

Exercises

1. Calculate the current and the momentum of the positive- and negative-frequency solutions of the Dirac equation.
2. Find the projection operators \mathcal{P}_\pm on the positive and negative frequency solutions of the Dirac equation.
3. The spinors $\phi \in (\frac{1}{2}, 0)$ and $\chi \in (0, \frac{1}{2})$ are often called “left” and “right”. Explain why.

Hint: investigate the behavior of the projection of the spin on the momentum, $\phi^\dagger \frac{1}{2} \vec{\sigma} \vec{p} \phi$, as function of the velocity with which the spinor moves relative to the observer.

What happens if the spinor has zero-mass and is thus doomed to forever move with the speed of light?

4. Show that $\frac{d^3 p}{2E_{\vec{p}}}$ is invariant.

Hint: consider $d^4 p \delta(p^2 - m^2)$.