General relativity: exam

Problems

1. A planet moves in a circular orbit around a black hole outside the Schwarzschild radius. What is the minimum possible value of the orbit's radius (in Schwarzschild coordinates)? Hint: $ds^2 > 0$

2. In Schwarzschild coordinates an observer, which is at rest at $r = r_0$, transmits a radio-signal with frequency ω_0 radially upwards. Another observer, which is at rest at $r = r_1 > r_0$, receives this signal at frequency ω_1. Calculate the frequency shift.

3. In Schwarzschild coordinates an observer under the Schwartzschild radius sends two light signal radially up and down. Calculate the proper times it takes for this two light signals to reach centrum.

4. Consider the closed isotropic universe with metric

$$ds^2 = a(\eta)^2(d\eta^2 - d\chi^2 - \sin\chi^2 d\Omega^2) \tag{1}$$

   If the matter density is equal μ, what is the total mass of the universe? What is the visible mass of the universe at time η? Explain what happens to the visible mass when $\eta > \pi$.

5. Estimate the order of magnitude of the relativistic effects on the surface of earth.

Constants

1. The mass of the earth is 6.0×10^{24} kg,
2. The mass of the sun is 2.0×10^{30} kg,
3. The radius of the earth is 6.4×10^{6} m,
4. The speed of light is 299792458 m/s,
5. The Newton's gravitational constant is $6.7 \times 10^{-11} m^3/kg/s^2$