Cavity QED with ion Coulomb crystals

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Abstract. We present a proposal for using ion Coulomb crystals in cavity QED experiments. Due to the high degree of spatial confinement of ions in Coulomb crystals, such ion ensembles will have some advantages over systems based on neutral atoms. Simulations of a system with up to 3200 ions coupled to a standing wave cavity with a finesse of about 4000 is presented, and show that our setup can be used for experiments within the strongly coupled regime of cavity QED. Furthermore, the potential of the system as a quantum memory device for light, is considered.

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INTRODUCTION

Cavity quantum electrodynamics (QED) provides a playground for light-matter interactions [1]. While such interactions are interesting merely from a fundamental physics perspective, recent years of experimental progress has promoted cavity QED as a tool for quantum information science where an efficient light-matter interface is in high demand for many applications [2, 3].

An attractive scenario for cavity QED experiments is that in which the atoms are tightly confined compared to the wavelength of the light (the Lamb-Dicke regime). Traditionally, experiments have been based on neutral atoms, making this scenario very challenging to realize. Trapped ions, on the other hand, can provide relatively easy access to the Lamb-Dicke regime and are thus an attractive alternative to the neutral atoms. Recent reports on such experiments with single ions [4, 5] has motivated us to engage in the field of cavity QED with a system based on an ensemble of several thousand ions.

Following a brief review of some of the basic theory needed for a description of our system, we present our experimental setup as well as simulations of some of its key parameters. Finally, we conclude and give and outlook on the potential of our setup in the context of quantum information science.

THEORY

Figure 1 shows the generic cavity QED experiment. In this particular case N ions are situated inside the TEM_{00} mode of an optical cavity. For simplicity we assume that one mirror is perfectly reflecting while the other is partially reflecting and treat the ions as two-level systems. We consider the following three processes by which excitation,

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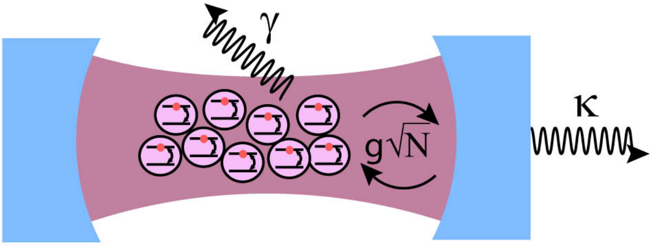

FIGURE 1. Cavity containing N two-level ions. g is the single ion-vacuum field coupling rate, γ is the spontaneous emission rate, and κ is the cavity decay rate.

within the system, can be exchanged. 1) Photon emission from an excited atomic state into the cavity mode and excitation of an ion by absorption of a cavity field quanta, which occurs at a rate $g(\mathbf{r})$ (for a single ion located at position $\mathbf{r}$). 2) Decay of an excited state to other field modes than the TEM$_{00}$ mode of the cavity, which occurs at a rate γ. 3) Decay of the cavity field through the partially reflecting mirror, which occurs at a rate κ. The two latter processes are determined by the spontaneous emission rate of the excited state of the ions and the mirror reflectivity, respectively, and comprise the excitation losses of the cavity-ion system, whereas the former process governs the coherent evolution of the system. The coupling strength between a single ion at position $\mathbf{r}$ and the cavity vacuum field is given by

$$g(\mathbf{r}) = g_0 \Psi(\mathbf{r}) , \tag{1}$$

where $\Psi(\mathbf{r})$ is the modefunction of the cavity field and where

$$g_0 = \frac{D}{\hbar}\sqrt{\frac{\hbar\omega}{2\varepsilon_0 V}} , \tag{2}$$

with D being the dipole moment, ω the frequency of the transition and V the volume of the TEM$_{00}$ mode defined as $V = \int |\Psi(\mathbf{r})|^2 d\mathbf{r}$.

The relative strength of the coherent dynamics to the dissipative processes can be expressed through the cooperativity parameter, C [6], defined as

$$C = \frac{g_{\text{coll}}^2}{2\kappa\gamma} , \tag{3}$$

where g_{coll} is the collective coherent coupling strength of all the ions given by

$$g_{\text{coll}}^2 = \int g^2(\mathbf{r}) n(\mathbf{r}) d\mathbf{r}, \tag{4}$$

with $n(\mathbf{r})$ being the density of the atomic medium. For the case where $C \gg 1$, the coherent dynamics dominate over the dissipative effects and the system is said to be in the so-called strong coupling regime [6].

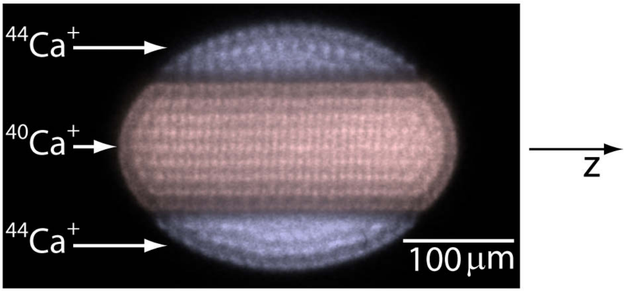

FIGURE 2. Image of bi-crystal consisting of ^{40}Ca$^+$ (inner component) and ^{44}Ca$^+$ (outer component). The image shows ions that are located near the imaging plane of the CCD camera.

