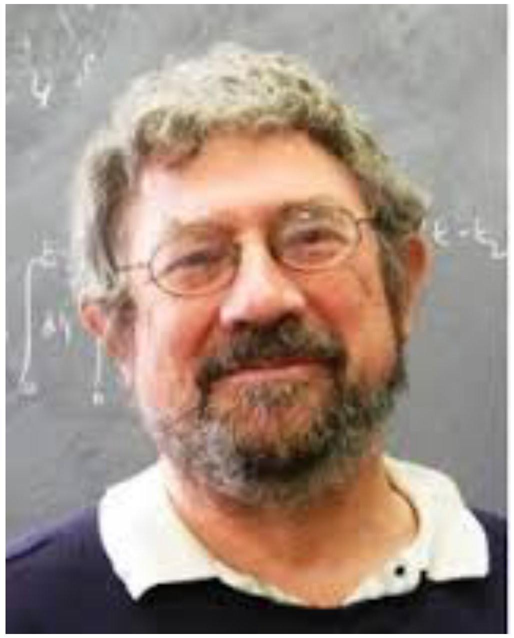
Nobel Price in Physics 2016

D. J. Thouless
PhD: Cornell
Univ. Washington



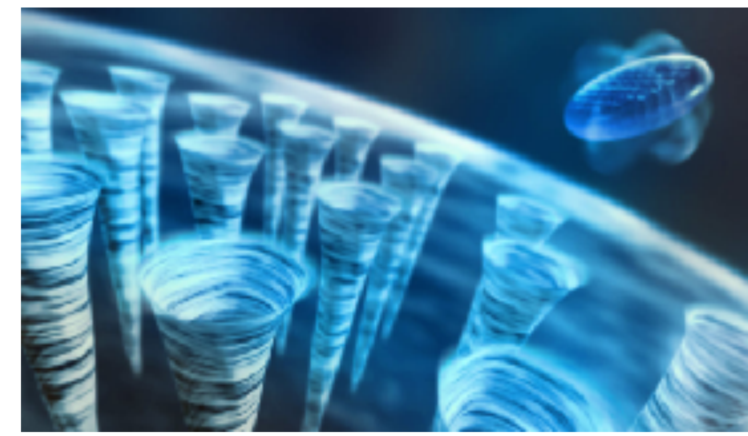
J. M. Kosterlitz
PhD: Oxford
Brown Univ.

F. D. M. Haldane
PhD: Cambridge
Princeton

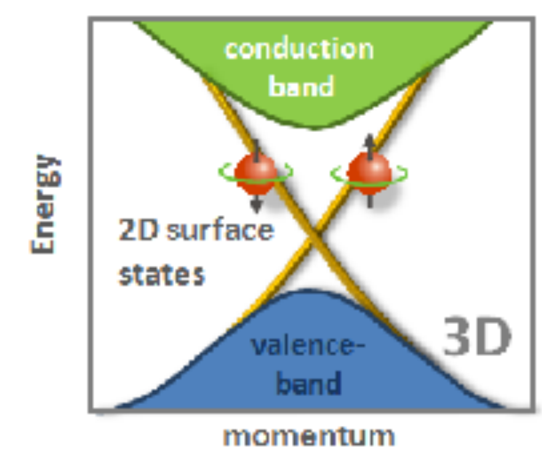
"for theoretical discoveries of topological phase transitions and topological phases of matter"

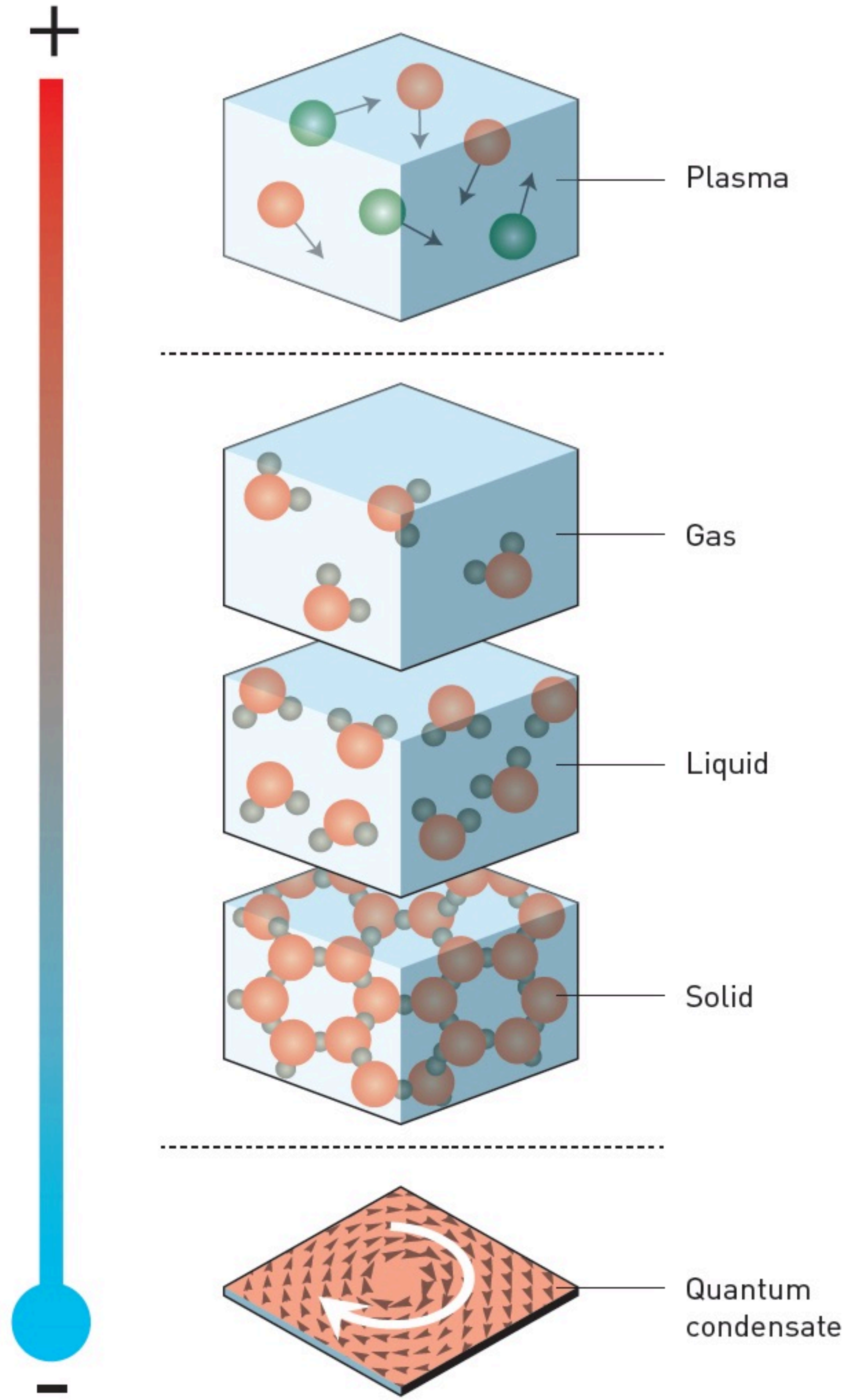
Outline

❶ Kosterlitz-Thouless phase transition.
Driven by topological defects.

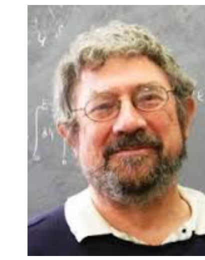


❷ Topological band theory
(Thouless + Haldane)



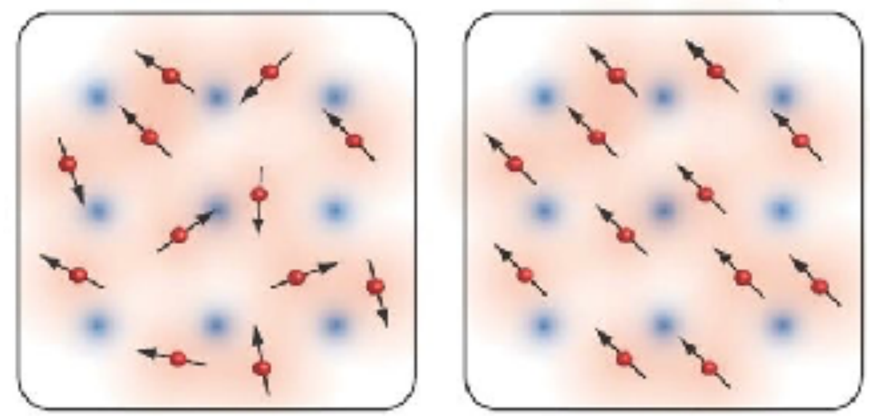

+
Plasma
Gas
Liquid
Solid
Quantum
condensate
−

Kosterlitz-Thouless phase transition

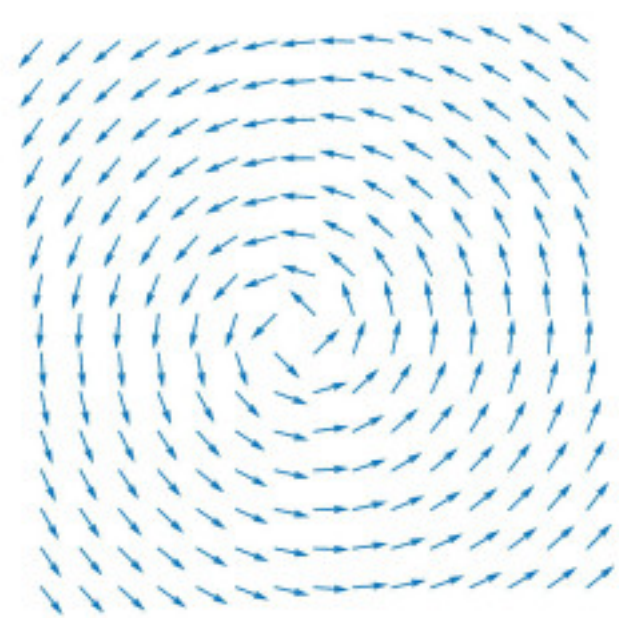
D. J. Thouless J. M. Kosterlitz

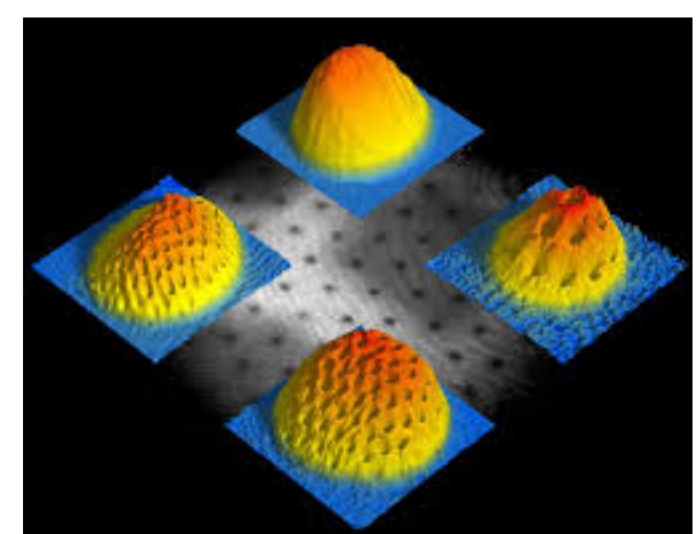
$T > T_c$ $T < T_c$



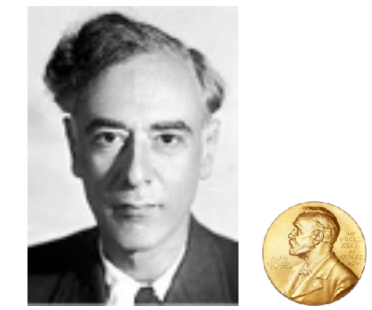
Usual phase transitions driven by *spontaneous broken symmetry*

A special kind of phase transition driven by topological defects
No broken symmetry



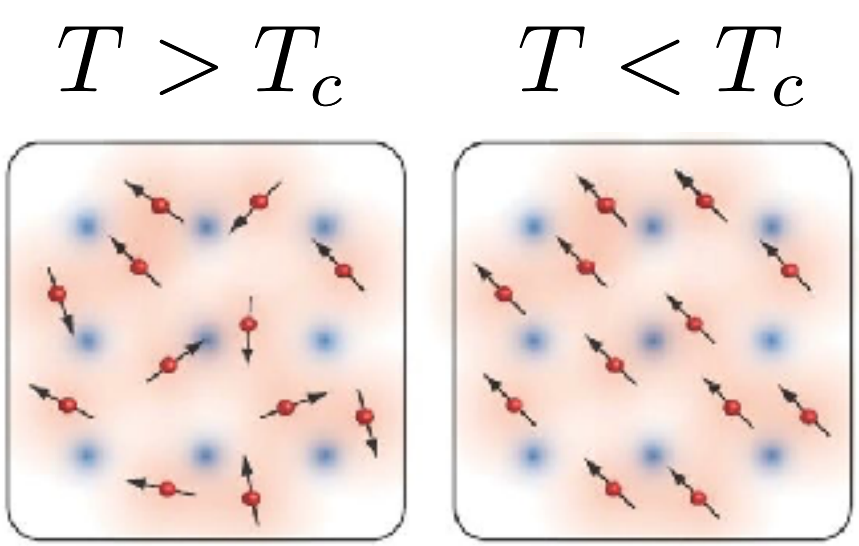


The “standard model” of phase transitions

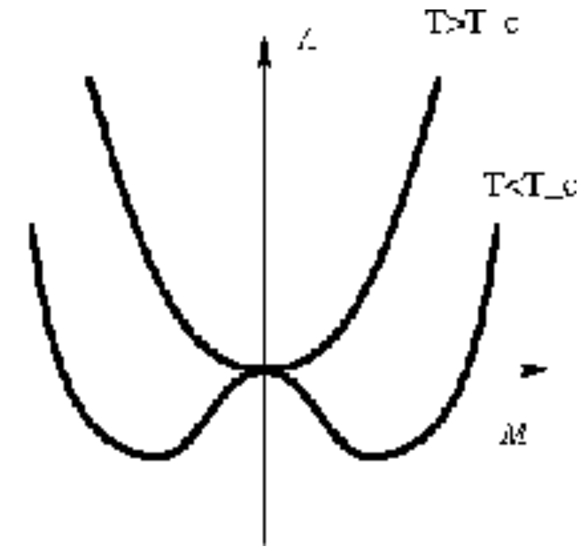


L. Landau

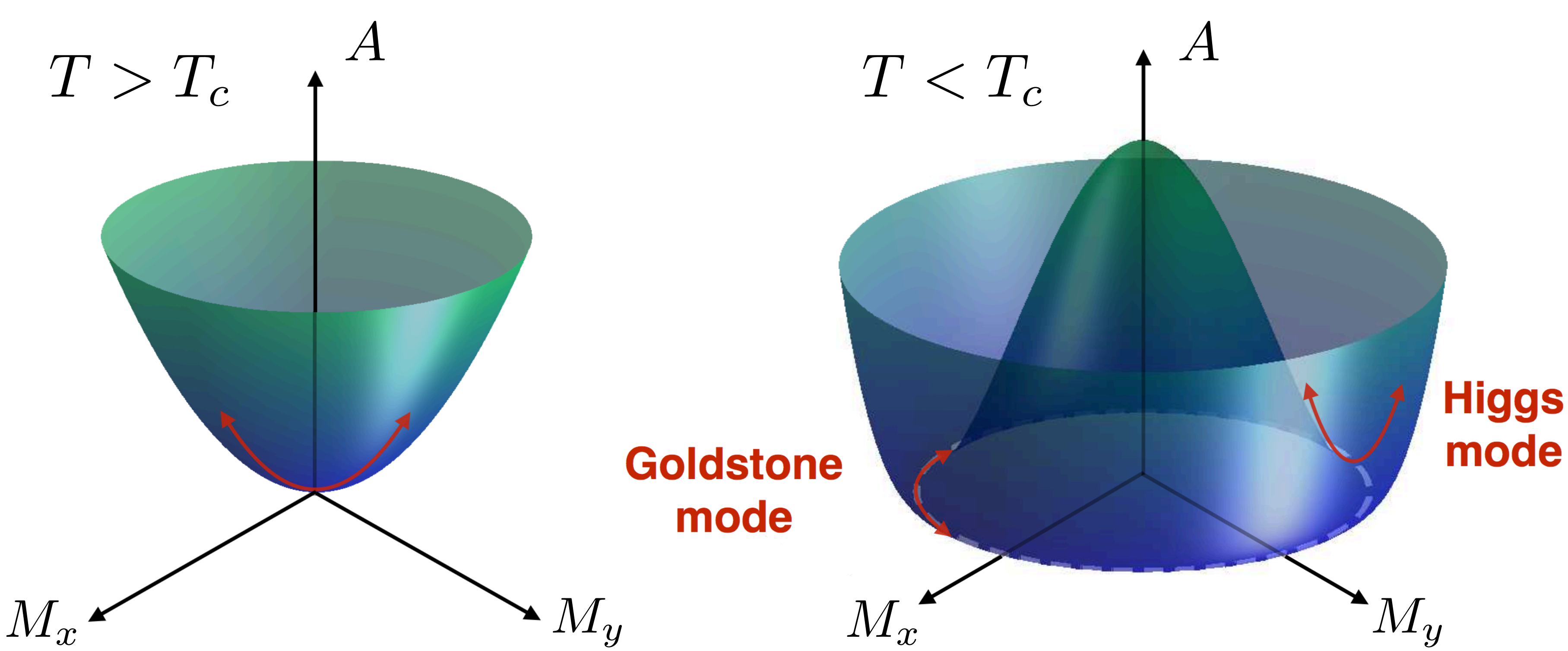
- Spontaneous symmetry breaking



Local order parameter M=⟨$\mathbf{s}_i$⟩



2D:



