General Scheme for the Construction of a Protected Qubit Subspace

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We present a new robust decoupling scheme suitable for levels with either half-integer or integer angular momentum states. Through continuous dynamical decoupling techniques, we create a protected qubit subspace, utilizing a multistate qubit construction. Remarkably, the multistate system can also be composed of multiple substates within a single level. Our scheme can be realized with state-of-the-art experimental setups and thus has immediate applications for quantum information science. While the scheme is general and relevant for a multitude of solid-state and atomic systems, we analyze its performance for the case composed of trapped ions. Explicitly, we show how single qubit gates and an ensemble coupling to a cavity mode can be implemented efficiently. The scheme predicts a coherence time of ~ 1 s, as compared to typically a few milliseconds for the bare states.

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Introduction.—Protecting quantum bits (qubits) from decoherence due to interactions with their environment is a prime issue of experimental quantum information science. In the case of solid-state and atomic qubit systems, the presence of ambient magnetic field fluctuations is in particular a problem. Consequently, several methods have been put forward to tackle this problem. The traditional solution is to utilize either a two-level subsystem of two integer total angular momentum states, which to first order has no Zeeman shifts [1–3], or a two-level system composed of two hyperfine states with identical first order shifts [4,5]. A third way is to use decoherence-free subspaces [6–8], which requires spatially separated physical qubits to represent a single logic qubit and thus incurs considerable overhead, and is potentially vulnerable to decoherence due to field gradients.

Dynamical decoupling is another general strategy to tackle this problem [9]. The pulsed version was proven to be extremely efficient [10,11]; however, it may require complex pulse sequences. The continuous version of dynamical decoupling [12] is based on spin locking [13], where a continuous drive protects the system from external noise and weaker continuous pulses improve its robustness [14]. Continuous dynamical decoupling could be combined in a natural way with gates [15] and could improve the coupling efficiency to superconducting cavities [16]. However, both versions require composite schemes to overcome both the external (magnetic) noise and the controller (optical, microwave, or rf) noise. A four-level structure composed of the magnetic substates of two hyperfine levels with $F = 0$ and $F = 1$ has been designed to be perfectly robust to control fluctuations in conjunction with composite schemes [17], but this method is only applicable for this particular spin system.

In this Letter we present a new and general method for the construction of a protected and robust qubit subspace. The method utilizes a multilevel structure, on which continuous dynamical decoupling fields are applied. Our method is suitable for a wide range of solid-state and atomic systems, and it is applicable to a variety of tasks in the field of quantum information science and quantum sensing, in particular, quantum magnetometery and quantum memories. The method can be implemented with state-of-the-art technology and should be able to push the T_2 time to the T_1 limit.

General scheme.—The general scheme defines the protected subspace which we denote by $\{|D_i\rangle\}$. In the following $\mathbf{J}$ is the angular momentum operator, H_d is the (continuous) driving Hamiltonian, $\mathcal{H}_D$ is the Hilbert subspace of the protected (and hence dark) states, and $\mathcal{H}_\perp$ is the complementary Hilbert space, that is, $\mathcal{H} = \mathcal{H}_D \oplus \mathcal{H}_\perp$. We define the protected subspace by

$$\langle D_j|J_z|D_i\rangle = 0 \quad \forall\ i, j, \qquad H_d|D_i\rangle = 0 \quad \forall\ i. \tag{1}$$

The first equation ensures that the noise does not operate within the protected subspace; the noise can only cause transitions between a state in the protected subspace and a state in the complementary subspace. We assume (by construction) that for any eigenstate $|\psi_i\rangle \in \mathcal{H}_\perp$ of H_d we have that $|\langle\psi_i|H_d|\psi_i\rangle|$ is much larger than the characteristic frequency of the power spectrum of the noise [18]. This ensures that the energy of all states in $\mathcal{H}_D$ is far from the energy of the states in $\mathcal{H}_\perp$, and thus the rate of transitions from $\mathcal{H}_D$ to $\mathcal{H}_\perp$ due to noise is negligible. The second equation indicates that the protected subspace is the kernel of H_d and, hence, the protected states do not collect a dynamical phase and are immune to the noise originating from H_d. Note that these conditions are

analogous to the error detection conditions in [19] since the errors are magnetic noise, which is represented by the J_z operator, and fluctuations in H_d.

