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Nuclear Instruments and Methods in Physics Research A 441 (2000) 203–208

NUCLEAR INSTRUMENTS & METHODS IN PHYSICS RESEARCH Section A

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Density distribution in laser-cooled bunched beams

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Abstract

Knowledge of the density distribution of a bunched beam is important for controlling and damping instabilities which often depend on the distribution function (A.W. Chao, in Physics of Collective Beam Instabilities in High Energy Accelerators, Wiley, New York, 1993). In this paper, experimental studies of the longitudinal bunch shapes and temperatures of laser-cooled bunched beams are presented. These studies are combined with measurements of the transverse density distribution of the beam using a novel technique (Madsen et al., Proceedings of the Sixth European Particle Accelerator Conference, EPAC'98, Stockholm, IOP Publishing, Bristol 1998, p. 1046). The bunch shapes for various temperatures comply with a model where the transverse temperature is assumed to be high. However, in the longitudinally space-charge limited case, it is observed that the maximum attainable density is much lower than it would be in a completely space-charge limited beam. Furthermore, it is observed that this maximum density decreases with increasing current. These observations might be important for the beam quality obtainable with laser-cooling.

PACS: 41.75.Ak; 29.20.Dh; 29.27.Bd; 52.25.Wz

Keywords: Laser-cooling; Bunched beams; Space-charge tune shift

Beam cooling is important for many storage ring applications. The introduction of laser-cooling in a storage ring has, because of its relatively short cooling times, and high cooling force, arisen much interest [1]. Especially in work on inertial confinement fusion, the ability of cooling bunched beams is of interest, due to the high currents and hence instability problems during acceleration in some of the proposed methods [2].

In this paper, recent results from studies of the longitudinal as well as transverse density profiles of a bunched beam during laser-cooling are presented. The most simple assumption is that the longitudinal density distribution is a Boltzmann distribution with only a weak (logarithmic) dependence on the transverse beam size [3]. The measurements do indeed agree with this assumption for beams which are not space charge limited in the longitudinal

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0168-9002/00/$ - see front matter
PII: S0168-9002(99)01134-1

dimension. However, in order to correctly model the measurements it is necessary to take into account that laser-cooling induces parabolic longitudinal velocity distributions and not Gaussian as in the Maxwell–Boltzmann distribution.

In the longitudinally space charge limited case deviations from the simple model, which seem to stem from variations in the transverse beam size, are observed. A novel technique for transverse beam profile diagnostics, which images the fluorescent light from the laser excited ion beam onto a high-resolution CCD [4], has made it possible to simultaneously monitor the transverse beam profiles. It is observed that at low current all three spatial dimensions of the bunches increase as a cube root in the number of particles, i.e. the density is constant. However, for higher currents the vertical dimension increases slightly faster in current, and the distributions become flat compared to their Gaussian shape at low current. Furthermore, the transverse sizes are far from the theoretical maximum density given by the confinement potential. These deviations and limitations are suspected to be due to tune resonances, and they might be important for the attainable beam quality with laser-cooling.

At the ASTRID storage ring a bunched beam of 100 keV $^{24}Mg^{+}$ ions has been laser-cooled by overlapping the ion beam with a counter-propagating laser-beam detuned slightly red from resonance with the mean ion beam velocity [5]. The beam is bunched by exciting a drift tube with a sinusoidally varying potential with a frequency of h times the revolution frequency. This splits the beam into h bunches by generating a longitudinal confinement force described in the bunch frame of reference by

$$F(x) = -F_0 \sin\left(\frac{2\pi h}{C}x\right), \quad F_0 = q\frac{2V_{\mathrm{rf}}}{C}\eta \sin\left(\frac{\pi h}{C}L\right) \tag{1}$$

where q is the charge of the circulating particles, V_{rf} is the amplitude of the applied sinusoidal voltage, C is the ring circumference, L is the tube length, x is the relative displacement of the particle from the center of mass of the bunch, and the slip factor $\eta = 1/\gamma^2 - \alpha$ where γ is the relativistic factor and α is the momentum compaction factor.

