Inertial (fictitious) forces are the forces which are thought to affect the bodies when looking at their motion from an non-inertial frame. Centrifugal, Coriolis and elevator forces are the examples.

Consider the motion of a free test body in an inertial frame K: its equation of motion is **x**''=0 which is an equation for a straight line. Frames, where free bodies move along a straight lines are called flat.

Consider the same body from a frame K° that accelerates with respect to K with acceleration **a**. In this frame the equation is **x**''=-**a** which is an equation for a parabola. Frames, where free bodies do not move along a straight lines are called curved. This equation can be written with the help of a fictitious inertial force $\mathbf{F}_I=-m\mathbf{a}$ as the Newton's law: $m\mathbf{x}''=\mathbf{F}_I$. Inertial forces are useful if one for some reason *must* work in a non-inertial frame.

Inertial forces have the following properties:

1. Inertial forces are proportional to the masses of bodies, or, in other words, under inertial forces all bodies move with the same acceleration.
2. Inertial forces disappear after a coordinate transformation to an inertial (flat) frame.
3. Inertial forces appear in the equations of motions because the frame is not inertial (curved).

Galileo's "Pisa experiment" showed that all bodies move in a gravitational field with the same acceleration (which is the first property of inertial forces). Einstein has postulated that all three properties of inertial forces are fulfilled also for gravitational forces. In other words, gravitational forces are physically equivalent to inertial forces. This is called the Einstein's equivalence principle. It can also be formulated as:

- In free fall the effects of gravity disappear in all possible local experiments and general relativity reduces locally to special relativity, that is inertial (flat) frame.
- An accelerated frame is locally equivalent to a frame in a gravitation field.
- Gravitation field is locally equivalent to a non-inertial (curved) frame.
- Gravitation forces are equivalent to inertial forces.

Exercises

Consider the motion of a particle with charge *e* and mass *m* in a constant uniform electric field **E** which is, say, in X direction.

1. Suppose, that at *t*=0 the particle was at rest, **v**=0, with **r**=0. Find x(t).
2. Non-obligatory: suppose, that at t=0 the particle had **r**=0, but $v_x\neq 0$ and $v_y\neq 0$. Find x(t), y(t) and x(y).

You might need the equation of motion of a charge in an electro-magnetic field **E**, **H**:

$$d\mathbf{p}/dt = e(\mathbf{E} + {}^1/_c\,\mathbf{v}\times\mathbf{H}),$$

where the (relativistic) momentum **p** and the velocity **v** are connected as

$$\mathbf{p} = m\mathbf{v}\,\left(1/\sqrt{[1-v^2/c^2]}\right)$$