From the definition of the protected subspace we can also study the evolution within the subspace. Transitions between dark states can be generated by only one of the operators J_x and J_y. Suppose that J_y transforms between dark states, $J_y|D_i\rangle = |D_j\rangle$ $(i \neq j)$. Together with $J_z|D_i\rangle = |\varphi_i\rangle \in \mathcal{H}_\perp$ we have that $J_yJ_z|D_i\rangle = |\tilde{\varphi}_i\rangle \in \mathcal{H}_\perp$ and $J_zJ_y|D_i\rangle = |\varphi_j\rangle \in \mathcal{H}_\perp$, and hence $J_x|D_i\rangle \in \mathcal{H}_\perp$. Whether it is J_x or J_y that transforms between the dark states is determined by H_d. Suppose again that J_y transforms between the dark states. It is then easy to show that $[H_d, J_y]|D_i\rangle = 0$ and that $[H_d, J_x]|D_i\rangle \in \mathcal{H}_\perp$. This limits the available *direct* operations on the dark state to rotations around one axis. However, general unitary operations can be implemented by various methods [20–22]. Since $J_z|D_i\rangle = |\varphi_i\rangle \in \mathcal{H}_\perp$ we can also conclude that $\lceil d_{\mathcal{H}}/2 \rceil \geq d_{\mathcal{H}_D}$, where the $d_{\mathcal{H}}$ and $d_{\mathcal{H}_D}$ are the dimensions of the total Hilbert space and the protected subspace, respectively.

Implementation with trapped ions.—Below we present an implementation of the scheme with a system of trapped ions. Although the suggested implementation is applicable to a variety of ionic systems, we focus on the calcium ion (see Fig. 1). Remarkably, the considered multistate system is composed of multiple substates within a single level, specifically, the $D_{3/2}$ sublevels. Since the $D_{3/2}$ states have a lifetime of $\sim$1 s, we consider their subspace to be the protected subspace. Please note that a very similar level structure exists for the barium ion with a longer lifetime of $\sim$20 s. For simplicity we will use the notation $|d_{3/2+m_i}\rangle \equiv |D_{3/2}; m_i\rangle$, $|p_{1/2+m_i}\rangle \equiv |P_{1/2}; m_i\rangle$, and $|s_{1/2+m_i}\rangle \equiv |S_{1/2}; m_i\rangle$. The definition of the protected subspace given by Eq. (1) results in the two dark states (see [22])

$$|D_1\rangle = \frac{\sqrt{3}}{2}|d_1\rangle - \frac{1}{2}|d_3\rangle, \qquad |D_2\rangle = \frac{1}{2}|d_0\rangle - \frac{\sqrt{3}}{2}|d_2\rangle, \tag{2}$$

where it can be seen that the average magnetic moment for each state vanishes.

These two orthonormal dark states can serve as a basis for a qubit memory. The $D_{3/2}$ degeneracy is removed by applying a constant magnetic field along the $\hat{z}$ axes which results in an energy gap of g_JB between any two adjacent energy levels, where $g_J = \frac{4}{5}$ is the Landé g factor. A large enough $|B|$ such that $|g_JB|$ is much larger than the characteristic frequency of the noise ensures that the dark states are also immune to J_x and J_y noise. We now describe the driving Hamiltonian, $H_d = H_{d1} + H_{d2}$. H_{d1} corresponds to the simultaneous on-resonance coupling of the $|d_1\rangle$ and $|d_3\rangle$ states to the $|p_1\rangle$ state, and results in the first dark state $|D_1\rangle$. H_{d2} corresponds to the on-resonance coupling of the $|d_0\rangle$ and $|d_2\rangle$ states to the $|p_0\rangle$ state, and results in the second dark state $|D_2\rangle$. However, the driving fields of each dark state can impact the other dark state since they operate on all of the $D_{3/2}$ states. We reduce this undesirable effect by creating an energy gap between the two $P_{1/2}$ states. This energy gap is achieved by the on-resonance coupling of the $|s_0\rangle$ and $|p_1\rangle$ states, and as a consequence, the driving fields of the first (second) dark state operate on the second (first) dark state with a detuning of $\Delta 2 = \Omega + (4B/5)$ $(\Delta 1 = -[\Omega + \frac{4B}{5}])$ (see Fig. 2).

