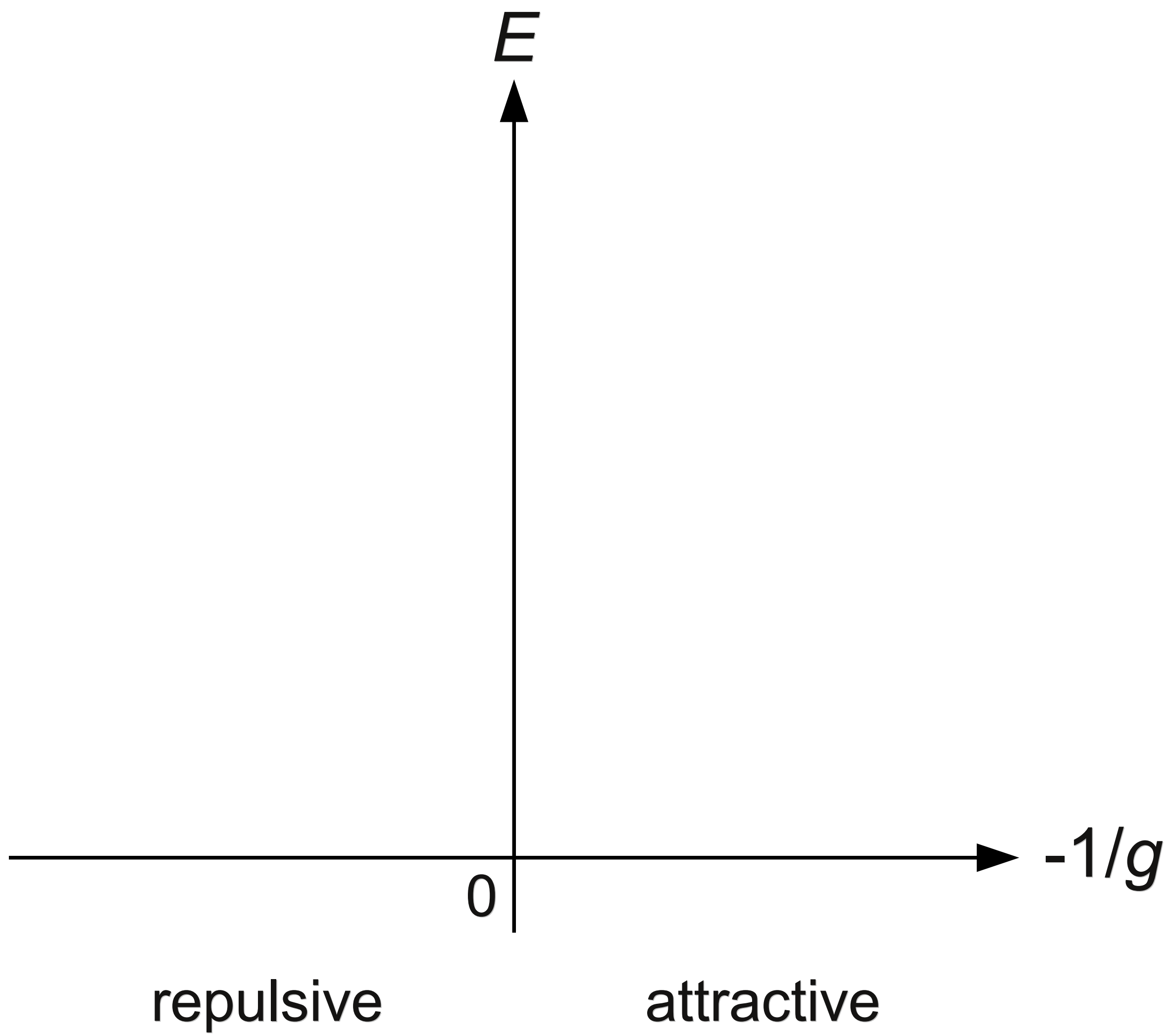
Large, but finite interactions



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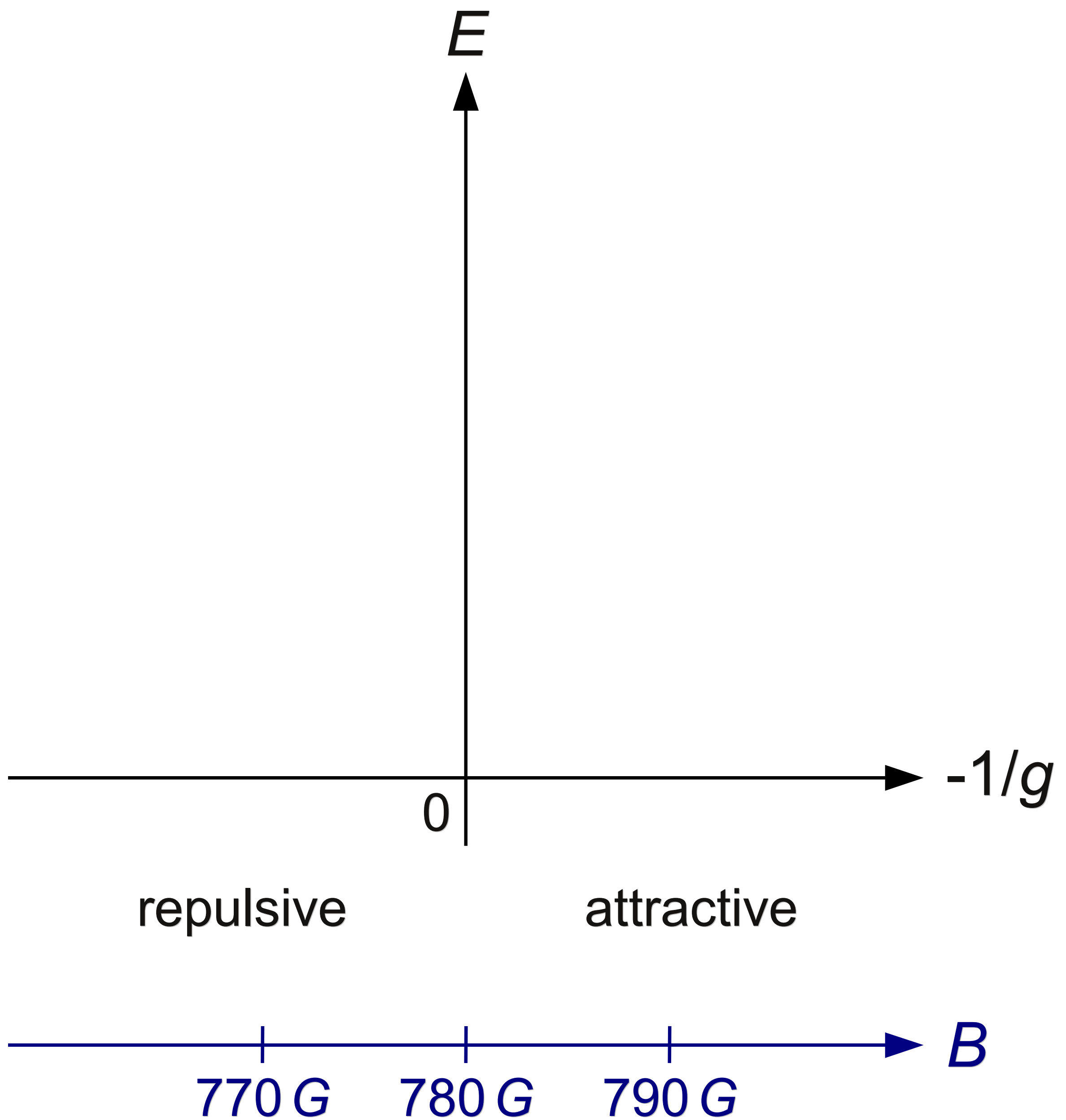

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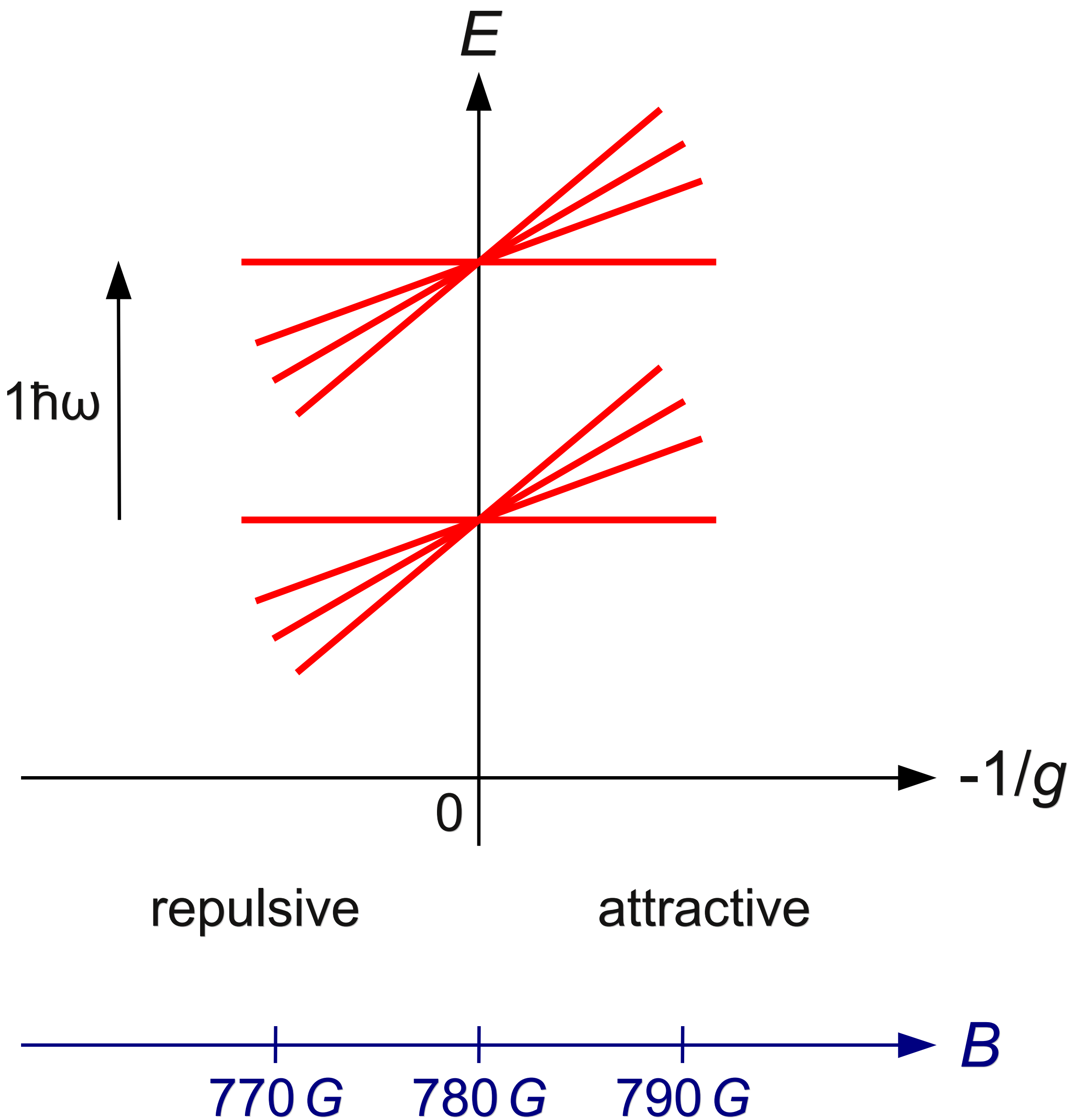

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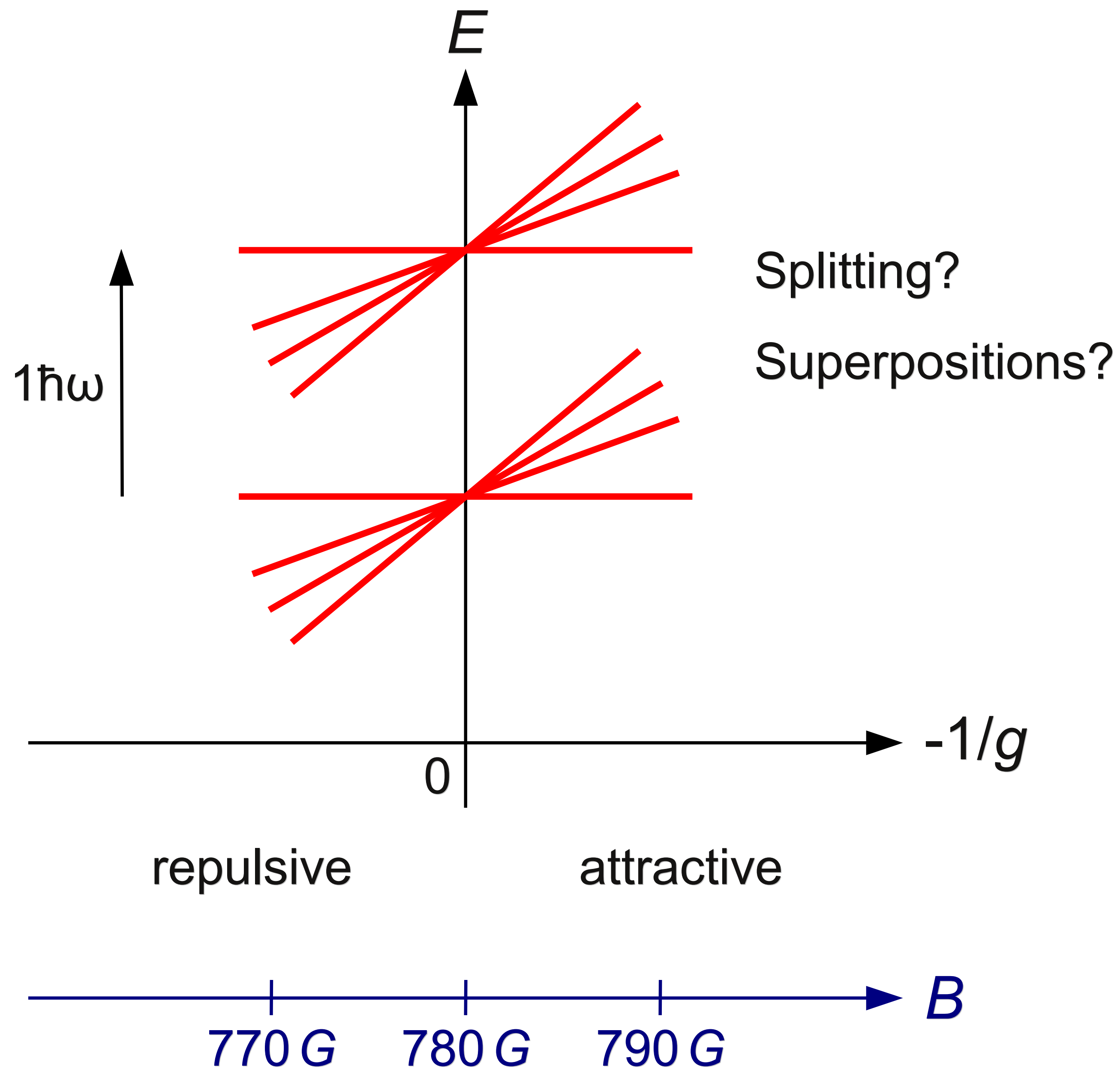