The transverse and the longitudinal beam profiles, as well as the longitudinal velocity distribution have been monitored. The transverse profiles were measured using a novel technique employing the fluorescent light from the laser-excited ion beam. This technique has been described elsewhere [4]. The longitudinal profiles where measured by measuring the induced sum signal on a beam position pickup. Finally, the longitudinal velocity distribution was measured by letting the beam pass through a drift tube which can be excited by a DC voltage. When excited the particles local velocity in the tube is shifted, thus the Doppler shifted resonance frequency is shifted. By sweeping the voltage on the tube and at the same time monitoring the fluorescence from the laser-excited beam with a photo-multiplier the longitudinal velocity distribution can be measured [6].

In Ref. [3], a self-consistent solution to the Maxwell–Boltzmann distribution, including the confinement and the space-charge potential is used to find the longitudinal bunch shapes. The model assumes that the transverse temperature is high, such that transverse space-charge forces are weak, and the longitudinal dimension therefore only weakly coupled to the transverse (through the geometric factor g_0). In Ref. [3], a harmonic potential is used. As long bunches are expected, the actual RF potential rather than a harmonic potential has been included in the calculations (extracted from Eq. (1)), and the longitudinal density distribution is found to be given by

$$\frac{\rho_{\mathrm{L}}(z)}{\rho_{\mathrm{L}}(0)} = \exp\left\{-\frac{F_0 C}{2\pi h k_{\mathrm{B}} T_{||}}\left[1-\cos\left(\frac{2\pi h}{C}z\right)\right] + \frac{g_0 q}{4\pi\varepsilon_0 k_{\mathrm{B}} T_{||}}(\rho_{\mathrm{L}}(0)-\rho_{\mathrm{L}}(z))\right\} \tag{2}$$

where $\rho_{\mathrm{L}}(z)$ is the line charge density in the beam, $T_{||} = \frac{1}{2}m\langle v_{||}^2\rangle$ is the longitudinal temperature (m the ion mass and $v_{||}$ the longitudinal velocity relative to the mean velocity of the beam) and g_0 is the geometric factor, which for long bunches (radius $\ll$ length) is given by $g_0 = 0.67 + 2\ln(b/a)$ [7], i.e. radius much smaller than length. Where $b \gg a$ is the radius of the vacuum chamber and a is the beam radius.

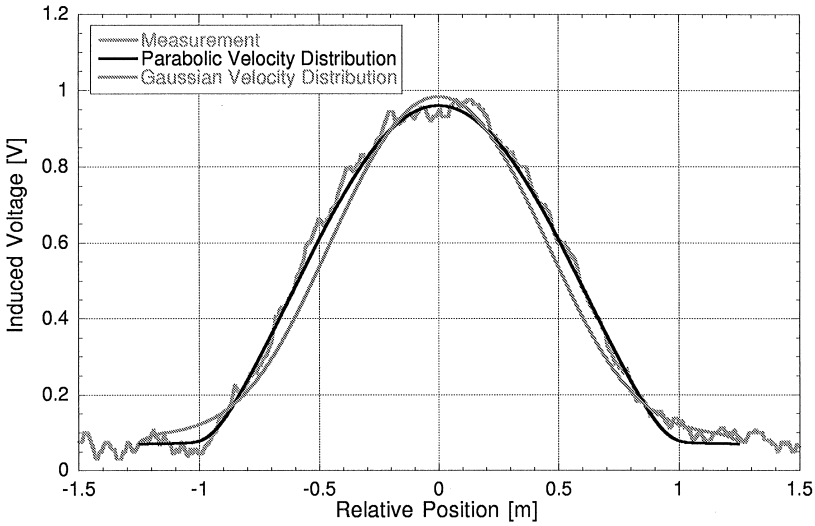

Fig. 1. Measured bunch profile together with the two discussed model distributions. The longitudinal rms velocity spread is 100 m/s.

