State-of-the-art N3LO chiral interactions

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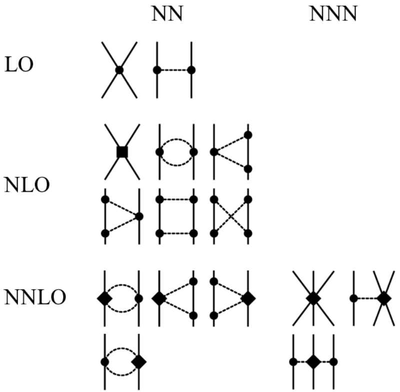
Phys. Rev. X 6, 011019 (2016)

Outline

χEFT

Controlled approximations

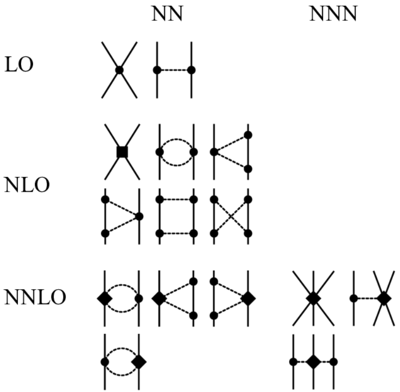
- Low-energy expansion **based on QCD**.
- It can be improved **order-by-order**.
- Links several **low-energy** nuclear physics processes.



χEFT

Expectations on χEFT

- Simultaneously give a good description of π**N, NN and many-nucleon** observables.
- Be able to estimate the **systematical error** in the model.
- Be able to propagate **statistical uncertainties** from fit of LECs.
- Fits and predictions should **improve with increased order** in the expansion.



Fit and results

Experimental data

πN data

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Goal

One model (χEFT) for all data, with error estimates.

Fit to experimental data

The χ^2 function

LECs must be determined from a **fit to experimental data**:

$$\chi^2(\mathbf{p}) \equiv \sum_i \left(\frac{O_i^{\text{theo}}(\mathbf{p}) - O_i^{\text{expr}}}{\sigma_{\text{tot},i}} \right)^2 \equiv \sum_i r_i^2(\mathbf{p})$$

Use **all available data** to constrain the model: πN, NN, NNN.

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Error budget

The total uncertainty σ_{tot} can be decomposed into:

$$\sigma_{\text{tot}}^2 = \sigma_{\text{exp}}^2 + \sigma_{\text{method}}^2 + \sigma_{\text{num}}^2 + \sigma_{\text{model}}^2$$

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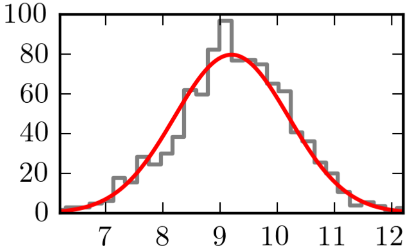
Exp. error

Provided by the **experimentalist**.



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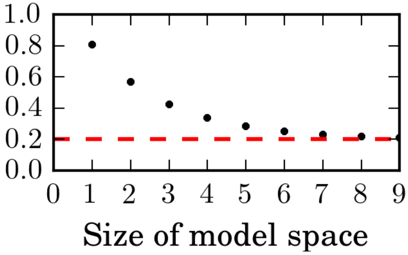
Method error

Relevant for the $\mathbf{A = 3}$ **bound-state** observables, due to limited model space and the isoscalar approximation.



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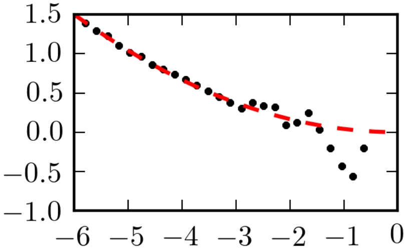
Num. error

Relevant for the **deuteron binding energy**, set to 0.01% of the experimental value.



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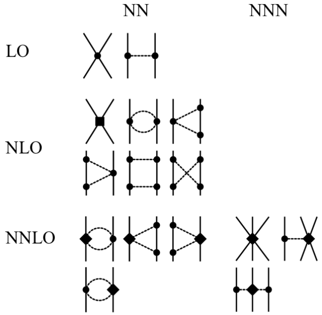
Model error

Applied to **scattering data**.