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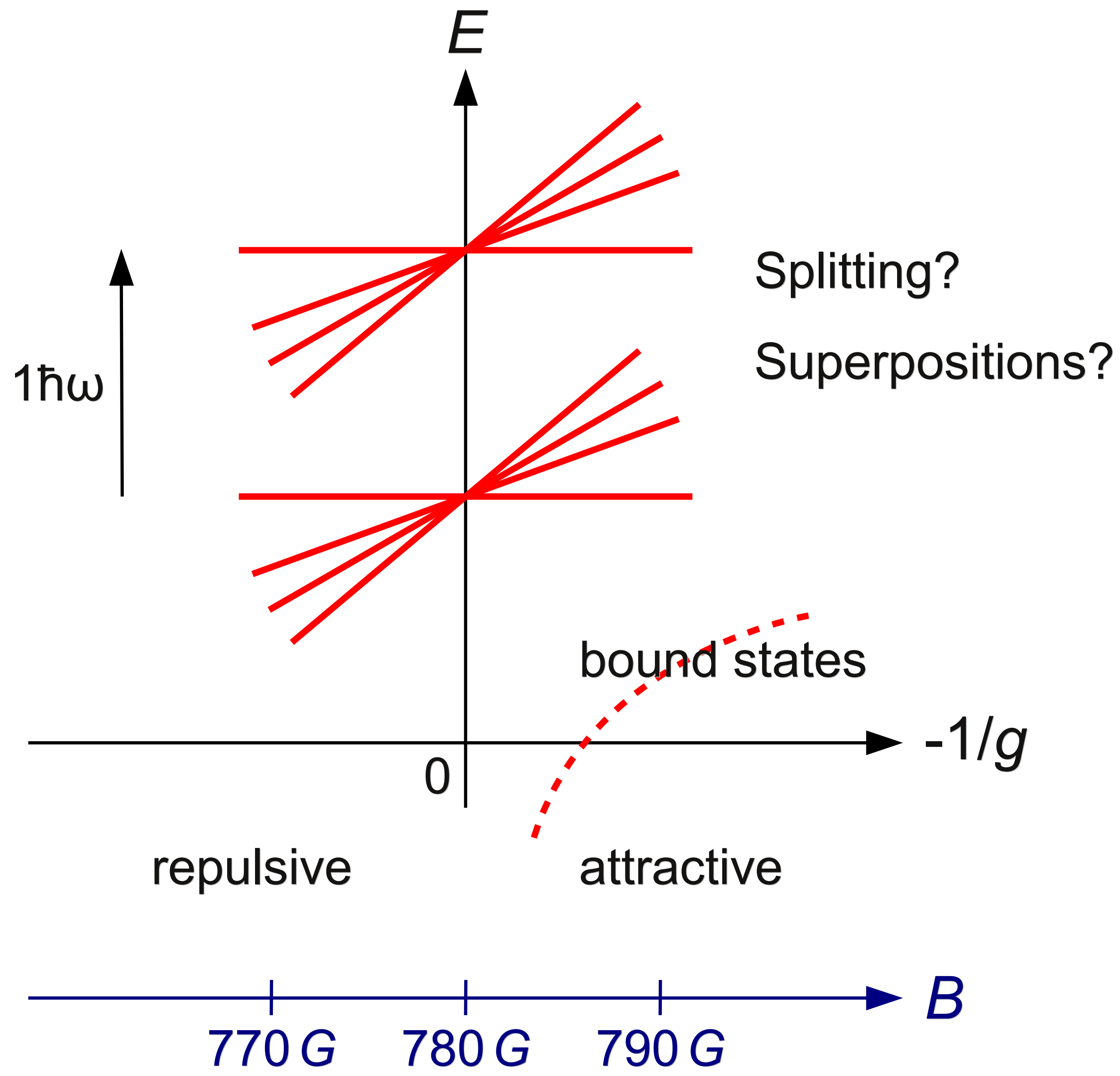
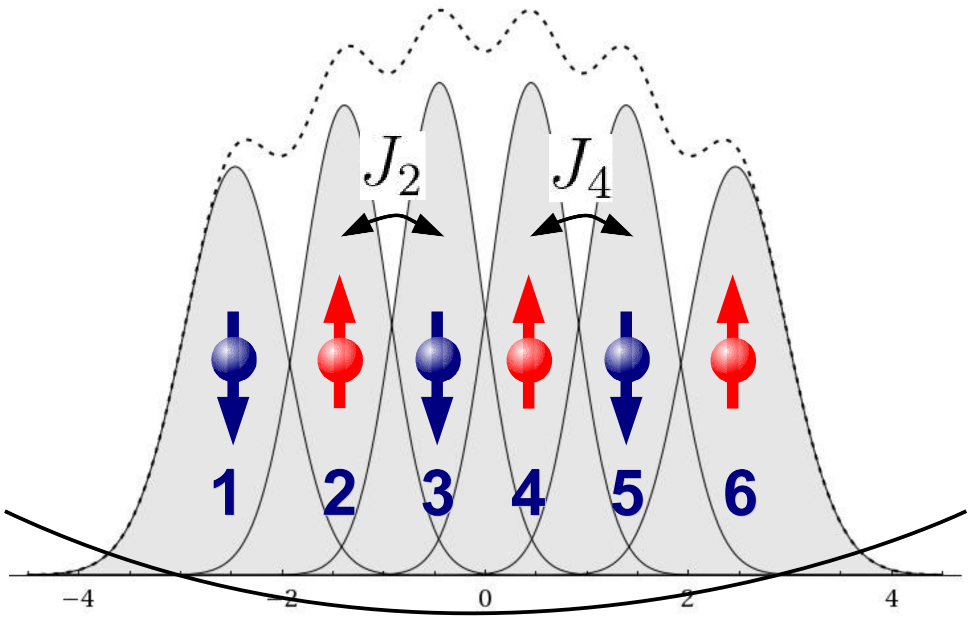


Antiferromagnetic Heisenberg spin chain of a few cold atoms in a 1D trap

Perturbative calculation around $1/g = 0$

Heisenberg model

$$H_{\text{eff}} \propto \pm \sum_{i=1}^{N-1} J_i \, \vec{S}_i \cdot \vec{S}_{i+1}$$



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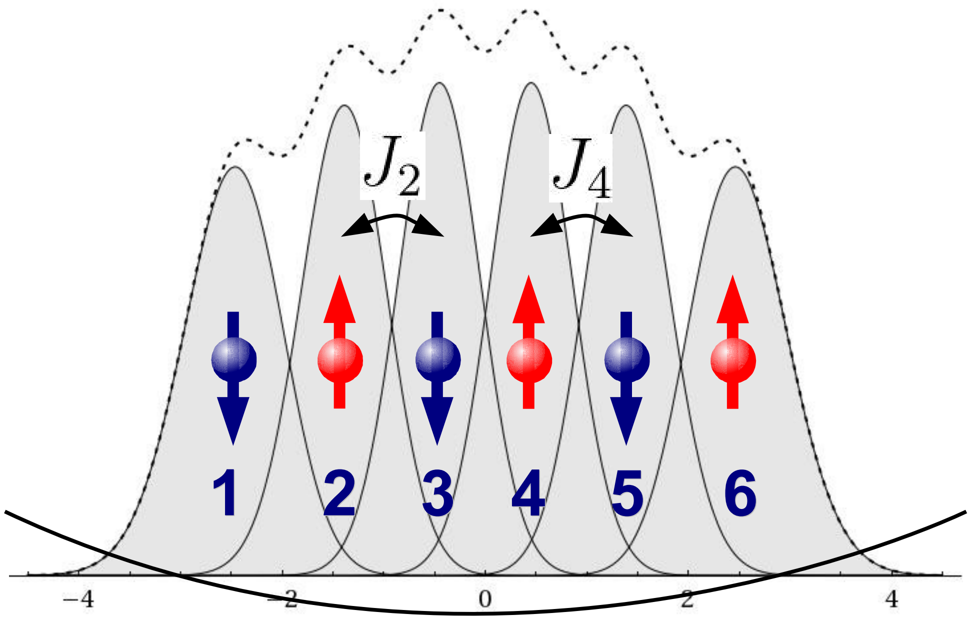
Perturbative calculation around $1/g = 0$

Heisenberg model

$$H_{\text{eff}} \propto \pm \sum_{i=1}^{N-1} J_i \, \vec{S}_i \cdot \vec{S}_{i+1}$$

$$J_i \propto \frac{n^3(z_i)}{g}$$



Antiferromagnetic Heisenberg spin chain of a few cold atoms in a 1D trap

Perturbative calculation around $1/g = 0$

effective spin-chain Hamiltonian

$$H_{\text{eff}} = \left(E_F - \sum_{i=1}^{N-1} J_i \right) \mathbb{1} \pm \sum_{i=1}^{N-1} J_i \hat{P}_{i,i+1}$$

SU(*N*) Sutherland model

Heisenberg model

$$H_{\text{eff}} \propto \pm \sum_{i=1}^{N-1} J_i \, \vec{S}_i \cdot \vec{S}_{i+1}$$

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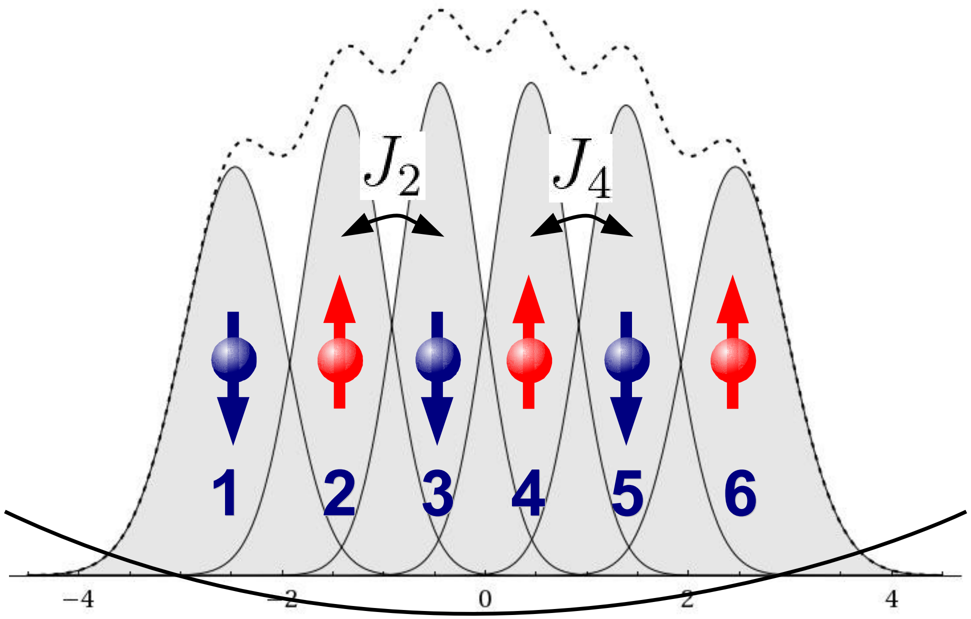
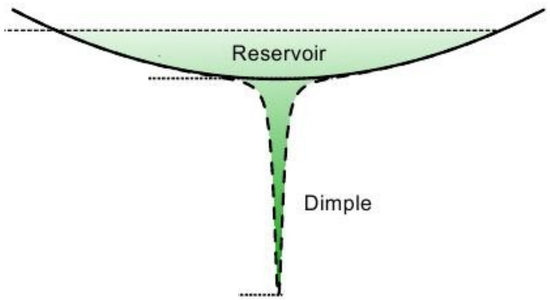


Antiferromagnetic Heisenberg spin chain of a few cold atoms in a 1D trap

Application to the experiment

Preparation of initial state

dimple trick



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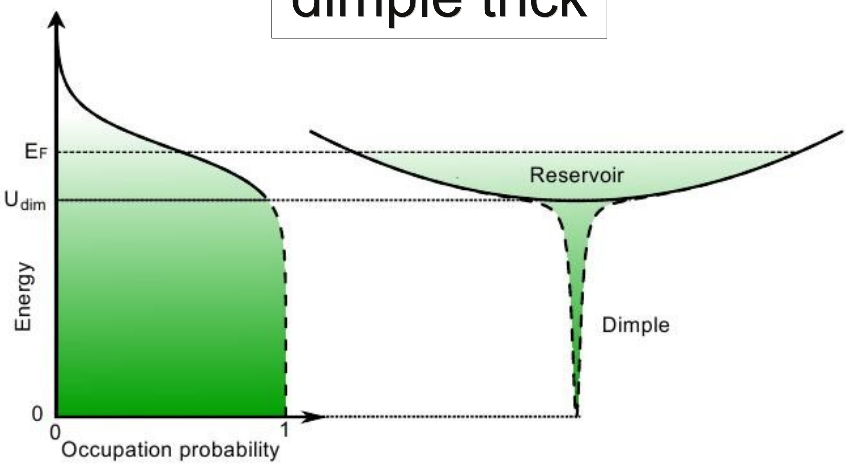

