# Trapped-ion quantum logic utilizing position-dependent ac Stark shifts 

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#### Abstract

We present a scheme utilizing position-dependent ac Stark shifts for doing quantum logic with trapped ions. By a proper choice of direction, position, and size, as well as power and frequency of a far-off-resonant Gaussian laser beam, specific ac Stark shifts can be assigned to the individual ions, making them distinguishable in frequency space. In contrast to previous all-optical based quantum gates with trapped ions, the present scheme enables individual addressing of single ions and selective addressing of any pair of ions for two-ion quantum gates, without using tightly focused laser beams. Furthermore, the decoherence rate due to offresonant excitations can be made negligible as compared with other sources of decoherence.


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In recent years, physical realizations of quantum computers have received growing interest. Several very different physical implementations have been considered [1], and schemes based on a string of trapped ions, as first introduced by Cirac and Zoller [2], are among the most promising and popular candidates for demonstrating large scale quantum logic. In these schemes two internal levels of an ion represent a quantum bit (qubit), which can be manipulated through laser interactions [3-8]. Two key requirements are the individual addressing of the ions for single-qubit manipulations, and the ability to make gate operations between any pair of ions. Although multi-ion entanglement was demonstrated in a recent experiment [9], individual and selective addressing remains a major experimental challenge for making an ion-trap quantum computer. The difficulties of addressing originates from the need for high trap frequencies, to ensure efficient motional ground-state cooling and high gate speeds, which leads to a small spatial separation of the ions. In addition, the ion separation decreases with an increasing number of ions [10]. In current experimental setups where ground-state cooling has been demonstrated, typical trap frequencies are $\omega_{z}=2 \pi \times 0.7 \mathrm{MHz}$ in Innsbruck [11] and $\omega_{z}=2 \pi \times 10 \mathrm{MHz}$ at NIST [12], which yields minimum spacings of $7.1 \mu \mathrm{~m}$ (two ${ }^{40} \mathrm{Ca}^{+}$ions) and $3.2 \mu \mathrm{~m}$ (two ${ }^{9} \mathrm{Be}^{+}$ions), respectively.

The most obvious method for individual addressing is simply to focus a laser beam onto a single ion. This was demonstrated by the Innsbruck group [13], but it is extremely demanding with the much stronger traps used at NIST or when more ions are involved. A few more complex methods for individual addressing of ions have been presented, which use either position-dependent micromotion $[14,15]$ or a position-dependent magnetic field [16], but they are either technically demanding or hard to generalize beyond two ions.

In this paper we propose to use a position-dependent energy shift, an ac Stark shift, of the qubit levels, to obtain a unique resonance frequency of each ion, such that the ions can be addressed individually just by tuning the frequency of a laser beam illuminating the whole ion string. In addition,

[^0]we demonstrate how a position-dependent ac Stark shift can be used for selecting any pair of ions in a multi-ion string for implementing a two-ion quantum gate, e.g., a MølmerSørensen gate $[4,5]$. Our scheme is technically not very demanding, since it relies on applying a far-off-resonant ac Stark-shifting laser beam focussed to a spot size larger than the ion spacing (see Fig. 1). This feature makes the scheme applicable even in experiments with relatively tightly confining traps ( $\omega_{z} / 2 \pi \sim 10 \mathrm{MHz}$ ).

First, we consider the criteria for performing single qubit operations between states of the type $|\downarrow, n\rangle$ and $\left|\uparrow, n^{\prime}\right\rangle$, where $|\downarrow\rangle$ and $|\uparrow\rangle$ are the two eigenstates of the ions and $n$ is the vibrational quantum number for one of the motional modes of the ions. To selectively manipulate such two states of a single ion in a string, the spectral resolution $\gamma_{\text {res }}$ of the laser performing the qubit operation has to be much better than the trap frequency $\omega_{z}$ and sufficiently high that transitions in any other ion are prohibited. For simplicity, in the


FIG. 1. The basic idea of individual addressing. (a) Left: Sketch of the ac Stark-shifting laser beam and two ions in a linear Paul trap. Right: Position of the ions with respect to the intensity distribution of the laser beam. (b) The associated ac Stark-shifted energy levels of the two ions (assuming two internal states, $|\downarrow\rangle$ and $|\uparrow\rangle$ ) in the harmonic trapping potential of oscillation frequency $\omega_{z}$ (external states $|0\rangle,|1\rangle$, etc.). Individual addressing is considered in two cases. Case A: $\hbar \omega_{z}>E_{1,2} \gg \gamma_{\text {res }}$. Case B: $E_{1,2} \gg \hbar \omega_{z} \gg \gamma_{\text {res }}$. Note: For convenience, the energy levels are shifted, such that the $|\downarrow\rangle$ state has the same energy for both ions.


