Physica B 460 (2015) 105–113

Contents lists available at ScienceDirect

Physica B

journal homepage: www.elsevier.com/locate/physb

Ion Coulomb crystals

Michael Drewsen

Aarhus University, Department of Physics and Astronomy, Ny Munkegade 120, DK8000 Aarhus C, Denmark

ARTICLE INFO

Available online 24 November 2014

Keywords:
Coulomb crystals
Ion trapping
Ion cooling

ABSTRACT

The following text will give a brief introduction to the physics of the spatially ordered structures, so-called Coulomb crystals, that appear when confined ions are cooled to sufficiently low temperatures. It will as well briefly comment on the very diverse scientific applications of such crystals, which have emerged the past two decades. While this document lacks figures, it includes a substantial number of references in which more detailed information can be found. It is the hope that the text will stimulate the reader to dig deeper into one or more of the discussed subjects and inspire her/him to think about new potential applications.

1. Introduction

Coulomb crystals constitute a special class of spatial ordered structures of matter. While usual solid matter ranging from simple molecules over clusters to solids with long range ordered structures form through the quantum delocalized nature of electrons and hence are governed by the laws of quantum mechanics, Coulomb crystals are exclusively based on classical electromagnetic interactions between confined charged particles of the same sign of charge, and hence relying only on the classical laws of physics.

The first elaborated considerations of Coulomb crystals was made by Wigner in the 1930s [1], where he was interested in understanding the change in the electronic structure and properties of matter when the density of electrons become so low that despite the rather large DeBroglie wavelength of electrons, their individual wavefunctions does not overlap. Under such conditions Wigner suggested the formation of regularly ordered structures with each electron taking up a particular position in space.

Wigner never managed to realize such crystals of electrons, but around 1990, Coulomb crystallization of two-dimensional electron gases was demonstrated within GaAs/GaAlAs quantum well structures [2]. Prior to these experiments, Wigner crystals had been produced on the surface of super fluid helium [3,4]. Three-dimensional Coulomb crystals of electrons have though still not been realized. When considering Coulomb crystals made up of electrons, such structures are also often referred to as Wigner-crystals.

Coulomb crystals can as well be composed of positively charged particles such as atomic ions at various charge stages. For decades people within the astrophysical community have been considerably interested in such objects. This is due to the fact that when a star dies, it will eventually turn into one of two very exotic stellar objects named White Dwarfs and Neuron Stars, which to various extents are assumed to incorporate Coulomb crystallized matters [5].

In case the mass of the original star is below a certain critical value after the nuclear burning processes have terminated, the remaining parts of the star will contract under the influence of gravity until the point where all the electrons form a degenerated gas which will be able to produce an equalizing quantum pressure. Such a remnant star with a radius of typically a few hundreds of kilometers will mainly consist of fully stripped oxygen and carbon atoms embedded in the degenerated electron gas. When eventually the so-called White Dwarf cools down by thermal radiation to a temperature, which becomes lower than the very high Fermi-energy of such a dense electron gas (typically equivalent to millions of kelvins), quantum mechanics prohibits collisional energy exchanges between the ions and electrons. The ions can though still exchange energy through their Coulomb interactions within the confining potential created by the degenerated electron gas, and eventually at sufficiently low temperatures, Coulomb crystals of the fully stripped ions will be formed.

In the case of initially more massive stars, the quantum pressure of the degenerated electron gas cannot counteract the gravitational contracting forces, and eventually most of the material of the star will be turned into a degenerated gas of neutrons which then provide a quantum pressure that can counteract a gravitational collapse. This situation leads to a so-called neutron star of the size of only few tens of kilometers in diameters. While nuclear burning in White Dwarfs typically stops with the formation of mainly oxygen and carbon, in Neutron Stars fusion processes continues until the formation of iron after which no further energy can be released through fusion. While the main part of neutron stars are made up of neutrons, it is believed that their crusts will

E-mail address: drewsen@phys.au.dk

http://dx.doi.org/10.1016/j.physb.2014.11.050
0921-4526/

still have fully stripped iron ions Fe^{26+} present, which even under the extreme high density conditions still is believed to form ion Coulomb crystals!

Due to the required conditions for formation of Coulomb crystals of ions, as will be revisited in the next chapter, it was first with the development of laser cooling techniques in the late 1970s and early 1980s that it became possible to form ion Coulomb crystals in laboratories. However, since these first demonstrations by the groups of Dr. Wineland, NIST, Boulder, USA [6], and Walther, Max-Planck Institute for Quantum Optics, Garching, Germany [7], the number of groups and the breath of scientific investigations based on ion Coulomb crystals created in laboratory have went through an amazing development.

The rest of this contribution is organized as follows: In Section 2 a discussion of the condition for Coulomb crystallization will be provided, and in Section 3 how these requirements can be met in the case of positively charged ions will be presented. Structural properties of Coulomb crystals of various dimensionalities will be the focus of Section 4 with examples in the form of crystals of atomic ion species. In Section 5, some of the dynamical properties of Coulomb crystals will be briefly discussed. Many applications of ion Coulomb crystals in a large variety of scientific investigations will be the theme of Section 6. Section 7 will finally conclude the presentation.

2. Conditions for Coulomb crystallization

The condition for Coulomb crystallization is often formulated in terms of the so-called plasma coupling parameter, Γ, which is defined in the following way:

$$\Gamma \equiv \frac{Q^2}{4\pi\epsilon_0 a k_B T}. \tag{1}$$

Here Q is the charge of the particle involved, a is the Wigner–Seitz radius, corresponding to the particle density, and T is the temperature of the particles. Essentially, Γ is just a measure of the ratio of the inter-particle Coulomb energy to their kinetic energy:

$$\Gamma \sim \frac{E_{\mathrm{Coul}}}{E_{\mathrm{kin}}}. \tag{2}$$

For infinitely large three-dimensional identical charged particle systems, often also referred to as one-component plasmas (OCPs), it can be shown that many of the thermodynamical properties scale with the value of Γ [8].

With respect to Coulomb crystallization, it has been proven by several groups (see, e.g., [9,10]) that the criterion is given by

$$\Gamma \geq 175 \tag{3}$$

or, as can be shown by introducing the local single ion oscillation frequency, or similarly the plasma frequency ω_{plas}:

$$\omega_{\mathrm{plas}} = \sqrt{\frac{nQ^2}{\epsilon_0 M}} \tag{4}$$

by the condition:

$$\Delta r_{\mathrm{exc}}/a < 0.1 \tag{5}$$

where Δr_{exc} is the root-mean-square single particle excursion around its equilibrium position due to its kinetic energy. This inequality is very similar to the Lindemann criterion [11] for usual solids with respect to the motion of the atomic nuclei.

For valence electrons in usual solids, even at the absolute zero-temperature, the quantum mechanical excursion of the individual electrons makes it impossible to fulfill inequality (5), and delocalizing electrons leading to specific energy band structures emerge as a consequence. More generally, one can derive a rather simple approximate condition for this 'quantum melting' to happen at absolute zero temperature (unpublished):

$$a \geq 10^4 \times \frac{h^2\epsilon_0}{Q^2 M} \tag{6}$$

where M is the mass of the charged particles.

For exotic stellar objects like White Dwarfs with interior ions being mainly C^{6+} and O^{8+}, and neutron star crusts with Fe^{26+} ions, the critical Wigner–Seitz radii for quantum melting become $a \sim 10^{-12}$ m and $a \sim 10^{-14}$ m, respectively. Surprisingly, despite the high density of such objects, quantum melting is not a problem, and classical Coulomb crystallization happens already at temperature in the million-kelvin range [5]!

In contrast to short-range interacting particles leading to closed packed crystal lattices, the ground state of a infinite Coulomb crystal is a body-centered cubic (*bcc*) structure [9]. Hexagonal closed packed (*hcp*) and face centered cubic (*fcc*) structures correspond, however, only to slightly excited states [12].

In the case of effective harmonic confinement potentials, it is straight forward through the application of Poisson's equation to prove that finite size one-component systems will also have constant particle density at low temperatures.

For such finite systems, one finds that the condition for reaching Coulomb crystallization, defined as the temperature at which there is a maximum in the specific heat, is first reached at larger values of Γ, or equivalently at lower temperatures. For instance, for isotropic confined systems ranging from ~100 to ~10000 particles, the corresponding critical values of Γ varies from ~500 to ~200 [13]. So far, criterions for crystallization under various anisotropic confinement conditions have not been studied in detail, but a variation in the values of Γ can be expected even for the same number of ions due to changes in the surface effects.

