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Tapered laser cooling of stored coasting ion beams

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Abstract

We propose a scheme for tapered laser cooling of coasting ion beams in storage rings. Tapered cooling has recently been shown to be crucial for attaining crystalline ion beams. The scheme proposed here, based on a relative displacement of a co- and a counterpropagating Gaussian laser beam, gives a radial variation in the equilibrium velocities to which particles are cooled. The variation is approximately linear in a relatively large range transverse to the laser beams. Expressions for the spatially dependent equilibrium velocities and the range of the tapered cooling forces are derived. We discuss the dependence on laser beam parameters as well as the limitations of this cooling scheme. © 1999 Published by Elsevier Science B.V. All rights reserved.

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Laser cooling of ion beams in storage rings have been subjected to studies for more than a decade (see, e.g., [1–3] and references therein). Laser cooling is regarded as a possible way to obtain Coulomb ordered states in fast ion beams as first suggested by Schiffer and Kienle [4] and later observed in molecular-dynamics simulations by Rahman and Schiffer [5]. Schiffer and Rahman [6] also questioned if a crystalline beam could withstand the shear arising in bending magnets and identified the need for graded cooling, where the ions are not cooled to exactly the same linear velocity, but rather to radially varying velocities in order to ensure particles to complete a revolution in phase. Although the situation in reality is somewhat more complicated, one can understand the problem of graded cooling by thinking of the cyclotron motion of a beam of charged particles in a uniform magnetic field. The outermost particles will here have to travel faster than the innermost ones if they are to move on concentric circles. In traps, where the ions are stationary, Coulomb crystallization by laser cooling has already been observed [7–9], but in contrast to storage rings, graded cooling is not needed here. The formation of a crystalline fast ion beam would be of great interest in beam physics, because it represents the ultimate in density, yielding hitherto unseen intensities in, e.g., collision experiments. Today, it is, however, an open question if it is possible to implement graded cooling experimentally [10].

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Recently, simulations by Wei, Okamoto and Sessler [11,12] showed that a tapered cooling force *is* of crucial importance in attaining a crystalline beam. The calculations were done for a 1 MeV $^{24}Mg^+$ beam in a specific lattice of the TARN II storage ring. It was shown, that even for low density 1D structures linearly varying graded cooling, so-called tapered cooling, is needed, although the demands here are less restrictive than for 3D crystals. For 1D structures shear is not a problem, but one still needs to cool the transverse degrees of freedom and due to the low number of ions in this regime, sympathetic cooling through intra-beam scattering ceases to be effective. It turns out that tapered cooling can provide damping of the transverse motion through a single particle mechanism [1]. Tapered cooling is described by a tapering coefficient C_{xs}, where a particle cooled to the right velocity obeys (in the nonrelativistic case)

$$\delta v = C_{xs} \frac{v_0}{\rho_m} x. \tag{1}$$

Here $\delta v = v - v_0$ is the deviation from the design velocity v_0, x is the horizontal distance from the design orbit, and ρ_m is the average radius of curvature in the bending sections of the storage ring. In the case of laser cooling it has been suggested that tapered cooling could be achieved by introducing a gradient in laser frequency across the ion beam by sending the laser beam through a light prism [11]. This is, however, not easily achieved with conventional laser light sources because of their extreme monochromatisities (in the calculated example [11], the wavelength would need to vary 0.01%). Another suggestion by Madsen et al. [2] uses the fringe field of a capacitor-like insertion device to shift the ions velocity locally across the beam when entering and leaving the cooling section, but the feasibility of this scheme has not yet been worked out in detail.

This Letter presents the principles of an additional tapered cooling scheme benefiting from the Gaussian intensity profile of a laser beam. It is shown that a relative horizontal displacement of a co- and counterpropagating laser beam leads to a tapered cooling force for a coasting ion beam.

For ions with a closed optical transition of frequency ω_0, it is well known [13], that a single near-resonant laser beam of frequency ω_L, will exert a velocity dependent mean scattering force on the ions described by

$$F(v) = \frac{1}{2}\hbar k \Gamma \left[\frac{S}{\left((\delta - vk) / \frac{1}{2}\Gamma \right)^2 + (1+S)} \right], \tag{2}$$

where $\delta = \omega_L - \omega_0$ is the laser detuning, $k = \omega_0 / c$ is the wavenumber of a resonant photon, and Γ is the spontaneous decay rate for the upper level. The saturation parameter $S = I/I_0 = 2\Omega^2/\Gamma^2$, is the ratio between the laser beam intensity I, and the saturation intensity I_0, with Ω being the Rabi frequency [1]. The mean scattering force becomes velocity dependent through the Doppler shift of the light seen by the moving ion, and it is directed along the laser beam due to the momentum kicks of absorbed photons. The spontaneous emissions occur in a center-symmetric pattern and do not contribute on average.