FIG. 2. Relevant energy levels and transitions in alkaline-earth ions (e.g., ${ }^{40} \mathrm{Ca}^{+},{ }^{88} \mathrm{Sr}^{+}$, and ${ }^{138} \mathrm{Ba}^{+}$) for calculating the ac Stark shifts of the qubit states $\left[|\downarrow\rangle={ }^{2} S_{1 / 2}\left(m_{J}=+1 / 2\right)\right.$ and $|\uparrow\rangle$ $\left.={ }^{2} D_{5 / 2}\left(m_{J}=+5 / 2\right)\right]$ in the case of a linearly polarized, far-offresonant laser beam. The Stark-shifting laser beam is assumed to be so far red detuned that the fine-structure splitting of the $P$ and $F$ levels can be neglected.
following, we consider a two-ion string, with one motional mode having the oscillation frequency $\omega_{z}$, and with the ac Stark shift induced energy difference between the two ions being $E_{1,2}$ (see Fig. 1). First, we treat the situation where $\hbar \omega_{z}>E_{1,2}>\hbar \gamma_{\text {res }}$ as sketched in Fig. 1(b) (case A). In this case, $E_{1,2}=\hbar \omega_{z} / 2$ is the optimum choice, since a laser resonant with a specific transition $|\downarrow, n\rangle \leftrightarrow\left|\uparrow, n^{\prime}\right\rangle$ in one ion is maximally off-resonant with all the transitions of the type $|\downarrow, n\rangle \leftrightarrow\left|\uparrow, n^{\prime}\right\rangle,|\downarrow, n\rangle \leftrightarrow\left|\uparrow, n^{\prime}+1\right\rangle$, or $|\downarrow, n\rangle \leftrightarrow\left|\uparrow, n^{\prime}-1\right\rangle$ in the other ion, leading to the highest possible gate speed. In the case $E_{1,2}>\hbar \omega_{z} \gg \gamma_{\text {res }}$ [case B in Fig. 1(b)], a laser resonant with a transition $|\downarrow, n\rangle \leftrightarrow\left|\uparrow, n^{\prime}\right\rangle$ in one ion is only resonant (or near resonant) with a transition $|\downarrow, n\rangle \leftrightarrow\left|\uparrow, n^{\prime}+m\right\rangle$ in the other ion, where $|m| \gg 1$. In the so-called Lamb-Dicke limit such a transition is strongly suppressed [10,17]. Case B is particularly interesting when more than two ions are present, since even in such cases the gate time will only be limited by the vibrational frequency $\omega_{z}$ instead of a fraction thereof as in case A.

An experimental realization of the above situation can be achieved, e.g., by a string of two ${ }^{40} \mathrm{Ca}^{+},{ }^{88} \mathrm{Sr}^{+}$, or ${ }^{138} \mathrm{Ba}^{+}$ ions, with the qubit states $|\downarrow\rangle$ and $|\uparrow\rangle$ represented by the ${ }^{2} S_{1 / 2}\left(m_{J}=+1 / 2\right)$ ground state and the ${ }^{2} D_{5 / 2}\left(m_{J}=+5 / 2\right)$ metastable state, respectively [24]. The far-off-resonant Stark-shifting laser beam is set to propagate perpendicular to the ion string, as indicated in Fig. 1(a), and its polarization is assumed to be linear along the axis defined by the ions. Assuming a Gaussian intensity profile with a waist $W$, a maximum difference in the ac Stark shift of the ions is obtained by displacing the laser beam by $W / 2$ with respect to the center of the ion string. The relevant internal levels of the considered ions, with respect to the Stark-shifting laser beam, are shown in Fig. 2. For simplicity, we assume that the Stark-shifting laser beam is so far red detuned from any transition frequency that fine-structure splitting can be neglected.