3. Ion Coulomb crystals in laboratories

By standard ion trapping techniques, either in the form of Penning traps or radiofrequency (rf) traps also named Paul traps [14], the highest practical reachable particle density of singly charged ions is $\sim 10^{15}$ m^{-3}, corresponding to a~10 μm. With this value of a, inequality (3) and the definition of Γ in Eq. (1) lead to the following requirement in terms of temperature to obtain Coulomb crystallization:

$$T \leq T_{\mathrm{CC}} \sim 10\ \mathrm{mK} \tag{7}$$

Consequently, before the advent of laser cooling techniques [15,16] it was impossible to reach the condition for ion Coulomb crystallization, since until then all other cooling techniques were limited by the temperature of liquid helium, and hence typically to temperatures in the range of a few kelvins. On the other hand, for standard laser Doppler cooling the minimum temperature is effectively given by [17]

$$T_{\mathrm{Dopp}} \sim \frac{\hbar\Gamma_{\mathrm{cool}}}{k_B} \sim 0.1 - 1\ \mathrm{mK}, \tag{8}$$

where Γ_{cool} is the spectral width of the cooling transition. This temperature is clearly below T_{CC}, and even low enough to create crystals of the smallest finite systems.

The production of ion Coulomb crystals through laser cooling of ions were pioneered by the groups of David Wineland (NIST, Boulder, Colorado, USA) and Prof. Walther (MPQ, Garching, Germany) in the 1980s, and the past decades an ever increasing

number of groups world wide have taken part in the further development, exploration and application of these exotic crystalline structures.

An interesting issue concerning Coulomb crystallization in both Penning and rf traps is the fact that in both cases the crystallized ions in addition to their random thermal motion perform synchronized driven motion, either in the form of rotation (Penning traps) or quadrupolar oscillations (rf traps). Interestingly, the corresponding averaged kinetic energy can easily exceeds the thermal energy by 10^4–10^6 orders of magnitude!

4. Structural properties. (1D, 2D and 3D structures)

While the lowest energy state of infinite constant density Coulomb crystals is a *bcc* structure [18,9], the structures of finite size crystals depend strongly on the number of ions as well as the shape of the trapping potential. Even in the case of trapping conditions leading to constant density (i.e., three-dimensional harmonic trapping potentials), three-dimensional crystals can have many different features.

For the isotropic confinement case, where the confining force can be written in terms of a single force constant κ:

$$F_{\mathrm{trap}}(\boldsymbol{r}) = -\kappa \boldsymbol{r}, \tag{9}$$

the over-all structure of the Coulomb crystals must be spherical due to spherical symmetry. However, what is not obvious, is the fact that for crystals (or more correctly clusters) containing up to a few thousand ions, the total potential energy is minimized by having the ions organized in concentric shells with a specific "magic" numbers of ions [19] as the temperature goes to zero. Within each shell the ions furthermore form closed two-dimensional structures, which to a high degree is like a hexagonal lattice, the minimum energy state for an infinite two-dimensional planar crystal [20].

Already with more that a few ions present, it is essentially impossible to predict the ground state configuration, and Monte Carlo or Molecular Dynamics (MD) simulations have to be introduced to find those [19]. However, since it is well known from cluster physics that the number of metastable configurations increases roughly exponentially with the number of particles forming the cluster, it soon becomes extremely difficult to ensure that a particular simulation have reached the minimum energetic state.

The uncertainty in the predicted magic numbers of ions from MD simulations within each shell is though so small that for a given spherical shell structure, the number of ions can be determined with an uncertainty much smaller than the square-root of the total number of ions [19]. Hence, comparing MD simulations with projection images of crystals recorded in the laboratories constitutes a reliable method to obtain information on the number of ions as well as the ion density in experiments.

Since the ions will always have a finite thermal energy in the laboratories, it is indeed possible also to observe the formation of energetically excited metastable configurations which may even have extended long range ordered structures with *bcc* or *fcc* character at ion numbers much smaller than ~10000, expected from theoretical investigations to be the minimum number in order to have such structures as the ground states [21]. Sometimes crystals composed of domains of both *fcc* and *bcc* structures can even appear [22].

In another special trapping situation, where the ions are confined by an isotropic harmonic potential in two dimensions while being freely to move along the third perpendicular direction, concentric cylindrically shell structures represent the minimum energy states. In this situation, the number of shells depends rather on the linear density of particles along the unconfined axis than on the total number of ions. In the laboratories this specific situation can effectively be realized by choosing a ring shaped two-dimensional rf quadrupole trap as has been pioneered by Prof. Walther, MPQ [23], or by letting the ion species of interest (the low mass species) be part of a two-species crystal in a standard linear rf quadrupole trap [24]. In the first case the ring shaped confinement will lead to an effective periodic boundary condition for the ions along the ring, which will not deviate substantially from the infinitely long 2D confinement situation, when the circumference is much larger than the typical ion–ion distance and diameter of the crystal. In the two-species case, the heavier ion species will feel an effective confining force which is weaker than for the lighter mass one in the plane with rf field confinement, and hence the lighter mass ions will segregate within a cylindrical volume along the rf field free axis [24]. Empirically, it has been shown that the shell structure of this inner component of the two-species crystal to a very high degree mimics the infinite case due to the effective electrical shielding by the surrounding heavier species [24]. In an extreme case a string of 43 equidistantly spaced $^{24}Mg^+$ ions was situated within a crystal of ~3000 $^{40}Ca^+$ ions [24].

In the more general case of harmonic confinement with an isotropic potential in two dimensions, but a different force constant in the third one ($\kappa_z \neq \kappa_x = \kappa_y$) the equipotential surfaces are described by concentric spheroids. While a weakly interacting gas of ions in thermal equilibrium would have a spatial distribution with contour surfaces of the density corresponding to the equipotential surfaces of the confining potential, this is not true for Coulomb crystallized ion ensembles [24–26]. In cases where the number of ions in a Coulomb crystal is so large that the extension in all three dimensions exceeds significantly the spacing of the individual ions, it can be expected that the shape of the crystal should be very similar to the case of a confined cold charged fluid with a constant charge density. For this latter situation it is indeed possible to find an analytic expression for the outer spheroidal shape of such a fluid in a potential with spheroidal equipotential surfaces [27]. However, due to the Coulomb repulsion, the shapes deviate from the equipotential surfaces of the trapping potential. In a series of experiments, it has been verified that the outer contour of Coulomb crystals indeed follow the same shape as the cold charged liquid model, except at extreme confinement cases leading to the formation of real 2D (pancake) or 1D (string) structures [24,25]. For the number of ions reaching up to a few thousand, the ions will as in the isotropic confining case furthermore be organized in concentric spheroidal shells [28,25].

The 2D planar structures [20,29,30] and 1D string structures [20,31,32] constitute special cases, where the liquid model does not suffice to describe the shape. Here, different approaches including MD simulations have to be used to obtain more detailed structural information.

Both in theory and by experiments it has been shown that under specific trapping conditions, Coulomb crystals can have negative Poisson coefficients, meaning the crystal when stretched along one direction, it will lead to a lateral expansion [33].

Another interesting situation is the one where two ion species with the same charge-to-mass ratio is simultaneously crystallized in, e.g., a linear rf trap. In this situation both ion species feel the same effective potential and *a priori* could be expected to mix. On a gross scale this is also true according to MD simulations, and for large systems one indeed finds a simple cubic lattice (as in CsCl salt) as the minimum energy state [34]. For smaller finite systems of a few thousand ions, common shell structures like for a single species is formed, however, with the common shells consisting of slightly displaced sub-shells of the two species [34]. It was also

found by MD simulations that the degree of mixing is very sensitive to the exact values of the charge-to-mass ratios of the two ion species. A difference of only one per mille is essentially enough to observe at least partial segregation of the two species [34]. As we will discuss briefly in Section 6, such segregation effects may be studied in details by experimenting with Highly Charged Ion (HCI), and may turn out to have relevance for the composition of Coulomb crystals within White Dwarfs, where $^{16}O^{8+}$ and $^{12}C^{6+}$ ions, which have nearly the same charge-to-mass ratio, are present.

Coulomb crystals with non-constant particle densities can as well be created and studied through the application of higher order pole rf traps, like hexapole and octopole traps. Recently, Coulomb crystallization of ions in particular octupole traps have been considered both theoretically [35–37] and experimentally [38,39]. In such higher order traps the ground state configuration of the ions will never be a single string, but rather ring/tube-like structures [36] or multi-string-like structures [37] depending on the trapping conditions. For few ions where near hexagonal lattice-like pancake structures appears for 2D systems under rf quadrupole confinement, in higher order traps, rings of ions are instead formed. These new types of structure may, for instance, find applications within the field of quantum simulations (see Section 6).

5. Dynamical properties

Like usual solids, Coulomb crystals have a series of vibrational modes (phonon modes). In general, if a Coulomb crystal consists of N ions then such a crystal will have $3N$ normal modes with corresponding distinct eigenmode frequencies [40].

For smaller crystals, the individual modes can be found either through direct computations [32] or through the analysis of the motions of ions in MD simulations after the introduction of a motional excitation [40].

