Introduction to General Relativity
Examination problems 2022

No aids allowed, formula collection is at the back side

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1. Describe shortly the following concepts:

   (i) (5%) Principle of stationary action;

   (ii) (5%) Equivalence principle;

   (iii) (5%) Metric;

   (iv) (5%) Geodesic;

   (v) (5%) The source of gravity in (the field equation of) general relativity;

   (vi) (5%) Schwarzschild metric;

   (vii) (5%) Event horizon;

   (viii) (5%) Friedmann's equations;

   (ix) (5%) Cosmological constant;

   (x) (5%) Gravitational wave;

2. (10%) Consider a non-relativistic Lagrangian of a free body with mass m written in curvilinear coordinates with metric $dl^2 = g_{\alpha\beta}dx^\alpha dx^\beta$ (where Greek indices run from 1 to 3) as

$$\mathcal{L} = \frac{1}{2} m g_{\alpha\beta} v^\alpha v^\beta \,,$$

   where $v^\alpha = dx^\alpha/dt$. Show that the corresponding Euler-Lagrange equation is equivalent to the geodesic equation for the given metric.

3. (10%) Consider the following equations of electrodynamics in special relativity,

$$F_{ab} = -A_{a,b} + A_{b,a} \,, \quad F^{ab}_{\ \ ,a} = 4\pi j^b \,, \quad m\frac{du_a}{ds} = eF_{ab}u^b \,.$$

   Name the equations and write down the corresponding equations in general relativity.

4. (15%) For the metric

$$ds^2 = dt^2 - a^2(t)(dx^2 + dy^2 + dz^2)$$

   calculate $\Gamma^x_{\ xt}$. What is this metric about?

5. (15%) Consider a flat dark-energy-dominated Friedmann universe where the cosmological constant is the dark energy. Write down the corresponding Friedmann equation and argue that the expansion of this universe is accelerating.