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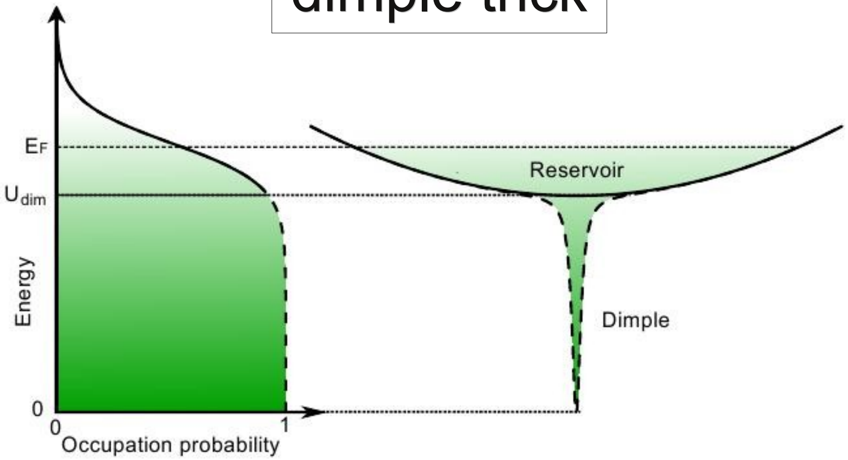
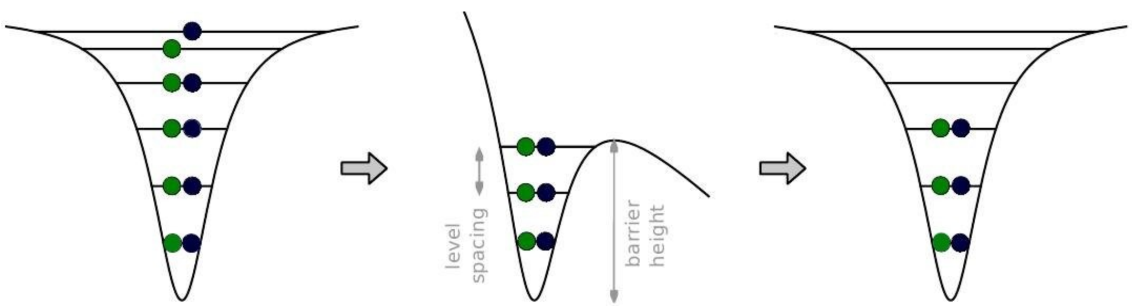


spilling technique



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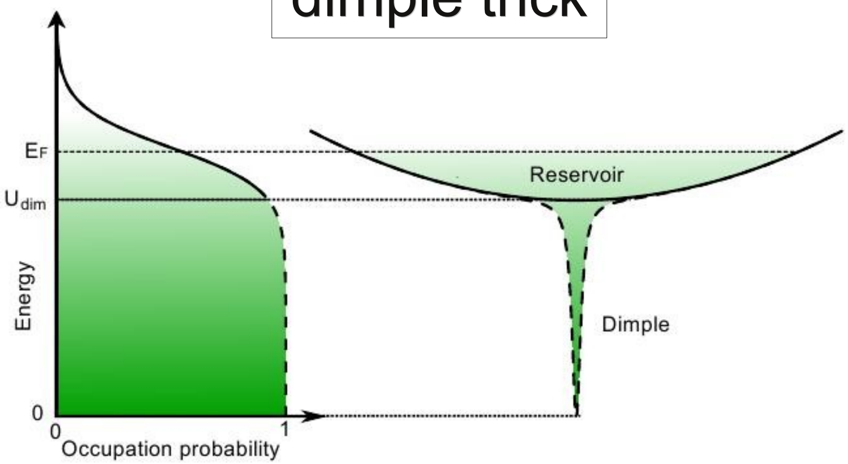

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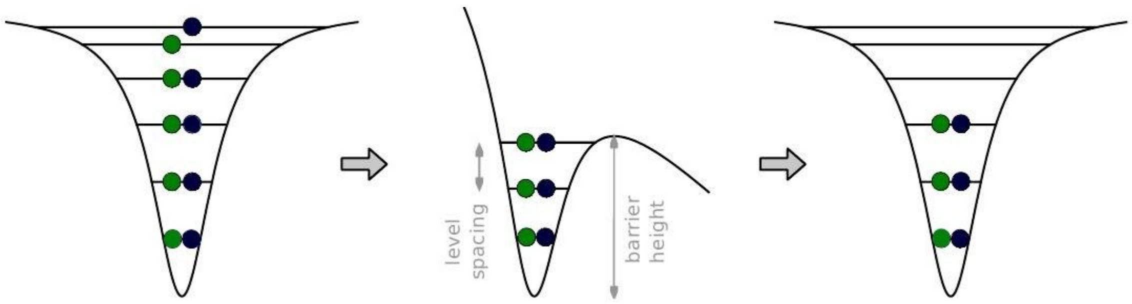
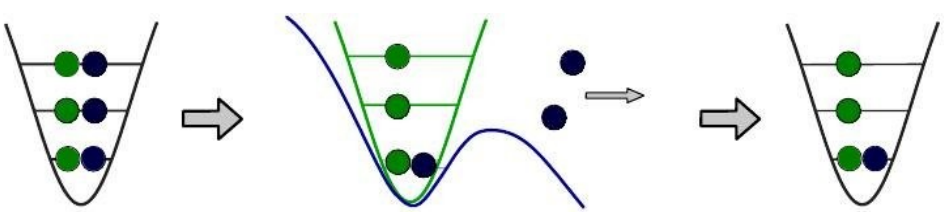


asymmetric spilling



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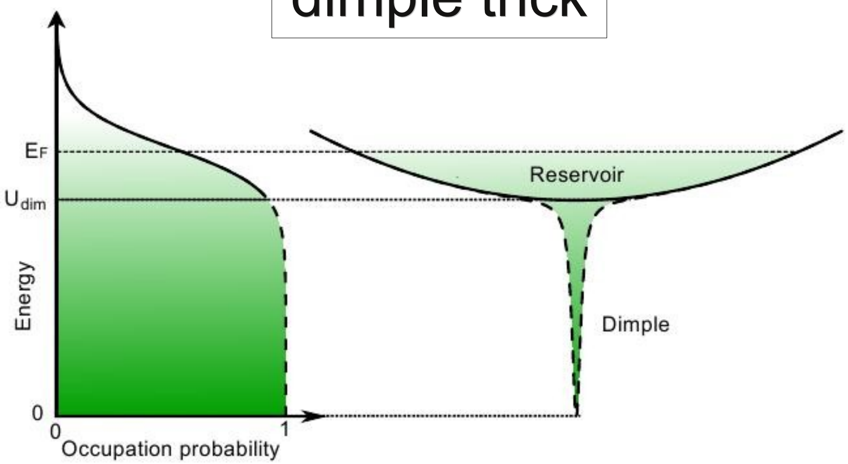

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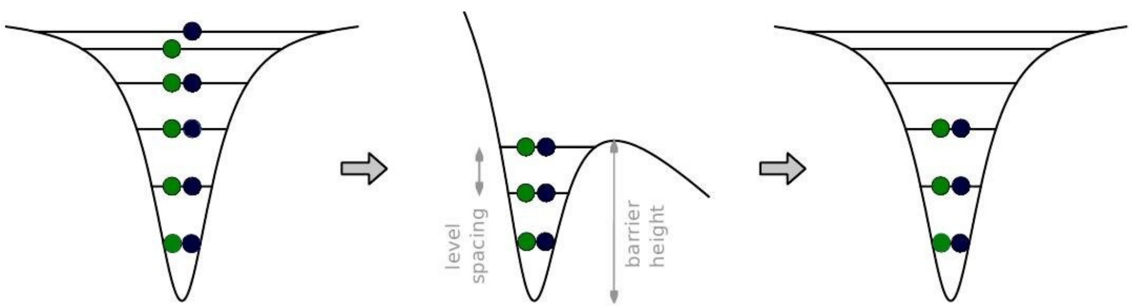

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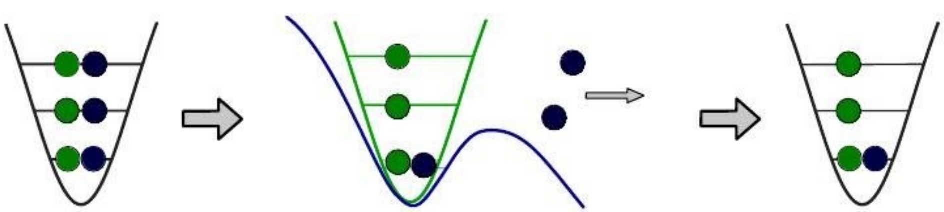
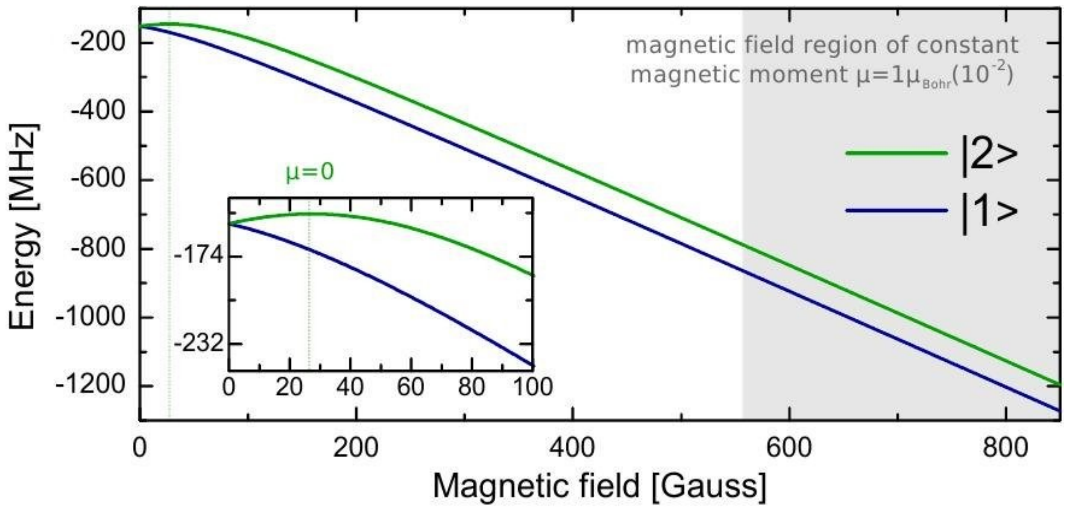
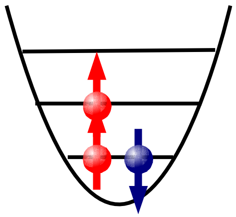