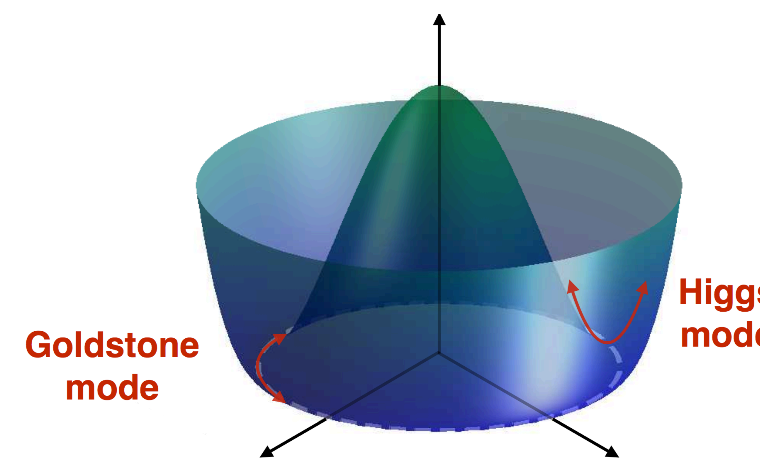
Low temperature quantum phases

❶ BCS superconductivity

Complex wave function $\psi = |\psi| \cdot e^{i\theta}$

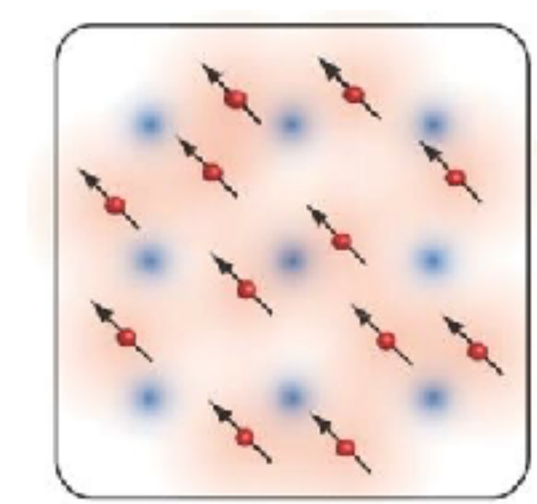
Ginzburg-Landau functional

$$\mathcal{F} = \frac{1}{2m}|\nabla\psi|^2 + \alpha|\psi|^2 + \frac{\beta}{2}|\psi|^4$$



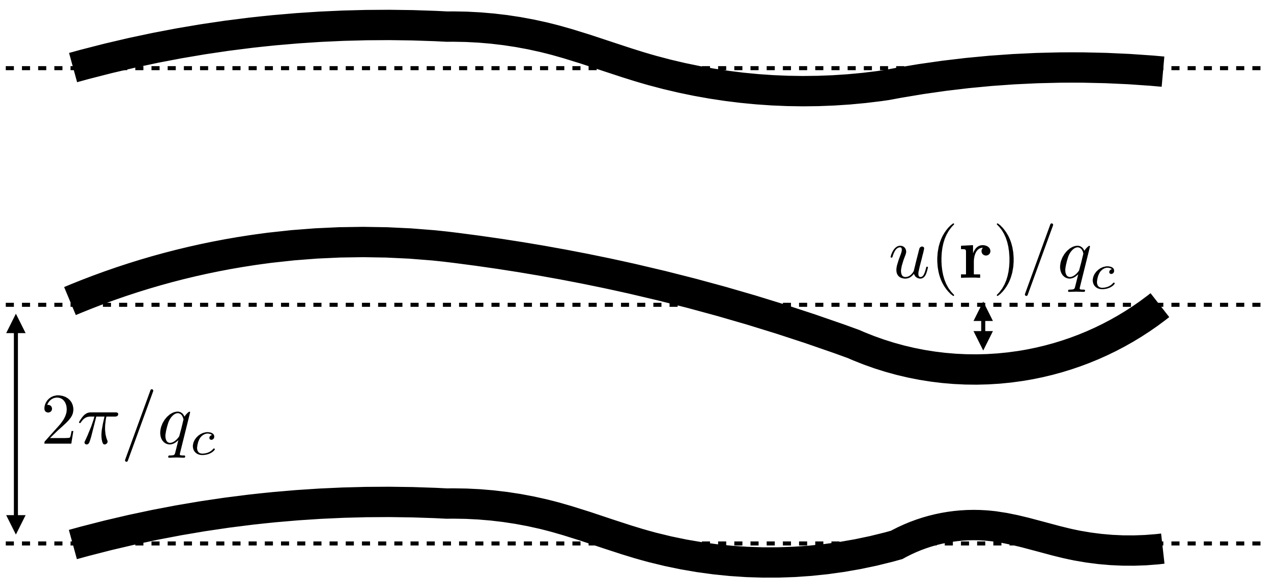
Broken symmetry: $\langle\psi\rangle \neq 0$

Angle θ like direction of magnetisation



❷ Bose-Einstein condensate
Gross-Pitaevskii functional

❸ Charge-density waves $n(\mathbf{r}) = n_0 + n_1 \cos[\mathbf{q}_c \cdot \mathbf{r} - u(\mathbf{r})]$



Same
universality
class

K. Wilson

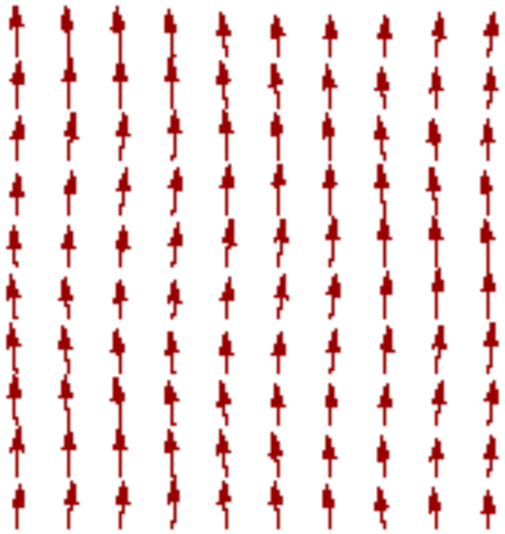
Systems have *same low-energy degree* of freedom: θ

The 2D XY model

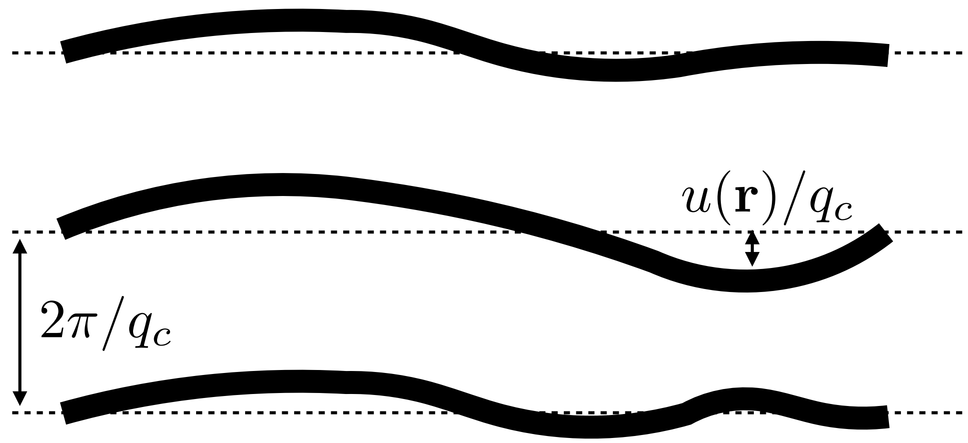
$$\boxed{H_{XY} = -J \sum_{\langle i,j \rangle} \cos(\theta_i - \theta_j) = -J \sum_{\langle i,j \rangle} \vec{s}_i \cdot \vec{s}_j}$$

Describes low energy degrees of freedom
of 2D magnets, superfluids, stripes …

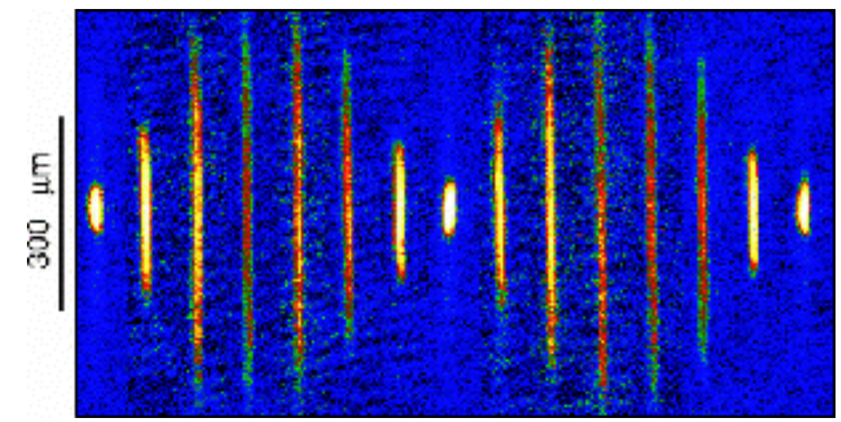
Phase fluctuations



Spin waves

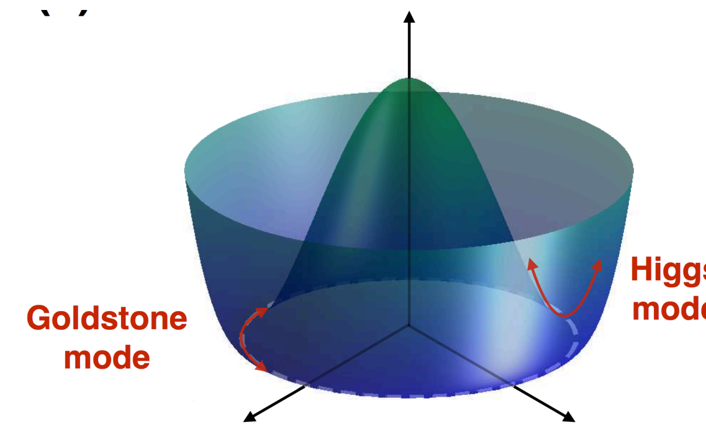


Stripe fluctuations



Supercurrents

Mermin & Wagner: Goldstone modes
destroy long range order in 2D



Low energy
DOS is too large

3D: $\int k^2 dk$ 2D: $\int k dk$

No broken symmetry $C(l,m) = \langle \mathbf{s}_l \cdot \mathbf{s}_m \rangle = \langle \cos(\theta_l - \theta_m) \rangle \rightarrow 0$
for $|\mathbf{r}_l - \mathbf{r}_m| \rightarrow \infty$

But still have a phase transition in 2D:
The Kosterlitz-Thouless transition

Low temperature - expand XY model in fluctuations:

$$H \simeq -J \sum_{\langle i,j \rangle} [1 - \frac{1}{2}(\theta_i - \theta_m)^2] = \frac{J}{2} \int d^2 r |\nabla \theta|^2$$

Gaussian model. Gives: $C(l,m) \sim \left(\frac{1}{|\mathbf{r}_l - \mathbf{r}_m|} \right)^{T/2\pi J}$

Power law = *algebraic order* = indicates fluid in *critical state*

Must be transition in between!

High temperature - expand in J/T:

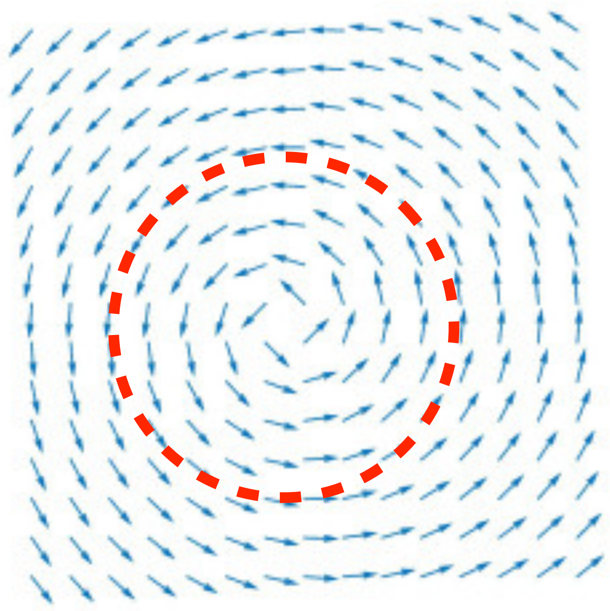
$$C(l,m) = \frac{1}{Z} \mathrm{Tr} \left[e^{\beta J \sum_{\langle i,j \rangle} \cos(\theta_i - \theta_j)} \cos(\theta_l - \theta_m) \right] \sim e^{-|\mathbf{r}_l - \mathbf{r}_m|/\xi}$$

$$\mathrm{Tr} = \int_0^{2\pi} d\theta_1 \ldots \int_0^{2\pi} d\theta_N$$

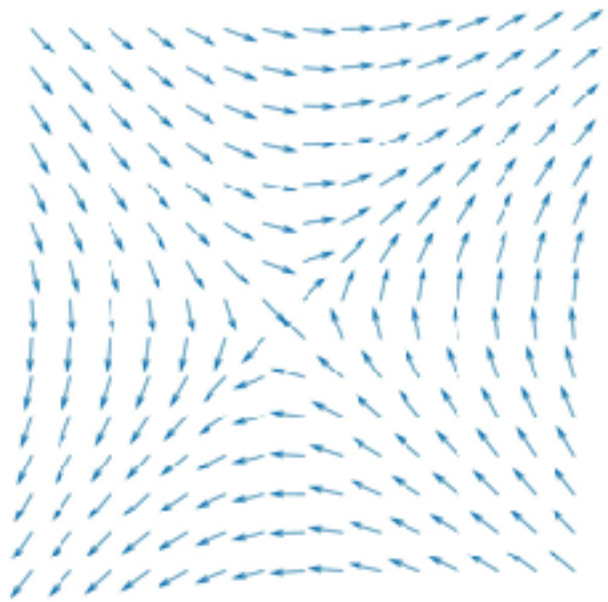
No order.

What is missing? Topological defects!

