

Introduction to General Relativity. Pensum.

1. Curvilinear coordinates

- (a) Einstein's equivalence principle
- (b) Curvilinear coordinates. Covariant differentiation and Christoffel symbols $DA^a = dA^a + \Gamma_{bc}^a A^b dx^c$. Geodesic as a "no-acceleration" trajectory $Du^a = 0$.
- (c) Metric tensor $ds^2 = g_{ab} dx^a dx^b$. Geodesics from the variational principle $\delta \int ds = 0$. Connection between Christoffel symbols and the metric tensor $\Gamma_{abc} = \frac{1}{2}(g_{ab,c} - g_{bc,d} + g_{cd,b})$.
- (d) Motion of a particle in a gravitational field. Maxwell equations in the presence of a gravitational field $F_{;a}^{ab} = 4\pi j^b$. Motion of a particle in the presence of both gravitational and electromagnetic field, $\frac{Du^a}{ds} = F^{ab} u_b$.

2. Gravitational field equations

- (a) Riemann curvature tensor R_{bcd}^a . Properties of the curvature tensor.
- (b) Invariant volume element $\sqrt{-g} d\Omega$.
- (c) Hilbert's action for the gravitation field $S = -\frac{1}{2\kappa} \int R \sqrt{-g} d\Omega$.
- (d) Energy-momentum tensor T_{ab} of the matter.
- (e) Einstein's equations for the gravitational field $R_{ab} - \frac{1}{2} R g_{ab} = \kappa T_{ab}$.

3. Solutions of the field equations

- (a) Newton's limit of the Einstein's equations.
- (b) Weak gravitational waves.
- (c) Centrally symmetric field. Schwarzschild metric.
- (d) Gravitational collapse. Lemaitre metric.
- (e) Mercury perihelion advance. Bending of light. Gravitational red shift.

4. Relativistic cosmology

- (a) Uniform and isotropic universe and its geometry.
- (b) Friedman's equation. Friedman-Lemaitre metric.
- (c) Red shift and the Hubble constant.