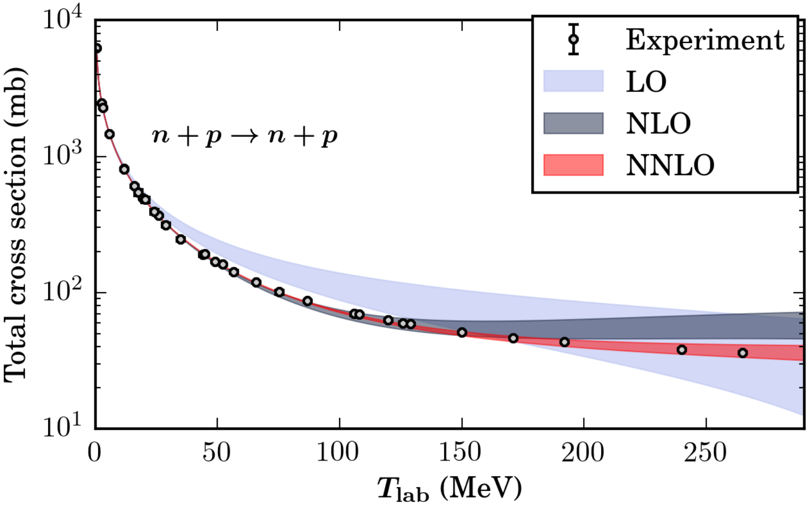
$$\sigma^{(\text{amp})}_{\text{model}} = C_x \left(q_{\text{cm}}/\Lambda_\chi\right)^{\nu+1}$$

Bound states

No model error for **bound state** data.



Total neutron-proton cross section **with model errors**



Results from **Phys. Rev. X 6, 011019 (2016)**

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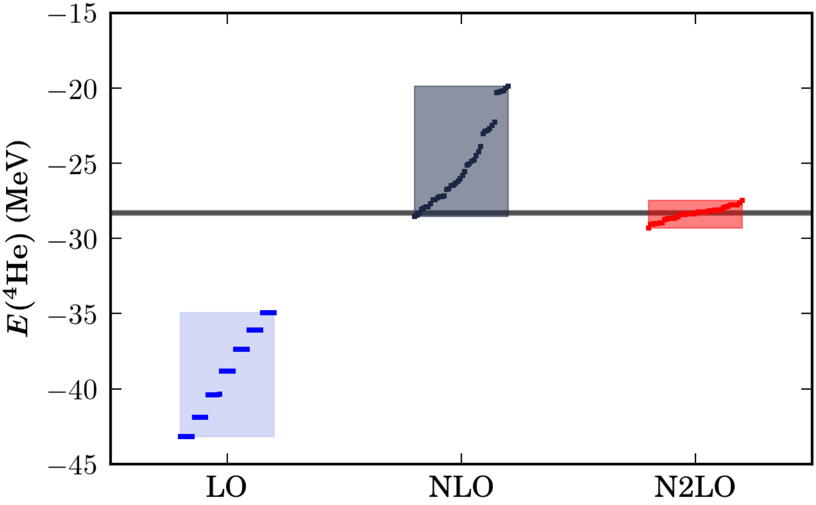
Model error

Estimate model error from **family of 42 potentials**. Uncertainty given by spread in predictions.

$$T_{\text{lab}}^{\text{max}} = 125 \ldots 290\,\mathrm{MeV}$$

$$\Lambda_\chi = 450 \ldots 600\,\mathrm{MeV}$$

Helium 4 binding energy



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N3LO

Moving on to N3LO

N3LO

- A total of 41 LECs.
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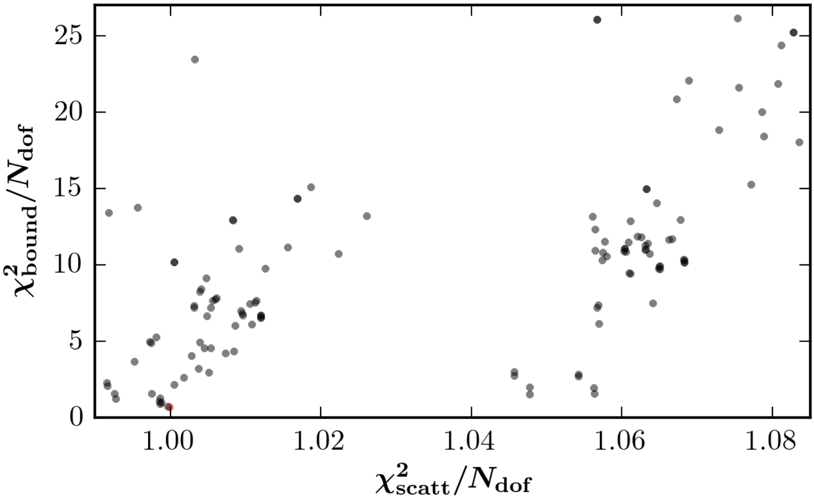
Complications

- Possible to find at least **over 100 minima** with good description of $A = 2, 3$ data.
- Could be due to a **lack of included data** in the fit.