Vortices

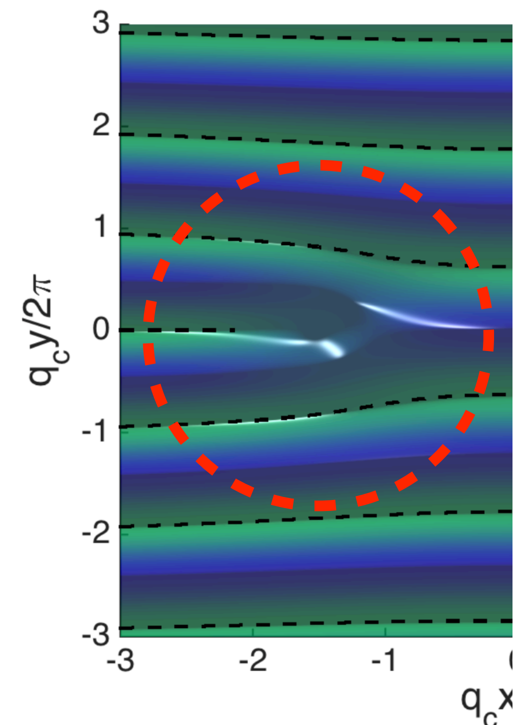


$n = 1$

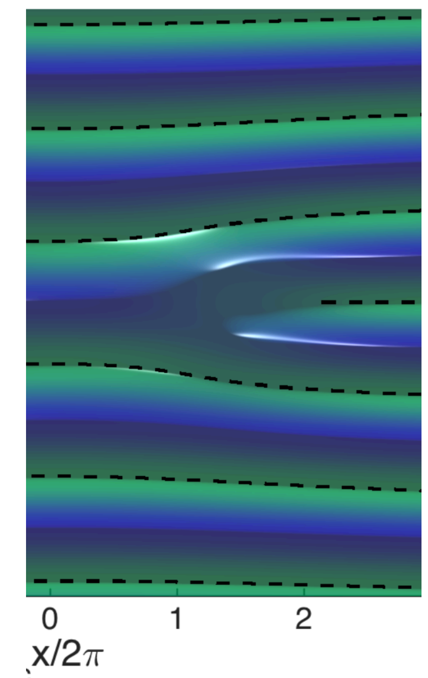


$n = -1$

Dislocations



$n = 1$



$n = -1$

"Charge" or "winding" number: $\oint d\theta = \oint d\mathbf{r} \cdot \nabla\theta = n2\pi$

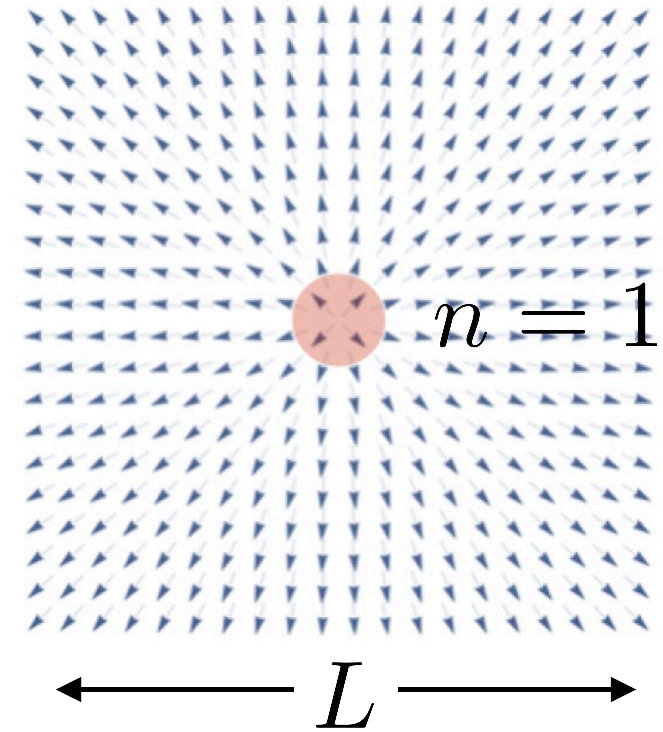
Integer. Cannot be changed by smooth deformation

$\boxed{\text{Topological property}}$

Energy of vortex: $\nabla\theta = \frac{1}{r}$

System size

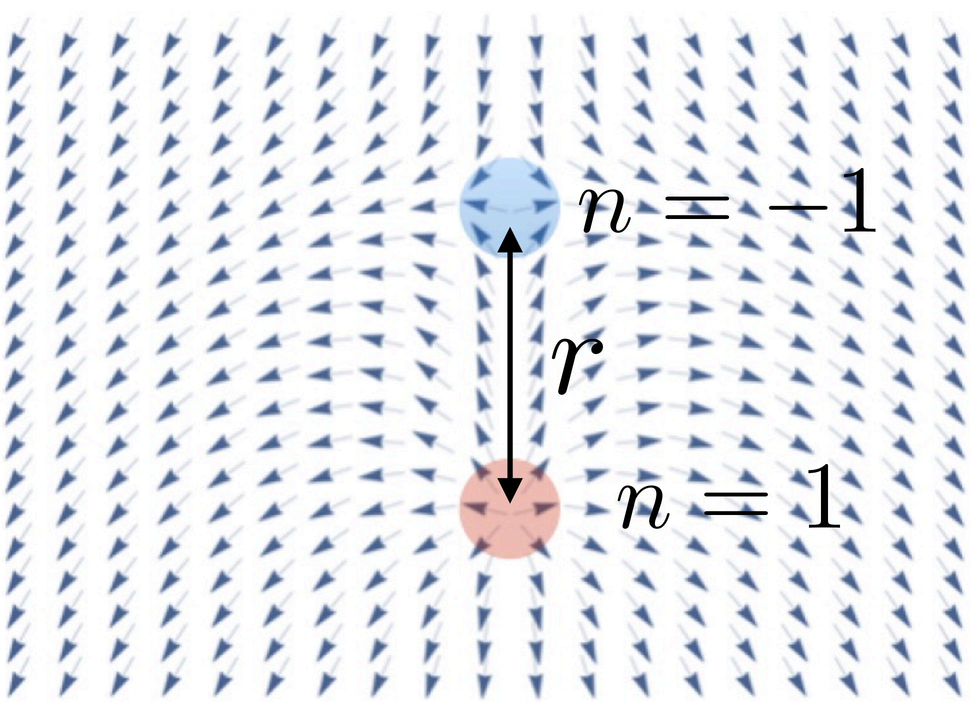
$$E_v = \frac{J}{2}\int d^2r|\nabla\theta|^2 = \frac{J}{2}\int d^2r\frac{1}{r^2} = J\pi\log(L/a)$$



Not excited thermally for large system

Can they be ignored? Not quite!

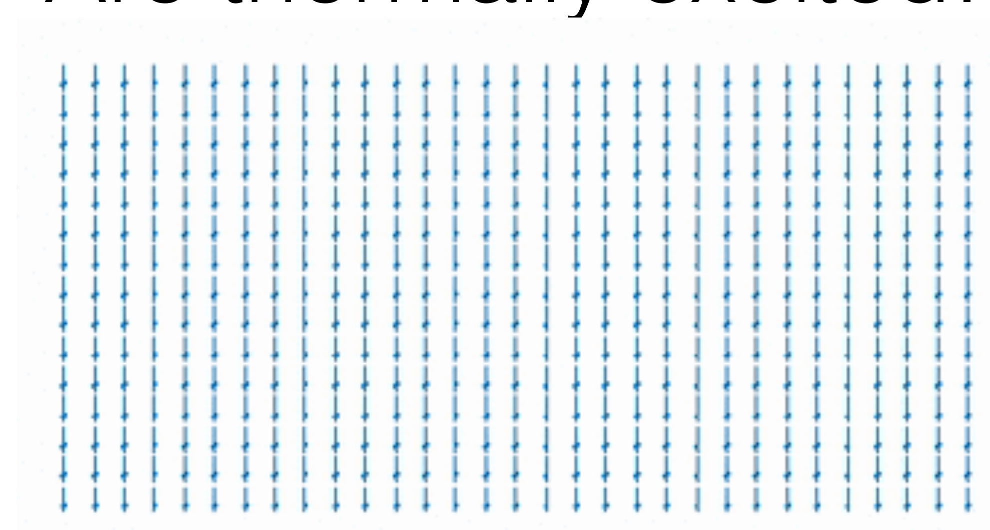
<u>Vortex-antivortex pair:</u>



$$E = J2\pi\log(r/a)$$

Local change of spins

Are thermally excited!



Kosterlitz-Thouless transition:

$$S = k_B \log \Omega$$

Free energy of defect: F=E-TS $= J\pi \log(L/a) - k_B T \log(L^2/a^2)$

Algebraically ordered phase melts when F=0

$$\boxed{T_{KT} = \frac{J\pi}{2k_B}}$$

Topological phase transition

No broken symmetry order parameter

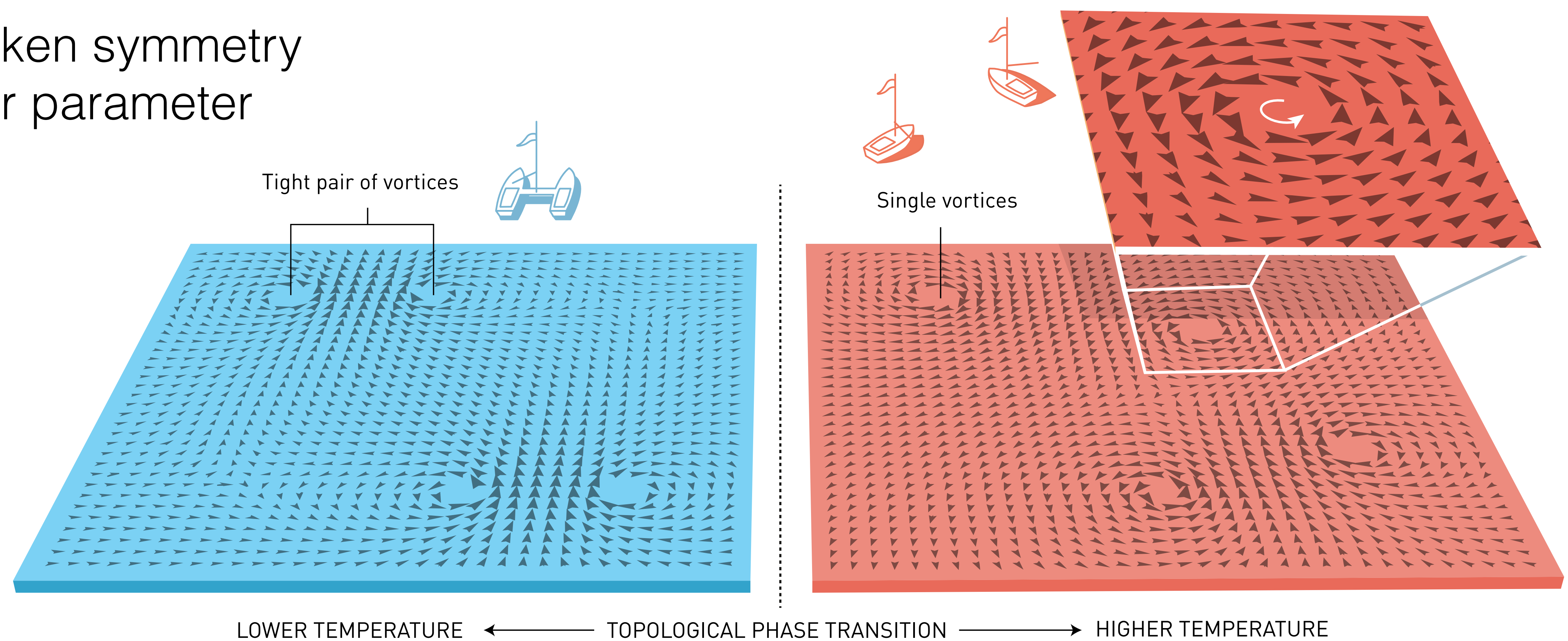
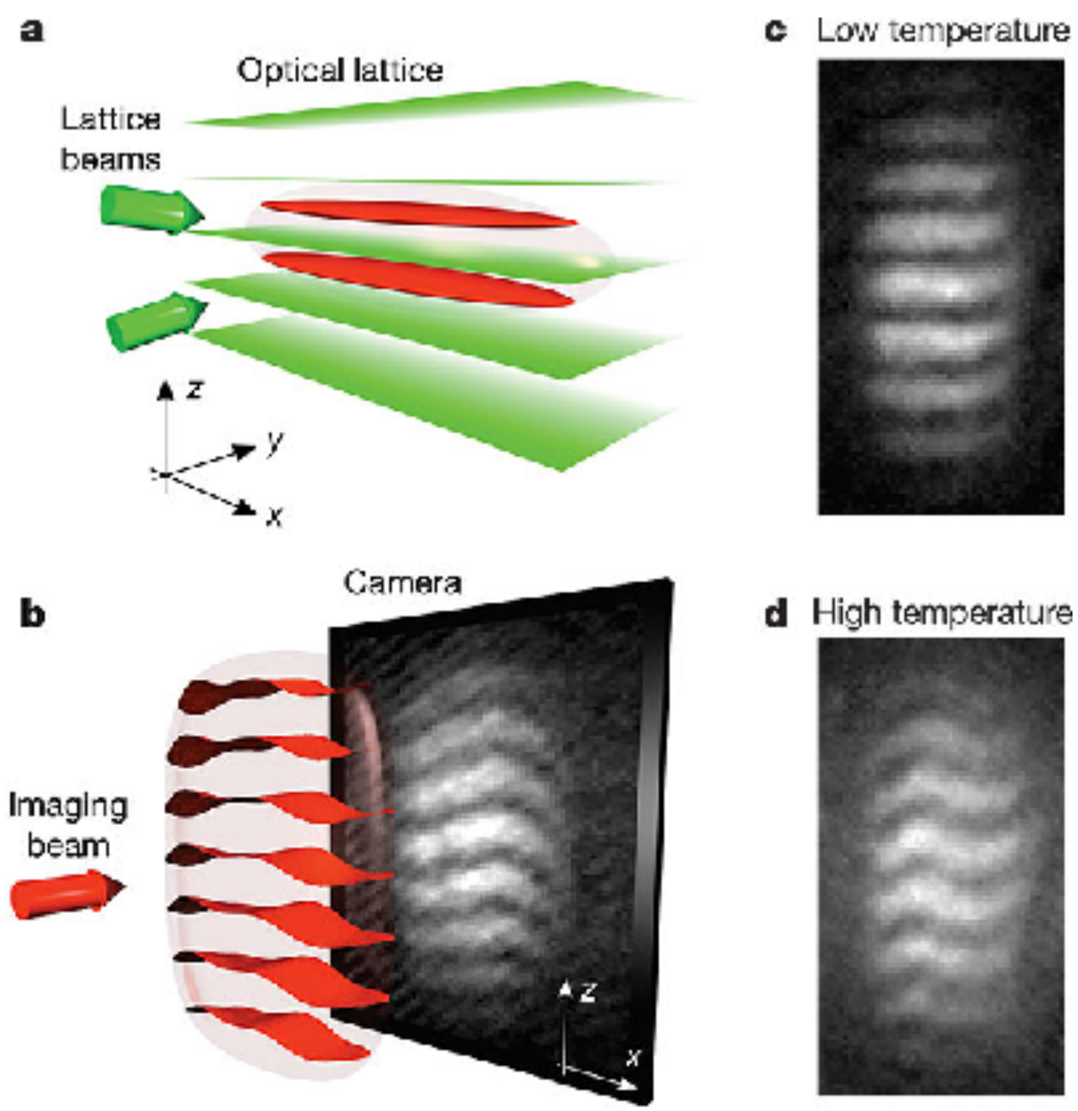
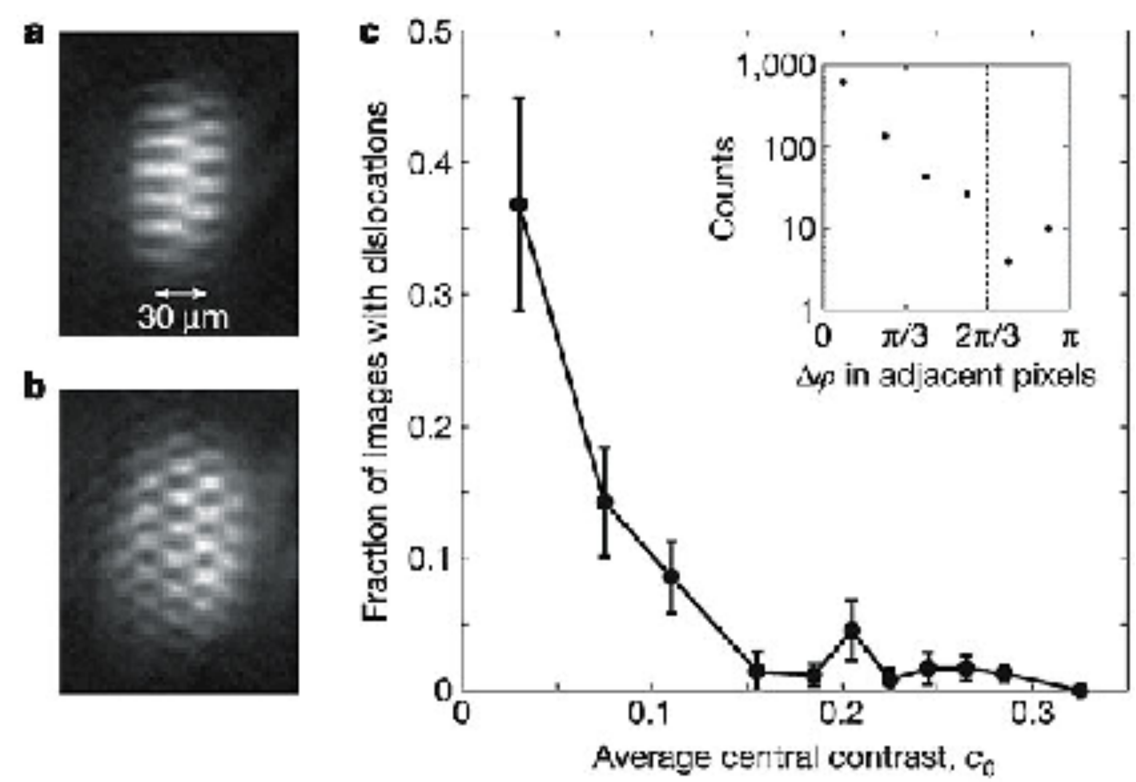