The ac Stark shift $\varepsilon_{\uparrow}-\varepsilon_{\downarrow}$ of the $|\uparrow\rangle-|\downarrow\rangle$ transition of a single ion can be calculated by summing the contributions from all relevant dipole-allowed couplings. The dominant shift of $|\downarrow\rangle$ is from the $n S-n P$ coupling, whereas the shift of $|\uparrow\rangle$ is composed of contributions from a series of $(n-1) D$ $-n^{\prime} F$ couplings. This gives rise to the following approxi-
mate expression for the ac Stark shift:

$$
\begin{align*}
\varepsilon_{\uparrow}-\varepsilon_{\downarrow}= & \frac{3 \pi c^{2}}{2}\left[\frac{1}{\omega_{P}^{3}}\left(\frac{\Gamma_{P}}{\omega_{P}-\omega}+\frac{\Gamma_{P}}{\omega_{P}+\omega}\right)\right. \\
& \left.-\sum_{n^{\prime}} \frac{1}{\omega_{n^{\prime} F}^{3}}\left(\frac{\Gamma_{n^{\prime} F}}{\omega_{n^{\prime} F}-\omega}+\frac{\Gamma_{n^{\prime} F}}{\omega_{n^{\prime} F}+\omega}\right)\right] I_{i o n} \\
\equiv & \psi \times I_{i o n}, \tag{1}
\end{align*}
$$

where $\omega$ is the laser frequency, $\omega_{P}$ and $\omega_{n^{\prime} F}$ are the $n S$ $-n P$ and $(n-1) D-n^{\prime} F$ transition frequencies, $\Gamma_{P}$ and $\Gamma_{n^{\prime} F}$ are the corresponding spontaneous decay rates, and $I_{i o n}$ is the intensity of the Stark-shifting laser beam at the position of the ion [18]. $\psi$ is a parameter that depends only on the properties of the ion and the laser frequency. With the two ions positioned at $r_{ \pm} \equiv W / 2 \pm \Delta z / 2$, where $\Delta z$ $=\left(e^{2} / 2 \pi \epsilon_{0} m \omega_{z}^{2}\right)^{1 / 3}$ is the equilibrium spacing of the ions of mass $m$, and given the laser intensity profile $I(r)=I_{0} \exp$ $\left[-2 r^{2} / W^{2}\right]$, Eq. 1 leads to the following difference in the transition frequency of the ions:

$$
\begin{equation*}
E_{1,2}=\kappa\left(\varepsilon_{\uparrow}-\varepsilon_{\downarrow}\right)=\kappa \psi I_{0}, \tag{2}
\end{equation*}
$$

where

$$
\begin{equation*}
\kappa=2 \sinh (\Delta z / W) \exp \left\{-\frac{1}{2}\left[1+(\Delta z / W)^{2}\right]\right\} . \tag{3}
\end{equation*}
$$

In Fig. 3(a), the laser power required to achieve an ac Stark shift difference $E_{1,2}=\hbar \omega_{z} / 2$ in the case of $\omega_{z}$ $=2 \pi \times 1 \mathrm{MHz}$ is presented for ${ }^{40} \mathrm{Ca}^{+},{ }^{88} \mathrm{Sr}^{+}$, and ${ }^{138} \mathrm{Ba}^{+}$as a function of the laser wavelength [25]. The waist of the laser beam is taken to be $30 \mu \mathrm{~m}$, which is much larger than the equilibrium spacing of $5.6 \mu \mathrm{~m}, 4.3 \mu \mathrm{~m}$, and $3.7 \mu \mathrm{~m}$ for the ${ }^{40} \mathrm{Ca}^{+},{ }^{88} \mathrm{Sr}^{+}$, and ${ }^{138} \mathrm{Ba}^{+}$ions, respectively. The required power, which approaches a constant in the long-wavelength limit, is well within reach of commercial lasers, e.g., a $\mathrm{CO}_{2}$ laser $(\lambda=10.6 \mu \mathrm{~m})$, a Nd:YAG (yttrium aluminum garnet) laser $(\lambda=1064 \mathrm{~nm})$, or a frequency-doubled Nd:YAG laser ( $\lambda=532 \mathrm{~nm}$ ).

