Geodesic from the variational principle (d'Inverno, 7.6; t'Hooft, page 29)
Geodesic is the straightest curve in a curved space. Along this curve the integral

$$S = \int_A^B ds,$$

between the two points A and B in space-time has an extremum:

$$\delta S = 0,$$

Maxwell equations in the gravitation field (d'Inverno, 12.5; t'Hooft, 10)
The first pair does not change its form

$$\partial_a F_{bc} + \partial_b F_{ca} + \partial_c F_{ab} = 0.$$

In the second pair we use the covariant derivative D/dx^k

$$DF^{ab}/dx^b = 4\pi/c\; j^a\;.$$

Equation of motion of a charged particle in the presence of both gravitational and electro-magnetic field

$$m\, Du^a/ds = e\, F^a{}_b u^b\;.$$

Exercises

1. Prove, that the no-acceleration trajectory,

   $$Du_a/ds \equiv du_a/ds - \Gamma_{bac}\, u^b u^c = 0,$$

   is equivalent to the shortest trajectory,

   $$du_a/ds - 1/2\; \left(dg_{bc}/dx^a \right)\; u^b u^c = 0\;.$$

2. In the Rindler space $ds^2 = (1 + g\xi/c^2)^2 (cd\eta)^2 - d\xi^2$ make the non-relativistic limit and prove, that the geodesic equation reduces to the Newtonian non-relativistic equation $d^2\xi/d\eta^2 = g$.
3. Prove that the Kronecker delta $\delta^a{}_b = \{1 \text{ if } a=b, 0 \text{ otherwise}\}$ is a tensor.
4. (non-obligatory) Explain the twins paradox from the point of view of the accelerating observer.
