# Electric Properties of One-Neutron Halo Nuclei in Halo EFT

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# One-Neutron Halo Nuclei

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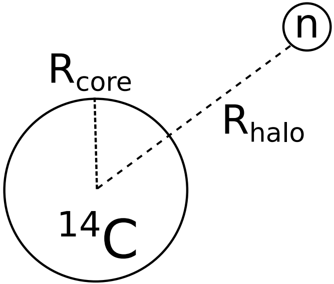
- study of electric properties of one-neutron Halo nuclei provide insights in **universal properties**
- neutron separation energy of $\frac{1}{2}^+$ $\left[\frac{5}{2}^+\right]$ state of $^{15}C$ is 1218 [478] keV
- first excitation of $^{14}C$ is 6.1 MeV above $0^+$ ground state



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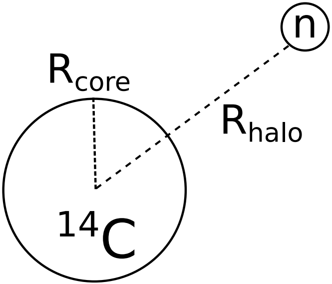
- first excitation of $^{14}C$ is 6.1 MeV above $0^+$ ground state
- exploit **separation of scales** in weakly-bound nuclei $\Rightarrow R_{core} \ll R_{halo}$
- compute observables in a **Halo EFT** in powers of $\mathbf{R_{core}/R_{halo} \approx 0.4}$
- relevant degrees of freedom: **core** and **halo neutron**



# Halo EFT Formalism

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- follow the approach of [Hammer and Phillips, 2011] for $^{11}$Be (*s*- & *p*-waves) and [Rupak et al., 2012, Fernando et al., 2015] for $^{15}$C (*s*-waves)
- include strong *s*-wave and *d*-wave interaction through incorporation of **auxiliary spinor fields** $\sigma$ and *d*, respectively

$$\mathcal{L} = c^\dagger \left( i\partial_t + \frac{\nabla^2}{2M} \right) c + n^\dagger \left( i\partial_t + \frac{\nabla^2}{2m} \right) n$$

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$$+d^\dagger\left[c_2\left(i\partial_t + \frac{\nabla^2}{2M_{nc}}\right)^2 + \eta_2\left(i\partial_t + \frac{\nabla^2}{2M_{nc}}\right) + \Delta_2\right]d$$

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$$
\begin{aligned}
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& + d^\dagger \left[ c_2 \left( i\partial_t + \frac{\nabla^2}{2M_{nc}} \right)^2 + \eta_2 \left( i\partial_t + \frac{\nabla^2}{2M_{nc}} \right) + \Delta_2 \right] d \\
& - g_0 \left[ c^\dagger n^\dagger \sigma + \sigma^\dagger n c \right] \; - \; g_2 \left[ d^\dagger n \overset{\leftrightarrow}{\nabla}^2 c + c^\dagger \overset{\leftrightarrow}{\nabla}^2 n^\dagger d \right]
\end{aligned}
$$

# Dressing the D-Wave State

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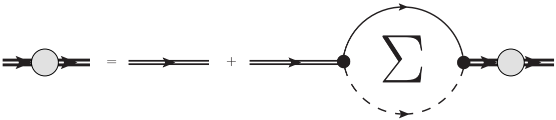


- **nc loops** must be **resummed** to compute the full *d* propagator
- use **Dyson equation** and calculate one-loop self-energy $\Sigma$ in **PDS**

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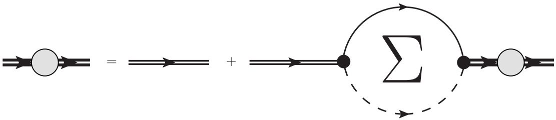


- **nc loops** must be **resummed** to compute the full $d$ propagator
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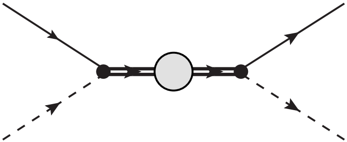
- match scattering amplitude to the **effective-range expansion** (ERE)



$$t_2(\mathbf{p}', \mathbf{p}; E) = \frac{15\pi}{m_R} \frac{\left(\mathbf{p}\cdot\mathbf{p}'\right)^2 - \frac{1}{3}\mathbf{p}^2\mathbf{p}'^2}{1/a_2 - \frac{1}{2}r_2k^2 + \frac{1}{4}\mathcal{P}_2k^4 + ik^5}$$

$$D_d(p) = Z_d \frac{1}{p_0 - \frac{\mathbf{p}^2}{2M_{nc}} + B_2} + R_d(p)$$

# Power-Counting Scheme for Shallow Bound States

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- power-counting scheme for arbitrary $l$-th partial wave shallow bound states
- **(l + 1) ERE parameters** needed at LO for matching due to higher divergences
- **minimal number** of fine tunings $\rightarrow$ **l fine tunings** for $l \geq 1$
- every additional fine tuning less likely in nature $\rightarrow$ proof that shallow bound states for higher partial waves **less likely to occur** in nature

$$\gamma_l \sim 1/R_{halo}$$

$$a_l \sim \begin{cases} R_{halo}, & l = 0 \\ R_{halo}^{2l}\, R_{core}, & l > 0 \end{cases}$$

$$r_l \sim \begin{cases} R_{core}^{1-2l}, & l = 0 \\ 1/\left(R_{halo}^{2l-2}\, R_{core}\right), & l > 0 \end{cases}$$

$$\mathcal{P}_l \sim \begin{cases} R_{core}^{3-2l}, & l \leq 1 \\ 1/\left(R_{halo}^{2l-4}\, R_{core}\right), & l > 1 \end{cases}$$

