Introduction to General Relativity
Examination problems 2021

1. Describe shortly the following concepts:
   (a) (5%) Equivalence principle;
   (b) (5%) Metric tensor;
   (c) (5%) Geometric theory of gravitation;
   (d) (5%) Geodesic;
   (e) (5%) Einstein's equation;
   (f) (5%) Schwarzschild metric;
   (g) (5%) Friedman's equation;
   (h) (5%) Cosmological red-shift;
   (i) (5%) Gravitational wave;
   (j) (5%) Cosmological constant;

2. (5%) The Schwarzschild radius is given in natural units as $R = 2M$ where M is the mass of the central body. Estimate the Schwarzschild radius of the Earth in SI units.

   Hints: $R_\oplus \approx 6.4$ megameters, $g \approx 9.8$ meters/second/second, $c \approx 300$ megameters/second.

3. (10%) The geodesic equation in a metric space with the metric tensor g_{ab} can be written as

$$\frac{du_c}{ds} = \frac{1}{2} g_{ab,c} u^a u^b \ .$$

   For the Schwarzschild metric[1] write down (do not solve, only write down) the geodesic equation where the index c corresponds to the radial coordinate.

4. In a weak stationary gravitational field $\phi(\mathbf{r}) \ll 1$ the metric can be approximated as

$$ds^2 \approx (1 + 2\phi)\, dt^2 - d\mathbf{r}^2 \ .$$

   (a) (10%) Calculate the Christoffel symbols in the lowest order in ϕ.
   (b) (10%) Derive the equation of motion for a slowly ($v \ll 1$) moving massive body; compare with the Newtonian equation of motion.

5. (15%) Argue that a ray of light can travel around a black hole in a circular orbit much like a planet. Is this orbit stable against small radial perturbations?

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[1]
$$ds^2 = \left(1 - \frac{2M}{r}\right) dt^2 - \frac{dr^2}{1 - \frac{2M}{r}} - r^2 \left(d\theta^2 + \sin^2\theta d\phi^2\right)$$