In Fig. 1, is an example of a measured longitudinal bunch profile. The profile is accompanied with two model results. One observes that the model assuming a Gaussian longitudinal velocity distribution deviates considerably from the measurement. If a parabolic model for the velocity distribution is used, Eq. (2) changes to

$$\frac{\rho_L(z)}{\rho_L(0)} = 1 - \left\{ \frac{F_0 C}{2\pi h \xi k_B T_{||}} \left[\cos\left(\frac{2\pi h}{C} z \right) - 1 \right] + \frac{g_0 q}{4\pi\varepsilon_0 \xi k_B T_{||}} (\rho_L(0) - \rho_L(z)) \right\} \quad (3)$$

where $\xi = \frac{5}{2}$. The ξ factor stems from a parabolic distribution being $(1 - x^2/x_0^2)$ whereas a Gaussian is $\exp(-x^2/2\langle x^2\rangle)$ and for a parabola $\langle x^2\rangle = 5x_0^2$ (details in Ref. [8]). Fig. 1 shows that Eq. (3) is in better agreement with the measurement.

Fig. 2 shows a series of velocity distributions and simultaneously measured bunch profiles. As discussed above the longitudinal velocity distributions induced by laser-cooling are parabolic. The reason is that laser-cooling tends to reduce the tails, as the synchrotron oscillations oscillate all particles which are too warm into resonance with the lasers at least once per oscillation, thus slowly cooling the particles that may have been lost by collisions. As the rate of collisions causing particles to be lost decreases fast with increasing capture range the tails at higher velocity spreads are vanishing [9]. Fig. 2 shows that the model predict the longitudinal bunch shapes very well.

Fig. 3 shows the results of a measurement of many bunches at different longitudinal velocity spreads and particle numbers. Each dot represents an average of bunches with the same number of particles and the same longitudinal velocity spread. The longitudinal velocity profile is measured as an average of several injections (typically 20) whereas the bunch profiles are measured in one shot.

The agreement of the parabolic modified distribution with the measurements is quite good. However, it is observed, as earlier [5], that for the coldest beams there is a slight tendency of the bunches to be shorter than expected in the high current regime. One possibility for the discrepancy could be that the beam was space charge limited in the transverse dimensions too, but then one would expect the bunches to become slightly longer than in the uncoupled case due to the increased space charge forces, thus this seems not to be the explanation. The maximum deviation observed is about 6%, which corresponds to a correction in g_0 of about 18%. This would correspond to a variation in either the beam size or the vacuum chamber

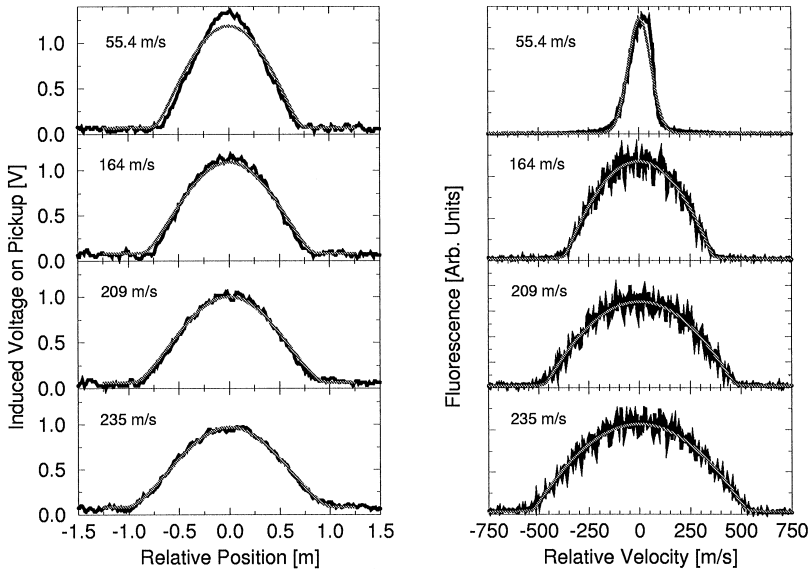

Fig. 2. Longitudinal bunch and velocity profiles for bunches of 4.06×10^6 particles. The gray lines in the velocity profiles are parabolic fit to the data (folded with the laser line shape), and in the bunch profiles it is the theoretical profile extracted from the parabolic modification of the Maxwell–Boltzmann distribution. The velocities in the upper left corners of each curve are the extracted rms velocity spreads.