We now consider a coasting ion beam in a storage ring interacting with both a co- and a counterpropagating laser beam denoted 1 and 2, respectively, in a straight section. In the following, reference is made to a frame co-moving with the ion beam at some ideal design velocity v_0, with x denoting the horizontal distance from the design orbit of the ion beam. Each laser is taken to have a Gaussian intensity distribution [2] leading to a position dependent saturation parameter

$$S_i(x) = S_{0i} \exp\left(- \frac{(x - \Delta x_i)^2}{\frac{1}{2} w^2} \right), \quad i = 1,2, \tag{3}$$

where w is the spot radius of the laser beam, and Δx_i is the horizontal laser beam displacement from the design orbit. If $S \ll 1$, multi-photon processes like absorption from one laser and subsequent stimulated emission from the other laser can be neglected,

[1] We note that the Rabi frequency is here defined to be $\Omega = dE/\hbar$, where d is the effective dipole-moment matrix element and E is the amplitude of the electric field. This might differ by a factor of 2 from other authors.

[2] This is, indeed, a very good approximation for most real lasers.

and the total force F_{cool} becomes just the sum of the forces from each laser beam. With the detuning of the two lasers chosen to be the same in the co-moving reference frame one gets

$$F_{\mathrm{cool}}(x,v) = \frac{1}{2}\hbar k\Gamma\left[\frac{S_1(x)}{\left((\delta - vk)/(\frac{1}{2}\Gamma)\right)^2+1} - \frac{S_2(x)}{\left((\delta + vk)/(\frac{1}{2}\Gamma)\right)^2+1}\right]. \quad (4)$$

Taking the lasers to be equally intense $S_{01}=S_{02}$, and displacing them by the same amount in opposite directions $\Delta x_{1,2} = \pm\Delta x$, gives the condition

$$\left[\frac{e^{4\Delta X X}}{4(\Delta - V)^2+1} - \frac{e^{-4\Delta X X}}{4(\Delta + V)^2+1}\right] = 0, \quad (5)$$

for laser force equilibrium $F_{\mathrm{cool}}=0$, where we have introduced the reduced variables $\Delta = \delta/\Gamma$, $V = v/(\Gamma/k)$, $\Delta X = \Delta x/w$, and $X = x/w$. This equation leads to an equilibrium velocity whenever

$$|X| < X_{\max} = \frac{\ln(16\Delta^2+1)}{8\Delta X}. \quad (6)$$

In the case when $|X|,|\Delta X|,|V/\Delta| \ll 1$, Eq. (5) gives rise to the linear relation for the equilibrium velocity V_{eq} as function of position

$$V_{\mathrm{eq}}(X) = -\frac{4\Delta^2+1}{2\Delta}\Delta X X \equiv d(\Delta,\Delta X)X, \quad (7)$$

so that any given linear variation $d(\Delta,\Delta X)$ can be obtained by choosing Δ and ΔX right. According to Eq. (1), this is exactly the kind of relation required for tapered cooling. We note, that at a vertical position y, different from the plane of the design orbit $y=0$ the derived tapering still applies [3].

Although in principle any tapering can be created in this way, there is of course some limitations to the feasibility of this laser cooling scheme. Expanding the cooling force from Eq. (4) to first order in V around V_{eq} for a given X, one have

$$F_{\mathrm{cool}}(X,V) = F_0(X,V_{\mathrm{eq}}) - \beta(X)(V - V_{\mathrm{eq}}), \quad (8)$$

with $F_0(X,V_{\mathrm{eq}}(X)) = 0$, and $\beta(X) = -\partial F_{\mathrm{cool}}/\partial V|_{V=V_{\mathrm{eq}}}$ being a viscous friction coefficient, which leads to cooling around $V_{\mathrm{eq}}(X)$ for given particle position X. At $X=0$, we get

$$\beta(0) = -8\hbar k\Gamma S_0 e^{-2\Delta X^2}\frac{\Delta}{(4\Delta^2+1)^2}, \quad (9)$$