magnetic moments



Preparation of antiferromagnetic spin chain

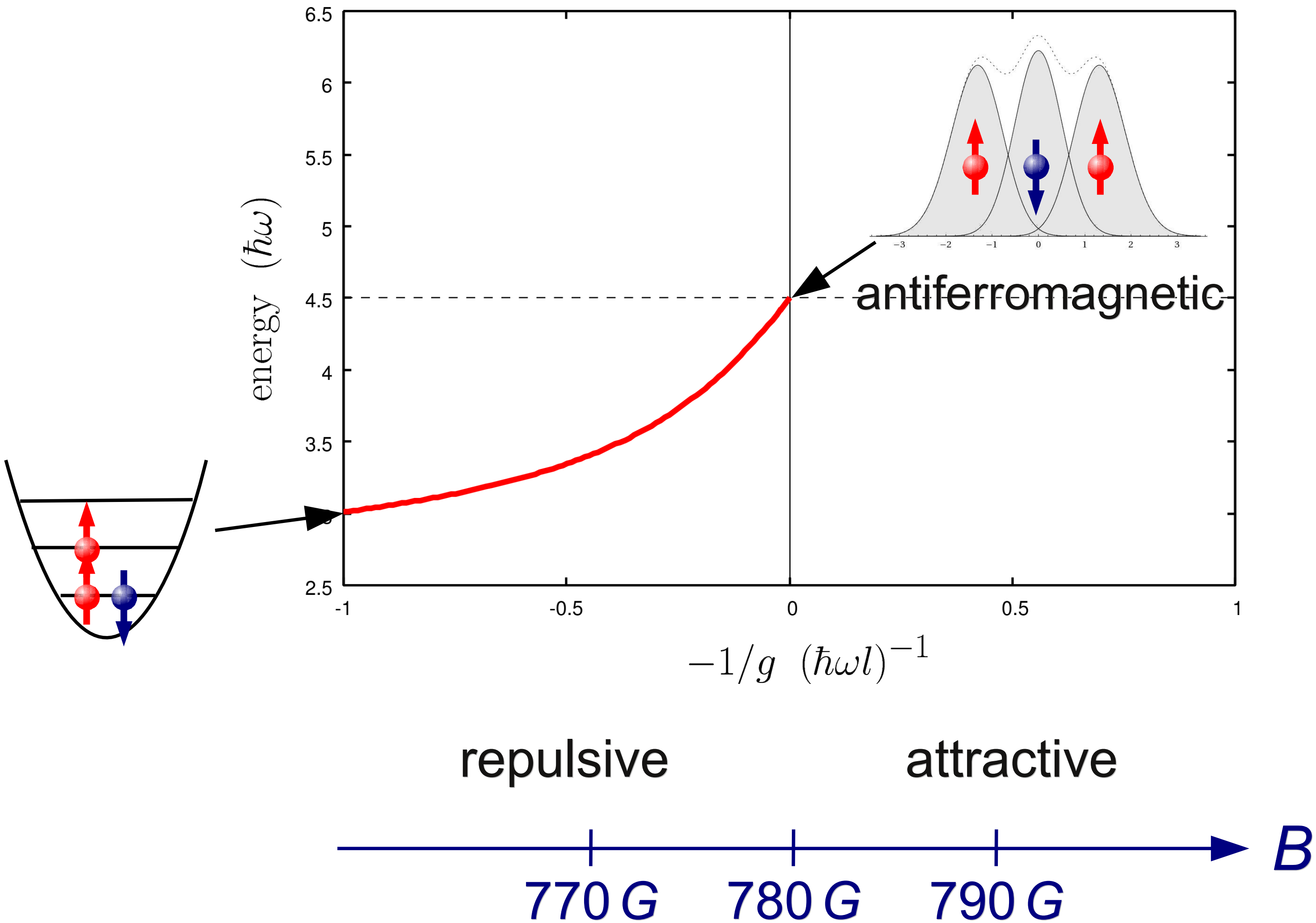
Noninteracting



Preparation of antiferromagnetic spin chain

Noninteracting

Fermionization



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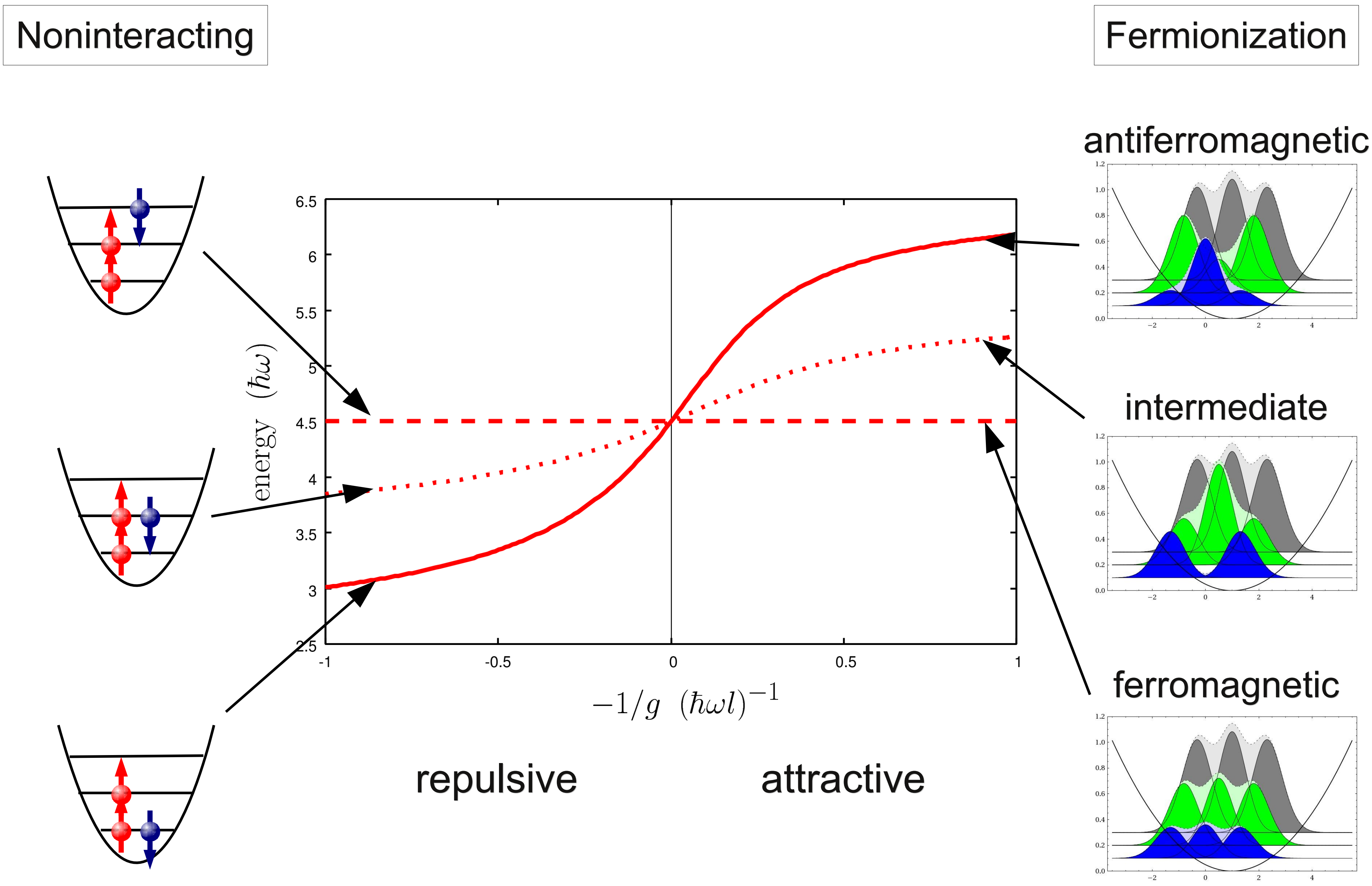
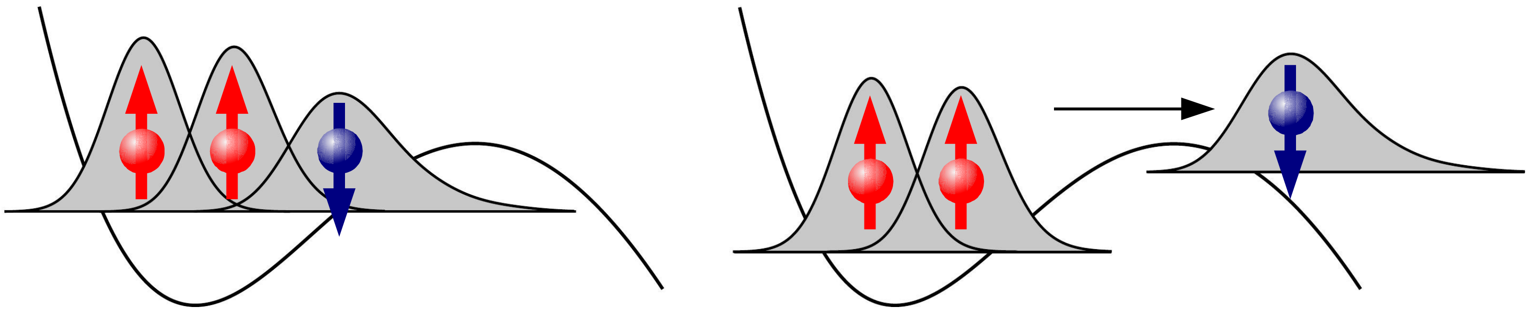
Multiplet structure

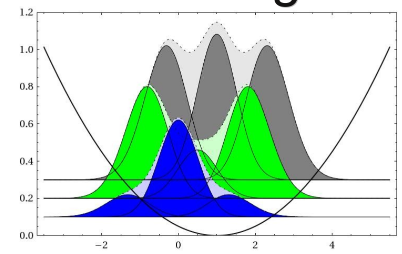


Antiferromagnetic Heisenberg spin chain of a few cold atoms in a 1D trap

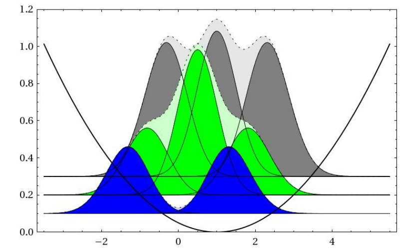
Measure orientation of rightmost spin



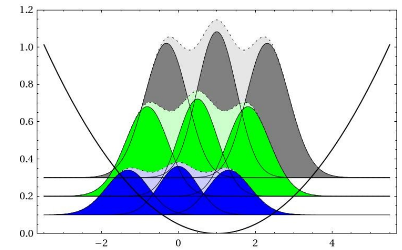
antiferromagnetic



intermediate



ferromagnetic



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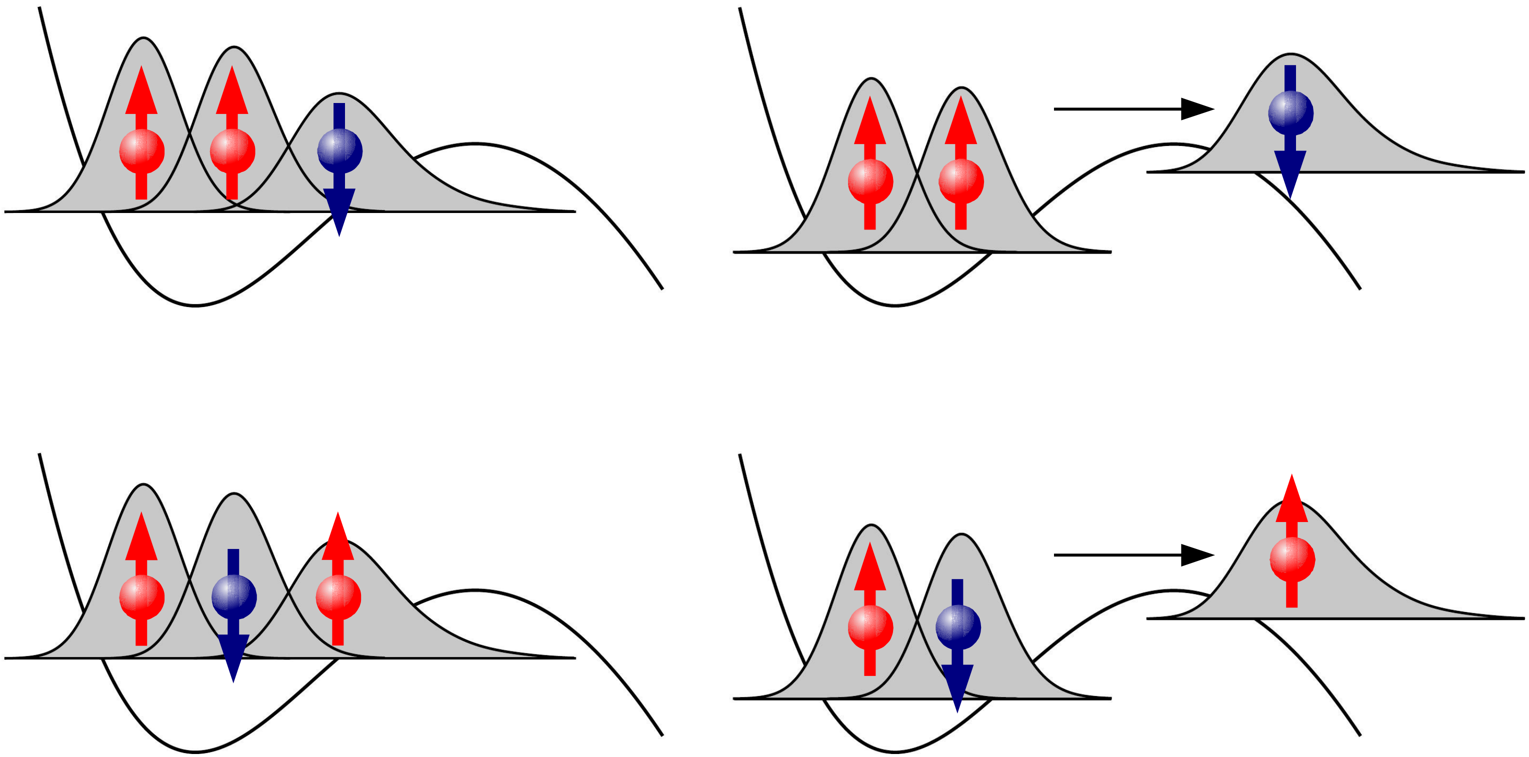
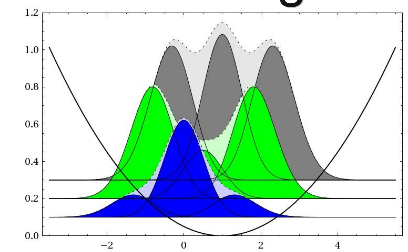


antiferromagnetic



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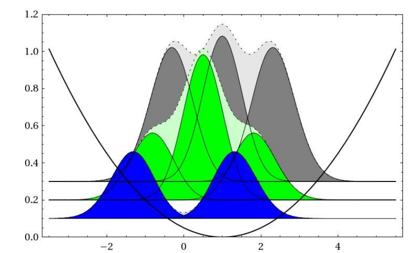

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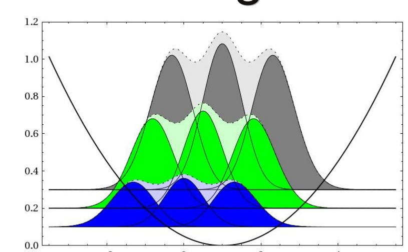

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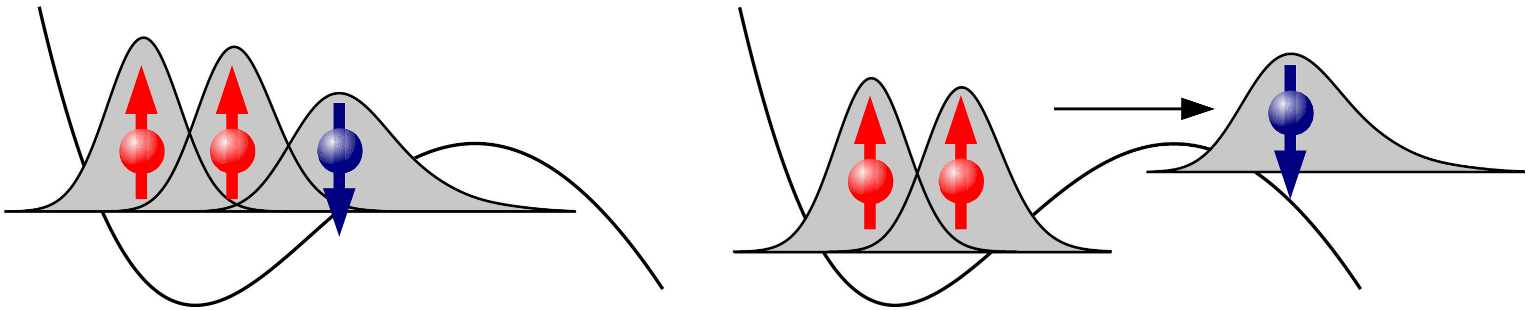
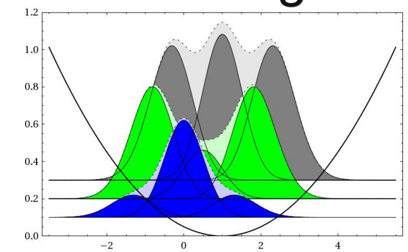
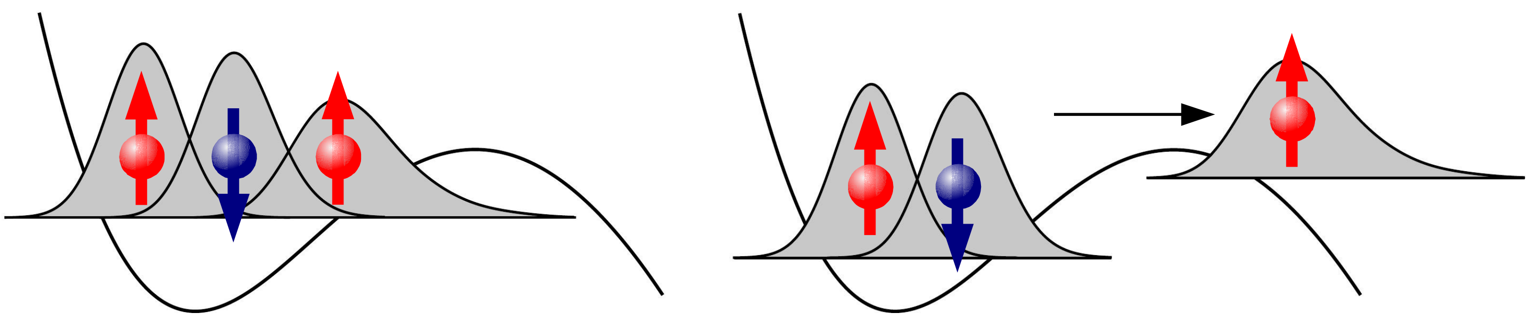


antiferromagnetic





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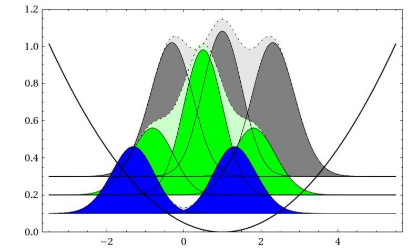
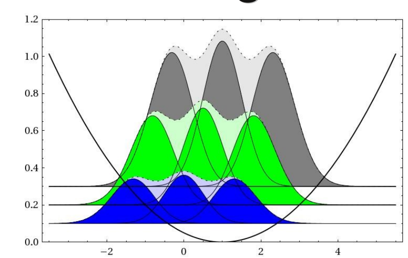