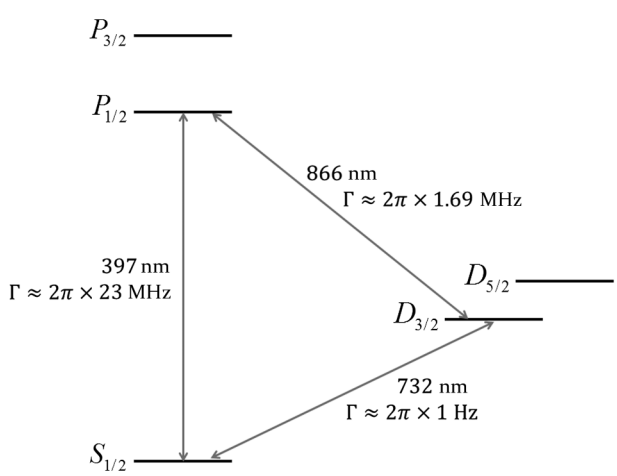

FIG. 1. Level structure of the calcium ion, $^{40}Ca^+$. The $D_{3/2}$ subspace, which has a lifetime of $\sim$1 s, serves as the protected subspace. The $S_{1/2} - P_{1/2}$ transitions and the $D_{3/2} - P_{1/2}$ transitions are used in the initialization and construction of the protected subspace.

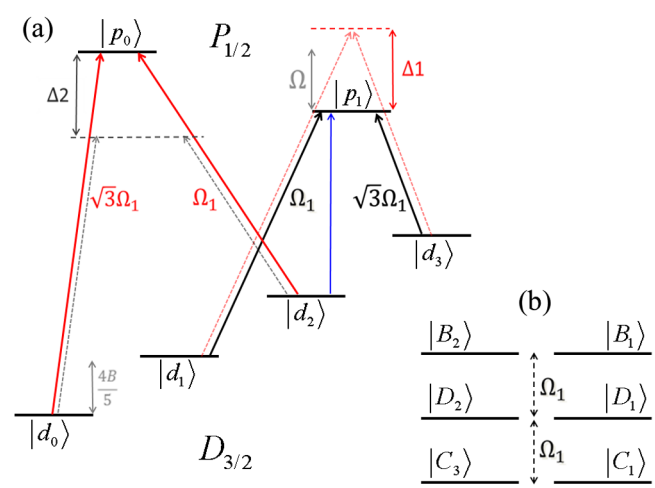

FIG. 2 (color online). Realization of dark states. (a) The black (red) driving fields result in the first (second) dark state. The driving fields of each dark state also operate on the subspace of the other dark state (dashed lines), resulting in small energy shifts. The detunings are given by $\Delta 2 = -\Delta 1 = \Omega + 4B/5$, where Ω is the energy gap between the two $P_{1/2}$ states, introduced by the $S_{1/2} - P_{1/2}$ coupling. Blue arrow shows optical pumping to the first dark state, $|D_1\rangle$. (b) Level structure in the dark states basis. The dark states $|D_1\rangle$ and $|D_2\rangle$ form the protected subspace.

In the interaction picture and in the rotating wave approximation the total driving Hamiltonian is given by

$$H_d = \left[\left(\frac{\Omega_1}{2}|p_1\rangle\langle d_1| + \frac{\sqrt{3}\Omega_1}{2}|p_1\rangle\langle d_3|\right) + \text{H.c.} + \left(\frac{\Omega_1}{2}|p_0\rangle\langle d_2| + \frac{\sqrt{3}\Omega_1}{2}|p_0\rangle\langle d_0|\right) + \text{H.c.}\right]. \quad (3)$$

This Hamiltonian has two eigenstates with zero eigenvalues, which are the desired dark states given by Eq. (2), and four bright eigenstates whose eigenvalues are equal to $\pm\Omega_1$ [22].