N3LO minima


25
20
15
10
5
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1.00
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$\chi^2_{\mathrm{bound}}/N_{\mathrm{dof}}$
$\chi^2_{\mathrm{scatt}}/N_{\mathrm{dof}}$

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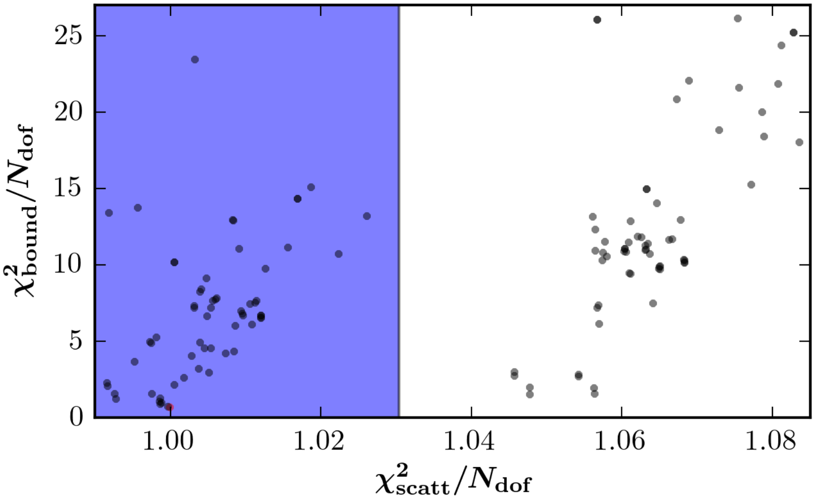
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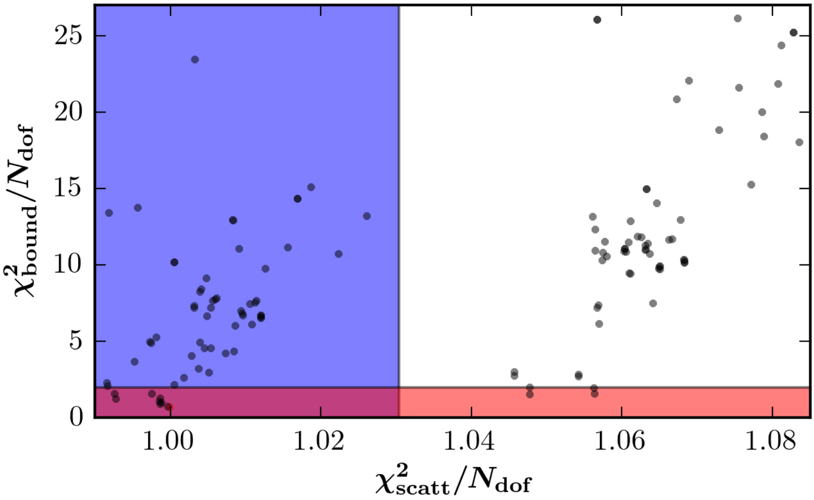

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Preliminary results

- We choose a minimum that looks promising.
- Construct a **family of 42 potentials** as for the lower orders.
- Results in the π**N and NN** sector looks **promising**.