For larger spheroidal crystals which may in real experiments actually be found to be a series of isomeric configuration with each their particular mode pattern and spectrum, a simpler way to gain information on mode structures and frequencies is once again to approximate the real granular Coulomb crystals by a cold charged liquids with the same shape and size, and find such object's eigenfrequencies and eigenmodes. As has been shown by Prof. Dan Dubin, UCSD, USA, the modes of such cylindrical symmetric objects can be characterized by two quantum numbers (l,m), where m describes the azimuthal part of the eigenmode, and l the longitudinal part with l-1 indicating the number of nodal planes along the symmetry axis [41].

As long as the characteristic length scale for the (l,m)-modes exceeds significantly the ion–ion spacing within the Coulomb crystals, these modes may provide a good description of the real crystal modes. Experiments conducted both in Penning [29] and linear rf traps [42] have in fact proved that the modes like the (l, m)-modes can be excited in ion Coulomb crystals.

For highly anisotropic trapping conditions leading to either 1D string structures or 2D pancake-like structures a sub-set of 1D [32] and 2D [43] modes can be found more easily. As we shall discuss in the next chapter, various modes of such dimensionally restricted crystals have found applications within quantum information processing as well as quantum simulation.

6. Applications

As should be clear from this section, the applications of Coulomb crystals in carrying out research beyond the studies of the basic properties of such crystals are extremely broad.

6.1. Non-linear dynamics

The fact that the interaction between confined ions by the nature of the Coulomb interaction is non-linear, made already around 1990 the smallest non-trivial ion systems consisting of two ions an interesting platform for the study of chaotic behavior in rf traps [44–47]. The appearance of bifurcation in the motional state of the ions as well as the existence of instability of two-ion crystals within the usual parameter range for single ion stable motion was studied [48]. Later, experiments with larger (~1000 ions) systems furthermore showed that when the plasma frequency of such crystals reached half the rf drive frequency, parametric excitation would lead to crystal melting [49]. In some cases the parametric resonance could be switched off by induced crystal rotation [49], which led to lower particle densities and consequently lower plasma frequencies.

6.2. Statistical physics and thermodynamics

Coulomb crystals in the form of ion-strings and so-called zig-zag configurations have recently also found applications within statistical physics [50]. For instance, several groups have recently investigated the formation of defects [51,52] (often referred to as kinks) in zig-zag configurations when formed through fast changes in the trapping parameters leading to a change in the equilibrium configurations from string to zig-zag structures. Through these experiments, universal scaling laws for the number of defects have been shown to follow predictions of the Kibble–Zurek model in the special inhomogeneous case [53,54].

From a thermodynamics perspective, ion Coulomb crystals constitute as well an interesting “playground”. Recently, e.g., the thermal diffusion of a single impurity ion in a chain of ions has been investigated experimentally [55], and models to study heat transport in such systems have been proposed [56]. Also recently, it has been proposed to use strings of trapped ions in combination with periodic corrugated potentials in the form of induced dipole forces to study fundamental issues related to friction [57] as well as quantum phase transitions [58] in connection with a Coulomb-version of the Frenkel–Kontorova model [59,57].

6.3. Solid-state physics

As mentioned already, Coulomb crystals constitute a special class of solids, which make them natural objects for studies of various structural formations and dynamics. In the future, the combination of trapping of ions in harmonic traps with periodic corrugated potentials in the form of standing wave dipole potentials, should make it possible not only to pin specific metastable structures by addressing selected lattice planes, but as well make it feasible to make controlled changes in the lattice structure, e.g., from a *bcc* to a *fcc* structure and vice versa, through only changes in the over-all trap potential [60]. While such experiments have not yet been carried out, first experiments of optical trapping and pinning of few ions within the nodal/antinodal planes of an optical standing wave field has recently been demonstrated [61–63].

6.4. Plasma physics

By nature Coulomb crystals are the solid-state representation of a one-component plasma (OCP) [8,64], and hence are in this respect an interesting special state of such plasma. By performing experiments in Penning as well as rf traps it becomes indeed possible to study OCPs under both magnetized and non-magnetized conditions [40,65]. By introducing different ion species in the same trap it becomes furthermore possible to study multi-species OCPs in or in the vicinity of Coulomb crystallization.

6.5. Nuclear fusion

When laser-cooled ions collide with hotter impurity ions within a magnetized plasma in a Penning trap, the correlated collision dynamics resembles to a high degree the processes governing thermal nuclear fusion in dense stellar interiors [66–68]. Hence, in the future, Penning trap experiments may actually be used to model nuclear fusion in hot dense plasmas.

6.6. Geoscience

Another not immediate connection of Coulomb crystals to other branches of science is the observed stick-slip motion of the orientation of ion Coulomb crystals in Penning traps with respect to the phase of a rotating dipolar or quadrupolar perturbation. However, it turns out that the probability distributions of the amplitude of the slips are well described by power law relations, similar to those found in experiments on avalanches and slips in granular systems, as well as in models of earthquakes [69].

6.7. Exotic stellar objects

Besides being of fundamental plasma physics interest, OCPs in the form of fully ionized atoms are expected to play an important role for the cooling mechanisms of White Dwarfs [5]. Since such stellar objects are supposed to consist mostly of $^{16}O^{8+}$ and $^{12}C^{6+}$ ions, which have nearly identical charge-to-mass ratios, it would be interesting to carry out experiments with two species with nearly the same charge-to-mass ratio. By performing experiments with Highly Charged Ions (HCIs) (e.g. $^{27}Al^{3+}$, $^{36}Ar^{4+}$, $^{45}Sc^{5+}$, $^{54}Fe^{6+}$, …, $^{180}Hf^{20+}$, $^{189}Os^{21+}$, …) cooled sympathetically through Coulomb interactions with simultaneously trapped and laser-cooled $^{9}Be^{+}$ ions, one can explore the effect of tiny differences in the charge-to-mass ratio on the segregation of the species. In such experiments, the fact that different species will crystallize at different temperatures (see Eq. (1)) may play an important role. In previous two-species MD simulations of ions in a linear rf trap, it was found that the degree of mixing is very sensitive to the exact values of the charge-to-mass ratios [34]. However, much more detailed work has to be carried out in order to provide results of high enough quality to be relevant for the investigations of the cooling of White Dwarfs.

6.8. Test of quantum mechanics

Ion Coulomb crystals in the form of few ion systems are quite unique objects for studying fundamental consequences of quantum mechanics. One of the first things to be studied was the concept of quantum jumps, i.e., the observation of the abrupt jumps in the quantum state of a single quantum system upon measurements during induced dynamics. In a series of experiments with single atomic ions, this concept was verified by driving narrow transitions between the ground and a metastable excited electronic state [70–72]. The fact that spontaneous decay is in fact also a very fundamental quantum phenomenon has beautifully been demonstrated by altering the boundary conditions for the light emission of a single ion, either by introducing a single mirror [73,74] or an optical cavity [75] to establish single photon self-interference. The interference of single photons in scattering processes involving two ions have furthermore led to the observation of super-/sub-radiant states [76] as well as Young's double slit like results [77]. Experiments focused on the investigation of the so-called quantum Zeno effect has additionally been carried out with ion Coulomb crystals [78,79].

6.9. Cavity QED related experiments

Ion Coulomb crystals are as well extremely interesting objects for the study of cavity QED effects. For a single atomic ion, laser-cooled to its quantum mechanically motional ground state of an harmonic trap potential with oscillation frequencies of 0.1–1 MHz, the resulting ground state wavefunction will have a spatial extension of only a few nanometers, and hence much smaller than the wavelength of light in the optical or near optical range (NIR–UV range). Consequently, it is possible to position a single ion very precisely with respect to the nodes and antinodes of a standing wave cavity field [80,75,81], and hence control the effective coupling between the ion and the cavity field [75]. In the recent past, there has been a series experiments demonstrating and exploiting the strong spatial localization even with ions cooled only to the Doppler limit [80,75,61–63,82,83]

Larger Coulomb crystals are as well interesting objects when the goal is the investigation and exploitation of the collective effect in the coupling of atoms to cavity fields. Recently, collective strong coupling, where the rate of exchange of a collective electronic excitation of ions in a Coulomb crystal with a photon in the cavity field exceeds both the excited state decay rate as well as the decay rate of photons out of the cavity, was demonstrated [84]. This is an interesting regime, since it enables the study of the fundamental quantum dynamics of the combined photon-ensemble system without decoherence, as well as open for, e.g., the realization of a quantum memory for light [85] and novel photon counters [86]. With the latter prospects in mind, recently, electromagnetically induced transparency (EIT) and an all-optical switch was demonstrated [87].

6.10. Quantum information processing

Due to the existence of common normal modes of vibration for the particles constituting a Coulomb crystal, quantum correlated states can be created between the internal states of ions in such crystals through the applications of spatially localized laser beams addressing narrow line transitions in the individual ions [88]. By detuning an excitation laser frequency up (blue) or down (red) by a common mode frequency with respect to the transition frequency, deterministic entanglement can be created between the internal states of an ion and the common motional state [89].

