General relativity: note12

Gravitational waves.

In a weak gravitational field the space-time is almost flat and the metric tensor g_{ab} is equal to the flat metric η_{ab} plus a small correction h_{ab}, $g_{ab} = \eta_{ab} + h_{ab}$. The Riemann tensor to the lowest order in h_{ab} is

$$R_{abcd} = \frac{1}{2}(h_{ad,bc} + h_{bc,ad} - h_{ac,bd} - h_{bd,ac}). \tag{1}$$

If we choose coordinates such that

$$(h^a_b - \frac{1}{2}h\delta^a_b)^{,b} = 0, \tag{2}$$

the Ricci tensor is simply

$$R_{ab} = -\frac{1}{2}h^{,c}_{ab,c} \tag{3}$$

and the vacuum Einstein's equations turn into the ordinary wave equations

$$(\frac{\partial^2}{\partial t^2} - \Delta)h_{ab} = 0. \tag{4}$$

The intensity of gravitational radiation by a system of slowly moving bodies is determined by its quadrupole moment $D_{\alpha\beta}$

$$-\frac{dE}{dt} = \frac{G}{45c^5}(D^{'''}_{\alpha\beta})^2 \tag{5}$$

Gravitational redshift.

Gravitational red shift is a change of the frequency of the electro-magnetic radiation as it passes through a gravitational field. It is a direct consequence of the equivalence principle. Gravitational red shift is one of the classical tests of general relativity (the others being Mercury perihelion advance and light bending).

The connection between the proper time interval $\Delta\tau$ and the world time interval Δt (here we only consider stationary gravitational fields where such world time can be introduced) is $\Delta\tau = \sqrt{g_{00}}\Delta t$.

Since frequencies are inversely proportional to the time intervals the corresponding connection between world frequency ω_0 and the locally measured frequency ω is $\omega = \frac{\omega_0}{\sqrt{g_{00}}}$. In a weak gravitational field $g_{00} = 1 + 2\phi$ and therefore $\omega = \omega_0(1-\phi)$. A photon emitted from a point with ϕ_1 and received at a point with ϕ_2 will be shifted by $\Delta\omega = (\phi_1 - \phi_2)\omega$.

Experimental verification of the gravitational redshift requires good clocks since at Earth the effect is small. The first experimental confirmation came as late as in 1959, in the Pound-Rebka experiment [R.V. Pound and G.A. Rebka, Apparent weight of photons, Phys. Rev. Lett. 4, 337 (1960)] later improved by Pound and Snider. The famous experiment is generally called the Pound-Rebka-Snider experiment. They used Mossbauer effect to accurately meauser the change of frequency of a photon travelling upwards 22 m in the Earth's field.

Exercises

1. (Non-oglig.) From the variational principle by varying A_a find the inhomogenius Maxwell equation in a gravitational field, $F^{ab}_{;b} = 4\pi j^a$.

2. (Non-oglig.) Prove, that it can be written as $(\sqrt{-g}F^{ab})_{,b} = 4\pi\sqrt{-g}j^a$ and that $j^a_{;a} = (\sqrt{-g}j^a)_{,a} = 0$.