which will be maximum for a detuning $\Delta = -1/(2\sqrt{3})$, and vanish for $\Delta \ll -1$. Although it is possible to create a large tapering coefficient by making Δ large and negative, as can be seen from Eq. (7), the friction force decreases rapidly with the detuning, which may make it impossible to keep the particle at its equilibrium velocity, when heating mechanisms are present. The tapering also increases with ΔX, but in order to maintain reasonable overlap between the two counterpropagating laser beams, the displacement of each laser beam must be kept less than half the spot radius, i.e., $\Delta X = \Delta x/w \leq 1/2$. For a fixed value of ΔX, we see that the relation Eq. (7) scales with w through $X = x/w$ so that by making the spot size small, the variation in the equilibrium velocities of the cooling force Eq. (7) can become arbitrarily large. Decreasing the spot size, however, also puts an upper limit to the transverse range of the cooling force as seen from Eq. (6), so this implementation of graded cooling may be limited to small ion beam sizes when a certain cooling rate is needed. Along the longitudinal dimension of the ion beam the tapering will change due to the divergence of the Gaussian beams. If both the Gaussian beams are focused to a waist w_0 at some point, their spot sizes will be larger before and after the focus. The distance to where a spot radius has increased by a factor $\sqrt{2}$ is given by the Rayleigh length $z_{\mathrm{R}} = \pi w_0^2/\lambda$, where λ is the laser wavelength. This means, that the length of the cooling region will need to be shorter or comparable to the Rayleigh length, unless large variations in the equilibrium velocities of the cooling force can be accepted.

As an example, we show in Fig. 1 the appearance of the unapproximated laser cooling force for

[3] Considering a vertical position $y \neq 0$ we have $S_i(x,y) = S_{0i}\exp\left\{-\left[(x-\Delta x_i)^2 + y^2\right]/(\frac{1}{2}w^2)\right\}$ and the effect is only to scale the saturation parameter down by a factor $\exp[-y^2/(\frac{1}{2}w^2)]$ as compared to the plane $y=0$.

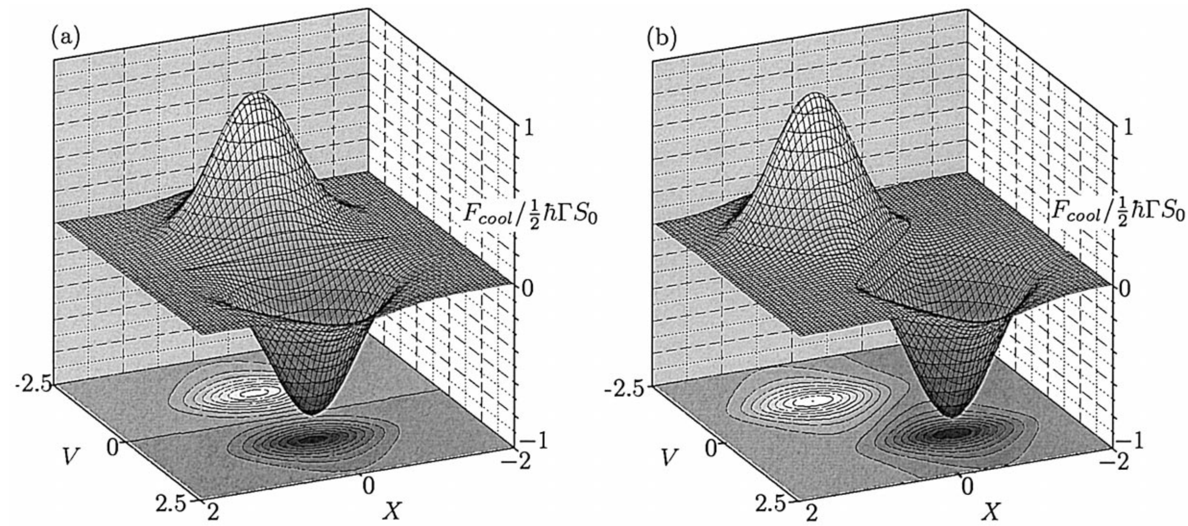

Fig. 1. Laser cooling force for (a) nondisplaced lasers, and (b) co- and counterpropagating lasers both displaced $\Delta X = 1/2$. The lasers have a peak saturation parameter of $S_0 = 1/10$ and are detuned $\Delta = -1$.

$\Delta = -1$,and $S_0 = 1/10$, for both zero displacement $\Delta X = 0$, and for a displacement $\Delta X = 1/2$. The effect of tapering is evident for the displaced beams and as expected the relation is linear in X for a finite region around $X = 0$ as can be seen more clearly in Fig. 2. From Eq. (6) one finds $X_{\max} = 0.71$ as a limit of the cooling region for the displaced lasers. Within this region, $|X| < X_{\max}$, the friction coefficient β is only slightly reduced compared to the case of nondisplaced lasers. The calculated friction coefficients is shown in Fig. 3.