By carefully addressing individual ions with appropriate laser pulses, it was shown by Cirac and Zoller in 1995 [90] that universal quantum gate operations can be performed between any two qubits represented by the internal states $|g\rangle \equiv |0\rangle$ and $|e\rangle \equiv |1\rangle$ of the ions. Hence, an ion Coulomb crystal in the form of a string of N ions, can be used as a quantum register of N qubits in a general purpose quantum computer. Since this first proposal for quantum computing with cold ions several other schemes have been proposed, often including the so-called Mølmer–Sørensen gates [91], and an impressive series of important results have been obtained experimentally regarding multi-particle entanglement [92,93] and quantum gate operations [94–96].

6.11. Quantum simulations

The possibility of performing universal gates between any pairs of ion qubit states in a N qubit system as briefly discussed above makes it in principle also possible to emulate the quantum dynamics of any Hamilton operator acting on a quantum system in a Hilbert space of dimension 2^N [97]. While such emulations can in principle be carried out on classical computers, the dimension of the Hilbert space increases exponentially with the number of qubits involved, and eventually becomes intractable. On the other hand using quantum gates to represent the dynamics, this

problem can be solved [97], though the concatenation of many gate operations can become rather inefficient. In some cases, like e.g. the study of magnetism and more generally spin–spin interactions at the quantum level, much more effective simulations can be carried out through emulating the two states of a magnetic dipole or spin by the two internal states of the ions, and apply appropriate light fields, which illuminate all the ions, to mimic magnetic fields [98–100,43]. By such quantum simulators one can simplify the number of operations needed considerable compared to "digital" quantum computing [101–103]. Whereas digital quantum processing procedure always will need some kind of individual addressing of all the ions, "analogue" quantum simulators have a larger potential to eventually be extended to two- and three-dimensional Coulomb crystals.

6.12. High precision spectroscopy

The fact that ions within Coulomb crystals are both spatially very localized and motionally cold, makes such crystals ideal for high-resolution spectroscopy. In particular, at such low temperatures where the quantized excitation of the normal modes is negligible, extremely narrow electronic transitions in the ions can be probed with ultra-high precision [104]. Today, one of the most precise measurements of an optical transition in an atom has been performed in a $^{27}Al^{+}$ ion being sympathetically cooled [105] by a single laser-cooled $^{24}Mg^{+}$ ion to form a two-ion Coulomb crystal [106–108]. Since a large range of ion species, including ions of exotic isotopes, super-heavy element ions [109], highly charged ions [110,111] as well as molecular ions [112–114], can be effectively sympathetically crystallized through Coulomb interactions with laser-cooled ions [115,112,116], such refined spectroscopy techniques may in the near future lead to a vast amount of new and improved spectroscopic data of importance for a variety of fundamental physics investigations ranging from nuclear to astrophysics.

6.13. Cold molecular ion research

Sympathetic cooling of molecular ions through the Coulomb interaction with simultaneously trapped laser-cooled atomic ions has the past years opened for a broad range of molecular investigations of both physical and chemical interest [112,117–129]. By bringing the molecular ions into a crystalline state, it is indeed possible to exploit the specific properties of two-species Coulomb crystals to learn about the number of molecular ions present without being able to observe them directly, since the fluorescence from the atomic ions will reveal the total size of the crystal [112,117,130]. By monitoring the growth in the number of dark ions in a reaction experiment, one can, e.g., infer reaction rates [112,117], and by recording the reduction in the size of the crystal through photodissociation of the molecular ions, one can, e.g., determine the internal state distribution of the molecular ions [120,121]. This latter method has been implemented to directly prove that though the translational motion of the molecular ions can be brought into the millikelvin range, the internal rovibrational temperature of the molecular ions is hardly changed due to the very distant (~10 μm) Coulomb interactions [131]. Indeed the cooling of the internal degrees of freedom of the molecules is so weak that they typically get in equilibrium with the radiation field in the trapping region, which for a room temperature trap means ~300 K [120,121]. The past years, several methods have though been devised to produce cold rovibrational molecular ions by the direct production of homonuclear molecular ions in specific internal states through Resonance Enhanced Multi-Photon Ionization [122], laser-induced rotational cooling [120,121] and buffer gas cooling either by laser-cooled atoms [128] or a cryogenic helium gas [129]. While the lowest internal temperatures have so far been obtained with a helium buffer gas (~8 K) [129], buffer gas cooling by an ensemble of laser-cooled atoms has the prospect to bring the internal temperature into the microkelvin range. Combined with sympathetic sideband cooling to translational temperatures in the microkelvin regime, this will enable investigations of ion chemistry in a whole new ultracold regime.

7. Conclusion

By this brief introduction to ion Coulomb crystals and their applications, I hope, I have managed to show how extremely versatile these objects are. Clearly, there is material enough for a whole book, however, the aim of the this work has primarily been to provide the reader with a text and references from which more detailed knowledge can be acquired elsewhere.

Remark

It should be noted that the present text will in a slightly different format as well be published in the Proceedings for the International School of Physics (Enrico Fermi), Course CLXXXIX, 2013.

References

[1] E. Wigner, On the interaction of electrons in metals, Phys. Rev. 46 (1934) 1002–1011. http://dx.doi.org/10.1103/PhysRev.46.1002.
[2] V.J. Goldman, M. Santos, M. Shayegan, J.E. Cunningham, Evidence for two-dimensional quantum Wigner crystal, Phys. Rev. Lett. 65 (1990) 2189–2192. http://dx.doi.org/10.1103/PhysRevLett.65.2189.
[3] C.C. Grimes, G. Adams, Evidence for a liquid-to-crystal phase transition in a classical, two-dimensional sheet of electrons, Phys. Rev. Lett. 42 (1979) 795–798. http://dx.doi.org/10.1103/PhysRevLett.42.795.
[4] H.-W. Jiang, A.J. Dahm, Conduction properties of a new two-dimensional sliding charge-density wave, Phys. Rev. Lett. 62 (1989) 1396–1399. http://dx.doi.org/10.1103/PhysRevLett.62.1396.
[5] H.M. Van Horn, Dense astrophysical plasmas, Science 252 (5004) (1991) 384–389.
[6] D.J. Wineland, J.C. Bergquist, W.M. Itano, J.J. Bollinger, C.H. Manney, Atomic-ion Coulomb clusters in an ion trap, Phys. Rev. Lett. 59 (1987) 2935–2938. http://dx.doi.org/10.1103/PhysRevLett.59.2935.
[7] F. Diedrich, E. Peik, J.M. Chen, W. Quint, H. Walther, Observation of a phase transition of stored laser-cooled ions, Phys. Rev. Lett. 59 (1987) 2931–2934. http://dx.doi.org/10.1103/PhysRevLett.59.2931.
[8] S. Ichimaru, Strongly coupled plasmas: high-density classical plasmas and degenerate electron liquids, Rev. Mod. Phys. 54 (1982) 1017–1059. http://dx.doi.org/10.1103/RevModPhys.54.1017.
[9] E.L. Pollock, J.P. Hansen, Statistical mechanics of dense ionized matter. II. Equilibrium properties and melting transition of the crystallized one-component plasma, Phys. Rev. A 8 (1973) 3110–3122. http://dx.doi.org/10.1103/PhysRevA.8.3110.
[10] W.L. Slattery, G.D. Doolen, H.E. DeWitt, Improved equation of state for the classical one-component plasma, Phys. Rev. A 21 (1980) 2087–2095. http://dx.doi.org/10.1103/PhysRevA.21.2087.
[11] F.A. Lindemann, The calculation of molecular vibration frequencies, Physik Z 11 (1910) 609–612.
[12] R.W. Hasse, A semiempirical mass formula for spherical Coulomb crystals, J. Phys. B: At. Mol. Opt. Phys. 36 (5) (2003) 1011, URL ⟨http://stacks.iop.org/0953-4075/36/i=5/a=320⟩.
[13] J.P. Schiffer, Melting of crystalline confined plasmas, Phys. Rev. Lett. 88 (2002) 205003. http://dx.doi.org/10.1103/PhysRevLett.88.205003.
[14] P. Ghosh, Ion Traps, Oxford Science Publications, Oxford University Press, Incorporated, 1995, URL ⟨http://books.google.dk/books?id=YX95QgAACAAJ⟩.
[15] H. Metcalf, P. van der Straten, Laser Cooling and Trapping, Graduate Texts in Contemporary Physics, Springer, New York, 1999, URL ⟨http://books.google.dk/books?id=i-40VaXqrj0C⟩.
[16] J. Eschner, G. Morigi, F. Schmidt-Kaler, R. Blatt, Laser cooling of trapped ions, J. Opt. Soc. Am. B 20 (5) (2003) 1003–1015. http://dx.doi.org/10.1364/JOSAB.20.001003, URL ⟨http://josab.osa.org/abstract.cfm?URI=josab-20-5-1003⟩.
[17] V.S. Letokhov, V.G. Moinogin, B.D. Pavlik, Cooling and capture of atoms and molecules by a resonant light field, Sov. Phys. JETP 45 (4) (1977) 698–705.
[18] W.M. Itano, J.J. Bollinger, J.N. Tan, B. Jelenković, X.-P. Huang, D.J. Wineland, Bragg diffraction from crystallized ion plasmas, Science 279 (5351) (1998) 686–689. http://dx.doi.org/10.1126/science.279.5351.686.

