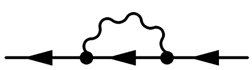
Exercises

1. Show that the S-matrix is unitary, $S^\dagger S = 1$.
2. Show that
$$\Delta\left(\phi(x)\phi^\dagger(x')\right) = \langle 0|T\left(\phi(x)\phi^\dagger(x')\right)|0\rangle \, .$$
3. Prove that the propagator
$$\Delta\Big(\phi(x)\phi^\dagger(x')\Big) = \begin{cases} \Delta^{(+)}(x-x')\,, & t > t' \\ -\Delta^{(-)}(x-x')\,, & t < t' \end{cases} ,$$
where
$$\begin{aligned} \Delta^{(\pm)}(x-x') &\doteq \Big[\phi^{(\pm)}(x), \phi^{(\pm)\dagger}(x')\Big] \\ &= \pm\sum_{\mathbf{k}} \frac{e^{\mp ik(x-x')}}{2\omega_{\mathbf{k}}} \, . \end{aligned}$$
can be written as
$$\Delta\Big(\phi(x)\phi^\dagger(x')\Big) = i\int \frac{d^4k}{(2\pi)^4}\frac{e^{-ik(x-x')}}{k^2-m^2+i0} \, ,$$
4. Show that the function $i\Delta(\phi(x)\phi^\dagger(x'))$ is the Green's function of the Klein-Gordon equation.
5. For $\mathcal{L}_v = -g\bar{\psi}\psi\phi$ apply the Wick's theorem to the second order term of the S-matrix. For each term draw the corresponding Feynman diagram and interpret the term.
6. Formulate Feynman rules for the interaction $\mathcal{L}_v = -g\bar{\psi}\psi(\phi + \phi^\dagger)$ between a fermionic and a complex scalar field.
7. For $\mathcal{L}_v = -g\bar{\psi}\psi\phi$ draw the lowest order Feynman diagram of the elastic scattering of two bosons and write down the corresponding term of the S-matrix.
8. (Extra) Formulate Feynman rules for $\mathcal{L}_v = -g\bar{\psi}\psi\phi$ in momentum space:
   (a) Consider the Compton scattering process and carry out the coordinate integrations.
   (b) Figure out the correspondence between the terms in the S-matrix element and the elements of the diagram.
9. Consider the loop diagram



Interpret it. Write down the matrix element in momentum space. Figure out the expression for the loop. Estimate the asymptotic behavior of the loop integral.