Thus far we have discussed the construction of the protected subspace. In the following we estimate the lifetime T_1 and the coherence time T_2 of the dark states. The lifetime can be affected by the energy shifts caused by the driving fields of the other dark state, and the coherence time can be affected by the fluctuations of these energy shifts. The fluctuations in the energy shifts cause dephasing at a rate equal to the power spectrum of the noise at zero frequency. For the first dark state $|D_1\rangle$ an energy shift fluctuation can also occur due to fluctuations of the driving field creating the energy gap between the two $P_{1/2}$ states. Calculation of these energy shifts and their fluctuations (assuming a maximal fluctuation of 1% in the intensity of the driving fields [23]) yields [22]

$$\Delta E_1 \leq \frac{\Omega_1^2}{4|\Delta 1|}\left(1 \pm \frac{3}{100}\right), \qquad \Delta E_2 \leq \frac{\Omega_1^2}{4|\Delta 2|}\left(1 \pm \frac{2}{100}\right). \quad (4)$$

Both ΔE_1 and ΔE_2 are of the order of Ω_1^2/Ω, which for typical experimental setups is $(\Omega_1^2/\Omega) \sim [(10^5)^2/10^9] = 10$ Hz. These energy shifts correspond to a small modification of the dark states, $|D_i\rangle \to \sqrt{1-\epsilon}|D_i\rangle + \sqrt{\epsilon}|\varphi_i\rangle$, where $|\varphi_i\rangle \in \mathcal{H}_\perp$, reducing the T_1 time from 1 s to approximately 0.9 s [22].

The T_2 time can be affected by the fluctuations of $\Delta E1$ and $\Delta E2$. For the above experimental parameters, we have that $T_2 \leq [\Delta(\Delta E1 - \Delta E2)]^{-1} \sim (\Omega_1^2/100\,\Omega)^{-1} = 10$ s [22,24]. As this bound is even larger than T_1, we conclude that the fluctuations in the driving fields do not reduce the T_2 time. In addition, relative amplitude and phase fluctuations will limit the T_2 time by T_2^*/η^2, where T_2^* is the coherence time of the bare states, and η is the rate of the relative amplitude fluctuations; since these are usually small we can neglect this correction.

Another source of noise comes from polarization imperfections. The typical experimental error in the polarization is ~1%. This means that ~1% of a σ^+ polarized beam is actually σ^- polarized and vice versa, causing an error within the driving of each dark state (for example, 1% of the σ^- beam which couples the $|d_3\rangle$ and $|p_1\rangle$ states is a σ^+ beam which couples the $|d_1\rangle$ and $|p_1\rangle$ states). The polarization errors also cause an energy shift and modify the dark state. However, an energy gap of ~10 MHz between the $D_{3/2}$ states (due to the Zeeman splitting) ensures that neither the T_1 time nor the T_2 time is reduced [22].

We have thus constructed a protected and robust qubit subspace with a lifetime and a coherence time which are almost identical to the $D_{3/2}$ lifetime, equaling approximately 0.9 s, while the T_2^* time is of the order of 1 ms [25,26].

Initialization and single qubit gates.—By adding two extra laser beams, one that couples the $|s_1\rangle$ state to the $|p_1\rangle$ state and the other that couples the $|d_2\rangle$ state to the $|p_1\rangle$ state (blues laser in Fig. 2), we can achieve optical pumping to the dark states $|D_1\rangle$. This way, the dark state $|D_2\rangle$ is taken out of the protected subspace, but because of H_d the state will eventually evolve to the dark state $|D_1\rangle$. Another method of initialization is to optically pump into the $|d_3\rangle$ state, and then conduct a STIRAP procedure via a Raman transition.

