Introduction

Einstein's general relativity is a geometrical[1] relativistic theory of gravitation. It describes relativistic motion of bodies[2] in strong gravitational fields. In the absence of gravitational fields it reduces to special relativity. For weak fields and slow motions it reduces to Newtonian mechanics.

Special relativity

Einstein's special relativity is a relativistic theory of motion of classical bodies (relativistic mechanics). In the slow motion limit it reduces to Newtonian mechanics. Special relativity is based on several postulates (which are deduced from experiments):

1. **Special principle of relativity**: the laws of physics are the same in all inertial[3] reference frames[4].

2. **Finiteness of the speed of light**: the fastest possible velocity (the speed of light in vacuum) is finite (and actually ridiculously small, 299792458m/s).

The special principle of relativity implies that the (cartesian) coordinate transformations between inertial frames form a goup and thus the transformation must have the form (exercise: prove it),

$$\begin{pmatrix} t' \\ z' \end{pmatrix} = \frac{1}{\sqrt{1-\frac{v^2}{c^2}}} \begin{bmatrix} 1 & -v/c^2 \\ -v & 1 \end{bmatrix} \begin{pmatrix} t \\ z \end{pmatrix}, \qquad (1)$$

where the frame with coordinates (t', z') moves relative to the frame with coordinates (t, z) with velocity v along the z (and z') axis. The c is apparently the fastest possible relative velocity (apparently the speed of light) which is experimentaly measured to be finite. Transformation (1) with finite c is called the Lorentz transformation.

The *interval*

$$ds^2 = dt^2 - dx^2 - dy^2 - dz^2 \qquad (2)$$

is invariant under Lorentz transformation (1) and thus defines a *metric*[5]. The (cartesian) space with metric (2) is called *Minkowski space*, which is the world of special relativity. In the limit $v \ll c$ it reduces to *Euclidian space*, which is the non-relativistic world of classical mechanics with Galilean transformation, where dt is invariant and the interval reduces to the spatial distance between points.

Exercises

1. Prove (1) using eg the following strategy:

   (a) assume a general form

   $$\begin{bmatrix} \gamma & \delta \\ \beta & \alpha \end{bmatrix}$$

   for the transformation matrix.

   (b) consider the motion of the origin of the frame K' (K) relative to frame K (K'): $\beta = -v\gamma$, $\beta = -v\alpha$;

   (c) cosider inverse transformation and isotropy: $\gamma^2 + v\gamma\delta = 1$;

   (d) consider composition of transformations: $v\gamma/\delta = -c^2$ = universal constant.

[1] *Geometrical* here means having to do with geometrical properties of the space-time rather than some physical fields like in electrodynamics.

[2] *Body* is a physical object whose spatial extension can be neglected. Example: the Earth when considering its motion around the sun.

[3] *Inertial reference frame* is a reference frame where free bodies travel with constant velocities.

[4] *Reference frame* (or just frame for brevity) is a system of real or virtual bodies and clocks which can be used to specify the spatial and temporal positions of events. Example: GPS satellites.

[5] *Metric* is a function of two infinitesimally close points in space which is used to measure distances and angles (and thus develope a geometry of space).