EXPERIMENTAL SETUP

Experiments within cavity QED have thus far been conducted in a standing wave configuration with neutral atoms for the most part [7, 8, 9], and have often suffered from a time-varying coupling strength $g(\mathbf{r})$ as well as a limited interaction time of the atoms with the cavity. This is due to the motion of the neutral atoms in their trapping potentials and despite much experimental progress in this field, tight confinement of neutral atoms remains a challenge [10, 11]. Our motivation for using cold trapped ions is that it allows for strong confinement and thus an easier access to the so called Lamb-Dicke regime in which the ions are well confined on a length scale defined by the wavelength of the optical transition in consideration. Furthermore, long storage times ($\sim$ hours) renders the issue of interaction time virtually non-existent.

When the ions are cooled below a certain critical temperature, which for our parameters is accessible through Doppler laser cooling [12, 13], they form a spatially ordered structure [14, 15] known as an ion Coulomb crystal (see e.g. contribution by A. Mortensen to the present proceedings and references therein). We envision the use ion Coulomb crystals in a linear Paul trap as our atomic medium. Specifically, bi-crystals composed of two different isotopes of calcium, namely ^{40}Ca$^+$ and ^{44}Ca$^+$. In Fig. 2 an image of such an ion Coulomb crystal, produced in our group, is shown. The two-dimensional image only shows the ions that are located near the imaging plane of the CCD camera. The real three-dimensional structure can be visualized by rotation around the z-axis which gives an overall spheroidal shaped object with a central cylindrical structure of ^{40}Ca$^+$ ions. The complete radial separation of the two isotopes is a consequence of the dynamical radial trapping in the linear Paul trap [16].

Cooling of the ^{44}Ca$^+$ ions through Doppler laser cooling will allow for sympathetic cooling of the ^{40}Ca$^+$ ions through mutual Coulomb interaction [17]. In this manner, experiments can be conducted continuously on the ^{40}Ca$^+$ ions without the need for intervals of cooling.

Figure 3 shows the energy levels of the calcium ions (applicable to both isotopes under consideration). The thick lines indicate the transitions used for Doppler laser cooling. The atomic transition used in the experiments is the 3d ^{2}D$_{3/2}\leftrightarrow$4p ^{2}P$_{1/2}$ transition at a wavelength of 866 nm where highly reflective cavity mirrors are readily available. With our present choice of one highly reflecting mirror (T $\simeq$ 5 ppm) and an input coupler with T $\simeq$ 1500 ppm the cavity has a decay rate of $\kappa = 1.7$ MHz at this wavelength and

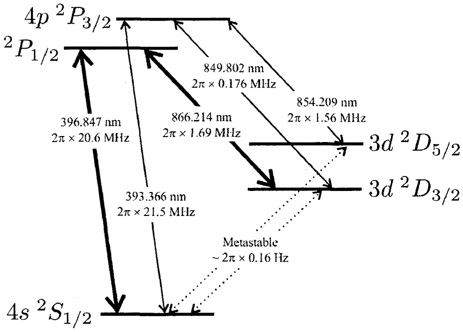

FIGURE 3. Energy level diagram of the $^{40}\mathrm{Ca}^+$ ion (also applicable to $^{40}\mathrm{Ca}^+$). Thick lines indicate the transitions used for Doppler laser cooling.

a finesse of about 4000.

Both the 3d $^2\mathrm{D}_{3/2}$ and the 4p $^2\mathrm{P}_{1/2}$ levels have magnetic substates and the transition used in the experiments is that between the $\mathrm{m_J} = 3/2$ of the 3d $^2\mathrm{D}_{3/2}$ state and the $\mathrm{m_J} = 1/2$ of the 4p $^2\mathrm{P}_{1/2}$ state, as it has the largest Clebsch-Gordan coefficient of the dipole transitions. On this transition, with our particular setup and an ion crystal of 3200 ions (assumed to couple identically to the cavity mode), a collective coherent coupling strength of $g_{\mathrm{coll}} = 23.1$ MHz should be feasible. Since the spontaneous emission rate of the 4p $^2\mathrm{P}_{1/2}$ state is $\gamma = 22.3$ MHz, this results in a value for the cooperativity parameter of $C \simeq 7$.

The numbers quoted above represent conservative estimates for parameters allowing for stable trapping and can be pushed some. An increase in coupling strength by a factor of 2 or 3 can in principle be achieved by increasing the density of ions in the Coulomb crystal. Furthermore, the current design is based upon a cavity length of about 10 mm, which gives a fairly large mode volume compared to experiments on neutral atoms, where mirror separations of the order of 10 μm has been achieved [10]. This larger mode volume gives rise to a lower coupling strength (c.f. Eq.(2)) but is necessary to allow for stable trapping of large ion Coulomb crystals between the mirror substrates. Despite the success of cavity QED experiments with single ions [4, 5], strong coupling with trapped ions has yet to be achieved, and for that reason, increasing the coupling strength through the use of large ion Coulomb crystals of several thousand ions (c.f. Eq.(4)) seems a promising route towards this goal.

