Cosmological redshift. Hubble constant.

In an isotropic universe the radial ($d\theta = d\phi = 0$) propagation of light ($ds^2 = 0$) is described by $\chi = \pm\eta + const$, from where one can deduce that along the light ray there remains a constant product $\omega a = const$. A light ray with frequency ω_0 emitted at a distance χ and observed at the origin ($\chi = 0$) at time η should then have the frequency $\omega = \omega_0 \frac{a(\eta-\chi)}{a(\eta)} \approx \omega_0(1 - \chi\frac{a'}{a})$, that is, redshifted, if the universe expands ($a' > 0$). The proper distance l to the source of light is $l = \chi a$. Thus the frequency shift z can be written as $z \equiv \frac{\omega_0-\omega}{\omega_0} = \frac{a'}{a^2} l \equiv Hl$, where H is the so called Hubble constant, $H = \frac{a'}{a^2} = \frac{1}{a}\frac{da}{dt}$. The current value of the Hubble constant is $H \approx \frac{1}{(13bil.years)}$.

Inserting $\frac{a'}{a^2} = H$ into Friedmann's equations leads to $\frac{1}{a^2} = H^2 - \frac{\kappa\mu}{3}$ for a closed universe, and to $\frac{1}{a^2} = \frac{\kappa\mu}{3} - H^2$ for an open universe. For the critical density μ_c, such that $\frac{\kappa\mu_c}{3} = H^2$, the universe is flat.

The current measurements show that the relative density $\Omega = \frac{\mu}{\mu_c}$ is close to one with an error about few per cent (flatness problem). About 30% of it is "dark matter" and about 70% is "dark energy". The visible matter constitutes only about 3% of the density.