Illustration: ©Johan Jarnestad/The Royal Swedish Academy of Sciences

Seen in: 4He films, superconducting films, melting of 2D solids, BEC’s …



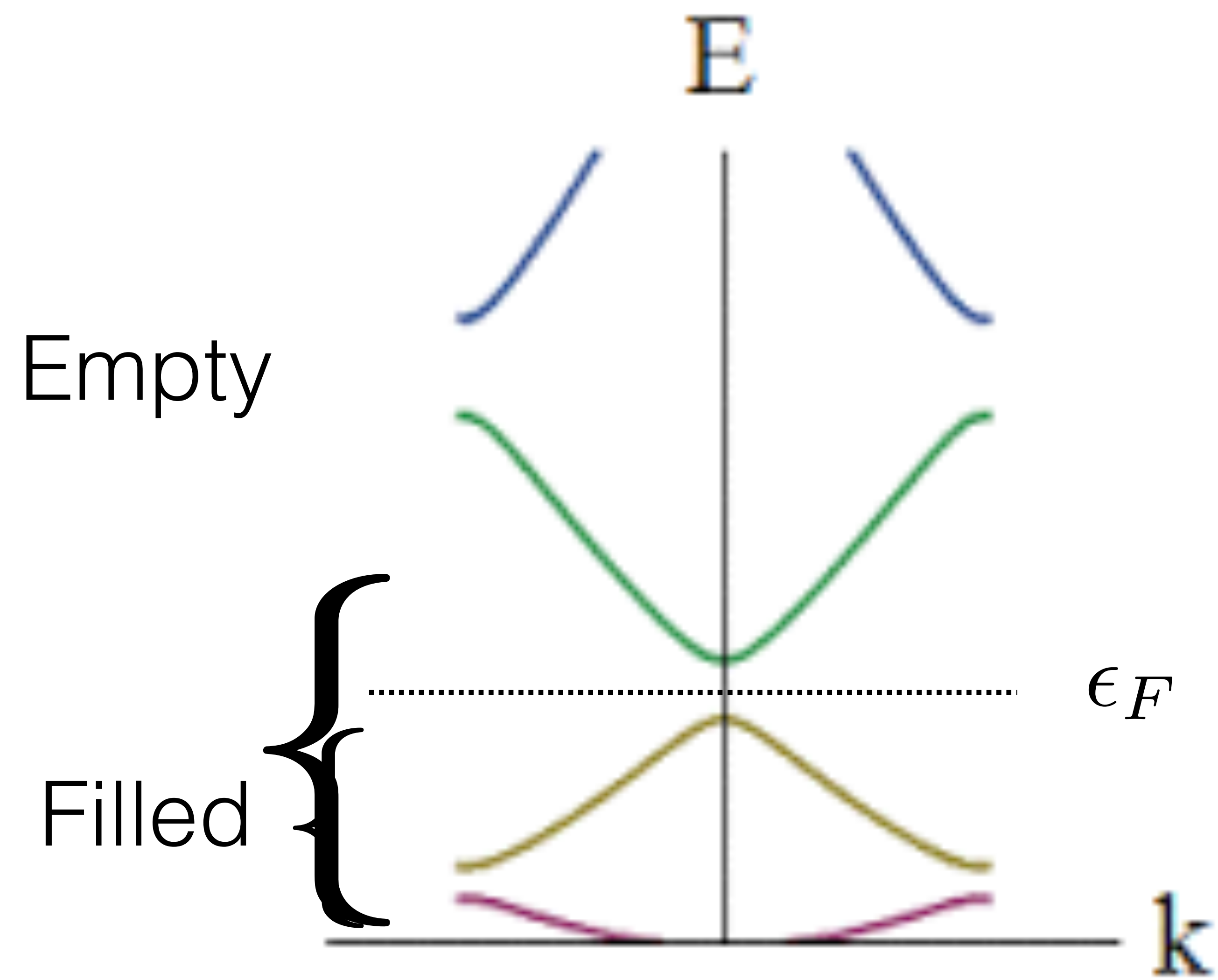


Zoran Hadzibabic, Peter Krüger, Marc Cheneau, Baptiste Battelier & Jean Dalibard, *Nature* **441**, 1118-1121 (2006)

Bloch band theory

Insulator:

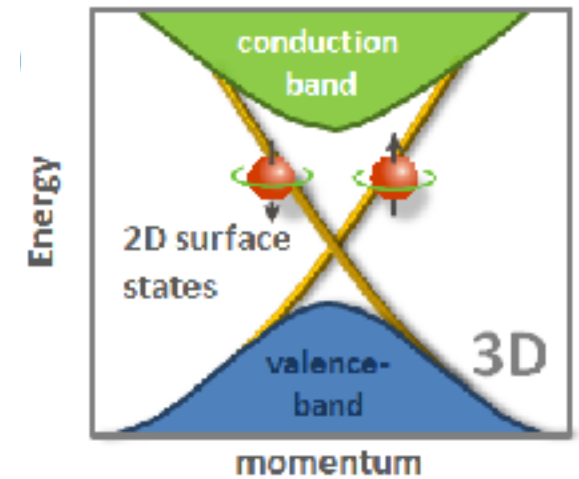
Metal:

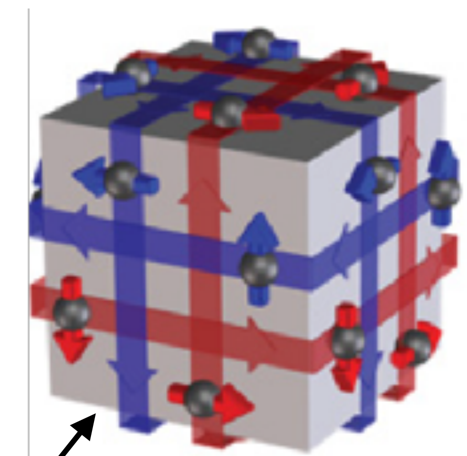


Full story for more than <u>90 years</u>

Topological insulator

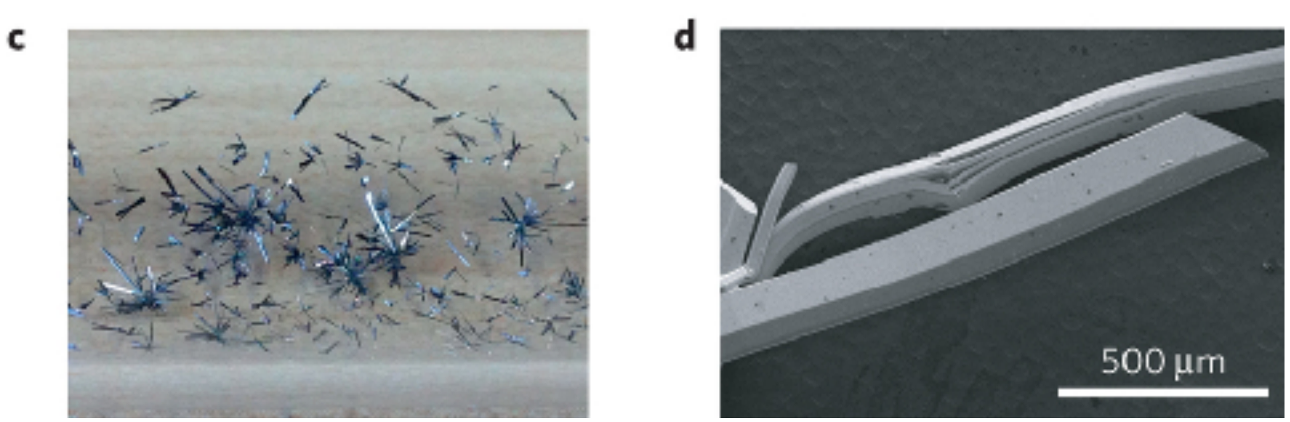
Insulating in the bulk
Conducting at the edge



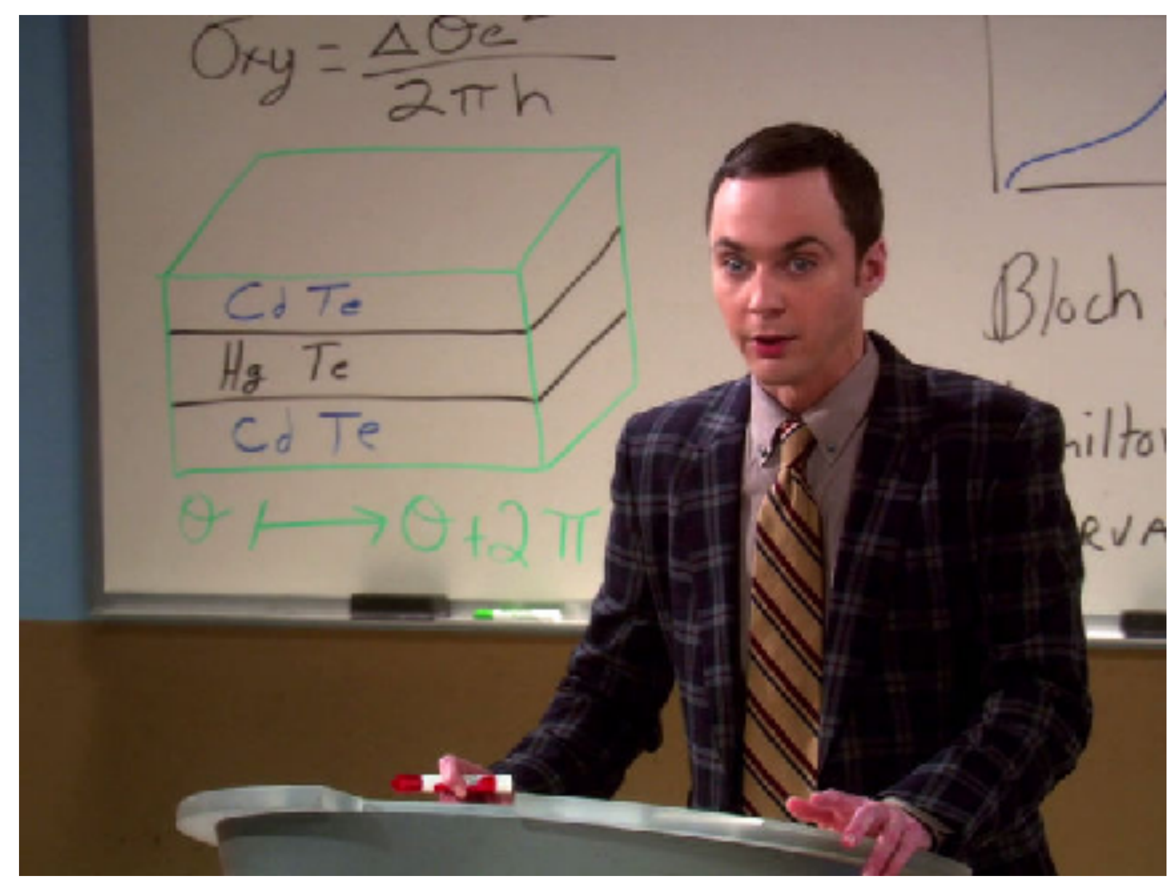


Topologically protected surface states

Very hot research area



β-Bi_4I_4

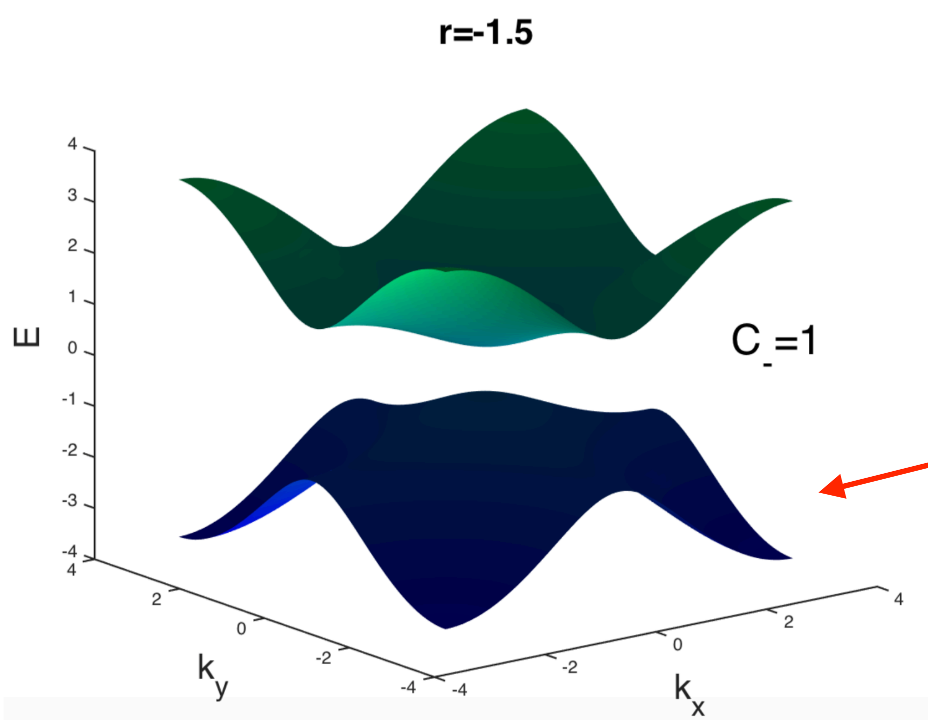


Example: Chern insulator

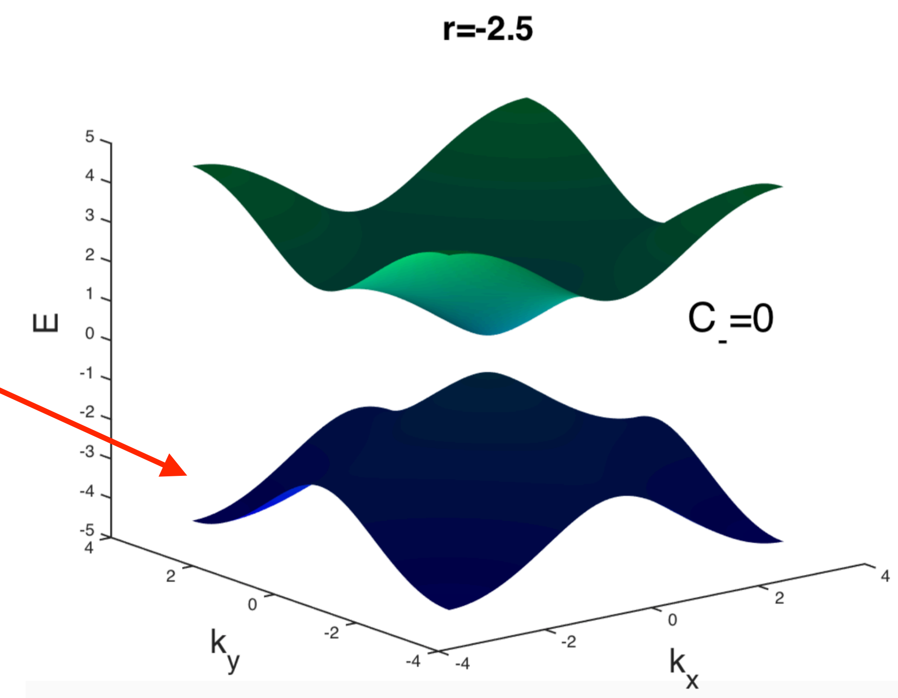
$$\mathcal{H}(\mathbf{k}) = \mathbf{h}(\mathbf{k}) \cdot \vec{\sigma} = \begin{bmatrix} r + \cos k_x + \cos k_y & \sin k_x - i \sin k_y \\ \sin k_x + i \sin k_y & -(r + \cos k_x + \cos k_y) \end{bmatrix}$$

Free parameter

<u>Two energy bands</u>



Topologically distinct!