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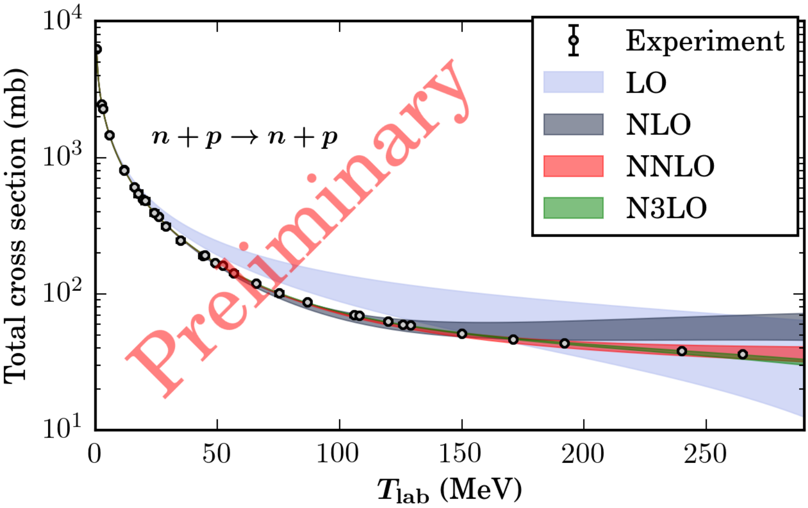
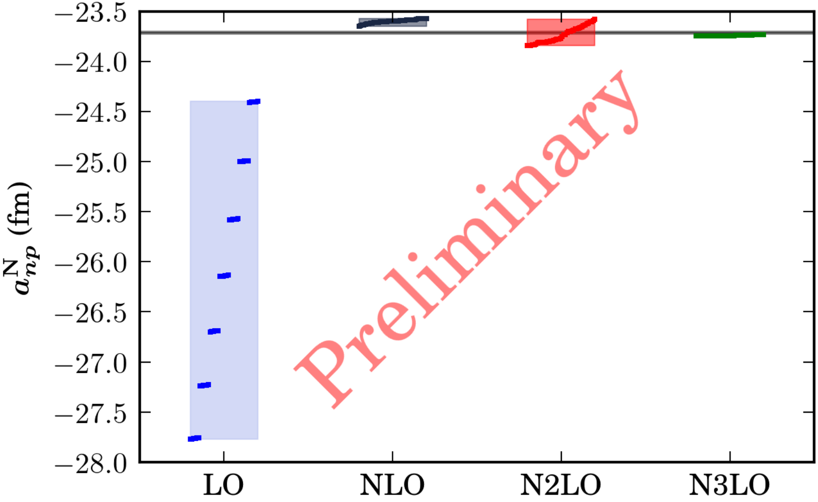
$$\Lambda_\chi = 450 \ldots 600\,\mathrm{MeV}$$

Total neutron-proton cross section **with model errors**



Neutron-proton scattering length



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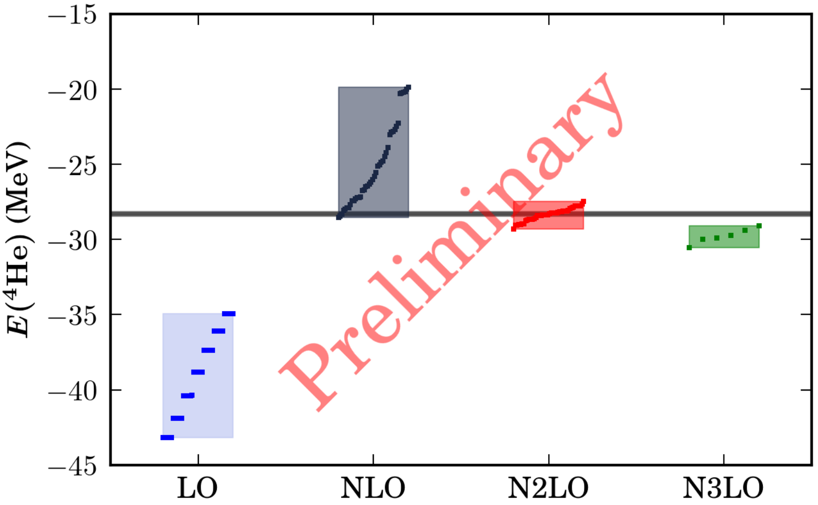
Few-body results

- **Large uncertainty** already at $A = 4$
- Removing high-energy scattering data from the fit makes the fit **poorly constrained**.

$$T_{\mathrm{lab}}^{\mathsf{max}} = 290\,\mathrm{MeV}$$

$$\Lambda_\chi = 450 \ldots 575\,\mathrm{MeV}$$

Helium 4 binding energy



Summary

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- First common **statistical regression** analysis of ab initio few-body physics and χEFT.
- **Simultaneous** optimization to πN-, NN- and NNN-data improves the model and is crucial in order to get small statistical errors.
- **Complications** at **N3LO** to be solved.