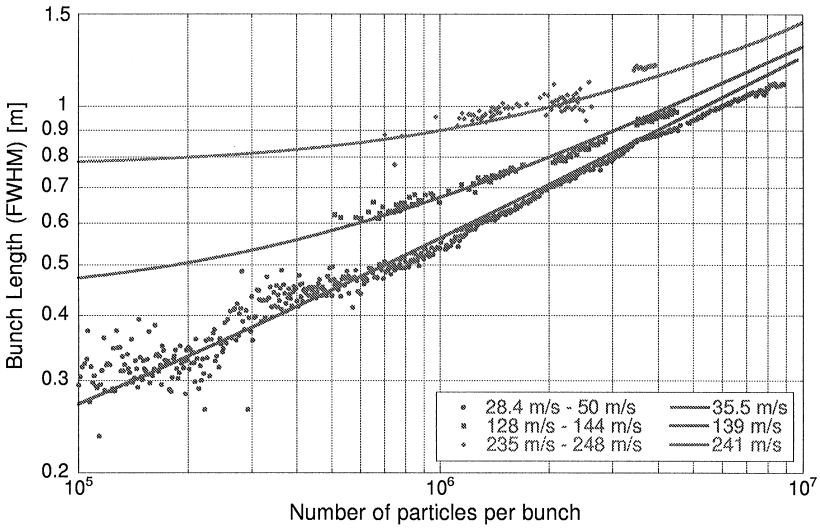

Fig. 3. The bunch length versus the number of particles in the bunch. The solid lines are results from the parabolic model. A transverse beam size of 2 mm has been assumed in the calculation ($b = 50$ mm). The velocities indicated in the caption are the RMS velocities measured simultaneously with the bunch measurements (data points) and used for the calculation in the parabolic model (solid lines).

radius of about 60%. Thus, large variations in the beam size would account for the changes.

In order to understand the deviations it is therefore necessary to look at the simultaneous measurements of the transverse profiles done during these studies. Fig. 4 shows all three spatial dimensions of the ion beam as a function of current. The variations in beam size are up to $\sim 60\%$ from the

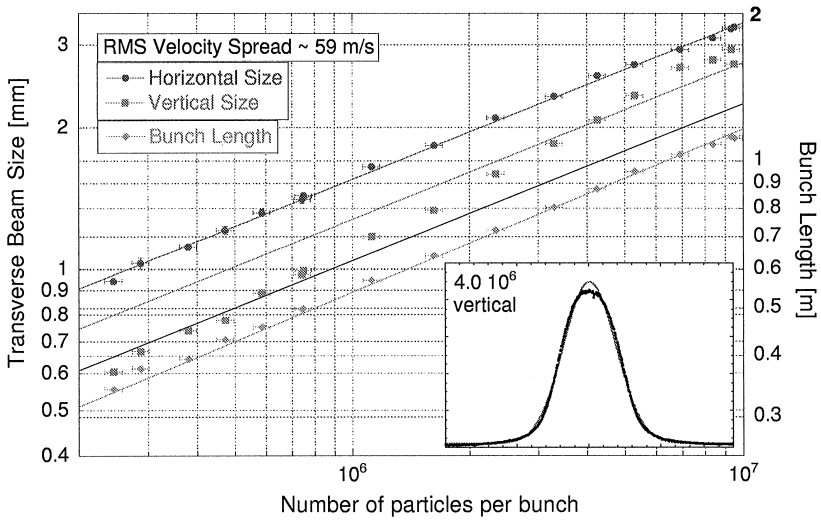

Fig. 4. The transverse and longitudinal dimensions of the laser-cooled beam as a function of the current. The colored lines are cube root fit to the data, whereas the black line is a parallel translation of a cube root fit to guide the eye. The inset shows an example of a vertical profile (the gray line is a Gaussian fit). The beta functions at the point of measurement are $\beta_x = 12.1$ m and $\beta_y = 2.6$ m.