[19] R.W. Hasse, V.V. Avilov, Structure and Madelung energy of spherical Coulomb crystals, Phys. Rev. A 44 (1991) 4506–4515. http://dx.doi.org/10.1103/PhysRevA.44.4506.

[20] Dubin, Theory of structural phase transitions in a trapped Coulomb crystal, Phys. Rev. Lett. 71 (17) (1993) 2753–2756.

[21] A. Mortensen, E. Nielsen, T. Matthey, M. Drewsen, Observation of three-dimensional long-range order in small ion Coulomb crystals in an rf trap, Phys. Rev. Lett. 96 (2006) 103001. http://dx.doi.org/10.1103/PhysRevLett.96.103001.

[22] M. Drewsen, T. Matthey, A. Mortensen, J.P. Hansen, Direct imaging of thermally excited metastable structures of ion Coulomb clusters, arXiv:1202.2544.

[23] G. Birkl, S. Kassner, H. Walther, Multiple-shell structures of laser-cooled $^{24}Mg^{+}$ ions in a quadrupole storage ring, Nature 357 (6376) (1992) 310–313, URL ⟨http://dx.doi.org/10.1038/357310a0⟩.

[24] L. Hornekær, N. Kjærgaard, A.M. Thommesen, M. Drewsen, Structural properties of two-component Coulomb crystals in linear Paul traps, Phys. Rev. Lett. 86 (2001) 1994–1997. http://dx.doi.org/10.1103/PhysRevLett.86.1994.

[25] M. Drewsen, I.S. Jensen, N. Kjærgaard, J. Lindballe, A. Mortensen, K. Mølhave, D. Voigt, Non-stationary Coulomb crystals in linear Paul traps, J. Phys. B: At. Mol. Opt. Phys.: At. 36 (3) (2003) 525, URL ⟨http://stacks.iop.org/0953-4075/36/i=3/a=310⟩.

[26] S. Mavadia, J.F. Goodwin, G. Stutter, S. Bharadia, D.R. Crick, D.M. Segal, R.C. Thompson, Control of the conformations of ion Coulomb crystals in a Penning trap, Nat. Commun. 4 (2013) 2571. http://dx.doi.org/10.1038/ncomms3571.

[27] L. Turner, Collective effects on equilibria of trapped charged plasmas (1958–1988), Phys. Fluids 30 (10) (1987) 3196–3203, URL ⟨http://scitation.aip.org/content/aip/journal/pof1/30/10/10.1063/1.866495⟩.

[28] M. Drewsen, C. Brodersen, L. Hornekær, J.S. Hangst, J.P. Schifffer, Large ion crystals in a linear Paul trap, Phys. Rev. Lett. 81 (1998) 2878–2881. http://dx.doi.org/10.1103/PhysRevLett.81.2878.

[29] Mitchell, Bollinger, Dubin, Huang, Itano, Baughman, Direct observations of structural phase transitions in planar crystallized ion plasmas, Science 282 (5392) (1998) 1290–1293.

[30] M. Block, A. Drakoudis, H. Leuthner, P. Seibert, G. Werth, M. Block, A. Drakoudis, H. Leuthner, P. Seibert, Crystalline ion structures in a Paul trap, J. Phys. B: At. Mol. Opt. Phys. 33 (11) (2000) L375, URL ⟨http://stacks.iop.org/0953-4075/33/i=11/a=101⟩.

[31] J.P. Schiffer, Phase transitions in anisotropically confined ionic crystals, Phys. Rev. Lett. 70 (1993) 818–821. http://dx.doi.org/10.1103/PhysRevLett.70.818.

[32] D.F.V. James, Quantum dynamics of cold trapped ions with application to quantum computation, Appl. Phys. B 66 (2) (1998) 181–190. http://dx.doi.org/10.1007/s003400050373.

[33] R.H. Baughman, S.O. Dantas, S. Stafström, A.A. Zakhidov, T.B. Mitchell, D.H. E. Dubin, Negative Poisson's ratios for extreme states of matter, Science 288 (5473) (2000) 2018–2022. http://dx.doi.org/10.1126/science.288.5473.2018.

[34] T. Matthey, J.P. Hansen, M. Drewsen, Coulomb bicrystals of species with identical charge-to-mass ratios, Phys. Rev. Lett. 91 (2003) 165001. http://dx.doi.org/10.1103/PhysRevLett.91.165001.

[35] F. Calvo, C. Champenois, E. Yurtsever, Crystallization of ion clouds in octupole traps: structural transitions, core melting, and scaling laws, Phys. Rev. A 80 (2009) 063401. http://dx.doi.org/10.1103/PhysRevA.80.063401.

[36] C. Champenois, M. Marciante, J. Pedregosa-Gutierrez, M. Houssin, M. Knoop, M. Kajita, Ion ring in a linear multipole trap for optical frequency metrology, Phys. Rev. A 81 (2010) 043410. http://dx.doi.org/10.1103/PhysRevA.81.043410.

[37] M. Marciante, C. Champenois, J. Pedregosa-Gutierrez, A. Calisti, M. Knoop, Parallel ion strings in linear multipole traps, Phys. Rev. A 83 (2011) 021404. http://dx.doi.org/10.1103/PhysRevA.83.021404.

[38] K. Okada, K. Yasuda, T. Takayanagi, M. Wada, H.A. Schuessler, S. Ohtani, Crystallization of Ca^{+} ions in a linear rf octupole ion trap, Phys. Rev. A 75 (2007) 033409. http://dx.doi.org/10.1103/PhysRevA.75.033409.

[39] K. Okada, T. Takayanagi, M. Wada, S. Ohtani, H.A. Schuessler, Observation of ion Coulomb crystals in a cryogenic linear octupole rf ion trap, Phys. Rev. A 80 (2009) 043405. http://dx.doi.org/10.1103/PhysRevA.80.043405.

[40] D.H.E. Dubin, J.P. Schiffer, Normal modes of cold confined one-component plasmas, Phys. Rev. E 53 (1996) 5249–5267. http://dx.doi.org/10.1103/PhysRevE.53.5249.

[41] D.H.E. Dubin, Theory of electrostatic fluid modes in a cold spheroidal non-neutral plasma, Phys. Rev. Lett. 66 (1991) 2076–2079. http://dx.doi.org/10.1103/PhysRevLett.66.2076.

[42] A. Dantan, J.P. Marler, M. Albert, D. Guénot, M. Drewsen, Noninvasive vibrational mode spectroscopy of ion Coulomb crystals through resonant collective coupling to an optical cavity field, Phys. Rev. Lett. 105 (2010) 103001. http://dx.doi.org/10.1103/PhysRevLett.105.103001.

[43] J.W. Britton, B.C. Sawyer, A.C. Keith, C.C.J. Wang, J.K. Freericks, H. Uys, M. J. Biercuk, J.J. Bollinger, Engineered two-dimensional ising interactions in a trapped-ion quantum simulator with hundreds of spins, Nature 484 (7395) (2012) 489–492, URL ⟨http://dx.doi.org/10.1038/nature10981⟩.

[44] J. Hoffnagle, R.G. DeVoe, L. Reyna, R.G. Brewer, Order-chaos transition of two trapped ions, Phys. Rev. Lett. 61 (1988) 255–258. http://dx.doi.org/10.1103/PhysRevLett.61.255.

[45] R. Blümel, C. Kappler, W. Quint, H. Walther, Chaos and order of laser-cooled ions in a Paul trap, Phys. Rev. A 40 (1989) 808–823. http://dx.doi.org/10.1103/PhysRevA.40.808.

[46] R.G. Brewer, J. Hoffnagle, R.G. DeVoe, Transient two-ion chaos, Phys. Rev. Lett. 65 (1990) 2619–2622. http://dx.doi.org/10.1103/PhysRevLett.65.2619.

[47] J. Hoffnagle, R.G. Brewer, Frequency-locked motion of two particles in a Paul trap, Phys. Rev. Lett. 71 (1993) 1828–1831. http://dx.doi.org/10.1103/PhysRevLett.71.1828.

[48] J. Hoffnagle, R. Brewer, Stability of two-ion crystals in the Paul trap: a comparison of exact and pseudopotential calculations, Appl. Phys. B 60 (2–3) (1995) 113–117. http://dx.doi.org/10.1007/BF01135851.

[49] N. Kjærgaard, K. Mølhave, M. Drewsen, Stability of Coulomb crystals in a linear Paul trap with storage-ring-like confinement, Phys. Rev. E 66 (2002) 015401. http://dx.doi.org/10.1103/PhysRevE.66.015401.