Chern number for Bloch band n $C_n = \frac{1}{2\pi} \int_{\mathrm{BZ}} \mathbf{B}_n(\mathbf{k}) \cdot d^2\mathbf{k} =$ integer

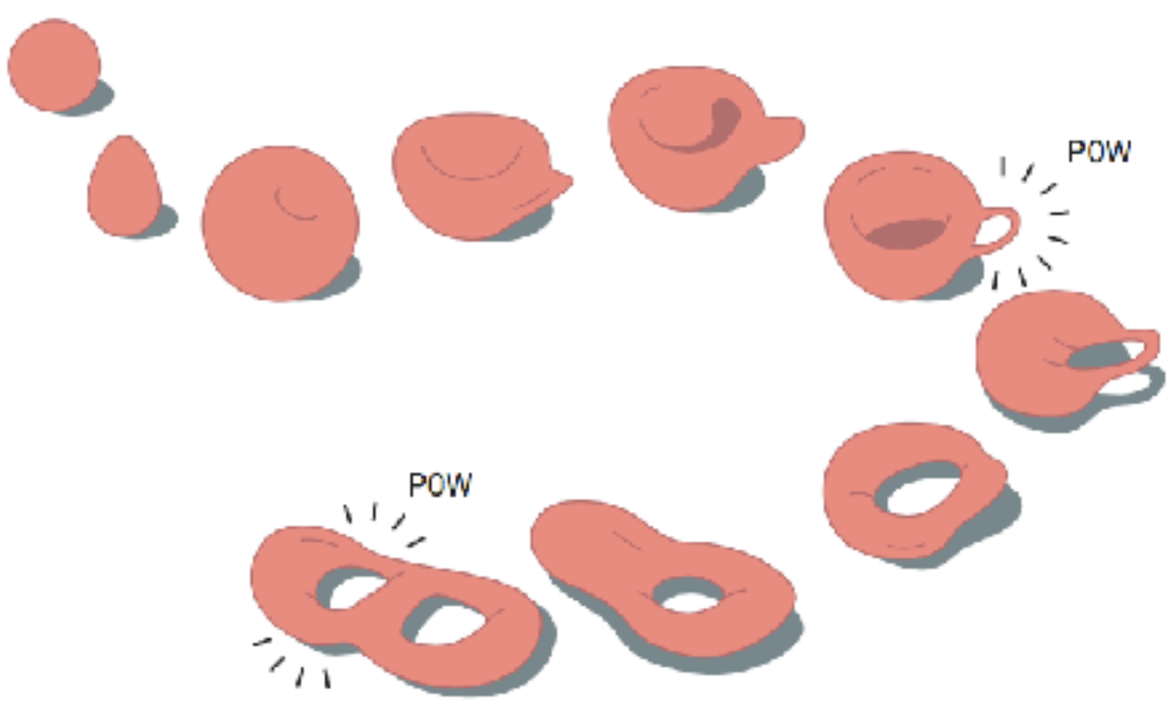
Topological property

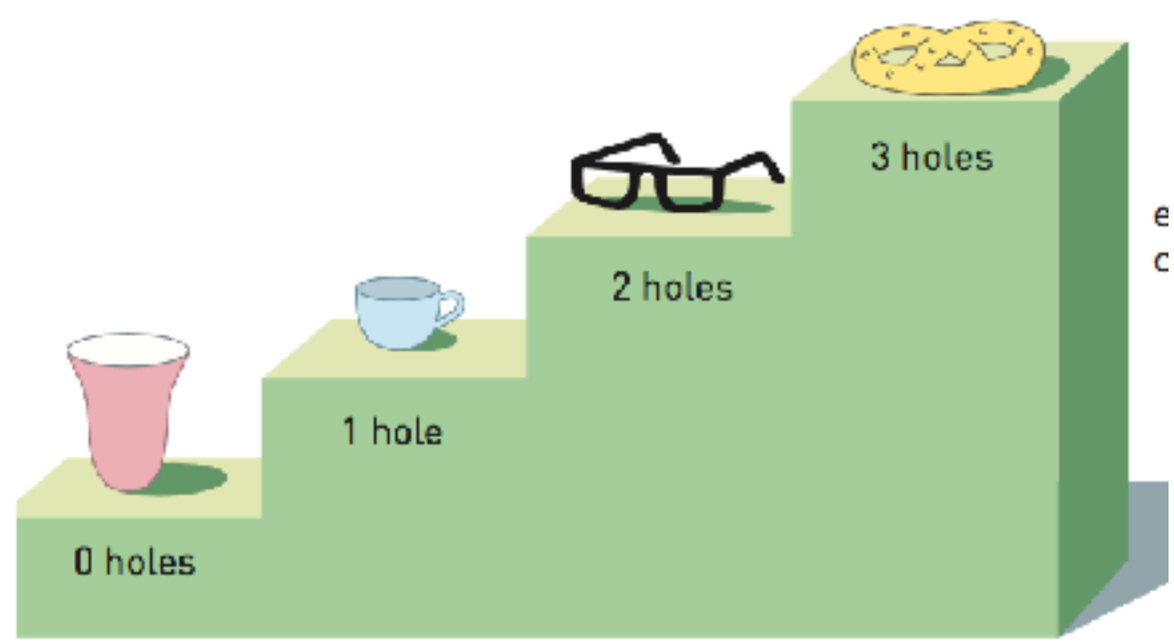
Gaussian curvature

\# holes

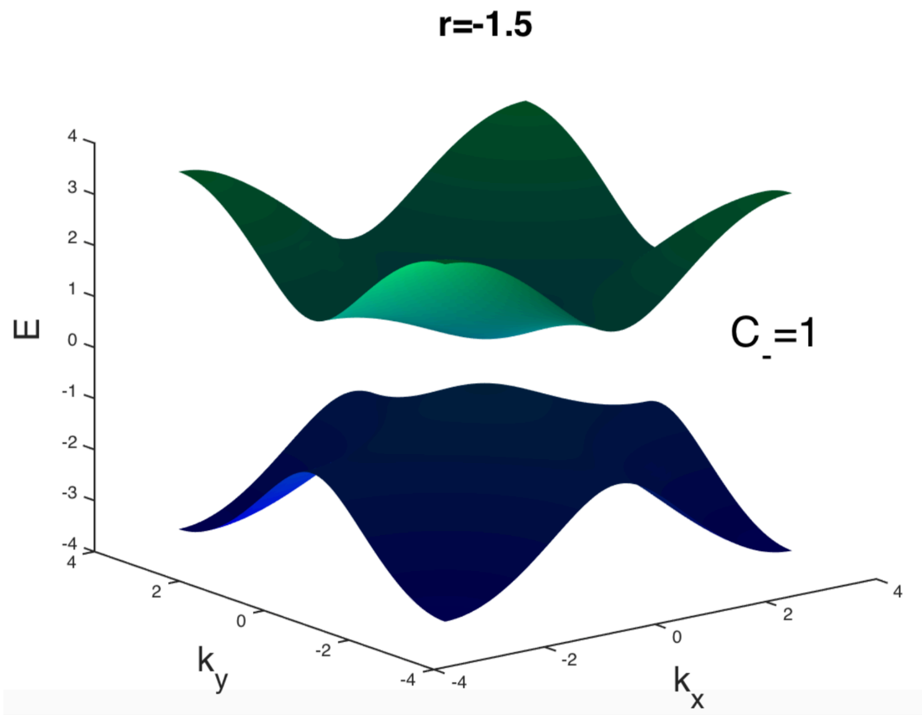
$$\boxed{\text{Gauss-Bonnet:} \quad \int_M K\,dA = 4\pi(1-g)}$$

Orientable manifold

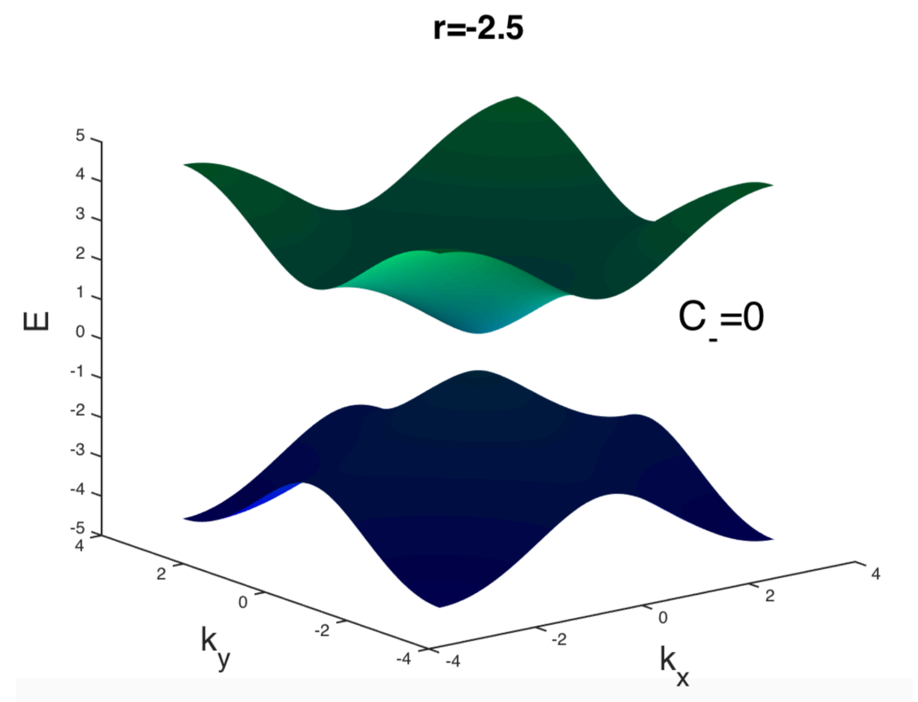




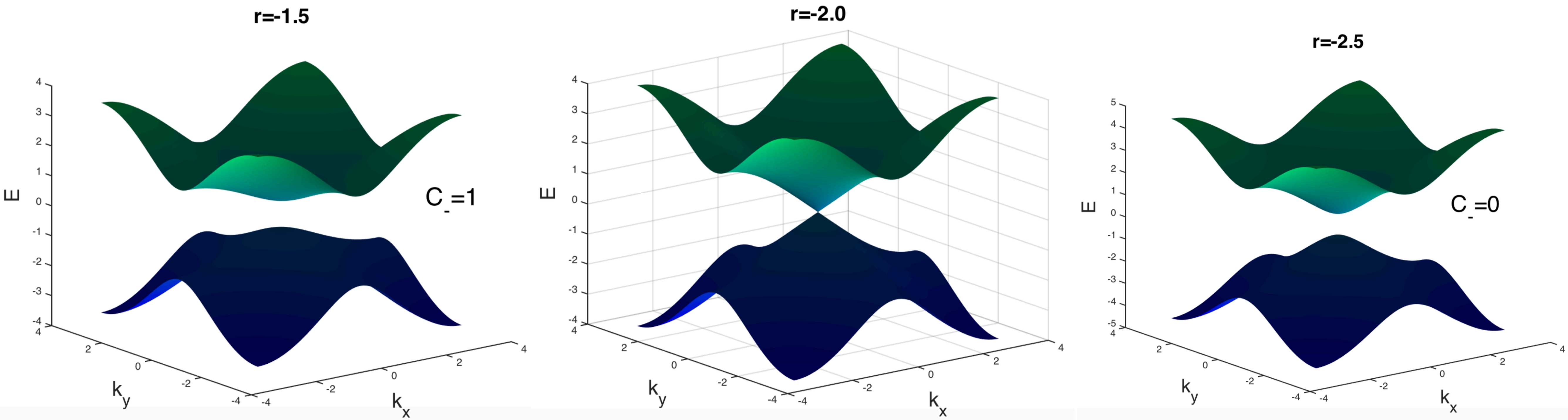
Chern number of given band cannot change by smooth infinitesimal change (like vorticity)



Something *singular* has to happen when r=-1.5→-2.5



What?

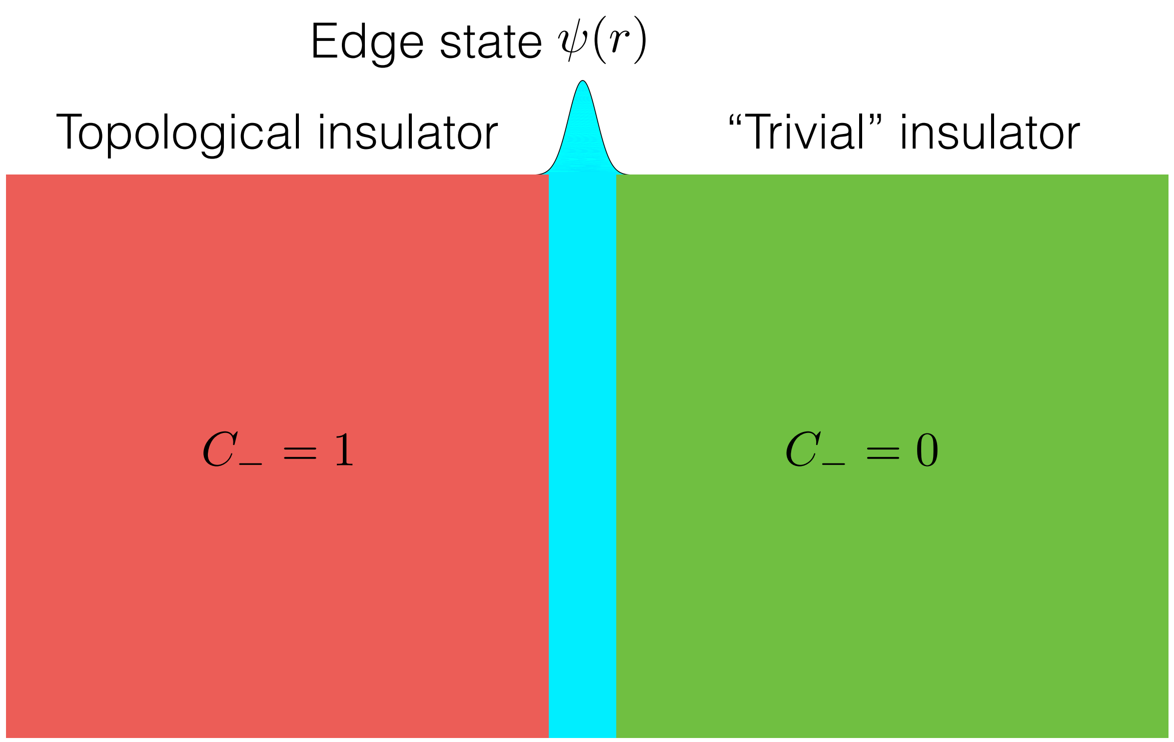


- Bands with different C_- look the same
- Topology of band given by eigenstates
- Chern numbers change by band gap closing
- Cannot go smoothly from one insulator state to the other. Semi-metal phase in-between.

$\boxed{\text{Topological phase transition}}$

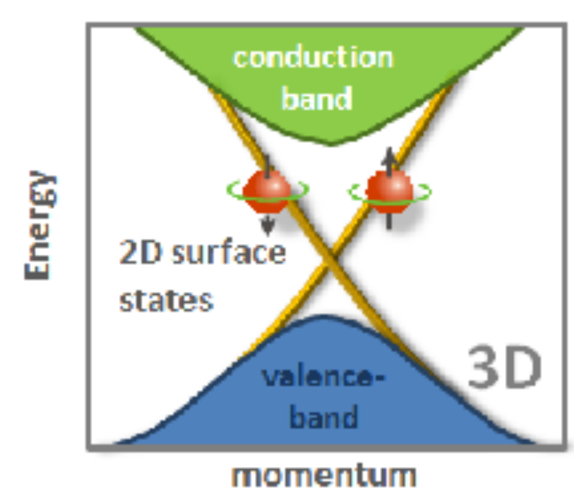
Not described by broken symmetry local order parameter