We propose an experimentally simple method for the implementation of a single qubit σ_y gate by applying a microwave field which is set to be on resonance with the energy gap between the $D_{3/2}$ states (see Fig. 3). More specifically, the microwave field is tuned to apply the J_y operator, as in our case $[H_d, J_y]|D_i\rangle = 0$. In the interaction picture and in the rotating wave approximation, the Hamiltonian of the single qubit gate is given by

$$H_g = i\Omega_g\left(\frac{\sqrt{3}}{2}|d_1\rangle\langle d_0| + |d_2\rangle\langle d_1| + \frac{\sqrt{3}}{2}|d_3\rangle\langle d_2|\right) + \text{H.c.}, \quad (5)$$

which corresponds to the operator $-(i3\Omega_g/2)|D_2\rangle\langle D_1|$ in the dark states basis [22]. In the Supplemental Material [22] we explicitly show how to construct σ_x and σ_z gates, which allow for the implementation of any single qubit unitary operation.

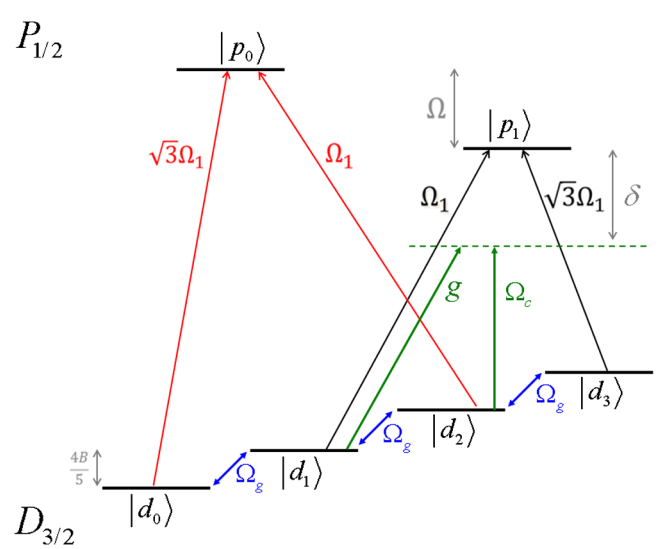

FIG. 3 (color online). Realization of (i) a single qubit gate (blue) (ii) coupling to a cavity mode (green).

Interaction with a cavity mode.—One of the most important applications of robust quantum states is the implementation of a quantum memory. For this purpose, it is also necessary to have an efficient interaction between the robust states of the quantum memory and the mediating system which delivers the data to be stored and retrieved from memory. Here, we focus on the interaction of ions with a cavity mode, as several experimental investigations are currently exploring this situation [25–30]. Such an interaction will not only allow for the implementation of a quantum memory but could also allow for multiqubit gates where the interaction between different qubits is mediated via the cavity modes.

We begin by setting the cavity mode such that its frequency and polarization correspond to the detuned coupling of the $|d_1\rangle$ state to the $|p_1\rangle$ state with the detuning δ to be specified below. In addition, we apply an external control field which corresponds to the detuned coupling of the $|d_2\rangle$ state to the $|p_1\rangle$ state with the same detuning δ and with a Rabi frequency Ω_c such that $\delta \gg \Omega_c \gg g$, where g is the rate describing the coupling between a single photon in the cavity mode to a single ion (see Fig. 3). This interaction couples the $|d_1\rangle$ and $|d_2\rangle$ states and results in the effective Hamiltonian $H_{\rm eff} = -\frac{g\Omega_c}{2\delta}(|d_2\rangle\langle d_1|a + {\rm H.c.})$, where a is the annihilation operator of the cavity mode. In the dark states basis the interaction, which is given by $H_{\rm eff} \approx -(3g\Omega_c/8\delta)(|D_2\rangle\langle D_1|a + {\rm H.c.})$, couples a cavity mode to a robust qubit [22]. However, the strength of this coupling is usually weak compared to the cavity and ion damping rates. That is, $\kappa, \gamma \gg (3g\Omega_c/8\delta)$, where κ is the cavity's damping rate and $\gamma = (\Omega_c^2/\delta^2)\Gamma_{p_1}$ is the ion's damping rate. This is known as the weak coupling regime in which transmission of quantum information is not possible. The problem can be circumvented by coupling a cavity mode to an ensemble of ions. The coupling strength is enhanced by $\sqrt{N}$, where N is the effective number of ions, and for a large enough ensemble this results in $\kappa, \gamma \ll \sqrt{N}(3g\Omega_c/8\delta)$, which are the conditions for the collective strong coupling regime. (Note that, since we consider N ions, the probability of emitting a photon is $\sim(\Omega_c^2/\delta^2)N$. However, the factor N is canceled out because the interaction results in a Dicke state.) From the condition of the strong coupling regime on the ion's damping rate, we must have that $\frac{\Omega_c}{\delta} \ll (3g\sqrt{N}/8\Gamma_{p_1})$. Substituting $\Gamma_{p_1} = 2\pi \times 23$ MHz, $g = 2\pi \times 0.5$ MHz, and $\sqrt{N} \sim 10$ (which could be achieved, e.g., as a string of one species of ions within another species [31]), we get that $(\Omega_c/\delta) \ll \frac{1}{10}$, and thus we set $(\Omega_c/\delta) \sim 10^{-2}$. The condition on the cavity's damping rate then implies that $\kappa \ll (g/10) \sim \pi \times 0.1$ MHz is required. Such damping rates are exhibited in current high-finesse cavities.

