Exercises

1. Show that the generator $I_{\vec{n}}$ for the rotation around axis $\vec{n}$ is given as $I_{\vec{n}} = \vec{I} \cdot \vec{n}$ (note the vector dot product, not a matrix acting on a vector).
   Hint: Show that rotation around $\vec{n}$ has to leave $\vec{n}$ unchanged, and therefore $I_{\vec{n}}\vec{n} = 0$ (here is a matrix acting on a vector). Prove the last equation by direct calculation.

2. Find the finite rotation matrix for a 2×2 representation of the rotation group.
   Hint: $\vec{I} = \frac{1}{2}\vec{\sigma}$, and $(\vec{\sigma} \cdot \vec{n})^2 = 1$, where $\vec{\sigma}$ are Pauli matrices.

3. Find the additive parameter φ of the velocity boost matrix (along the x-axis) and the corresponding generator.
   Hint: $v = \tanh\varphi$.

4. Using the additive parameter $\varphi = \frac{1}{2}\ln\frac{1+v}{1-v}$ find the velocity-boost matrices for the $(\frac{1}{2}, 0)$ and $(0, \frac{1}{2})$ representations of the Lorentz group.