Bulk-edge correspondence



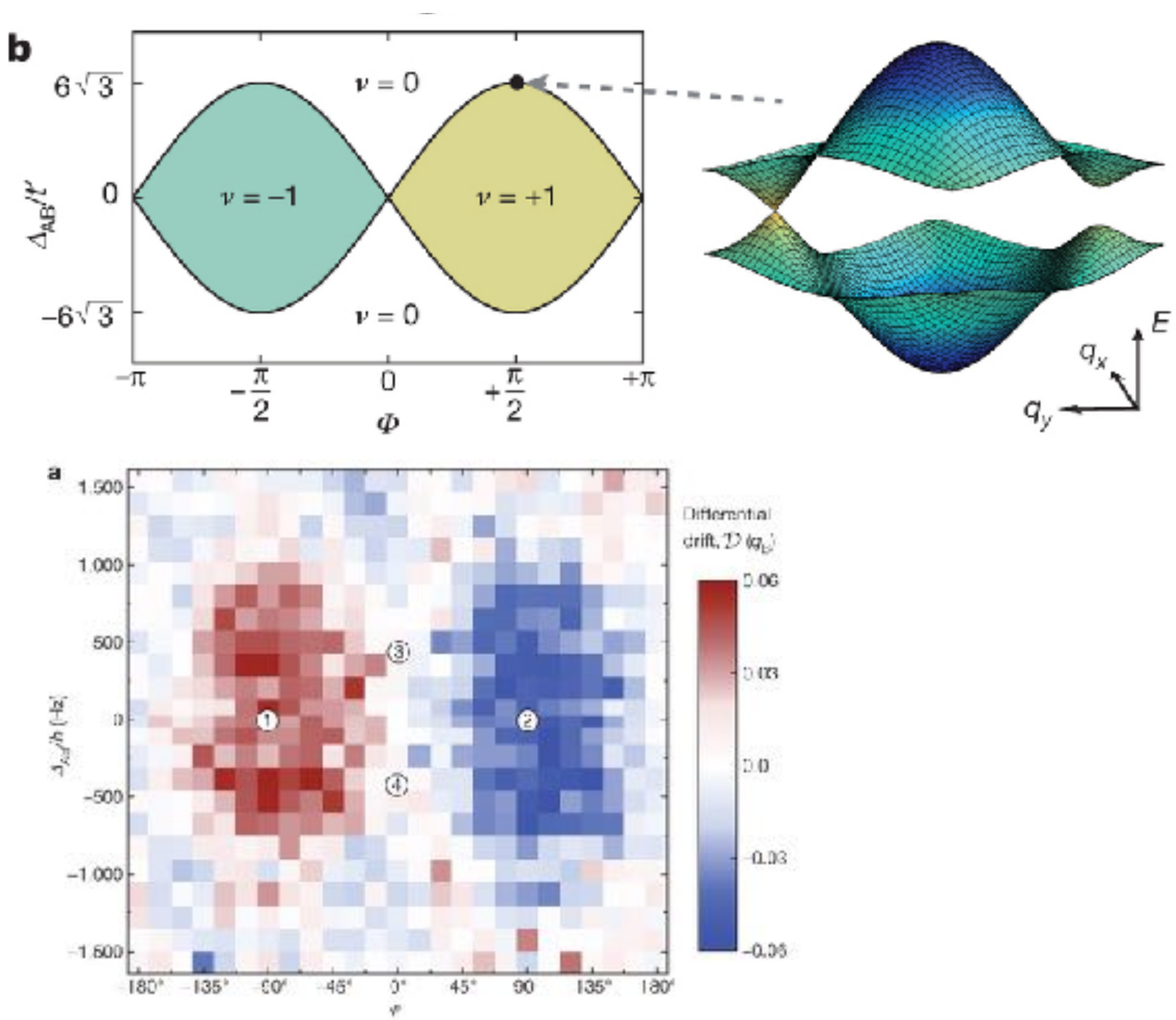
Topological distinct phases adjacent ⇒

topologically protected edge modes

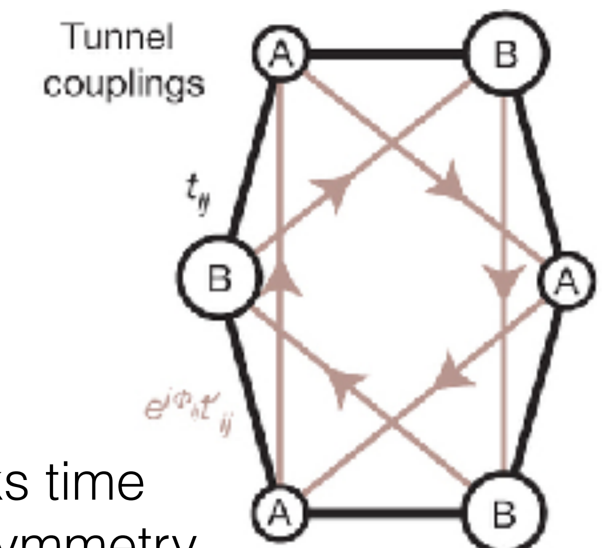


Experimental realisation of Haldane model

Atoms hopping in honeycomb lattice



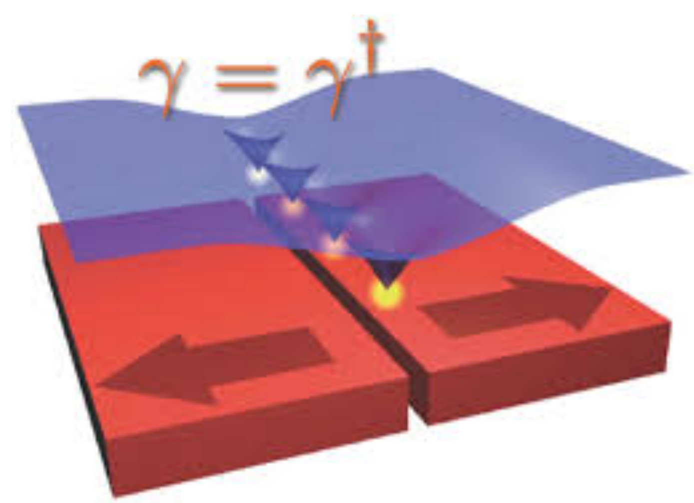
A≠B breaks inversion symmetry

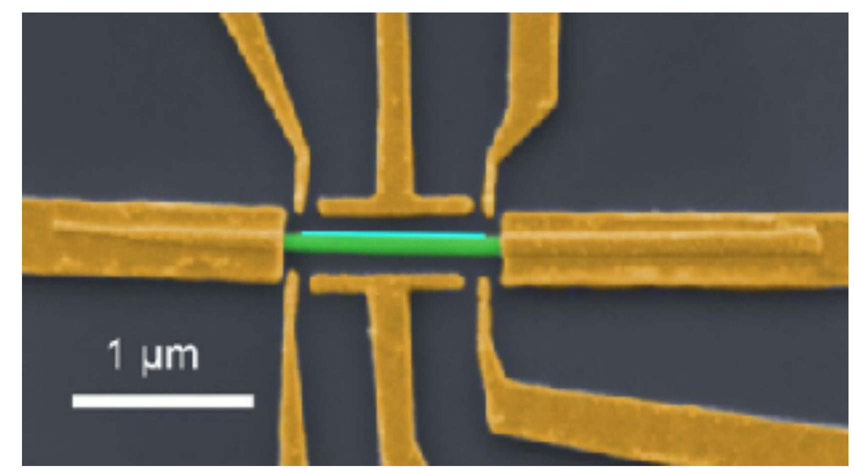


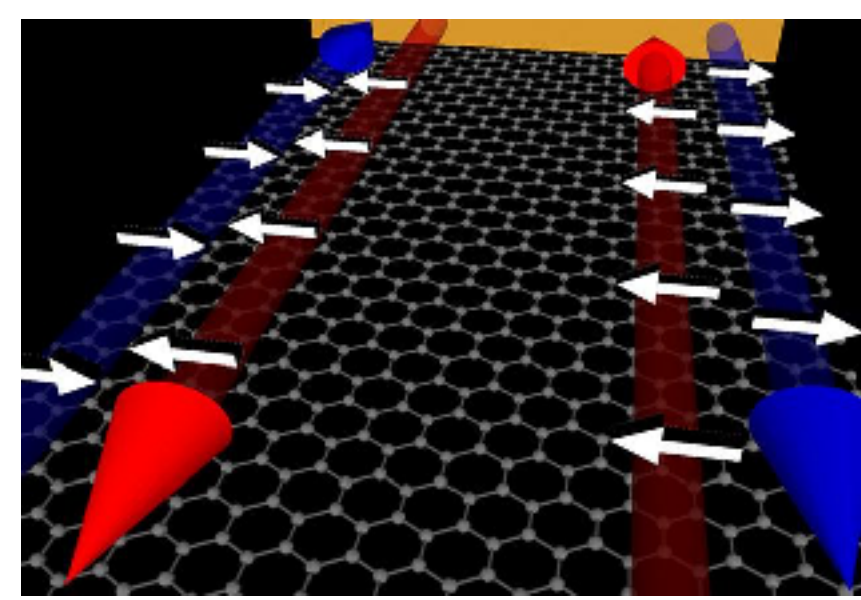
ϕ breaks time reversal symmetry

Extremely active research topic

- Different classes of topological insulators
- Topological superconductors/superfluids
- Majorana modes
- Interactions?
- Detection?
- Use in fast electronics, computers, spintronics, quantum computing …