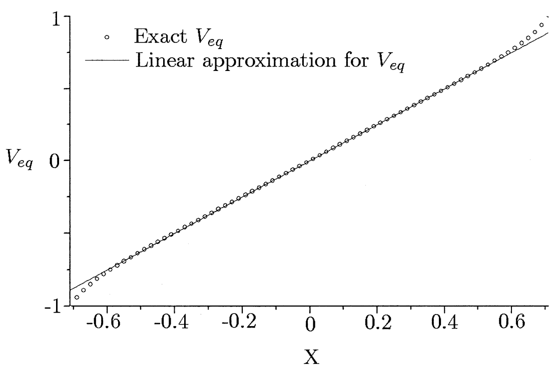

Fig. 2. (∘) The equilibrium velocity V_{eq} for the cooling force as function of the horizontal distance X to the design orbit, when the co- and counterpropagating both have displaced $\Delta X = 1/2$. The line shows the linear approximation for V_{eq} as given by Eq. (7).

In Ref. [11], a typical optimum value of C_{xs} is given to be 0.26 (although in the bunched beam case) for $^{24}Mg^{+}$-ions stored at 1 MeV in TARN II. We estimate the laser beam parameters that would give such a tapering coefficient for our scheme, using these TARN II parameters and considering $^{24}Mg^{+}$-ions. Assuming the cooling region to be 4 m long, we use Gaussian beams with Rayleigh length $z_R = 2$ m, corresponding to a waist $w_0 = 422$ μm. Displacing the lasers such that $\Delta X = 1/2$ at the

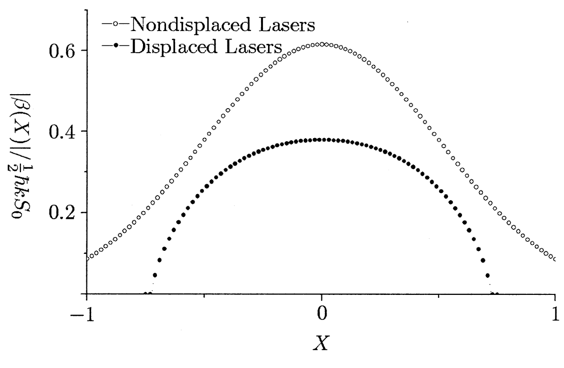

Fig. 3. Magnitude of the friction coefficent β for (○) nondisplaced lasers, and (●) co- and counterpropagating both displaced $\Delta X = 1/2$. The lasers have a peak saturation parameter of $S_0 = 1/10$ and are detuned $\Delta = -1$. At $X = 0$ the friction for displaced lasers is $e^{-1/2} \approx 0.6$ of what is achieved in the nondisplaced case.

waist location, we find that the laser detunings need to be $\Delta = -6.85$ to get the desired tapering coefficient here. Furthermore, the laser power of each laser beam has only to be some a few mW to achieve $S \approx 1/10$. The tapering coefficient C_{xs}, will fall off to 0.13 at the ends of the cooling region. This does not seem to pose a problem since C_{xs} (at least if the number of particles is not to high), according to [11], is allowed to vary within a range around the optimum value.

In the above presented two-level ion cooling scheme, the maximum cooling force and hence also the maximum cooling rate is proportional to the saturation parameter S, which had to be much smaller than unity to avoid unwanted multi-photon processes. By using the natural multilevel structure of real ions, one can, however, avoid such limiting processes if the co- and counterpropagating laser beams interact with different internal levels of the ions. In such way values of S of the order of unity or larger can be applied for obtaining larger cooling rates. This also leads to a possibility of making the detuning large in order to create a large tapering coefficient, instead of making the laser spot size small. In the case of $^{24}Mg^{+}$-ions, which has previously been laser cooled in the ASTRID storage ring [2], one can achieve the above non-coupling situation by, e.g., using the $3s^2S_{1/2} \leftrightarrow 3p^2P_{3/2}$ transition and apply counterpropagating circular polarized laser beams with opposite helicity. Another realistic way to avoid multi-photon processes for $^{24}Mg^{+}$-ions would be to have the one laser beam resonant with the $3s^2S_{1/2} \leftrightarrow 3p^2P_{3/2}$ transition while the other would be tuned to the $3s^2S_{1/2} \leftrightarrow 3p^2P_{1/2}$ transition. The latter scheme makes it possible to use only one frequency stable laser if the ion beam energy is around 20 keV. For this particular energy, the Doppler shifted two counterpropagating laser beams can actually be in resonance with one of the two fine-structure transitions.

In conclusion, we have proposed a tapered cooling scheme for coasting ion beams. By displacing a co- and a counterpropagating laser beam we obtain, in a simple way, cooling to different equilibrium velocities across the ion beam. The equilibrium velocities are shown to vary linearly with position to a good approximation in accordance with the concept of tapering cooling. As the tapered cooling force is limited to a finite transverse range, this scheme may turn out only to be applicable to small ion beam sizes, but this may at least be of interest in the quest of forming a 1D crystalline structure. However, by benefiting from the multilevel structure of ions, it may be possible to extend the transverse range for effective tapered cooling.

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