COLLECTIVE COUPLING STRENGTH

The collective coupling strength, g_{coll}, can be found through spectroscopic measurements of the eigenfrequencies, of the composite ion Coulomb crystal-cavity system. In practice, what is mostly considered in such measurements is the transmission spectrum of the cavity, tuned on resonance with a particular transition in the atomic medium, in this case the 3d $^2\mathrm{D}_{3/2} \leftrightarrow$ 4p $^2\mathrm{P}_{1/2}$ transition of $^{40}\mathrm{Ca}^+$. It can be shown [18, 19], that this is given by

$$T = \left| \frac{\kappa(\gamma - i\Delta)}{(\lambda_+ + i\Delta)(\lambda_- + i\Delta)} \right|^2 , \tag{5}$$

where Δ is the detuning between the probe field and the ion-cavity resonance and $\lambda_\pm$ are the eigenvalues of the composite system, given by

$$\lambda_\pm = -\left(\frac{\kappa+\gamma}{2}\right) \pm \sqrt{\left(\frac{\kappa-\gamma}{2}\right)^2 - g_{\mathrm{coll}}^2} \, . \tag{6}$$

Such transmission spectra have been measured previously in neutral atom systems, even for single atoms [20]. Numerical simulations, shown in Fig. 4, reveal that, similar to experiments performed on neutral atoms [21], we should be able to see a signature of the so-called vacuum Rabi splitting, which manifests itself as splitting of the transmission spectrum into two peaks. This is clearly visible in both Figs. 4c) and d), where Fig. 4d) is based on the parameters quoted previously in this section for our setup with 3200 ions. This type of spectrum indicates a strong coupling between the atomic medium and the cavity field and as such, the numerical simulations presented here, strongly motivates the use of ion Coulomb crystals for cavity QED experiments.

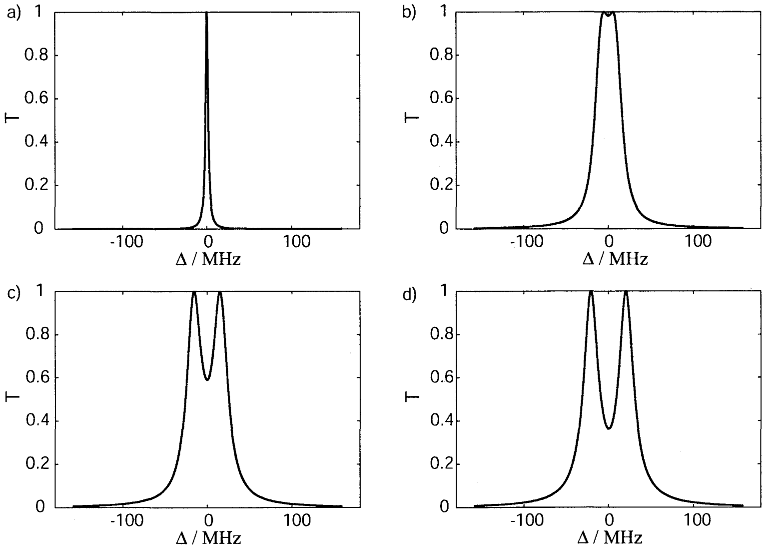

FIGURE 4. Cavity transmission vs. detuning, Δ. Parameters used in the simulations are $(\gamma, \kappa, g_{\mathrm{coll}})$ = a) $2\pi \cdot (22.3, 1.7, 5.8)$ MHz, b) $2\pi \cdot (22.3, 1.7, 14.1)$ MHz, c) $2\pi \cdot (22.3, 1.7, 19.2)$ MHz, and d) $2\pi \cdot (22.3, 1.7, 23.1)$ MHz.

CONCLUSION AND OUTLOOK

Here we have treated the ions as simple two-level systems, however, one could easily construct scenarios where they constitute Λ-level systems, defined by two magnetic substates of the 3d $^2D_{3/2}$ state each coupled to a certain magnetic substate of the 4p $^2P_{1/2}$ state (c.f. Fig. 3). The use of STImulated Raman Adiabatic Passage (STIRAP) techniques in such a system has been proposed as a means for mapping of quantum states of light onto the atomic medium [22], and in this manner ion Coulomb crystals could be employed as a quantum memory for light [23].

In conclusion, the work presented here indicates that systems based on ion Coulomb crystals may well be employed for cavity QED studies. A simple analysis showed that the coherent collective coupling strength, g_{coll}, could indeed be made high enough for experiments to enter a regime of strong coupling and it is our firm belief that ion Coulomb crystals will be able to add constructively to the merging fields of cavity QED and quantum information science.

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