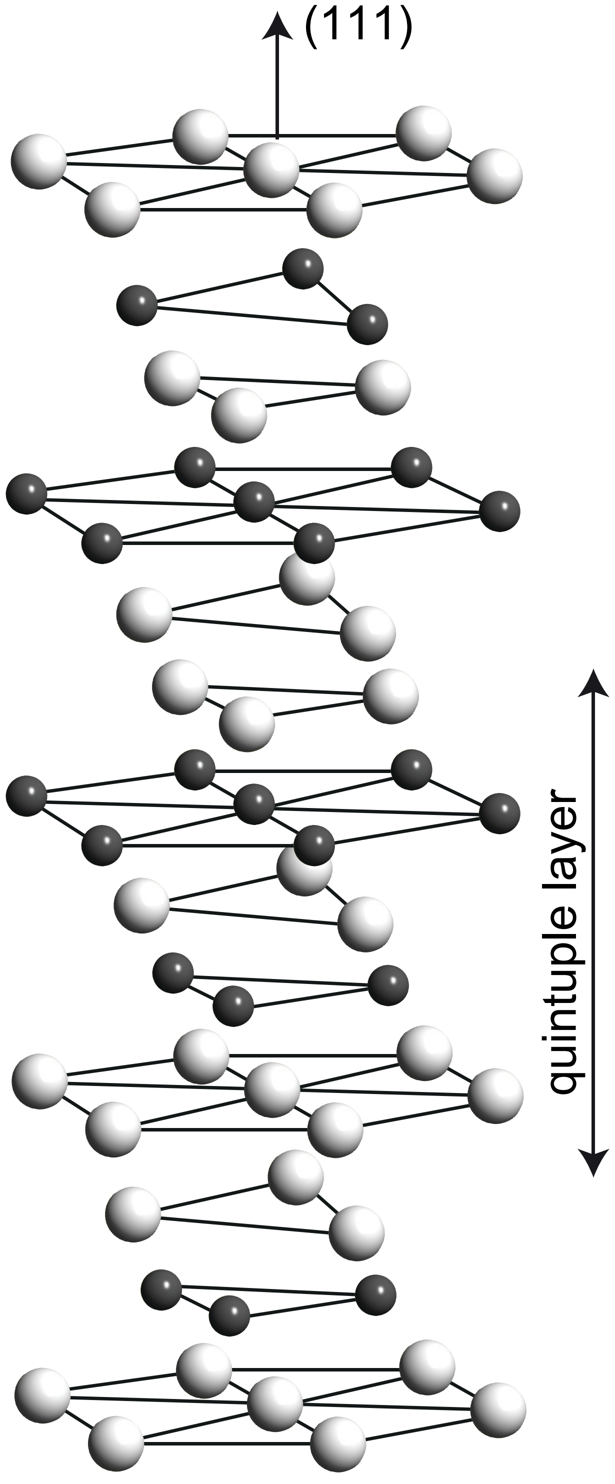


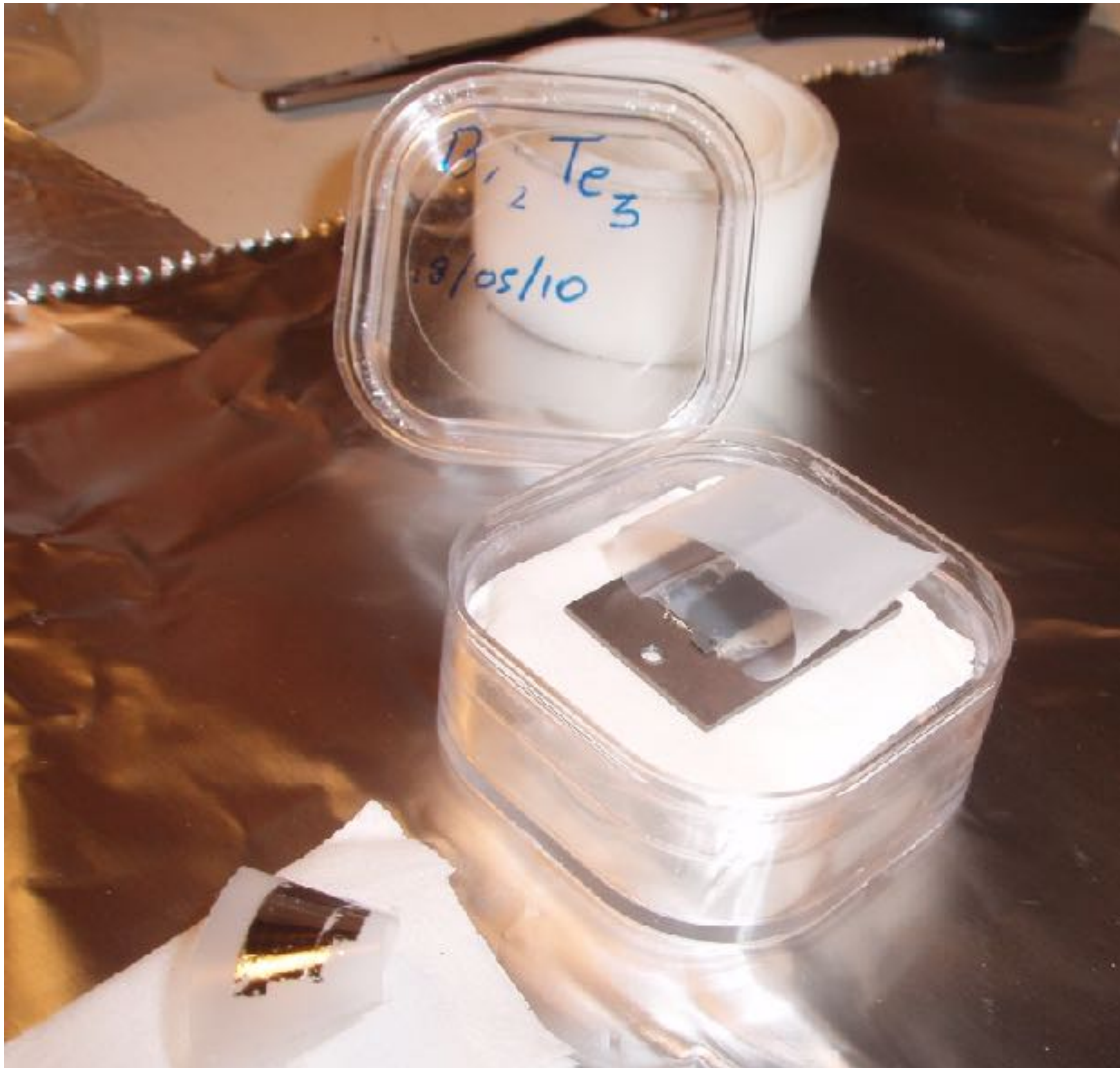




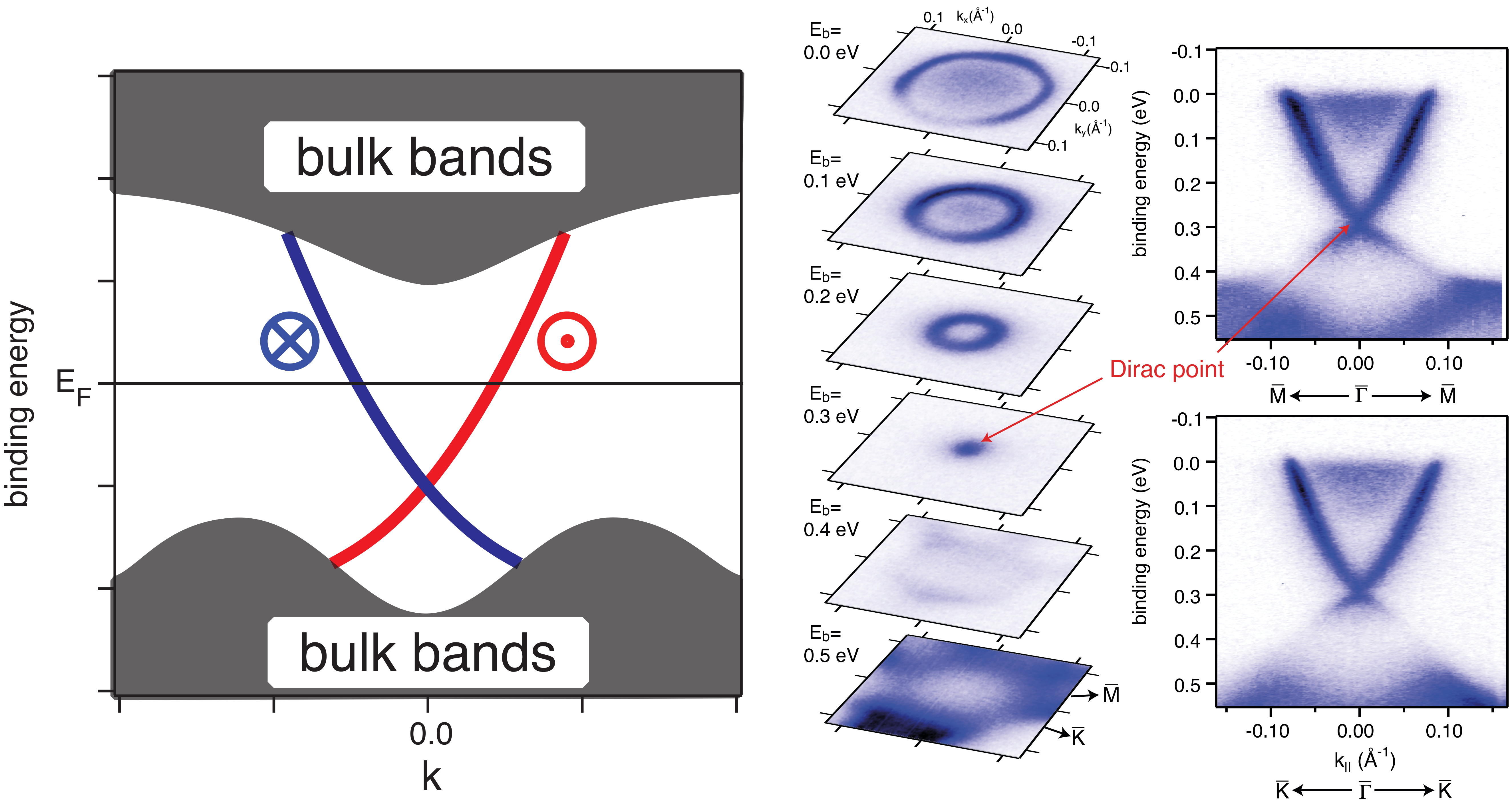
Philip Hofmann

Bi_2Se_3

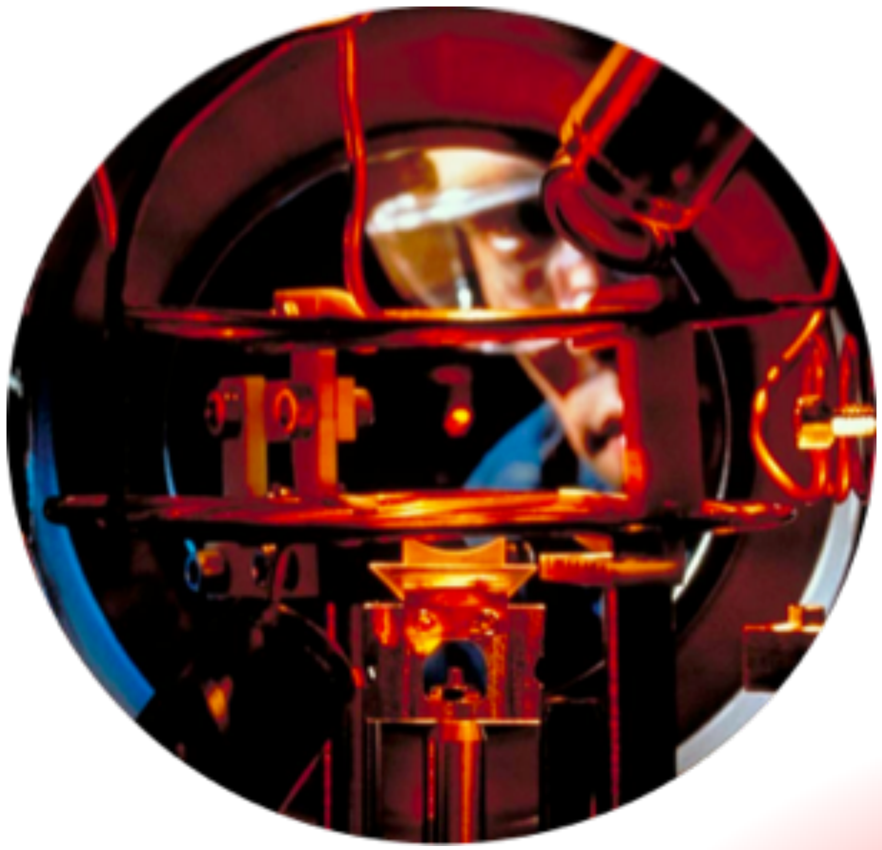




topological insulators
typical surface bands



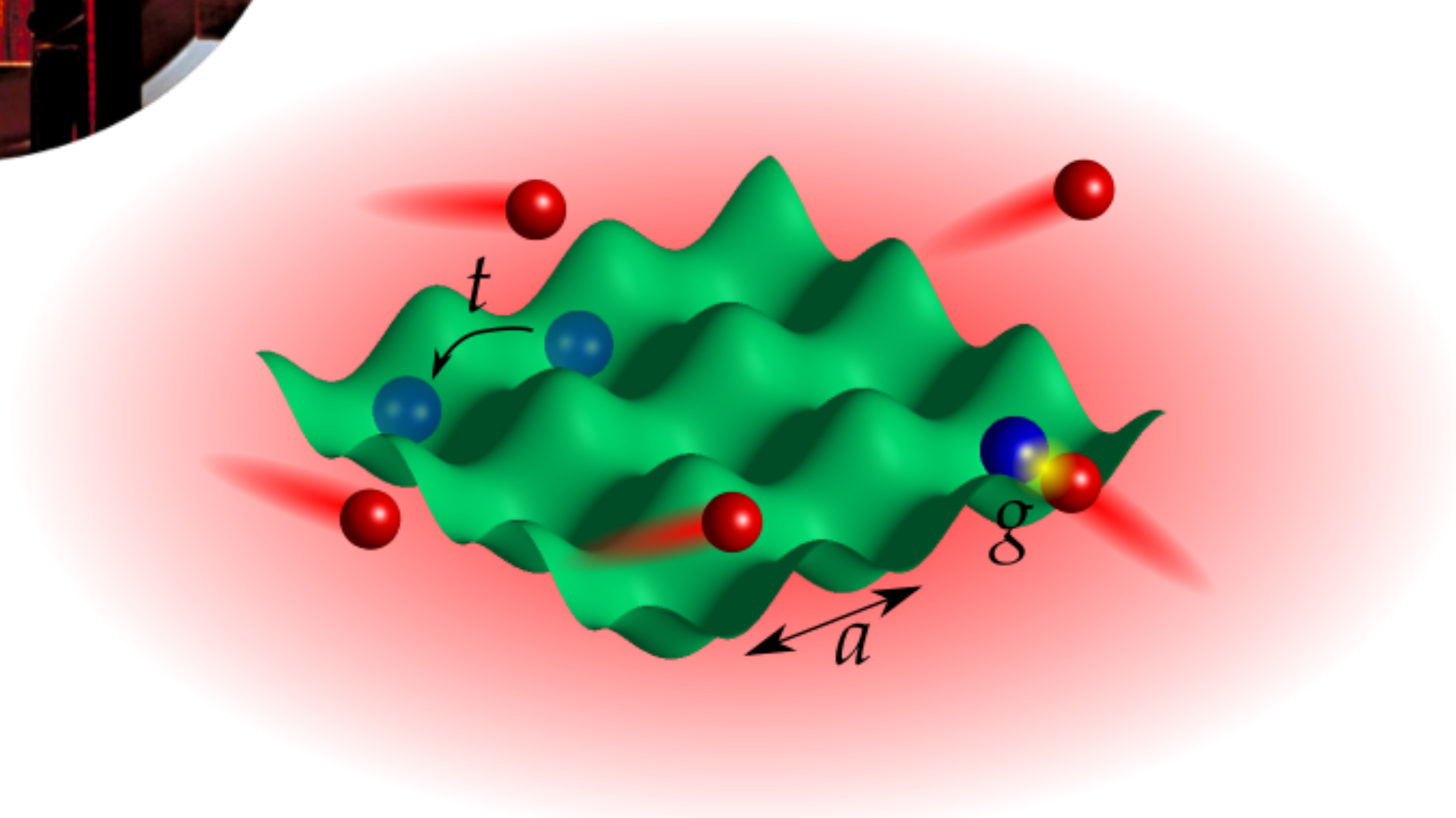
Cold atom systems

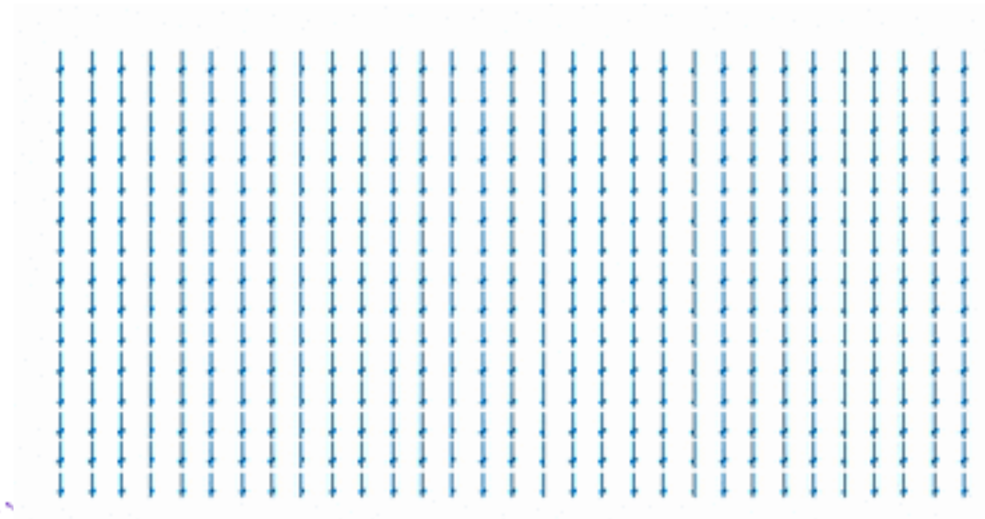


n~10^{12}cm^{-3}

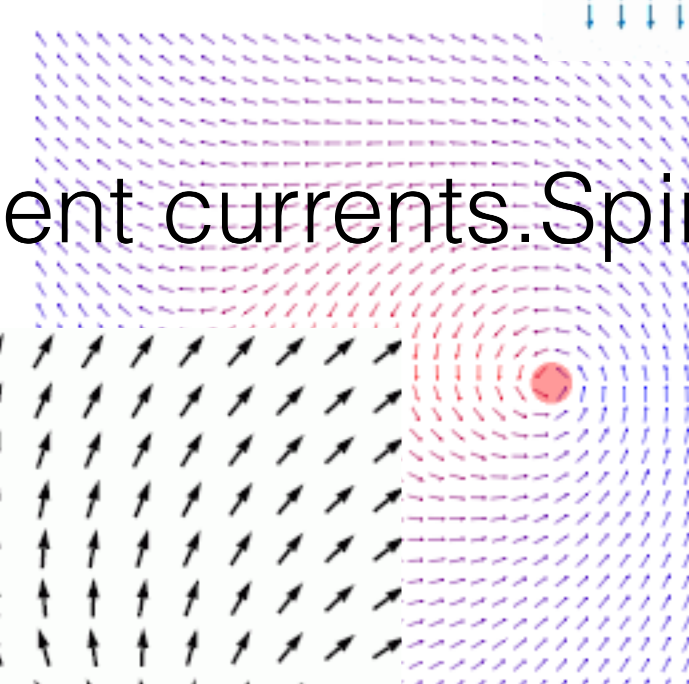
T~nK

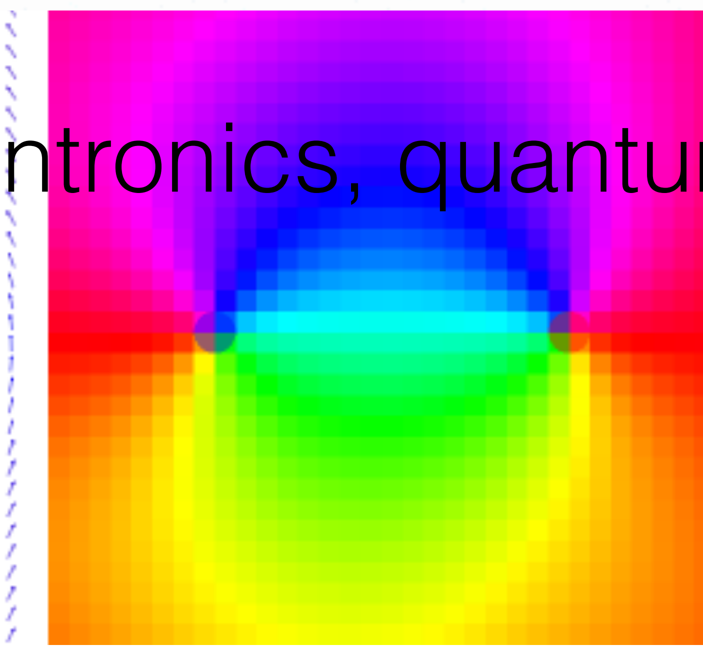
Highly flexible

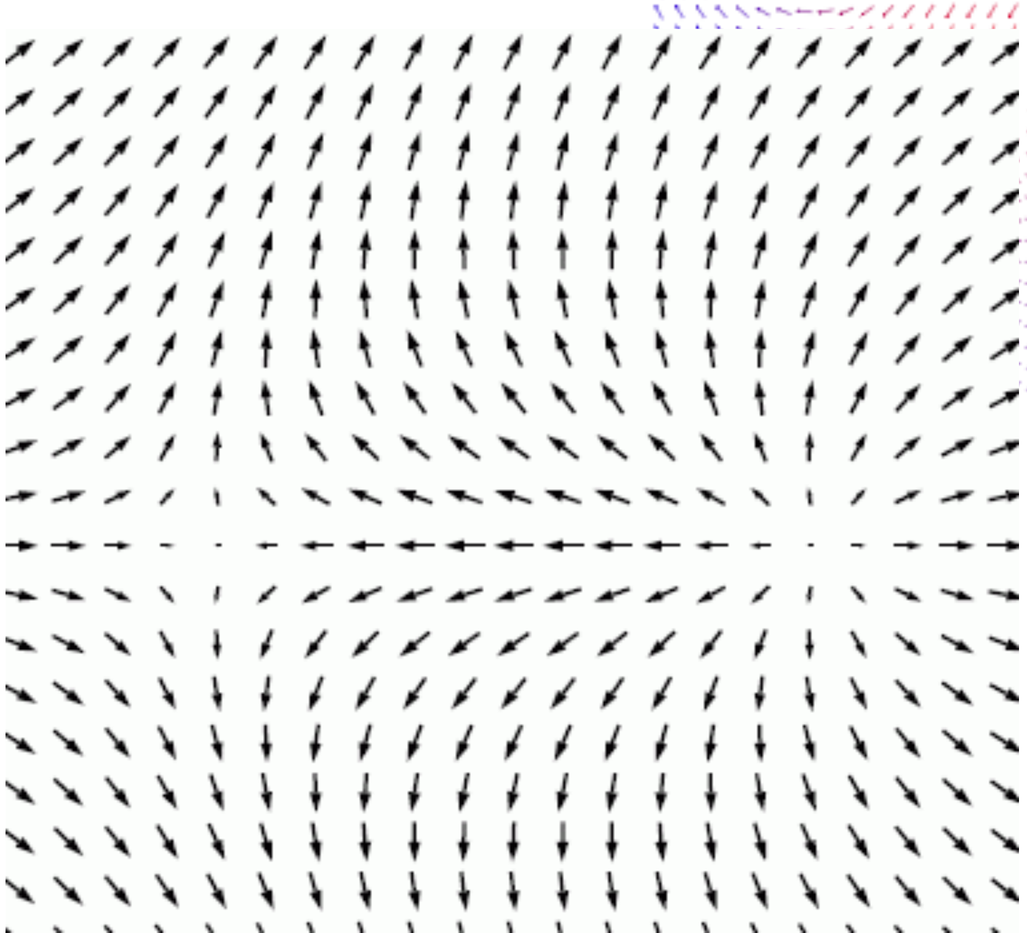


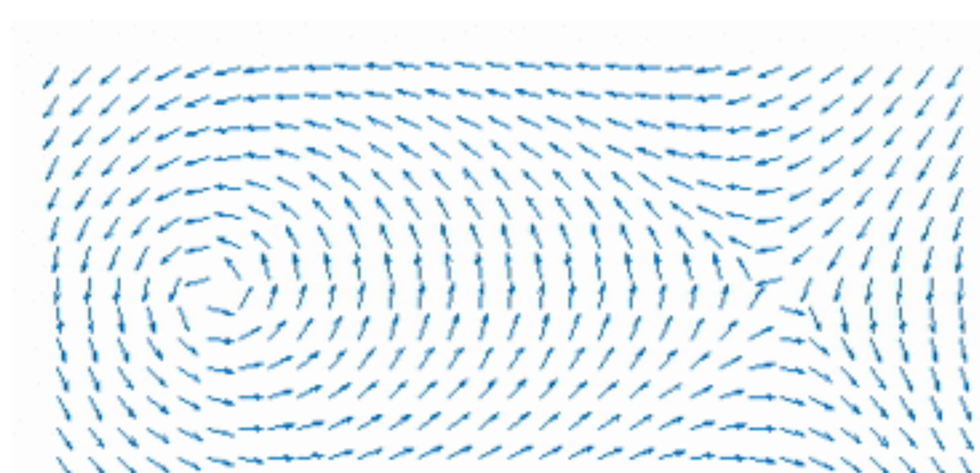


Spin dependent currents.Spintronics, quantum computing.