antiferromagnetic

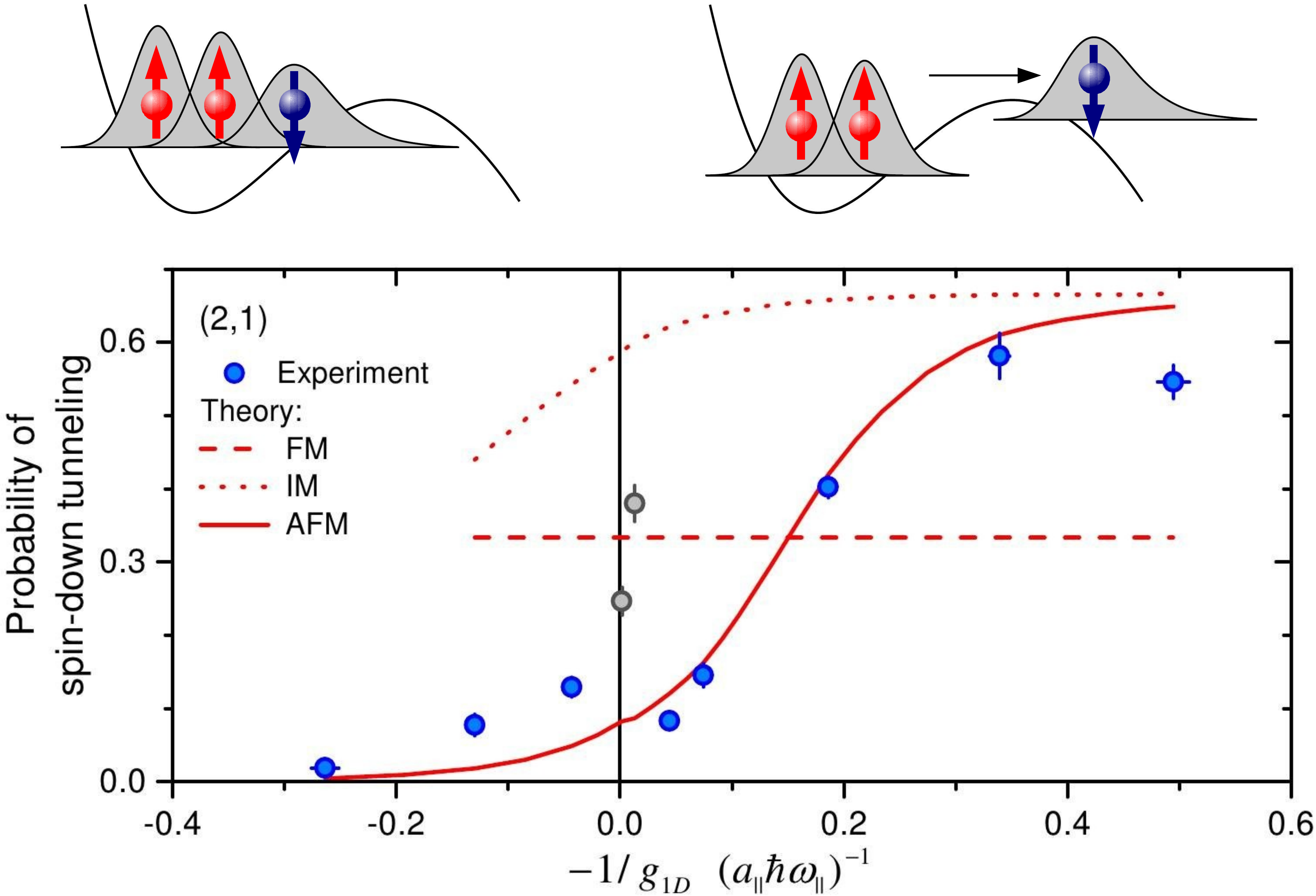
$$\frac{1}{\sqrt{6}}\Big(|\uparrow,\uparrow,\downarrow\rangle - 2|\uparrow,\downarrow,\uparrow\rangle + |\downarrow,\uparrow,\uparrow\rangle\Big)$$

ferromagnetic



Antiferromagnetic Heisenberg spin chain of a few cold atoms in a 1D trap

Probability of spin-down tunneling



Antiferromagnetic Heisenberg spin chain of a few cold atoms in a 1D trap

Tunneling theory

Initial AFM state

$$\frac{1}{\sqrt{6}}\Big(|\uparrow,\uparrow,\downarrow\rangle - 2|\uparrow,\downarrow,\uparrow\rangle + |\downarrow,\uparrow,\uparrow\rangle\Big)$$

Tunneling theory

Final states

Initial AFM state

$$\frac{1}{\sqrt{6}}\Big(|\uparrow,\uparrow,\downarrow\rangle - 2|\uparrow,\downarrow,\uparrow\rangle + |\downarrow,\uparrow,\uparrow\rangle\Big)$$

FM $\quad |\uparrow,\uparrow\rangle_{\text{in-trap}} \otimes |\downarrow\rangle_{\text{outside}}$

FM $\quad \frac{1}{\sqrt{2}}\Big(|\uparrow,\downarrow\rangle + |\downarrow,\uparrow\rangle\Big) \otimes |\uparrow\rangle$

AFM $\quad \frac{1}{\sqrt{2}}\Big(|\uparrow,\downarrow\rangle - |\downarrow,\uparrow\rangle\Big) \otimes |\uparrow\rangle$

Tunneling theory

Final states

Initial AFM state

$$\frac{1}{\sqrt{6}}\Big(|\uparrow,\uparrow,\downarrow\rangle - 2|\uparrow,\downarrow,\uparrow\rangle + |\downarrow,\uparrow,\uparrow\rangle\Big) \xrightarrow{T_{i,f}}$$

FM $|\uparrow,\uparrow\rangle_{\text{in-trap}} \otimes |\downarrow\rangle_{\text{outside}}$

FM $\frac{1}{\sqrt{2}}\Big(|\uparrow,\downarrow\rangle + |\downarrow,\uparrow\rangle\Big) \otimes |\uparrow\rangle$

AFM $\frac{1}{\sqrt{2}}\Big(|\uparrow,\downarrow\rangle - |\downarrow,\uparrow\rangle\Big) \otimes |\uparrow\rangle$

Tunneling rate

$$T_{i,f} \propto |\langle i|f\rangle|^2$$

Tunneling theory

Final states

Initial AFM state

$$\frac{1}{\sqrt{6}}\Big(|\uparrow,\uparrow,\downarrow\rangle - 2|\uparrow,\downarrow,\uparrow\rangle + |\downarrow,\uparrow,\uparrow\rangle\Big) \xrightarrow{T_{i,f}}$$

FM $|\uparrow,\uparrow\rangle_{\text{in-trap}} \otimes |\downarrow\rangle_{\text{outside}}$

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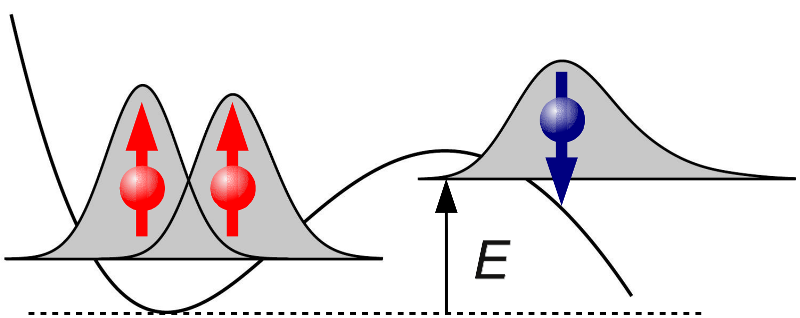
Tunneling rate

$$T_{i,f} \propto |\langle i|f\rangle|^2$$



Tunneling theory

Final states

Initial AFM state

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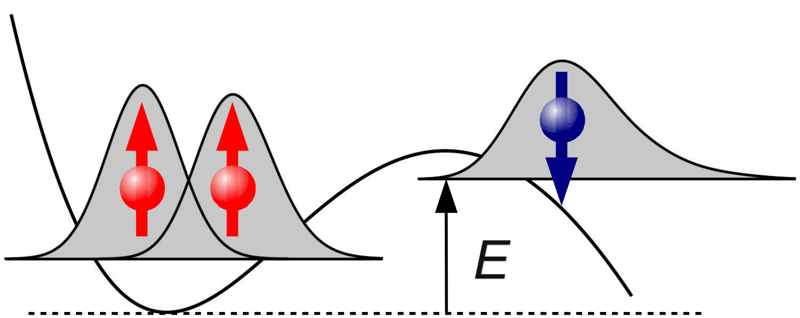

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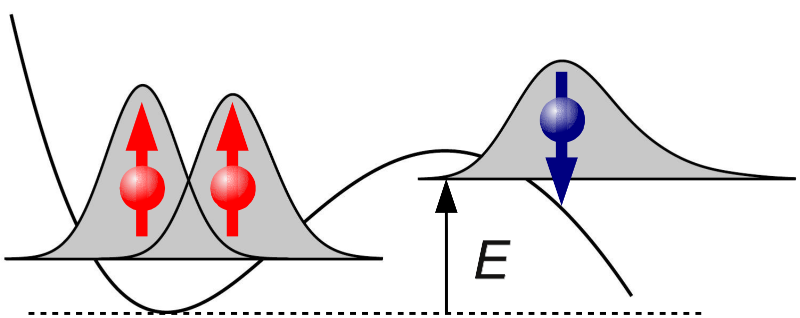

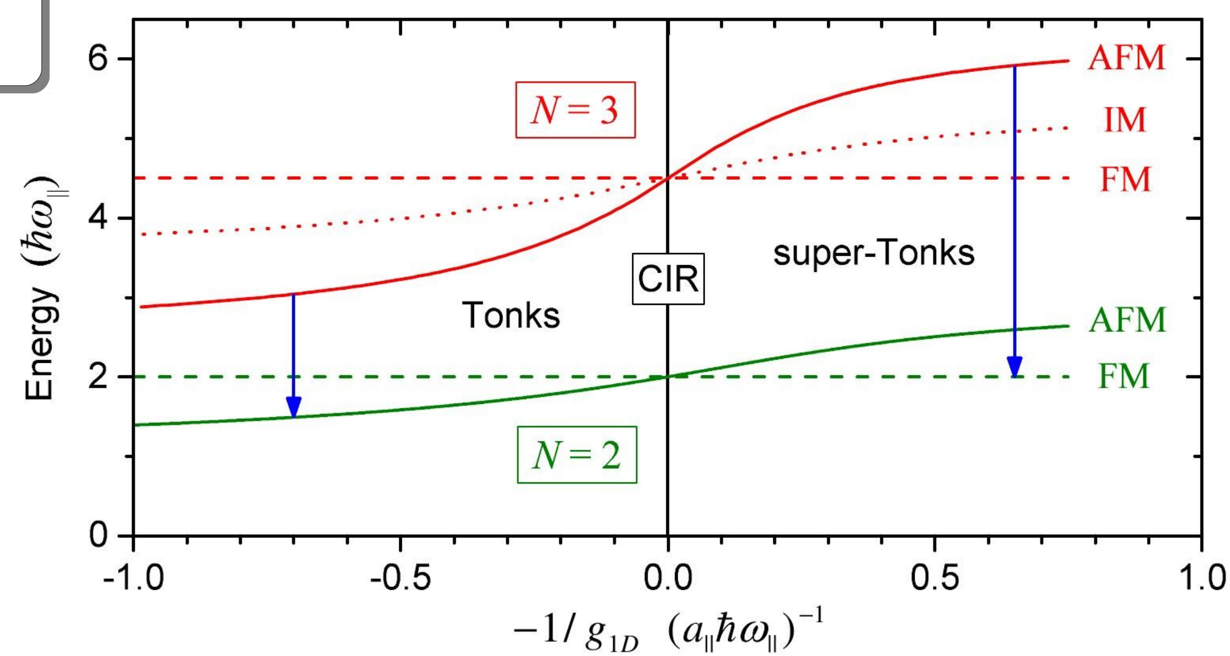

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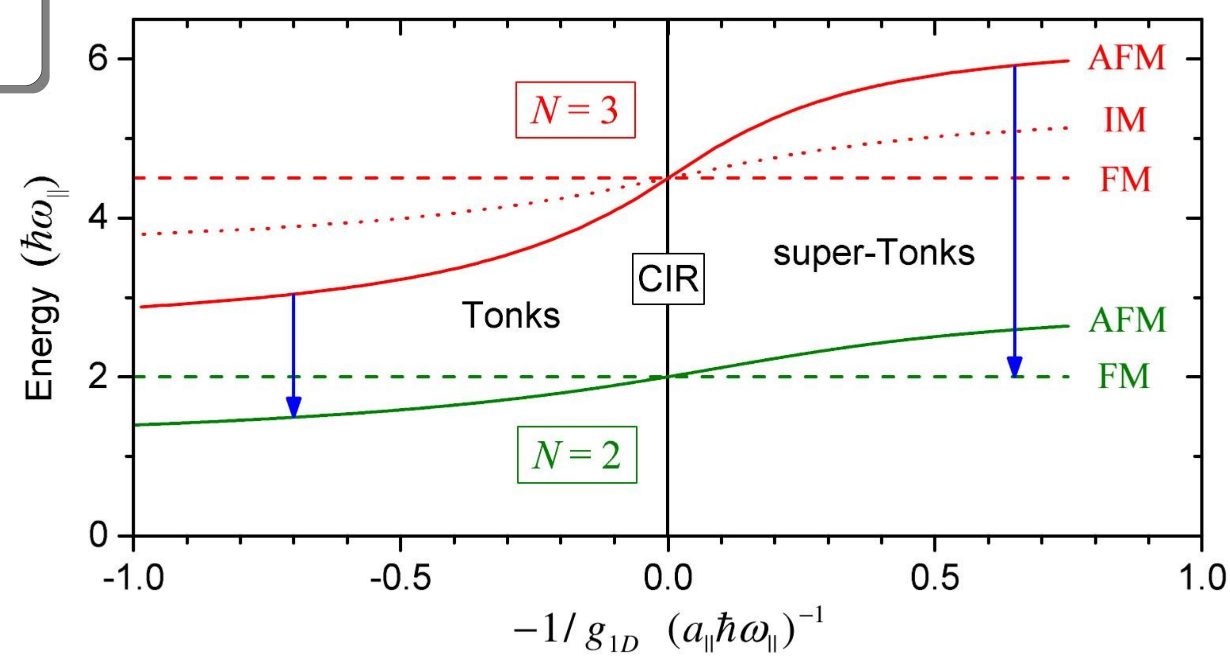
Tunneling probability