[50] S. Fishman, G. De Chiara, T. Calarco, G. Morigi, Structural phase transitions in low-dimensional ion crystals, Phys. Rev. B 77 (2008) 064111. http://dx.doi.org/10.1103/PhysRevB.77.064111.

[51] A. del Campo, G. De Chiara, G. Morigi, M.B. Plenio, A. Retzker, Structural defects in ion chains by quenching the external potential: the inhomogeneous Kibble–Zurek mechanism, Phys. Rev. Lett. 105 (2010) 075701. http://dx.doi.org/10.1103/PhysRevLett.105.075701.

[52] M. Mielenz, J. Brox, S. Kahra, G. Leschhorn, M. Albert, T. Schaetz, H. Landa, B. Reznik, Trapping of topological-structural defects in Coulomb crystals, Phys. Rev. Lett. 110 (2013) 133004. http://dx.doi.org/10.1103/PhysRevLett.110.133004.

[53] S. Ulm, J. Roßnagel, G. Jacob, C. Degüther, S.T. Dawkins, U.G. Poschinger, R. Nigmatullin, A. Retzker, M.B. Plenio, F. Schmidt-Kaler, K. Singer, Observation of the Kibble–Zurek scaling law for defect formation in ion crystals, Nat Commun 4 http://dx.doi.org/10.1038/ncomms3290, in press.

[54] K. Pyka, J. Keller, H.L. Partner, R. Nigmatullin, T. Burgermeister, D.M. Meier, K. Kuhlmann, A. Retzker, M.B. Plenio, W.H. Zurek, A. del Campo, T.E. Mehlstäubler, Topological defect formation and spontaneous symmetry breaking in ion Coulomb crystals, Nat Commun 4, http://dx.doi.org/10.1038/ncomms3291, in press.

[55] J. Liang, P.C. Haljan, Hopping of an impurity defect in ion crystals in linear traps, Phys. Rev. A 83 (2011) 063401. http://dx.doi.org/10.1103/PhysRevA.83.063401.

[56] A. Bermudez, M. Bruderer, M.B. Plenio, Controlling and measuring quantum transport of heat in trapped-ion crystals, Phys. Rev. Lett. 111 (2013) 040601. http://dx.doi.org/10.1103/PhysRevLett.111.040601.

[57] A. Benassi, A. Vanossi, E. Tosatti, Nanofriction in cold ion traps, Nat. Commun. 2 (2011) 236. http://dx.doi.org/10.1038/ncomms1230.

[58] E. Shimshoni, G. Morigi, S. Fishman, Quantum zigzag transition in ion chains, Phys. Rev. Lett. 106 (2011) 010401. http://dx.doi.org/10.1103/PhysRevLett.106.010401.

[59] I. García-Mata, O.V. Zhirov, D.L. Shepelyansky, Frenkel–Kontorova model with cold trapped ions, Eur. Phys. J. D 41 (2) (2007) 325–330. http://dx.doi.org/10.1140/epjd/e2006-00220-2.

[60] P. Horak, A. Dantan, M. Drewsen, Optically induced structural phase transitions in ion Coulomb crystals, Phys. Rev. A 86 (2012) 043435. http://dx.doi.org/10.1103/PhysRevA.86.043435.

[61] M. Enderlein, T. Huber, C. Schneider, T. Schaetz, Single ions trapped in a one-dimensional optical lattice, Phys. Rev. Lett. 109 (2012) 233004. http://dx.doi.org/10.1103/PhysRevLett.109.233004.

[62] R.B. Linnet, I.D. Leroux, M. Marciante, A. Dantan, M. Drewsen, Pinning an ion with an intracavity optical lattice, Phys. Rev. Lett. 109 (2012) 233005. http://dx.doi.org/10.1103/PhysRevLett.109.233005.

[63] J.D. Thompson, T.G. Tiecke, A.S. Zibrov, V. Vuletić, M.D. Lukin, Coherence and Raman sideband cooling of a single atom in an optical tweezer, Phys. Rev. Lett. 110 (2013) 133001. http://dx.doi.org/10.1103/PhysRevLett.110.133001.

[64] D.H.E. Dubin, T.M. O'Neil, Trapped nonneutral plasmas, liquids, and crystals the thermal equilibrium states, Rev. Mod. Phys. 71 (1999) 87–172. http://dx.doi.org/10.1103/RevModPhys.71.87.

[65] D.H.E. Dubin, Effect of correlations on the thermal equilibrium and normal modes of a non-neutral plasma, Phys. Rev. E 53 (1996) 5268–5290. http://dx.doi.org/10.1103/PhysRevE.53.5268.

[66] J.J. Bollinger, M.J. Delaney, T. Hasegawa, D.H.E. Dubin, Proceedings of the workshop on non-neutral plasmas, in: M. Drewsen, U. Uggerhøj, H. Knudsen (Eds.), Non-Neutral Plasma Physics VI, AIP, New York, 2006, p. 183.

[67] F. Anderegg, D.H.E. Dubin, T.M. O'Neil, C.F. Driscoll, Measurement of correlation-enhanced collision rates, Phys. Rev. Lett. 102 (2009) 185001. http://dx.doi.org/10.1103/PhysRevLett.102.185001.

[68] D.H.E. Dubin, Measurement of screening enhancement to nuclear reaction rates using a strongly magnetized and strongly correlated non-neutral plasma, Phys. Rev. Lett. 94 (2005) 025002. http://dx.doi.org/10.1103/PhysRevLett.94.025002.

[69] T.B. Mitchell, J.J. Bollinger, W.M. Itano, D.H.E. Dubin, Stick-slip dynamics of a stressed ion crystal, Phys. Rev. Lett. 87 (2001) 183001. http://dx.doi.org/10.1103/PhysRevLett.87.183001.

[70] W. Nagourney, J. Sandberg, H. Dehmelt, Shelved optical electron amplifier: observation of quantum jumps, Phys. Rev. Lett. 56 (1986) 2797–2799. http://dx.doi.org/10.1103/PhysRevLett.56.2797.

[71] T. Sauter, W. Neuhauser, R. Blatt, P.E. Toschek, Observation of quantum jumps, Phys. Rev. Lett. 57 (1986) 1696–1698. http://dx.doi.org/10.1103/PhysRevLett.57.1696.

[72] J.C. Bergquist, R.G. Hulet, W.M. Itano, D.J. Wineland, Observation of quantum jumps in a single atom, Phys. Rev. Lett. 57 (1986) 1699–1702. http://dx.doi.org/10.1103/PhysRevLett.57.1699.

[73] J. Eschner, C. Raab, F. Schmidt-Kaler, R. Blatt, Light interference from single atoms and their mirror images, Nature 413 (6855) (2001) 495–498, URL ⟨http://dx.doi.org/10.1038/35097017⟩.

[74] L. Slodička, G. Hétet, N. Röck, S. Gerber, P. Schindler, M. Kumph, M. Hennrich,

