

Exercises

1. Find the infinitesimal gauge-transformation rule for the gauge field A_j^a .
2. Argue that the covariant derivative D_μ transforms under $U(1)$ gauge transformation as

$$D_\mu \rightarrow D'_\mu = e^{i\alpha(x)} D_\mu e^{-i\alpha(x)}$$

3. Find the transformation law of the covariant derivative under a general gauge transformation $\psi \rightarrow U\psi$.
4. Find the transformation law of the Yang-Mills field tensor

$$F_{ab} = \frac{1}{ig} [D_a, D_b]$$

5. Argue that the Yang-Mills Lagrangian

$$L_{YM} = -\frac{1}{2} \text{Tr}(F_{ab} F^{ab})$$

is gauge invariant.