$$P_{i,f} = \frac{T_{i,f}}{\left(\sum_{f'} T_{i,f'}\right)}$$



Probability of spin-down tunneling

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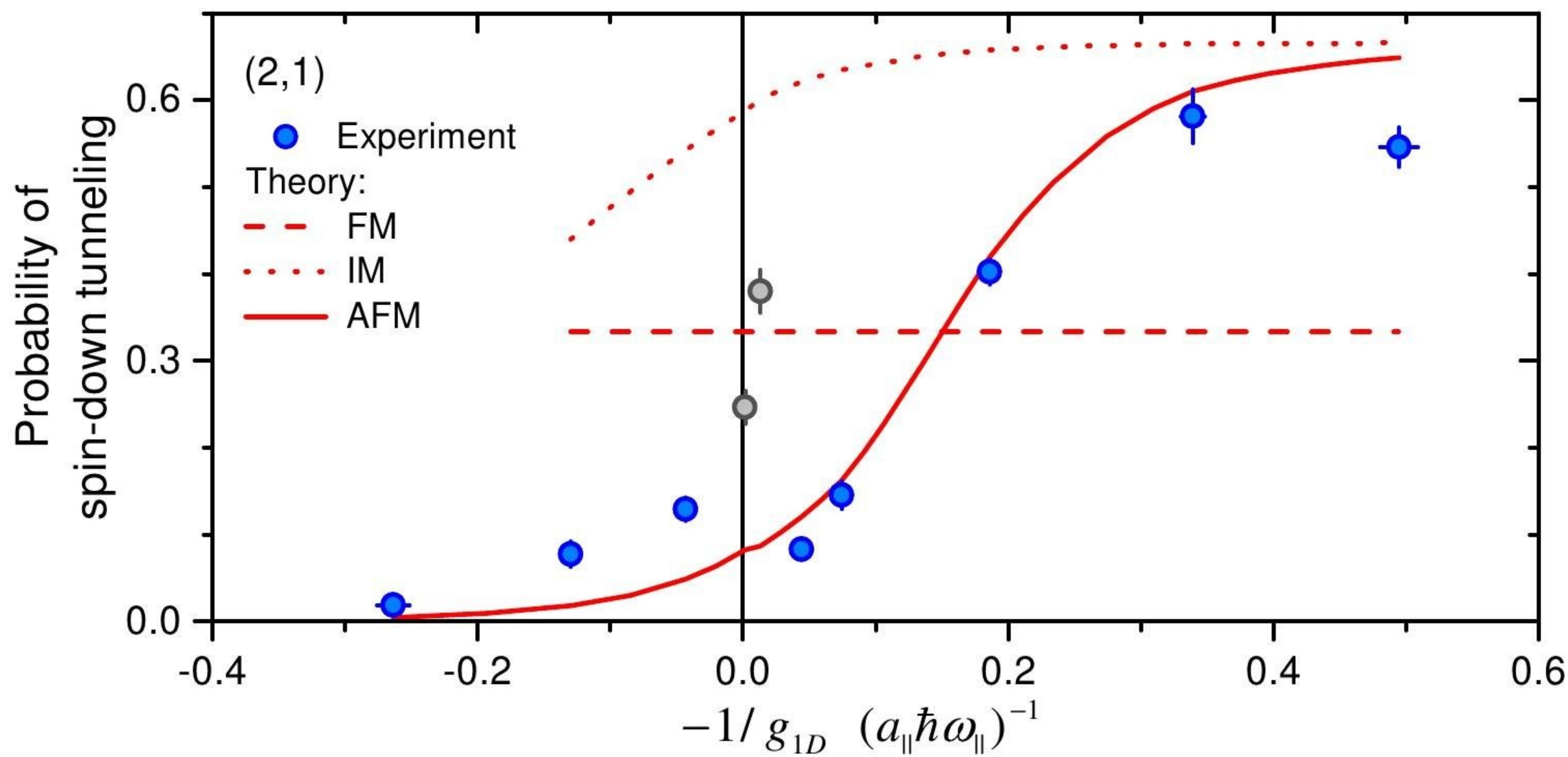

Antiferromagnetic Heisenberg spin chain of a few cold atoms in a 1D trap

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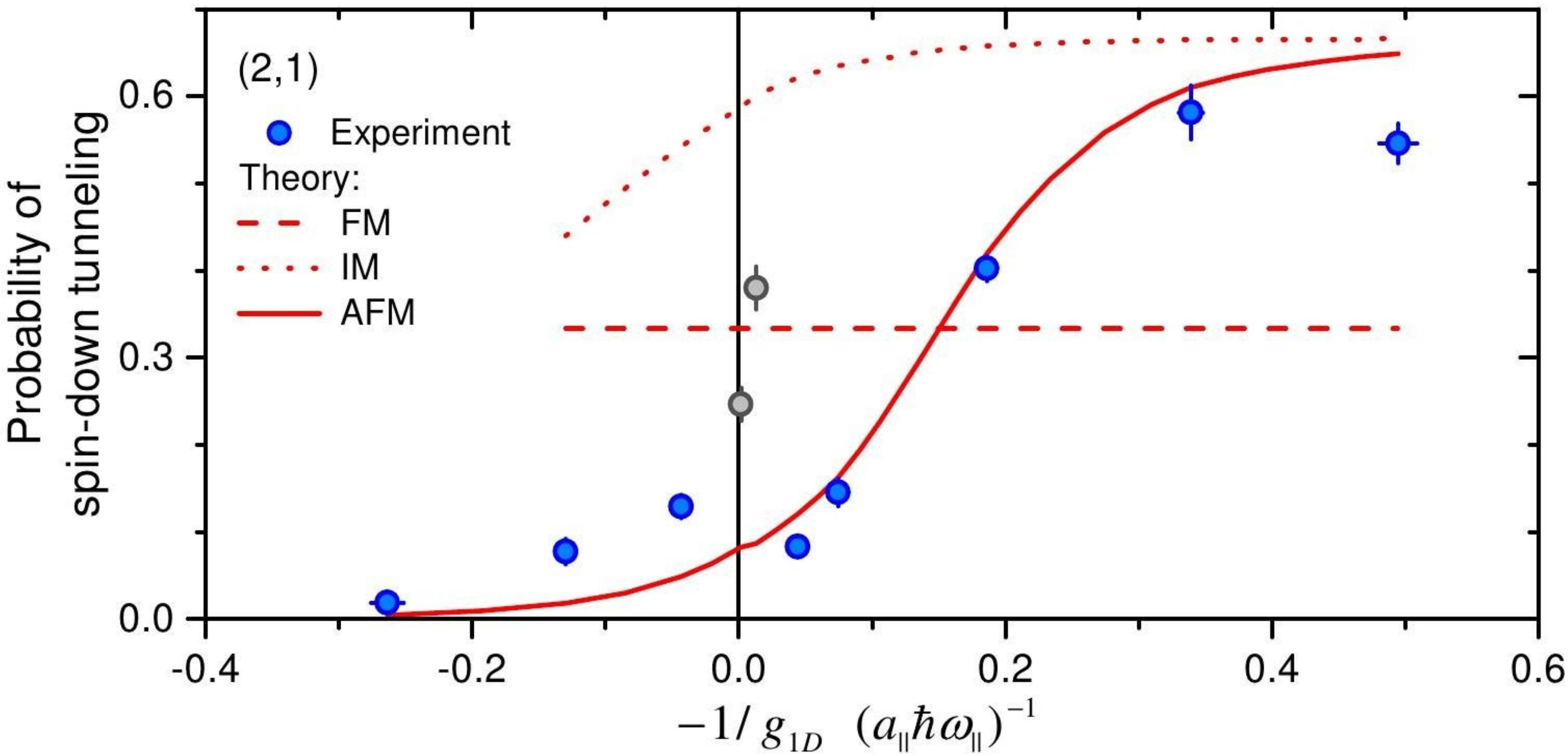

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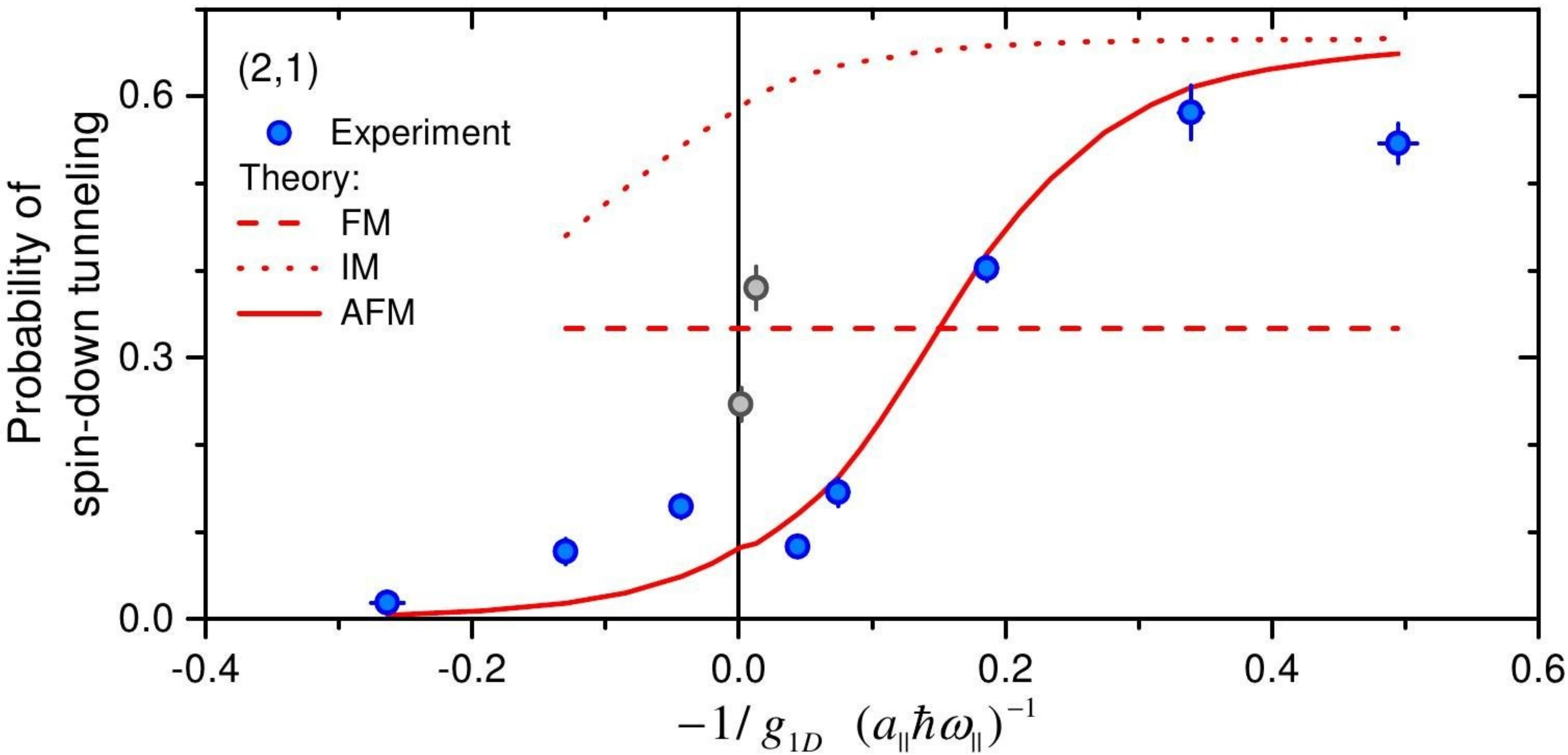
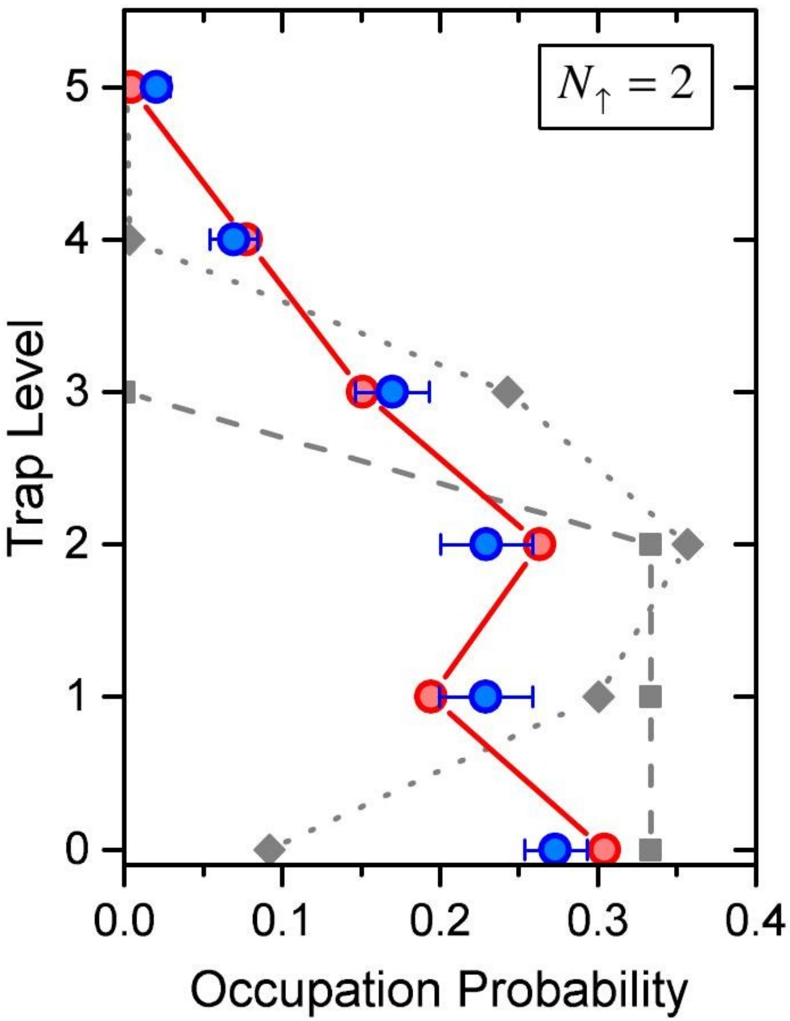