Another very important parameter to consider in the present scheme is the spontaneous scattering rate $\Gamma_{s c}$ of light from the Stark-shifting laser beam, since it will limit the ultimate coherence time. Under the assumptions made above in calculating the ac Stark shifts, an upper limit to the sum of the scattering rates of both ions can be expressed as

$$
\begin{align*}
\Gamma_{s c, \max }= & \frac{E_{1,2}}{\kappa \psi} \frac{e^{-1 / 2} 3 \pi c^{2} \omega^{3}}{\hbar}\left[\frac{1}{\omega_{P}^{6}}\left(\frac{\Gamma_{P}}{\omega_{P}-\omega}+\frac{\Gamma_{P}}{\omega_{P}+\omega}\right)^{2}\right. \\
& \left.+\sum_{n^{\prime}} \frac{1}{\omega_{n^{\prime} F}^{6}}\left(\frac{\Gamma_{n^{\prime} F}}{\omega_{n^{\prime} F}-\omega}+\frac{\Gamma_{n^{\prime} F}}{\omega_{n^{\prime} F}+\omega}\right)^{2}\right] \tag{4}
\end{align*}
$$

where $\Delta z / W \ll 1$, as obeyed by the parameters used in Fig. 3, is assumed [18].

In Fig. 3(b) the minimum coherence time (or rather $\Gamma_{s c, \text { max }}^{-1}$ ) is plotted as a function of laser wavelength, and we


FIG. 3. (a) The required laser power as a function of wavelength for obtaining an ac Stark shift difference $E_{1,2}=\hbar \omega_{z} / 2$, when $\omega_{z}$ $=2 \pi \times 1.0 \mathrm{MHz}$ and $W=30 \mu \mathrm{~m}$ for ${ }^{40} \mathrm{Ca}^{+},{ }^{88} \mathrm{Sr}^{+}$, and ${ }^{138} \mathrm{Ba}^{+}$. (b) The corresponding minimum coherence time $\left(\Gamma_{s c, \text { max }}^{-1}\right)$.
see that in the long-wavelength limit, the minimum coherence time grows as the wavelength to the third power, which is also readily deduced from Eq. (4). Hence, at first, a $\mathrm{CO}_{2}$ laser seems to be favorable. However, since the lifetime of the ${ }^{2} D_{5 / 2}$ level is only $1.0 \mathrm{~s}, 345 \mathrm{~ms}$, and 47 s for ${ }^{40} \mathrm{Ca}^{+}$, ${ }^{88} \mathrm{Sr}^{+}$, and ${ }^{138} \mathrm{Ba}^{+}$, respectively, the use of the fundamental wavelength of a Nd:YAG laser might be more attractive, since this will be much easier to focus to the required spot size. Actually, in current experimental setups the maximal coherence time is limited by heating of the ions on a time scale of $1-100 \mathrm{~ms}[12,19]$, hence even a continuously operated frequency-doubled Nd:YAG laser can be used without introducing significant additional decoherence.

There are several reasons for not choosing $W$ too large compared with $\Delta z$. First, the required power to achieve a certain energy difference $E_{1,2}$ grows as $W^{3}$. Second, the total scattering rate for a fixed $E_{1,2}$ also increases with $W$ ( $\Gamma_{s c}$ $\propto W)$. Furthermore, it should be noted that although a large $E_{1,2}$ implies a short coherence time, it also allows a high gate speed.

