http://www.phys.au.dk/subatom/nucltheo/QFT

Introduction

The subject of quantum field theory

Quantum field theory is a theory of elementary (i.e. structureless) particles. Elementary particles exhibit wave-particle duality: on the one hand they diffract and interfere as waves (in certain fields); on the other hand they appear and disappear as whole entities (called quanta). Hence the name of the theory.

The Standard Model – the theory of (some of) the most fundamental particles of our universe and their interactions – is a quantum field theory[1].

A quantum field theory seeks to explain certain fundamental experimental observations (like the existence of antiparticles; the connection between the spin of a particle and its statistics; the CPT symmetry) as well as predict the results of any given experiment (like the cross-section of the Compton scattering, or the anomalous magnetic moment of the electron).

Fields

Diffraction and interference are phenomena related to propagation of waves. A wave is a (propagating) disturbance of a physical quantity, continuously distributed through space and time. This quantity is referred to as field. For example, the sound waves in the air are disturbances in the field of the local pressure of the air; the electromagnetic waves are disturbances of the electromagnetic field.

Since elementary particles are waves (as shown by the double-slit experiment) there is a field associated with each particle. A quantum field theory is built as a classical Lagrangian field theory with subsequent canonical quantization[2].

Relativity and covariance

The (special) relativity principle, based on a great number of different observations, states that all laws of nature are identical in all inertial systems of reference (inertial frames). As a generalization of the relativity principle, the covariance principle states that the mathematical equations, describing the laws of nature, must have the same form in all inertial frames. In other words the equations must be invariant under the (Lorentz) transformation from one inertial frame to another.

Conventions for 4-vectors

An event in an inertial frame is marked with four coordinates $x^\mu \equiv \{t, \mathbf{r}\}$, where t is the time of the event, and $\mathbf{r} \equiv \{x, y, z\}$ are the three spatial coordinates of the event.

The connection between the coordinates of an event in one frame, x^μ, and another frame, x'^μ, is a linear Lorentz transformation

$$x'^\mu = a^\mu_\nu x^\nu \ , \tag{1}$$

where[3], the matrix a^μ_ν is given e.g. as

$$\begin{pmatrix} t' \\ z' \end{pmatrix} = \frac{1}{\sqrt{1-\frac{v^2}{c^2}}} \begin{bmatrix} 1 & -v/c^2 \\ -v & 1 \end{bmatrix} \begin{pmatrix} t \\ z \end{pmatrix}, \tag{2}$$

where the frame with coordinates (t', z') moves relative to the frame with coordinates (t, z) with velocity v along the z (and z') axis, and c is the speed of light[4].

The Lorentz transformation conserves the invariant

$$s^2 = t^2 - \mathbf{r}^2 \equiv x_\mu x^\mu \ , \tag{3}$$

where x_μ are the co-variant coordinates,

$$x_\mu \equiv \{t, -\mathbf{r}\} = g_{\mu\nu} x^\nu \ , \tag{4}$$

where the diagonal tensor $g_{\mu\nu}$ with the diagonal $\{1, -1, -1, -1\}$ is the metric tensor of the (flat pseudo-Euclidian) Minkowski space of special relativity. The co-variant coordinates transform with the inverse Lorentz matrix (the one where v is substituted with $-v$),

$$x'_\mu = (a^{-1})^\nu_\mu x_\nu ; , \tag{5}$$

A four-vector is a set of four numbers, $A^\mu = \{A^0, \mathbf{A}\}$, which transform (from one frame to another) the same way as contr-variant coordinates, eq. (5).

The partial derivatives with respect to coordinates apparently form a contra-variant four-vector,

$$\partial_\mu \equiv \frac{\partial}{\partial x^\mu} = \{\frac{\partial}{\partial t}, \frac{\partial}{\partial x}, \frac{\partial}{\partial y}, \frac{\partial}{\partial z}\} \equiv \{\frac{\partial}{\partial t}, \nabla\} \ , \tag{6}$$

with

$$\partial_\mu x^\nu \equiv \frac{\partial x^\nu}{\partial x_\mu} = \delta^\nu_\mu \tag{7}$$

[1] specifically, a Yang-Mills gauge theory with the symmetry group $U(1) \times SU(2) \times SU(3)$.

[2] sometimes erroneously called "second quantization".

[3] Note the "implicit summation" notation, $a^\mu_\nu x^\nu \equiv \sum_{\nu=0}^{3} a^\mu_\nu x^\nu$.

[4] In the following the notation with $\hbar = c = 1$ shall be mostly used.