Antiferromagnetic Heisenberg spin chain of a few cold atoms in a 1D trap

Level occupation of spin-down particle



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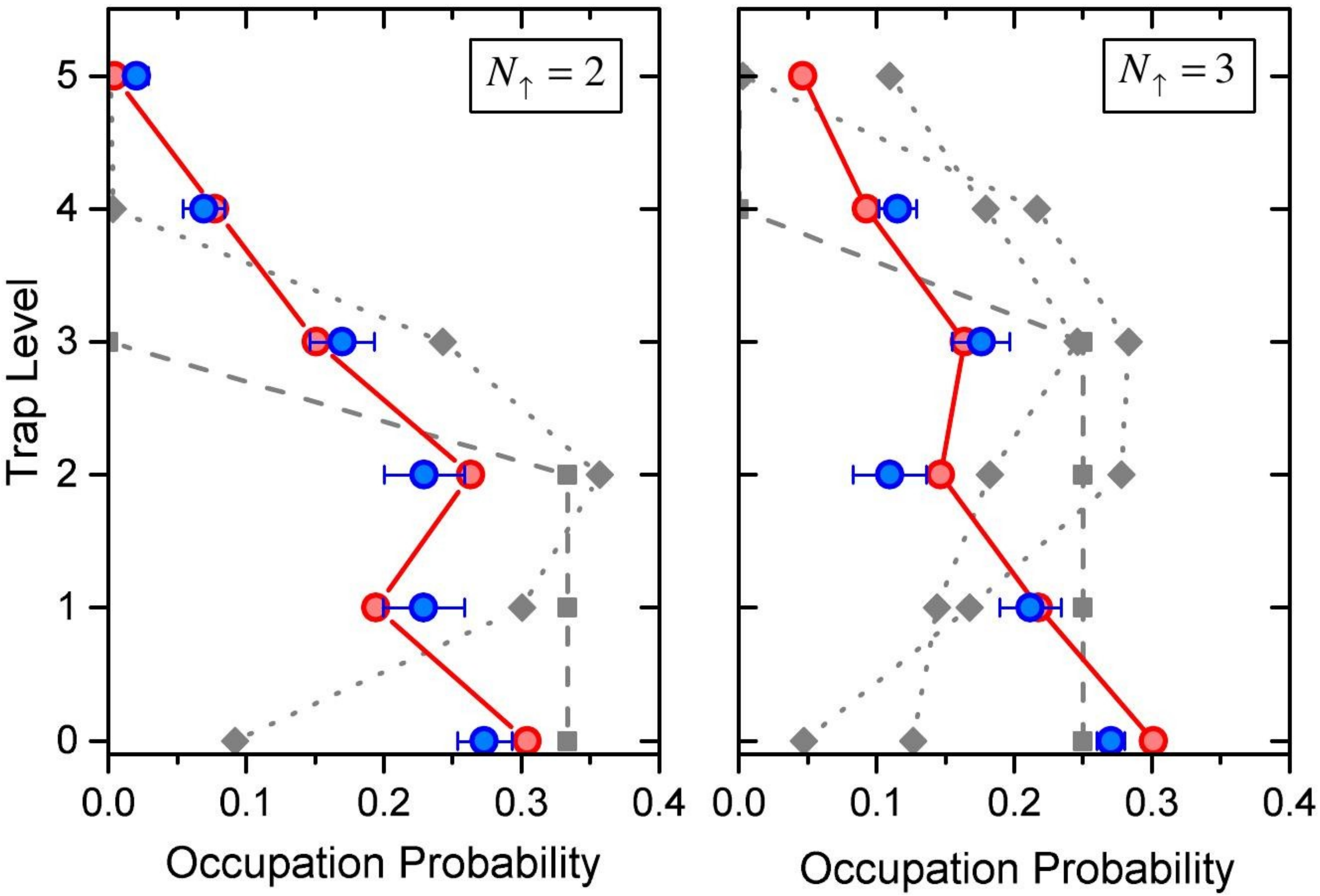

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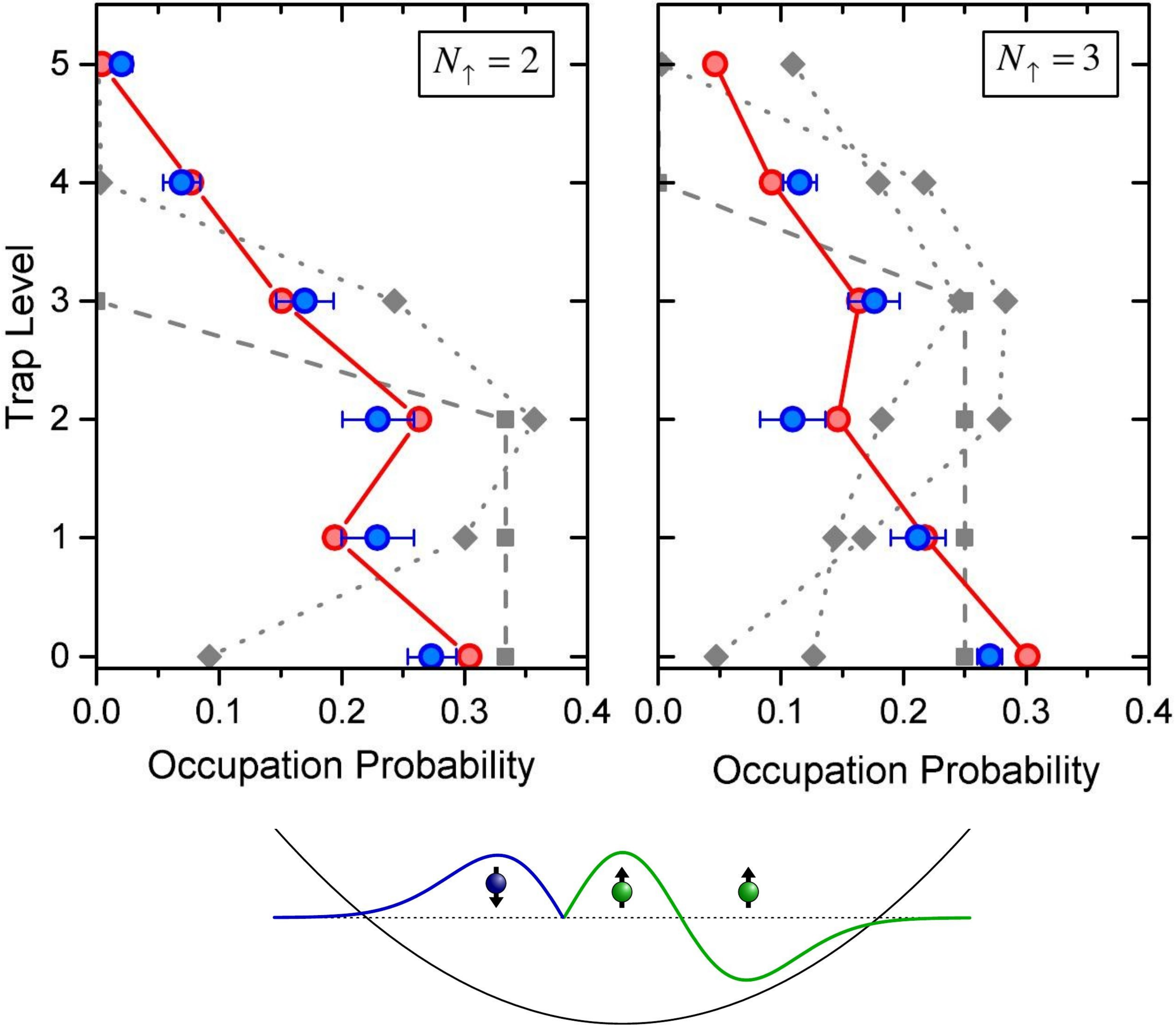

Antiferromagnetic Heisenberg spin chain of a few cold atoms in a 1D trap

Numerical methods

- Calculation of some properties of the atom chain is complicated

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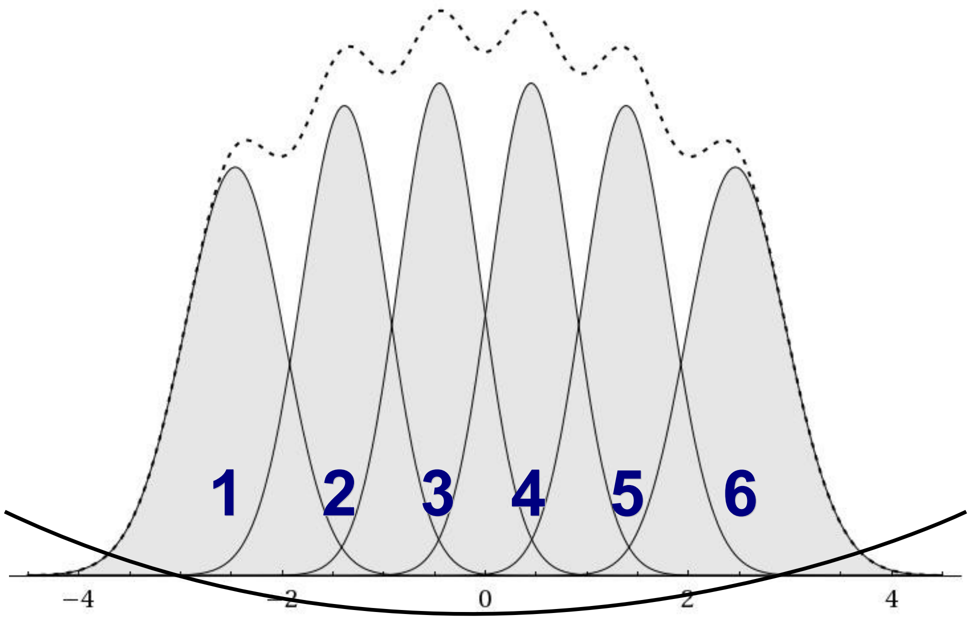

$$\rho^{(i)}(z) = N! \int_{z_1<\cdots<z_{i-1}<z<z_{i+1}<\cdots<z_N} dz_1 \cdots dz_{i-1} dz_{i+1} \cdots dz_N \left|\psi_F(z_1,\ldots,z_{i-1},z,z_{i+1},\ldots,z_N)\right|^2$$

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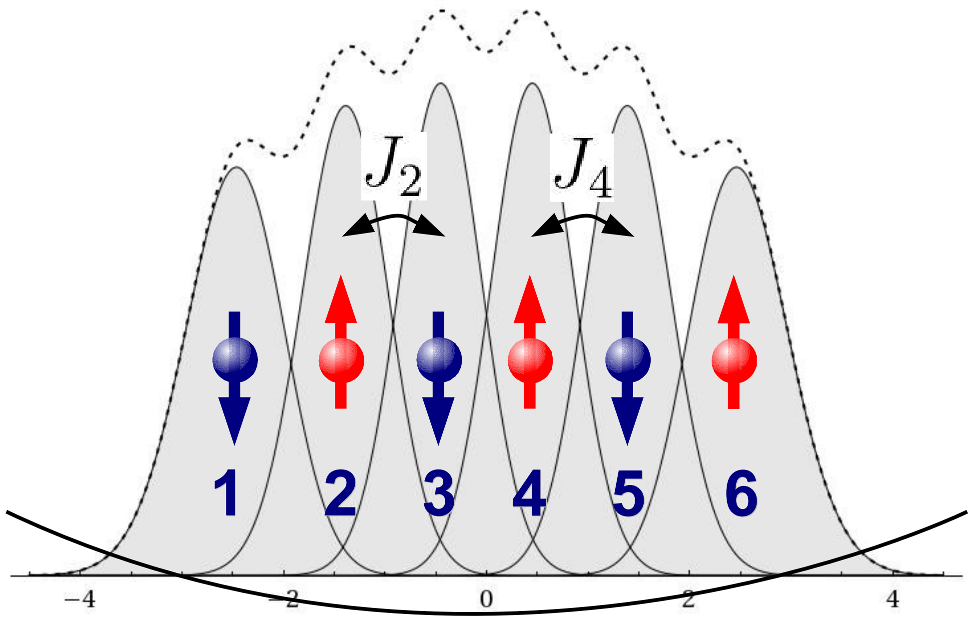

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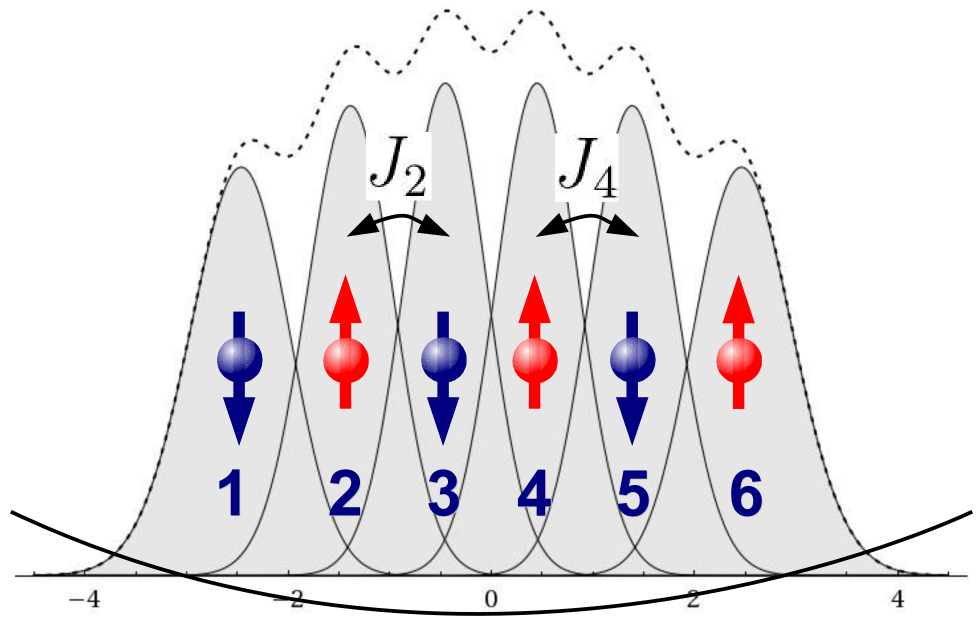
Numerical methods

- Calculation of some properties of the atom chain is complicated
- Mathematica script available



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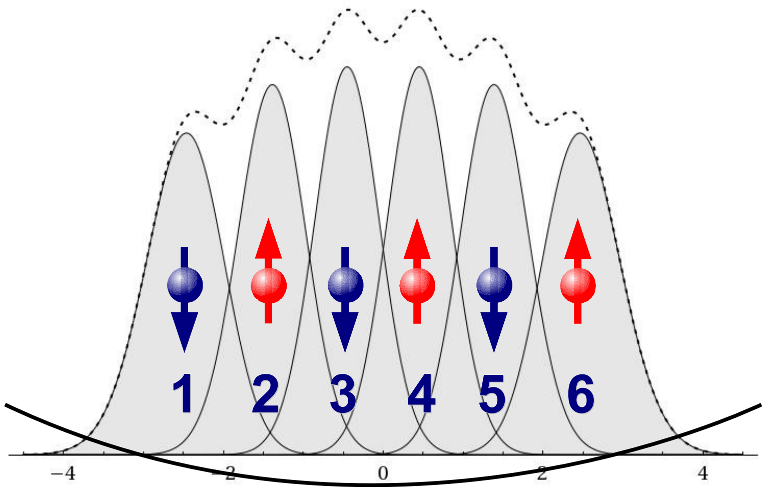
Summary