R. Blatt, Interferometric thermometry of a single sub-Doppler-cooled atom, Phys. Rev. A 85 (2012) 043401. http://dx.doi.org/10.1103/PhysRevA.85.043401.
[75] A. Kreuter, C. Becher, G.P.T. Lancaster, A.B. Mundt, C. Russo, H. Häffner, C. Roos, J. Eschner, F. Schmidt-Kaler, R. Blatt, Spontaneous emission lifetime of a single trapped Ca^+ ion in a High Finesse cavity, Phys. Rev. Lett. 92 (2004) 203002. http://dx.doi.org/10.1103/PhysRevLett.92.203002.
[76] R.G. DeVoe, R.G. Brewer, Observation of superradiant and subradiant spontaneous emission of two trapped ions, Phys. Rev. Lett. 76 (1996) 2049–2052. http://dx.doi.org/10.1103/PhysRevLett.76.2049.
[77] U. Eichmann, J.C. Bergquist, J.J. Bollinger, J.M. Gilligan, W.M. Itano, D.J. Wineland, M.G. Raizen, Young's interference experiment with light scattered from two atoms, Phys. Rev. Lett. 70 (1993) 2359–2362. http://dx.doi.org/10.1103/PhysRevLett.70.2359.
[78] W.M. Itano, D.J. Heinzen, J.J. Bollinger, D.J. Wineland, Quantum zeno effect, Phys. Rev. A 41 (1990) 2295–2300. http://dx.doi.org/10.1103/PhysRevA.41.2295.
[79] K. Mølhave, M. Drewsen, Demonstration of the continuous quantum zeno effect in optical pumping, Phys. Lett. A 268 (1–2) (2000) 45–49. http://dx.doi.org/10.1016/S0375-9601(00)00166-3.
[80] G.R. Guthohrlein, M. Keller, K. Hayasaka, W. Lange, H. Walther, A single ion as a nanoscopic probe of an optical field, Nature 414 (6859) (2001) 49–51, URL ⟨http://dx.doi.org/10.1038/35102129⟩.
[81] M. Steiner, H.M. Meyer, C. Deutsch, J. Reichel, M. Köhl, Single ion coupled to an optical fiber cavity, Phys. Rev. Lett. 110 (2013) 043003. http://dx.doi.org/10.1103/PhysRevLett.110.043003.
[82] A. Stute, B. Casabone, B. Brandstatter, K. Friebe, T.E. Northup, R. Blatt, Quantum-state transfer from an ion to a photon, Nat. Photon 7 (3) (2013) 219–222, URL ⟨http://dx.doi.org/10.1038/nphoton.2012.358⟩.
[83] B. Casabone, A. Stute, K. Friebe, B. Brandstätter, K. Schüppert, R. Blatt, T.E. Northup, Heralded entanglement of two ions in an optical cavity, Phys. Rev. Lett. 111 (2013) 100505. http://dx.doi.org/10.1103/PhysRevLett.111.100505.
[84] P.F. Herskind, A. Dantan, J.P. Marler, M. Albert, M. Drewsen, Realization of collective strong coupling with ion Coulomb crystals in an optical cavity, Nat. Phys. 5 (7) (2009) 494–498, URL ⟨http://dx.doi.org/10.1038/nphys1302⟩.
[85] K.R. Zangenberg, A. Dantan, M. Drewsen, Spatial mode effects in a cavity EIT-based quantum memory with ion Coulomb crystals, J. Phys. B: At. Mol. Opt. Phys.: At. 45 (12) (2012) 124011, URL ⟨http://stacks.iop.org/0953-4075/45/i=12/a=124011⟩.
[86] C. Clausen, N. Sangouard, M. Drewsen, Analysis of a photon number resolving detector based on fluorescence readout of an ion Coulomb crystal quantum memory inside an optical cavity, N. J. Phys. 15 (2) (2013) 025021, URL ⟨http://stacks.iop.org/1367-2630/15/i=2/a=025021⟩.
[87] M. Albert, A. Dantan, M. Drewsen, Cavity electromagnetically induced transparency and all-optical switching using ion Coulomb crystals, Nat. Photon. 5 (10) (2011) 633–636, URL ⟨http://dx.doi.org/10.1038/nphoton.2011.214⟩.
[88] D. Leibfried, R. Blatt, C. Monroe, D. Wineland, Quantum dynamics of single trapped ions, Rev. Mod. Phys. 75 (2003) 281–324. http://dx.doi.org/10.1103/RevModPhys.75.281.
[89] C. Monroe, D.M. Meekhof, B.E. King, W.M. Itano, D.J. Wineland, Demonstration of a fundamental quantum logic gate, Phys. Rev. Lett. 75 (1995) 4714–4717. http://dx.doi.org/10.1103/PhysRevLett.75.4714.
[90] J.I. Cirac, P. Zoller, Quantum computations with cold trapped ions, Phys. Rev. Lett. 74 (1995) 4091–4094. http://dx.doi.org/10.1103/PhysRevLett.74.4091.
[91] K. Mølmer, A. Sørensen, Multiparticle entanglement of hot trapped ions, Phys. Rev. Lett. 82 (1999) 1835–1838. http://dx.doi.org/10.1103/PhysRevLett.82.1835.
[92] H. Haffner, W. Hansel, C.F. Roos, J. Benhelm, D. Chek-alkar, M. Chwalla, T. Korber, U.D. Rapol, M. Riebe, P.O. Schmidt, C. Becher, O. Guhne, W. Dur, R. Blatt, Scalable multiparticle entanglement of trapped ions, Nature 438 (7068) (2005) 643–646, URL ⟨http://dx.doi.org/10.1038/nature04279⟩.
[93] C.A. Sackett, D. Kielpinski, B.E. King, C. Langer, V. Meyer, C.J. Myatt, M. Rowe, Q.A. Turchette, W.M. Itano, D.J. Wineland, C. Monroe, Experimental entanglement of four particles, Nature 404 (6775) (2000) 256–259, URL ⟨http://dx.doi.org/10.1038/35005011⟩.
[94] C. Ospelkaus, U. Warring, Y. Colombe, K.R. Brown, J.M. Amini, D. Leibfried, D. J. Wineland, Microwave quantum logic gates for trapped ions, Nature 476 (7359) (2011) 181–184, URL ⟨http://dx.doi.org/10.1038/nature10290⟩.
[95] P. Schindler, J.T. Barreiro, T. Monz, V. Nebendahl, D. Nigg, M. Chwalla, M. Hennrich, R. Blatt, Experimental repetitive quantum error correction, Science 332 (033) (2011) 1059–1061. http://dx.doi.org/10.1126/science.1203329 ⟨http://arxiv.org/abs/http://www.sciencemag.org/content/332/6033/1059.full.pdf⟩ URL ⟨http://www.sciencemag.org/content/332/6033/1059.abstract⟩.
[96] F. Schmidt-Kaler, H. Haffner, M. Riebe, S. Gulde, G.P.T. Lancaster, T. Deuschle, C. Becher, C.F. Roos, J. Eschner, R. Blatt, Realization of the Cirac–Zoller controlled-NOT quantum gate, Nature 422 (6930) (2003) 408–411, URL ⟨http://dx.doi.org/10.1038/nature01494⟩.
[97] B.P. Lanyon, C. Hempel, D. Nigg, M. Müller, R. Gerritsma, F. Zähringer, P. Schindler, J.T. Barreiro, M. Rambach, G. Kirchmair, M. Hennrich, P. Zoller, R. Blatt, C.F. Roos, Universal digital quantum simulation with trapped ions, Science 334 (6052) (2011) 57–61. http://dx.doi.org/10.1126/science.1208001 ⟨http://arxiv.org/abs/http://www.sciencemag.org/content/334/6052/57.full.pdf⟩, URL ⟨http://www.sciencemag.org/content/334/6052/57.abstract.
[98] D. Porras, J.I. Cirac, Effective quantum spin systems with trapped ions, Phys. Rev. Lett. 92 (2004) 207901. http://dx.doi.org/10.1103/PhysRevLett.92.207901.
[99] A. Friedenauer, H. Schmitz, J.T. Glueckert, D. Porras, T. Schaetz, Simulating a quantum magnet with trapped ions, Nat. Phys. 4 (10) (2008) 757–761, URL ⟨http://dx.doi.org/10.1038/nphys1032⟩.
[100] R. Islam, C. Senko, W.C. Campbell, S. Korenblit, J. Smith, A. Lee, E.E. Edwards, C.-C.J. Wang, J.K. Freericks, C. Monroe, Emergence and frustration of magnetism with variable-range interactions in a quantum simulator, Science 340 (6132) (2013) 583–587. http://dx.doi.org/10.1126/science.1232296, URL ⟨http://arxiv.org/abs/http://www.sciencemag.org/content/340/6132/583.full.pdf⟩, ⟨http://www.sciencemag.org/content/340/6132/583.abstract⟩.
[101] J.T. Barreiro, M. Muller, P. Schindler, D. Nigg, T. Monz, M. Chwalla, M. Hennrich, C.F. Roos, P. Zoller, R. Blatt, An open-system quantum simulator with trapped ions, Nature 470 (7335) (2011) 486–491, URL ⟨http://dx.doi.org/10.1038/nature09801⟩.
[102] R. Blatt, C.F. Roos, Quantum simulations with trapped ions, Nat. Phys. 8 (4) (2012) 277–284, URL ⟨http://dx.doi.org/10.1038/nphys2252⟩.
[103] C. Monroe, J. Kim, Scaling the ion trap quantum processor, Science 339 (6124) (2013) 1164–1169. http://dx.doi.org/10.1126/science.1231298 ⟨http://arxiv.org/abs/http://www.sciencemag.org/content/339/6124/1164.full.pdf⟩, URL ⟨http://www.sciencemag.org/content/339/6124/1164.abstract⟩.
[104] P.O. Schmidt, T. Rosenband, C. Langer, W.M. Itano, J.C. Bergquist, D.J. Wineland, Spectroscopy using quantum logic, Science 309 (5735) (2005) 749–752. http://dx.doi.org/10.1126/science.1114375 ⟨http://arxiv.org/abs/http://www.sciencemag.org/content/309/5735/749.full.pdf⟩, URL ⟨http://www.sciencemag.org/content/309/5735/749.abstract⟩.
[105] D.J. Larson, J.C. Bergquist, J.J. Bollinger, W.M. Itano, D.J. Wineland, Sympathetic cooling of trapped ions: a laser-cooled two-species nonneutral ion plasma, Phys. Rev. Lett. 57 (1986) 70–73. http://dx.doi.org/10.1103/PhysRevLett.57.70.
[106] T. Rosenband, D.B. Hume, P.O. Schmidt, C.W. Chou, A. Brusch, L. Lorini, W.H. Oskay, R.E. Drullinger, T.M. Fortier, J.E. Stalnaker, S.A. Diddams, W.C. Swann, N.R. Newbury, W.M. Itano, D.J. Wineland, J.C. Bergquist, Frequency ratio of Al^+ and Hg^+ single-ion optical clocks; metrology at the 17th decimal place, Science 319 (71) (2008) 1808–1812. http://dx.doi.org/10.1126/science.1154622 ⟨http://arxiv.org/abs/http://www.sciencemag.org/content/319/5871/1808.full.pdf⟨, URL ⟨http://www.sciencemag.org/content/319/5871/1808.abstract⟩.
[107] C.W. Chou, D.B. Hume, J.C.J. Koelemeij, D.J. Wineland, T. Rosenband, Frequency comparison of two high-accuracy Al^+ optical clocks, Phys. Rev. Lett. 104 (2010) 070802. http://dx.doi.org/10.1103/PhysRevLett.104.070802.
[108] C.W. Chou, D.B. Hume, T. Rosenband, D.J. Wineland, Optical clocks and relativity, Science 329 (5999) (2010) 1630–1633. http://dx.doi.org/10.1126/science.1192720 ⟨http://arxiv.org/abs/http://www.sciencemag.org/content/329/5999/1630.full.pdf⟩, URL ⟨http://www.sciencemag.org/content/329/5999/1630.abstract⟩.
[109] M. Drewsen, Cooling, identification and spectroscopy of super-heavy element ions, Eur. Phys. J. D 45 (1) (2007) 125–127. http://dx.doi.org/10.1140/epjd/e2007-00162-1.
[110] L. Gruber, J.P. Holder, J. Steiger, B.R. Beck, H.E. DeWitt, J. Glassman, J.W. McDonald, D.A. Church, D. Schneider, Evidence for highly charged ion Coulomb crystallization in multicomponent strongly coupled plasmas, Phys. Rev. Lett. 86 (2001) 636–639. http://dx.doi.org/10.1103/PhysRevLett.86.636.
[111] S. Schiller, Hydrogenlike highly charged ions for tests of the time independence of fundamental constants, Phys. Rev. Lett. 98 (2007) 180801. http://dx.doi.org/10.1103/PhysRevLett.98.180801.
[112] K. Mølhave, M. Drewsen, Formation of translationally cold MgH^+ and MgD^+ molecules in an ion trap, Phys. Rev. A 62 (2000) 011401. http://dx.doi.org/10.1103/PhysRevA.62.011401.
[113] J.C.J. Koelemeij, B. Roth, A. Wicht, I. Ernsting, S. Schiller, Vibrational spectroscopy of HD^+ with 2-ppb accuracy, Phys. Rev. Lett. 98 (2007) 173002. http://dx.doi.org/10.1103/PhysRevLett.98.173002.
[114] O.O. Versolato, M. Schwarz, A.K. Hansen, A.D. Gingell, A. Windberger, L. Kłosowski, J. Ullrich, F. Jensen, J.R. Crespo López-Urrutia, M. Drewsen, Decay rate measurement of the first vibrationally excited state of MgH^+ in a cryogenic Paul trap, Phys. Rev. Lett. 111 (2013) 053002. http://dx.doi.org/10.1103/PhysRevLett.111.053002.
[115] P. Bowe, L. Hornekær, C. Brodersen, M. Drewsen, J.S. Hangst, J.P. Schiffer, Sympathetic crystallization of trapped ions, Phys. Rev. Lett. 82 (1999) 2071–2074. http://dx.doi.org/10.1103/PhysRevLett.82.2071.
[116] H. Rohde, S.T. Gulde, C.F. Roos, P.A. Barton, D. Leibfried, J. Eschner, F. Schmidt-Kaler, R. Blatt, Sympathetic ground-state cooling and coherent manipulation with two-ion crystals, J. Opt. B: Quantum Semiclass. Opt. 3 (1) (2001) S34, URL ⟨http://stacks.iop.org/1464-4266/3/i=1/a=357⟩.
[117] M. Drewsen, I. Jensen, J. Lindballe, N. Nissen, R. Martinussen, A. Mortensen, P. Staanum, D. Voigt, Ion Coulomb crystals: a tool for studying ion processes, Int. J. Mass Spectrom. 229 (1-2) (2003) 83–91. http://dx.doi.org/10.1016/S1387-3806(03)00259-8, mass Spectrometry Contributions to Nanosciences and Nanotechnology. http://www.sciencedirect.com/science/article/pii/S1387380603002598.
[118] P.F. Staanum, K. Højbjerre, R. Wester, M. Drewsen, Probing isotope effects in chemical reactions using single ions, Phys. Rev. Lett. 100 (2008) 243003. http://dx.doi.org/10.1103/PhysRevLett.100.243003.
[119] K. Højbjerre, D. Offenberg, C.Z. Bisgaard, H. Stapelfeldt, P.F. Staanum, A. Mortensen, M. Drewsen, Consecutive photodissociation of a single complex molecular ion, Phys. Rev. A 77 (2008) 030702. http://dx.doi.org/10.1103/PhysRevA.77.030702.
[120] P.F. Staanum, K. Hojbjerre, P.S. Skyt, A.K. Hansen, M. Drewsen, Rotational