2 mm used for the calculation in Fig. 3, in agreement with the expectations from the bunch length measurements. Thus, the model corresponds very well to the observations.

Furthermore, Fig. 4 shows that all the three spatial dimensions of the bunches scale as a cube root in the number of particles, except that the vertical dimension grows faster for particle numbers above $\sim 10^6$. This means that the central density of the bunches is approximately constant, which might indicate a zero emittance beam whose density would be given purely by the confinement forces. However, the transverse spatial distributions are Gaussian, and the measured peak density is about an order of magnitude lower than what would be expected from the confinement forces. Thus, something else limits the beam density. It is likely that it is a tune resonance which limits the density, as a constant density means a constant first-order space charge tune shift, a situation similar to the one observed with electron cooled bunched beams in Ref. [10]. For a beam with a Gaussian transverse distribution and parabolic bunch shapes, the peak density in the bunches is $n_b = 3\sqrt{2}N/8\pi\sigma_\perp^2 L$, where N is the number of particles in the bunch, $\sigma_\perp$ the beam size and L the FWHM bunch length. In this case the maximum (Laslett) space-charge tune shift is [10]

$$\Delta Q = \frac{e^2}{4\varepsilon_0 m\omega_0^2\gamma^3 Q} n_b \tag{4}$$

which, inserting the peak densities calculated from the measurements presented in Fig. 4 and an average tune $Q = 2.5$, calculates to tune shifts in the range 0.085–0.054 for low to high currents, respectively (corresponding peak densities: 2.3×10^5 and $1.5 \times 10^5\,\mathrm{cm}^{-3}$). The operating (bare) tunes are $Q_x = 2.27$ and $Q_y = 2.83$, which is about 0.07 away from the nearest resonance ($Q_x + Q_y = 5$). Thus, the maximum tune shift brings the tune beyond the nearest resonance. This phenomenon is in agreement with earlier observations, from simulations [11] and experiments [10] that the tune shift which determines the influence of an instability is actually the RMS tune shift which is a factor of 2 less than the maximum tune shift (and in this case the RMS tune shift therefore only brings the tune close to the resonance, but not across it). Furthermore, simulations by Machida [11] and experiments by Chanel [12] have shown that when approaching a resonance, the transverse profile of the beam changes

from being Gaussian to a more uniform shape such that the tune shift of the particles tend to be constant throughout the beam. This is in agreement with the observation in these studies, that the vertical beam size becomes non-Gaussian at particle numbers above $\sim 10^6$ (see inset in Fig. 4). Thus, the presented observations indicates that the beam density is tune resonance limited.

However, there is no explanation for the decrease of the tune shift with increasing current, which is the opposite effect of what was observed in Ref. [10]. As it is the vertical dimension alone which blows up it is speculated that this is due to some extra heating mechanism, perhaps higher-order driving terms in the resonance, which are amplitude dependent. This will affect the vertical dimension the most, as the longitudinal to horizontal coupling (and thereby the horizontal cooling) is expected to be stronger than the longitudinal to vertical coupling due to the horizontal dispersion, as earlier observed in Ref. [13].

The longitudinal and transverse spatial profile of a laser-cooled bunched beam in a storage ring have been studied. The longitudinal shapes showed good agreement with a Maxwell–Boltzmann distribution modified to account for the parabolic longitudinal velocity distributions induced by laser-cooling, and taking into account the weak (logarithmic) dependence on the transverse beam size.

As also observed in coasting beams [4] it was furthermore observed that the densities reached corresponds to space-charge tune shifts which bring the betatron tune close to resonances, these therefore seem to limit the maximum density to be lower than that given by the confinement potential alone. This corresponds to observations done in electron-cooled bunched beams [10]. These results might indicate important limitations to the beam quality obtainable by longitudinal laser-cooling of bunched beams.

This work was supported by the Danish National Research Foundation through the Aarhus Center for Atomic Physics, and by the US Department of Energy, Nuclear Physics Division, under Contract W-31-109-Eng-38.

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