Spin chain without lattice



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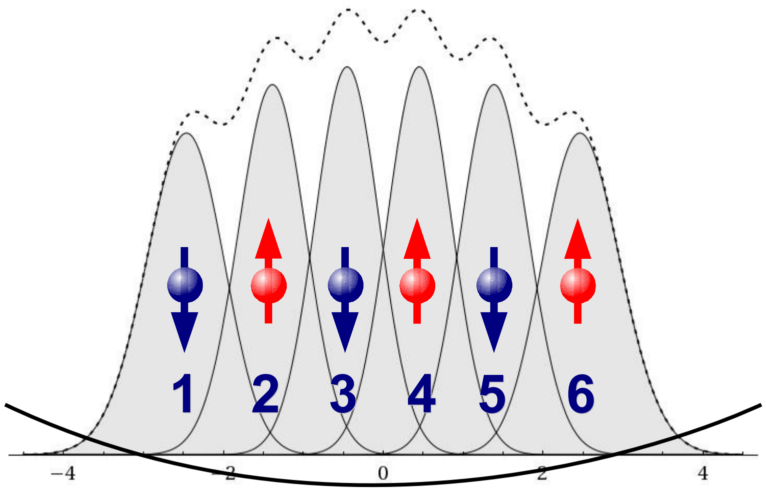
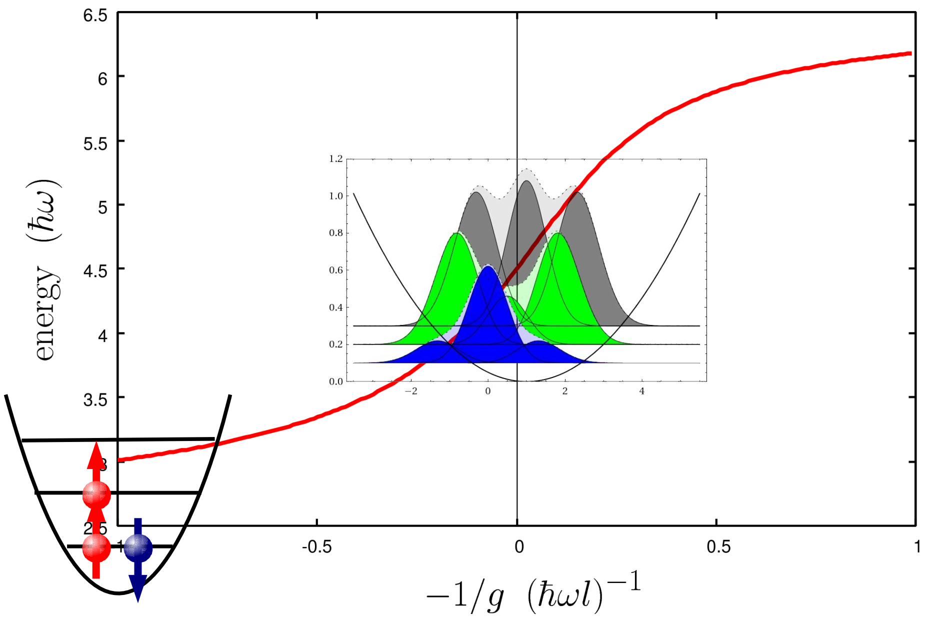


Preparation of antiferromagnetic Heisenberg spin chain



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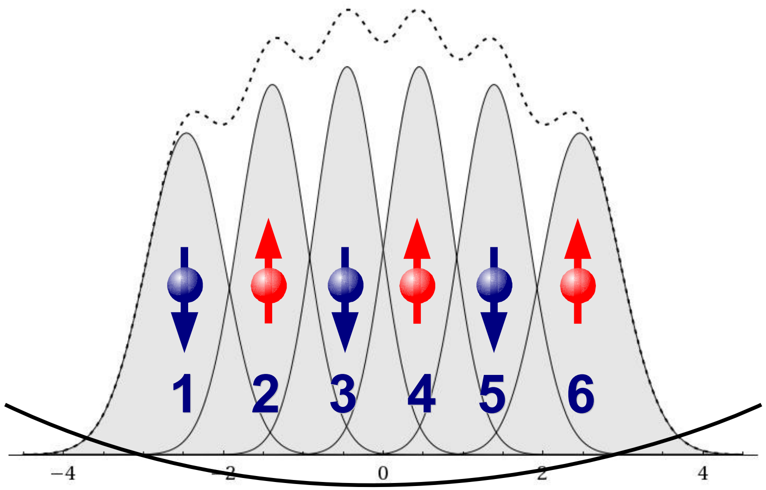
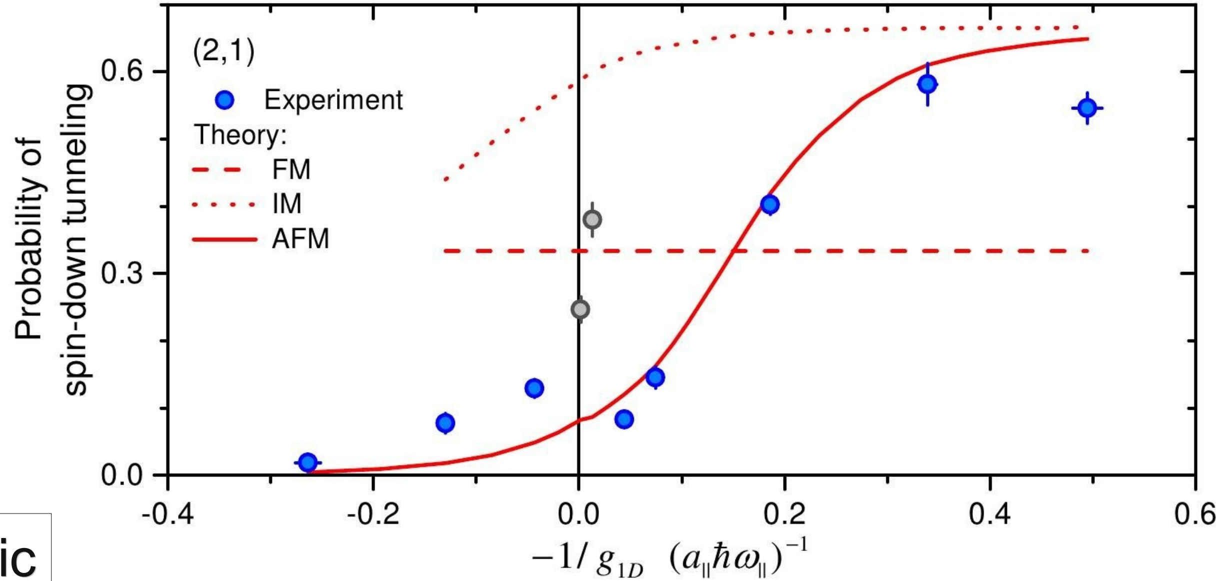


Orientation of rightmost spin



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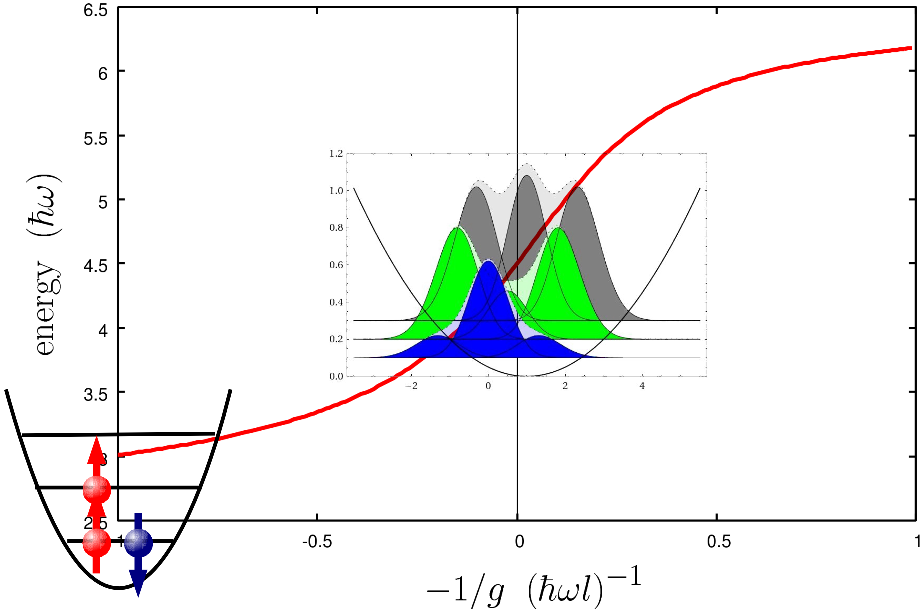

Antiferromagnetic Heisenberg spin chain of a few cold atoms in a 1D trap

Summary

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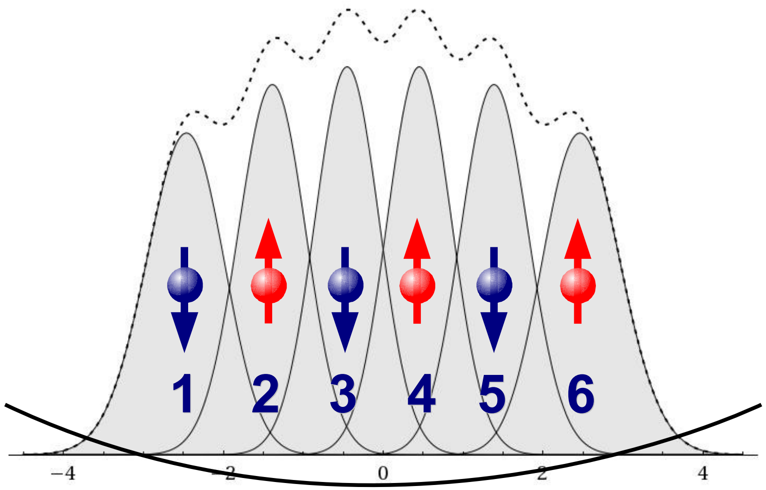

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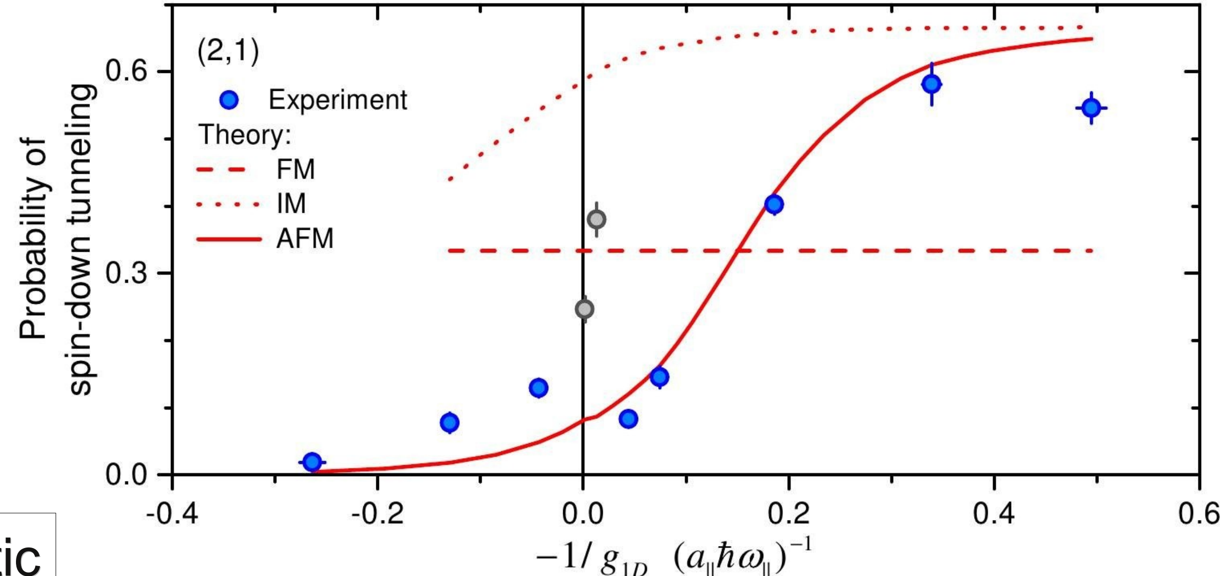

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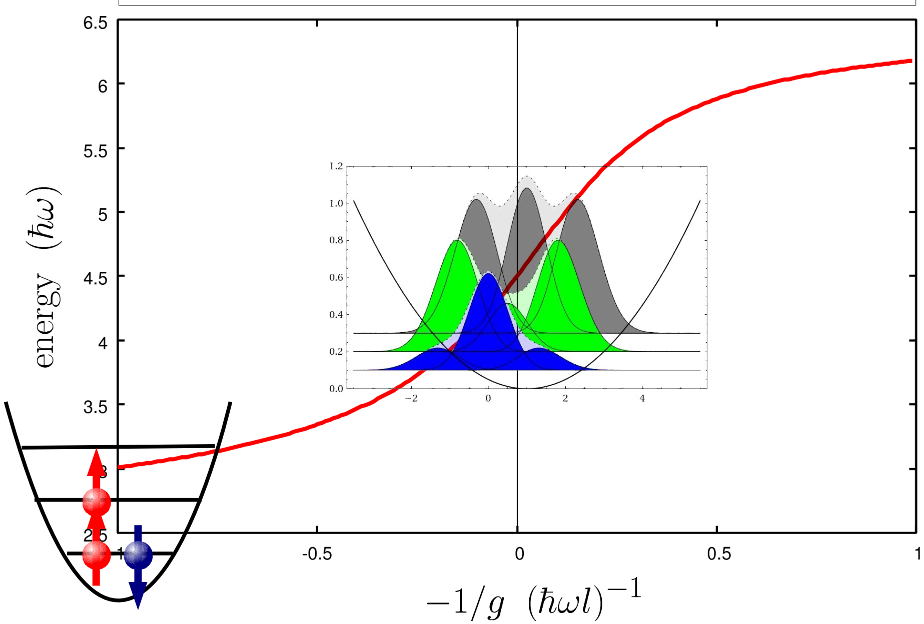

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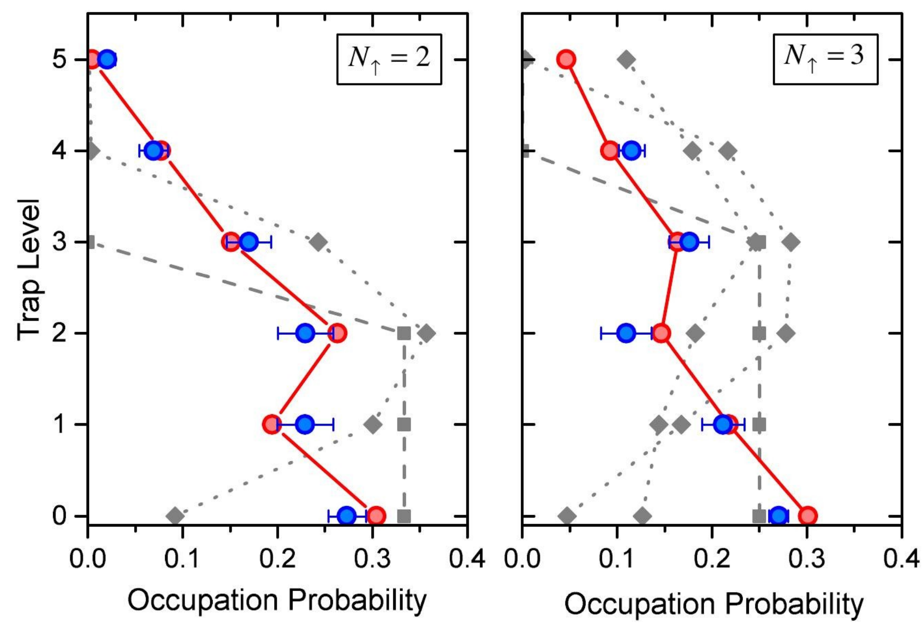

Antiferromagnetic Heisenberg spin chain of a few cold atoms in a 1D trap

Thank you for your attention!

Daniel

Johannes