The effect of the (internal state dependent) gradient force exerted on the ions by the Stark-shifting laser beam has to be considered. The maximal gradient force will be on the order of $F_{g r a d}=-\partial \varepsilon_{\downarrow} / \partial z \approx E_{1,2} / \Delta z$. Taking the example of ${ }^{40} \mathrm{Ca}^{+}$, and using the same parameters as above, the maximal gradient force will be $\sim 10^{5}$ times smaller than the confining force exerted by the trap, and the associated change in the equlibrium distance between the ions, $\delta z$, is $\sim 300$ times


FIG. 4. A Stark-shifting laser beam making two ions have the same unique resonance frequency. This allows for selective addressing of any pair of ions for two-qubit operations.
smaller than the spread of the vibrational wave function. This displacement is totally negligible. Nevertheless, when the Stark-shifting laser beam is turned on, an ion obtains a speed $v \approx \delta z / t_{\text {rise }}$, where $t_{\text {rise }}$ is the "rise time" of the Starkshifting laser beam. The associated kinetic energy must be much smaller than $\hbar \omega_{z}$, which is fulfilled if $t_{\text {rise }} \gg 1 \mathrm{~ns}$. In practice, this is no limitation.

Above we considered in detail the simple case of two ions and one motional mode. If we take both motional modes, i.e., the so-called center-of-mass mode at frequency $\omega_{z}$ and the stretch mode at frequency $\sqrt{3} \omega_{z}$ [10], into account, the optimal value of $E_{1,2}$ is slightly changed, but our conclusions remain valid. Further, we can generalize case A and case B of Fig. 1(b) up to at least five ions, which is sufficient for applying our proposal in combination with a recent proposal for large scale quantum computation with trapped ions [3]. To go much beyond five ions is difficult in case A due to the additional energy levels, while in case B the only limit is that $E_{1,2} / \hbar$ should not coincide with the frequency of one of the higher motional modes.

It should be noted that the different ionic transition frequencies, while applying the Stark-shifting laser beam, leads to a differential phase development of the various ions. Since the frequency differences are known, this can be accounted for by controlling the phase of the addressing light field.

In addition to individual addressing, a Stark-shifting laser beam can be used for realizing two-ion quantum logic operations with a single bichromatic laser pulse, as proposed by Mølmer and Sørensen [4,5], between any two ions in a string. As an example, we show in Fig. 4 how one can make two ions in a three-ion string have the same unique resonance frequency, needed for making a Mølmer-Sørensen gate between these two ions. Two neighboring ions, e.g., ions 1 and 2 in Fig. 4 can have the same resonance frequency, if the center of the Stark-shifting beam is positioned halfway between ions 1 and 2.

The position-dependent ac Stark shift method discussed above is also applicable to other qubit levels and ions. For example, the two Zeeman sublevels of the ground state in ${ }^{40} \mathrm{Ca}^{+},{ }^{88} \mathrm{Sr}^{+}$, or ${ }^{138} \mathrm{Ba}^{+}$can be used as qubit levels $[|\downarrow\rangle$ $\left.={ }^{2} S_{1 / 2}\left(m_{J}=-1 / 2\right),|\uparrow\rangle={ }^{2} S_{1 / 2}\left(m_{J}=+1 / 2\right)\right]$ with qubit operations performed by two-photon stimulated Raman transitions. An ac Stark shift can be induced by a circularly polarized Stark-shifting laser beam with wavelength $\lambda$ tuned in between the two fine-structure levels of the excited $P$ state. If we take $\omega_{z}=2 \pi \times 1 \mathrm{MHz}$ and $W=30 \mu \mathrm{~m}$, as in the previous discussion, an ac Stark shift difference $E_{1,2}=\hbar \omega_{z} / 2$ can be obtained for ${ }^{40} \mathrm{Ca}^{+}$with a maximum scattering rate of
about 160 Hz for the optimal choice of laser parameters ( $\lambda$ $=395.2 \mathrm{~nm}$ and 64 mW laser power). This scattering rate allows only for a very limited number of gate operations, even in the case where the Stark-shifting laser beam is only present during the quantum logic processing. For ${ }^{88} \mathrm{Sr}^{+}$and ${ }^{138} \mathrm{Ba}^{+}$somewhat lower scattering rates can be obtained, owing to their larger fine-structure splitting. Applying the same approach to ${ }^{25} \mathrm{Mg}^{+}$or ${ }^{9} \mathrm{Be}^{+}$(with hyperfine levels of the ground state as qubit levels [9]) is impracticable, due to their relatively small fine-structure splitting.