Note that by removing the control field we are left only with the coupling to the cavity mode which results in the Hamiltonian $H_R = -(g^2/\delta)|d_1\rangle\langle d_1|a^\dagger a$, corresponding to $H_R \approx -(3g^2/4\delta)|D_1\rangle\langle D_1|a^\dagger a$ in the dark states basis. As H_R takes $\frac{1}{\sqrt{2}}(|D_2\rangle + |D_1\rangle)$ to $\frac{1}{\sqrt{2}}(|D_2\rangle + e^{i(3g^2t/4\delta)a^\dagger a}|D_1\rangle)$, a nondemolition measurement of the photon number in the cavity can be done by a Ramsey spectroscopy experiment on the dark states [22]. This constitutes an alternative strategy to electron shelving based methods [32].

Discussion.—A scheme for robust qubits based on continuous dynamical decoupling was presented. The scheme is general in the sense that it can be applied to all systems satisfying Eq. (1), but otherwise the systems can have different characteristics. Unlike most commonly used methods, our scheme is applicable to systems with half-integer total angular momentum.

Although our example utilizes the $D_{3/2}$ subspace, in principle, the scheme can also be applied to subspaces of a different total angular momentum, such as the $D_{5/2}$ subspace of the calcium ion. In this case, a protected qubit subspace can be achieved by first an on-resonance J_x coupling of all $P_{3/2}$ states (which results in four J_x eigenstates), and second, by the on resonance coupling of the $|d_0\rangle$ and $|d_5\rangle$ states to one of the above eigenstates (resulting in one dark state), and by the on-resonance coupling of the $|d_2\rangle$ and $|d_3\rangle$ states to another J_x eigenstate (resulting in a second dark state). The ability to couple negative angular momentum states with positive angular momentum states constitutes a necessary condition for satisfying Eq. (1).

The scheme was analyzed in detail for a system of trapped ions based on optical control, in which the quantum memory consists of a string of ions that could either exist on its own or inside a larger crystal of a different species [31]. The simplicity of the scheme, which does not require complex laser pulses, enlarges the scope of quantum memories to laser control and provides new perspectives for laser manipulations.

By combining the setup with a stripline resonator, a conversion between an optical photon to a microwave photon could be achieved. Our scheme can also be realized with barium ions which have a lifetime of ~20 s. Such a long lifetime would enable a relaxation of the requirements on the number of ions and the cavity damping rate, resulting in a simpler experimental realization, and would also increase the storage time by one further order of magnitude.

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[23] However, it is possible to keep the intensity of the laser beams stable at a level considerably below 1%, including at frequencies of 1 Hz or below, by the use of commercial power stabilizers (liquid crystal noise eaters, http://www.thorlabs.com).
[24] The actual T_2 due to a Markovian noise is $1/g^2\tau_c$, where g is the rate of the noise and τ_c is the correlation time of the noise. A lower bound on T_2 is given by $1/g$, which is achieved for a long τ_c ($\tau_c = T_2$).
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