laser cooling of vibrationally and translationally cold molecular ions, Nat. Phys. 6 (4) (2010) 271–274, URL ⟨http://dx.doi.org/10.1038/nphys1604⟩.
[121] T. Schneider, B. Roth, H. Duncker, I. Ernsting, S. Schiller, All-optical preparation of molecular ions in the rovibrational ground state, Nat. Phys. 6 (4) (2010) 275–278, URL ⟨http://dx.doi.org/10.1038/nphys1605⟩.
[122] X. Tong, A.H. Winney, S. Willitsch, Sympathetic cooling of molecular ions in selected rotational and vibrational states produced by threshold photoionization, Phys. Rev. Lett. 105 (2010) 143001. http://dx.doi.org/10.1103/PhysRevLett.105.143001.
[123] C. Zipkes, S. Palzer, C. Sias, M. Kohl, A trapped single ion inside a Bose–Einstein condensate, Nature 464 (7287) (2010) 388–391, URL ⟨http://dx.doi.org/10.1038/nature08865⟩.
[124] S. Schmid, A. Härter, J.H. Denschlag, Dynamics of a cold trapped ion in a Bose–Einstein condensate, Phys. Rev. Lett. 105 (2010) 133202. http://dx.doi.org/10.1103/PhysRevLett.105.133202.
[125] A.K. Hansen, M.A. Sørensen, P.F. Staanum, M. Drewsen, Single-ion recycling reactions, Angew. Chem. Int. Edition 51 (32) (2012) 7960–7962. http://dx.doi.org/10.1002/anie.201203550.
[126] F.H.J. Hall, S. Willitsch, Millikelvin reactive collisions between sympathetically cooled molecular ions and laser-cooled atoms in an ion-atom hybrid trap, Phys. Rev. Lett. 109 (2012) 233202. http://dx.doi.org/10.1103/PhysRevLett.109.233202.
[127] Y.-P. Chang, K. Długołęcki, J. Küpper, D. Rösch, D. Wild, S. Willitsch, Specific chemical reactivities of spatially separated 3-aminophenol conformers with cold Ca^+ ions, Science 342 (6154) (2013) 98–101. http://dx.doi.org/10.1126/science.1242271 ⟨http://www.sciencemag.org/content/342/6154/98.abstract⟩, URL ⟨http://arxiv.org/abs/http://www.sciencemag.org/content/342/6154/98.full.pdf⟩.
[128] W.G. Rellergert, S.T. Sullivan, S.J. Schowalter, S. Kotochigova, K. Chen, E.R. Hudson, Evidence for sympathetic vibrational cooling of translationally cold molecules, Nature 495 (7442) (2013) 490–494. http://dx.doi.org/10.1038/nature11937.
[129] A.K. Hansen, O.O. Versolato, L. Kłosowski, S.B. Kristensen, A. Gingell, M. Schwarz, A. Windberger, J. Ullrich, J.R.C. Lopez-Urrutia, M. Drewsen, Efficient rotational cooling of Coulomb-crystallized molecular ions by a helium buffer gas, Nature 508 (2014) 76–79. http://dx.doi.org/10.1038/nature12996.
[130] A. Ostendorf, C.B. Zhang, M.A. Wilson, D. Offenberg, B. Roth, S. Schiller, Sympathetic cooling of complex molecular ions to millikelvin temperatures, Phys. Rev. Lett. 97 (2006) 243005. http://dx.doi.org/10.1103/PhysRevLett.97.243005.
[131] A. Bertelsen, S. Jørgensen, M. Drewsen, The rotational temperature of polar molecular ions in Coulomb crystals, J. Phys. B: At. Mol. Opt. Phys. 39 (5) (2006) L83, URL ⟨http://stacks.iop.org/0953-4075/39/i=5/a=L02⟩.