In conclusion, we have shown that individual or selective addressing of trapped ions can be achieved, without intro-
ducing significant decoherence, by utilizing a Stark-shifting laser beam with modest focusing and power requirements. By combining our proposal with the proposal of Ref. [3], it is even applicable in large scale quantum computation. The presented scheme readily makes it possible to perform single- qubit as well as multi-qubit gates.

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[1] M.A. Nielsen and I.L. Chuang, Quantum Computation and Quantum Information (Cambridge University Press, Cambridge, England, 2000).
[2] J.I. Cirac and P. Zoller, Phys. Rev. Lett. 74, 4091 (1995).
[3] D. Kielpinski, C. Monroe, and D.J. Wineland, Nature (London) 417, 709 (2002).
[4] K. Mølmer and A. Sørensen, Phys. Rev. Lett. 82, 1835 (1999).
[5] A. Sørensen and K. Mølmer, Phys. Rev. Lett. 82, 1971 (1999).
[6] D. Jonathan, M.B. Plenio, and P.L. Knight, Phys. Rev. A 62, 042307 (2000).
[7] D. Jonathan and M.B. Plenio, Phys. Rev. Lett. 87, 127901 (2001).
[8] J.F. Poyatos, J.I. Cirac, and P. Zoller, Phys. Rev. Lett. 81, 1322 (1998).
[9] C.A. Sackett et al., Nature (London) 404, 256 (2000).
[10] D.F.V. James, Appl. Phys. B: Lasers Opt. 66, 181 (1998).
[11] D. Leibfried et al., in Atomic Physics 17, edited by E. Arimondo, P.D. Natale, and M. Inguscio, AIP Conf. Proc. No. 551, (AIP, Melville, NY, 2001).
[12] Q.A. Turchette et al., Phys. Rev. A 61, 063418 (2000).
[13] H.C. Nägerl, D. Leibfried, H. Rohde, G. Thalhammer, J. Eschner, F. Schmidt-Kaler, and R. Blatt, Phys. Rev. A 60, 145 (1999).
[14] D. Leibfried, Phys. Rev. A 60, R3335 (1999).
[15] Q.A. Turchette, C.S. Wood, B.E. King, C.J. Myatt, D. Leib-
fried, W.M. Itano, C. Monroe, and D.J. Wineland, Phys. Rev. Lett. 81, 3631 (1998).
[16] F. Mintert and C. Wunderlich, Phys. Rev. Lett. 87, 257904 (2001).
[17] S. Stenholm, Rev. Mod. Phys. 58, 699 (1986).
[18] R. Grimm, M. Weidemüller, and Y.B. Ovchinnikov, Adv. At., Mol., Opt. Phys. 42, 95 (2000).
[19] C. Roos, T. Zeiger, H. Rohde, H.C. Nägerl, J. Eschner, D. Leibfried, F. Schmidt-Kaler, and R. Blatt, Phys. Rev. Lett. 83, 4713 (1999).
[20] NIST, http://physics.nist.gov
[21] C.E. Moore, Atomic Energy Levels Natl. Bur. Stand. (U.S.) Circ. No. 467 (GPO, Washington, D.C., 1952), Vol. II.
[22] C.E. Moore, Atomic Energy Levels Natl. Bur. Stand. (U.S.) Circ. No. 467 (GPO, Washington, D.C., 1958), Vol. III.
[23] A. Lindgård and S.E. Nielsen, At. Data Nucl. Data Tables 19, 533 (1977).
[24] Indeed the ${ }^{2} S_{1 / 2}$ and ${ }^{2} D_{5 / 2}$ states in ${ }^{40} \mathrm{Ca}^{+}$are used as qubit levels in Innsbruck [11].
[25] In Fig. 3 we do take the fine-structure splitting and the different couplings to the fine-structure levels into account. For the $(n-1) D-n^{\prime} F$ transitions we sum over $n^{\prime}=4-10$. The data used are from Ref. [20] ( $\left.{ }^{40} \mathrm{Ca}^{+}\right)$and Refs. [21-23] $\left({ }^{88} \mathrm{Sr}^{+}\right.$and ${ }^{138} \mathrm{Ba}^{+}$). For a good overview of the $S, P$, and $D$ levels, see Ref. [10].


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