# Electromagnetic Interactions

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- include electromagnetic interactions via **minimal substitution** in the Lagrangian $\partial_\mu \to D_\mu = \partial_\mu + ie\hat{Q}A_\mu$
- add **local gauge-invariant** operators involving the electric field $\mathbf{E} = \nabla A_0 - \partial_0 \mathbf{A}$

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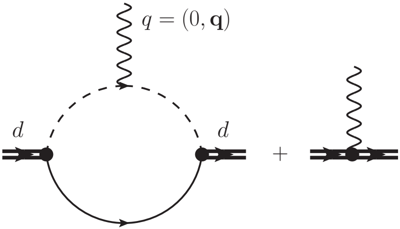
- compute electric form factors in the **Breit frame** $q = (0, \mathbf{q})$
- electric form factors get contributions only from irreducible **$\Gamma_0$ vertex** up to NLO
- for $d$-wave obtain $G_E(|\mathbf{q}|)$, $G_Q(|\mathbf{q}|)$ & $G_H(|\mathbf{q}|)$ form factors



# Electric Form Factors



- need additional local gauge-invariant operators for $r_E \sim L^{(d)}_{C0E}$ and $\mu_Q \sim L^{(d)}_{C0Q}$ at LO to treat arising divergences for $G_E(|\mathbf{q}|)$ and $G_Q(|\mathbf{q}|)$

$$G_E(|\mathbf{q}|) \approx 1 - \frac{1}{6}\left\langle r_E^2 \right\rangle |\mathbf{q}|^2 + ... \quad \xrightarrow{\text{LO}} \quad \left\langle r_E^2 \right\rangle^{(d)} = -\frac{6\tilde{L}^{(d)\,\text{LO}}_{C0E}}{r_2 + \gamma_2^2 \mathcal{P}_2 - 5\gamma_2^3}$$

$$G_Q(|\mathbf{q}|) \approx \mu_Q \left(1 - \frac{1}{6}\left\langle r_Q^2 \right\rangle |\mathbf{q}|^2 + ...\right) \quad \xrightarrow{\text{LO}} \quad \mu_Q^{(d)} = \frac{40\tilde{L}^{(d)\,\text{LO}}_{C0Q}}{3\left(r_2 + \mathcal{P}_2\gamma_2^2 - 5\gamma_2^3\right)}$$

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$$\langle r_Q^2\rangle^{(d)} = \frac{90 f^4}{7\gamma_2\left(r_2 + \mathcal{P}_2\gamma_2^2 - 5\gamma_2^3\right)}$$

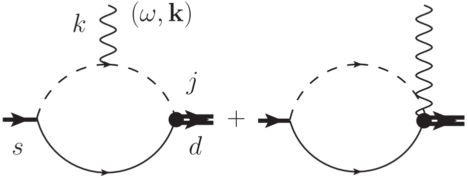
$$G_H(|\mathbf{q}|) \approx \mu_H \left(1 - \frac{1}{6}\langle r_H^2\rangle |\mathbf{q}|^2 + \dots\right) \quad \xrightarrow{\text{LO}} \quad \mu_H^{(d)} = -\frac{2 f^4}{3\gamma_2\left(r_2 + \mathcal{P}_2\gamma_2^2 - 5\gamma_2^3\right)}$$

- **obtain correlations** between different electric observables: $\langle r_Q^2\rangle^{(d)} \sim \mu_H^{(d)}$

# E2 Transition



- calculate irreducible $\mathbf{\Gamma_{j\mu}}$ **vertex** for E2 transition from the $1/2^+$ to the $5/2^+$ state at LO
- neutron spin unaffected
- **divergences cancel** each other



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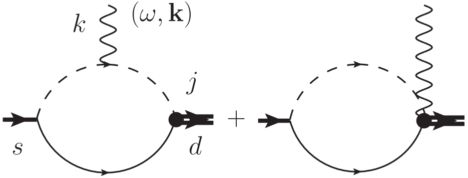


$$B(E2) = \frac{6}{\pi}\left(\frac{\bar{\Gamma}_E}{\omega}\right)^2 = \frac{6}{15\pi} Z_{eff}^2 e^2 \frac{\gamma_0}{-r_2 - \mathcal{P}_2\gamma_2^2 + 5\gamma_2^3}\left[\frac{3\gamma_0^2 + 9\gamma_0\gamma_2 + 8\gamma_2^2}{(\gamma_0+\gamma_2)^3}\right]^2$$

- **experimental result** for B(E2) = 0.967(22) $e^2$ fm$^4$
  $\rightarrow$ extract $1/\left(r_2 + \mathcal{P}_2\gamma_2^2 - 5\gamma_2^3\right)$
- **numerical predictions** for $\langle r_Q^2\rangle^{(d)} = -0.578$ fm$^4$ and $\mu_H^{(d)} = 0.030$ fm$^4$

# Summary & Outlook

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## Summary

- Halo EFT to calculate electric observables of weakly-bound *d*-wave states
- number of matching parameters increases
- shallow bound states in lower partial waves more likely
- local gauge-invariant operators get more important
- obtain correlations between electric observables

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## Outlook

- compute magnetic observables
- compare results with *ab initio* calculations

# References




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