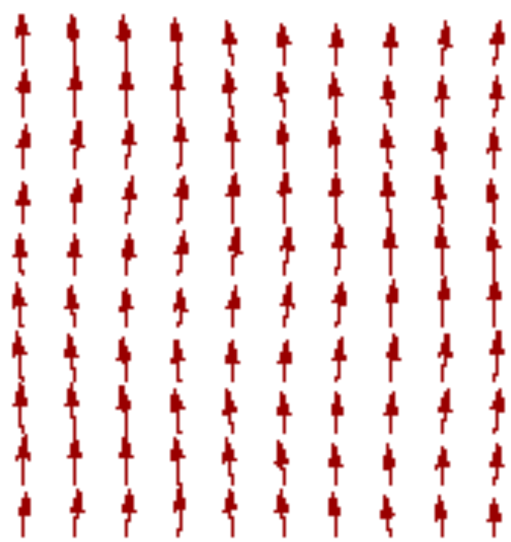






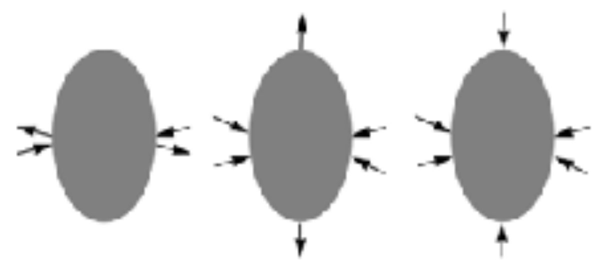
Phase fluctuations

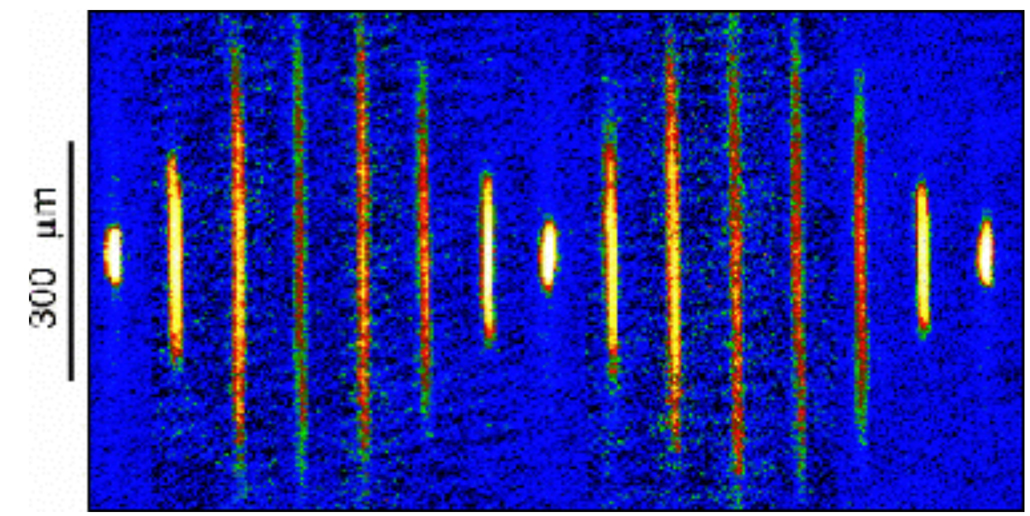
Spin waves



Supercurrents & sound

$$\vec{v}_s = \frac{\hbar}{m}\nabla\theta(\vec{r},t)$$





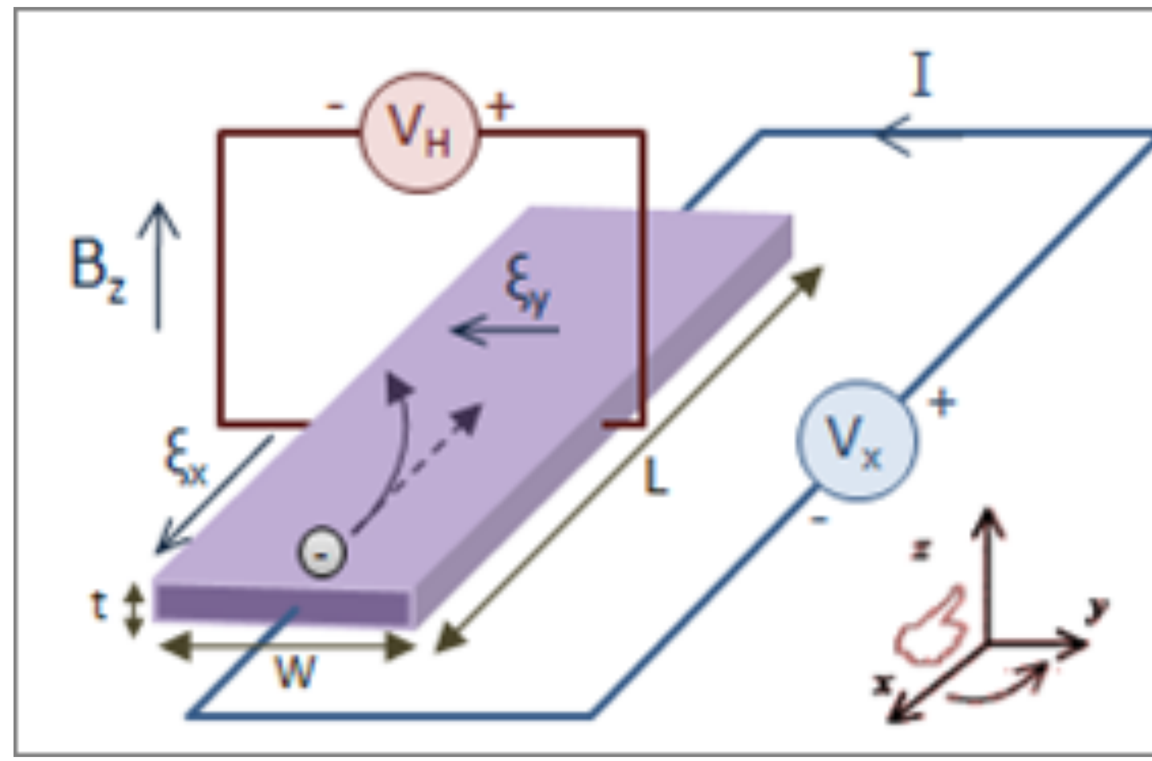
Stripe fluctuations

Quantum Hall effect as a topological effect

D. J. Thouless

Hall effect (Edwin Hall 1879):

$$\begin{bmatrix} E_x \\ E_y \end{bmatrix} = \begin{bmatrix} \rho_{xx} & \rho_{xy} \\ \rho_{xy} & \rho_{yy} \end{bmatrix} \begin{bmatrix} j_x \\ j_y \end{bmatrix}$$



Drude model:

$$m\frac{d\mathbf{v}_d}{dt} = -e(\mathbf{E} + \mathbf{v} \times \mathbf{B}) - \frac{1}{\tau}m\mathbf{v}_d = 0$$

$$\begin{bmatrix} E_x \\ E_y \end{bmatrix} = \begin{bmatrix} m/e^2n\tau & B/en \\ -B/en & m/e^2n\tau \end{bmatrix} \begin{bmatrix} j_x \\ j_y \end{bmatrix}$$

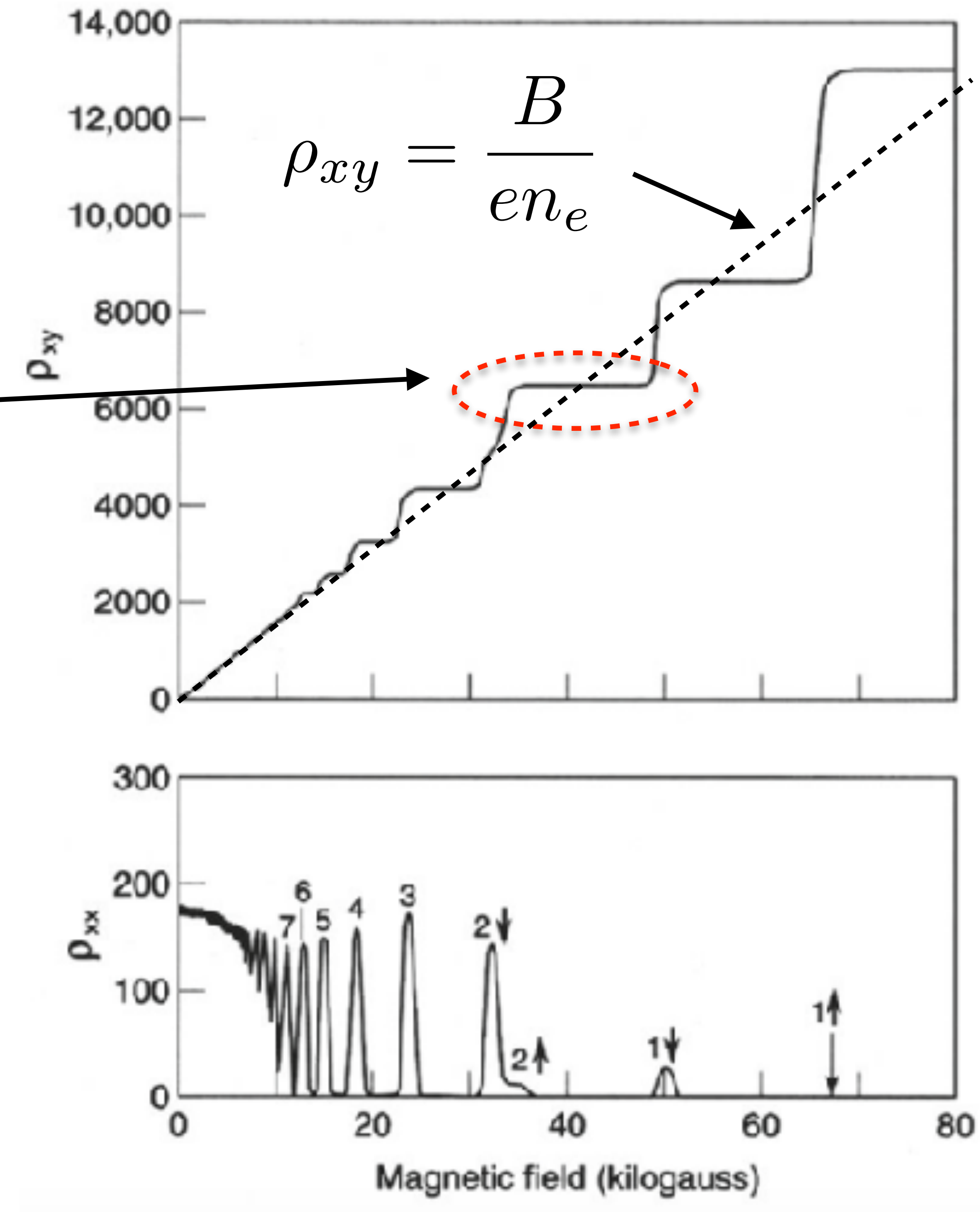
$$\boxed{\rho_{xy} = \frac{B}{en}}$$

Hall resistance

Has been measured in 2DEG

Integer quantum Hall effect

$$\rho_{xy} = \frac{1}{n}\frac{h}{e^2}$$ Error < 10^{-9}!

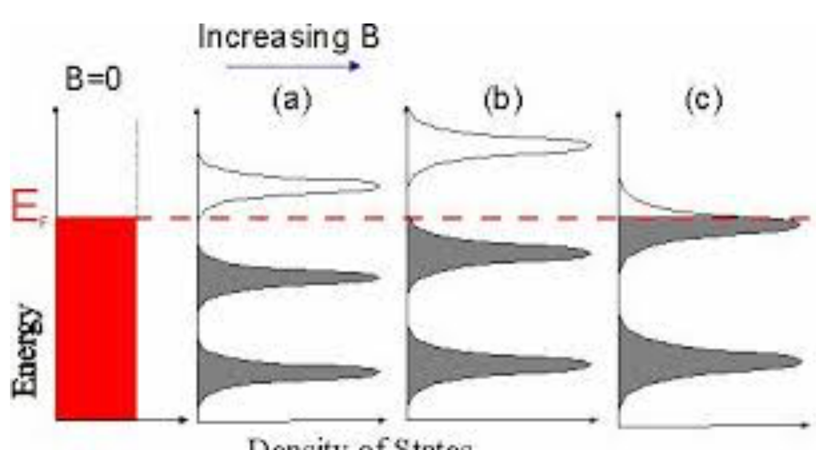


<u>Electron in magnetic field:</u>

$$E_n = (n + 1/2)\hbar\omega_c \qquad \omega_c = \frac{eB}{mc}$$

Landau levels

Macroscopic degeneracy $N_L(B) = \frac{\Phi(B)}{\Phi_0}$



$$\Phi_0 = \frac{hc}{e}$$

von Klitzing

Wave function in n'th Landau level: $u_{n\mathbf{k}}(\mathbf{r})$

Reciprocal lattice vector of magnetic translation lattice

Thouless:

$$\boxed{\sigma_H = \frac{e^2}{h}\sum_n \frac{1}{2\pi}\int_{\mathrm{BZ}} \mathbf{B}_n(\mathbf{k})\cdot d^2\mathbf{k}}$$

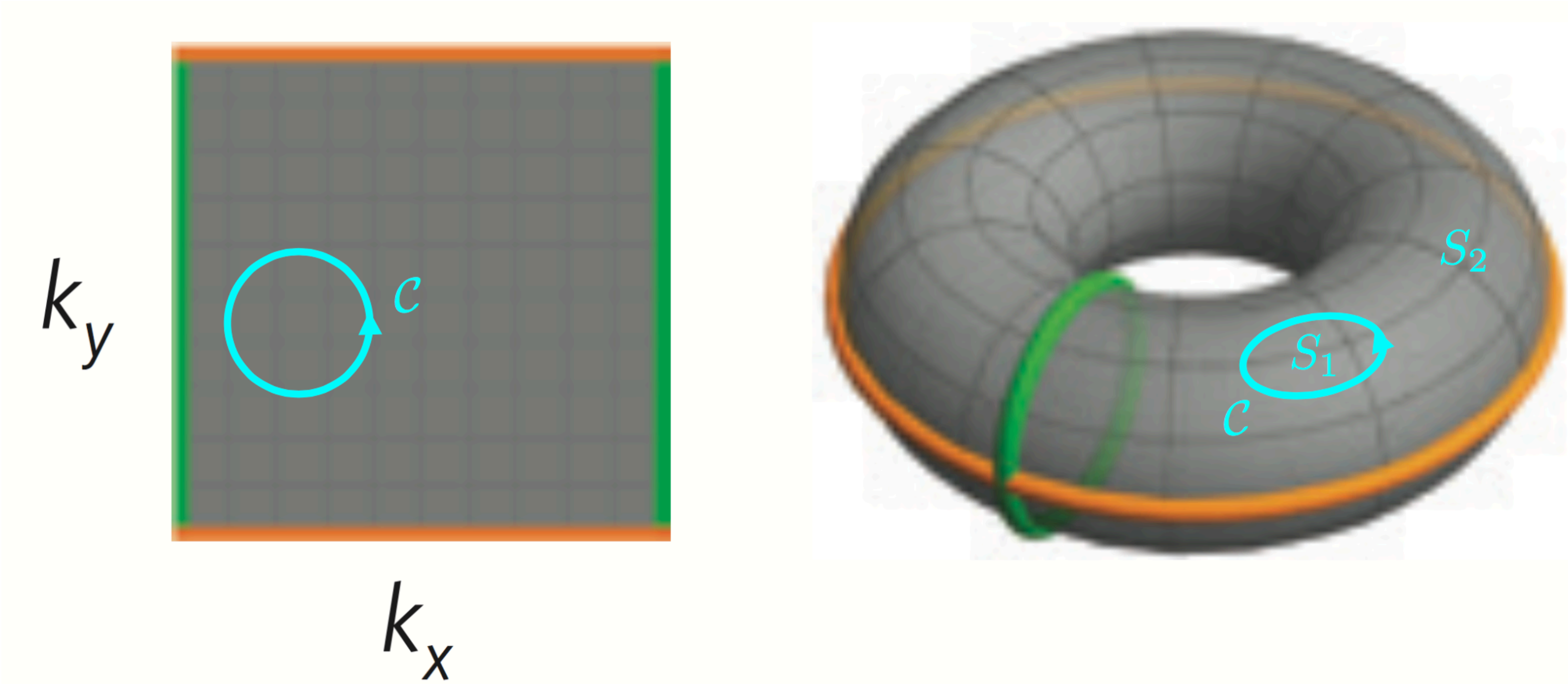
Berry field: $\mathbf{B}_n(\mathbf{k}) = \nabla_{\mathbf{k}} \times \mathbf{A}_n(\mathbf{k})$

Berry vector potential: $\mathbf{A}_n(\mathbf{k}) = i\langle u_{n\mathbf{k}}|\nabla_{\mathbf{k}}|u_{n\mathbf{k}}\rangle$

Chern number: $\boxed{C_n = \frac{1}{2\pi}\int_{\mathrm{BZ}} \mathbf{B}_n(\mathbf{k})\cdot d^2\mathbf{k} = \text{integer}}$

A topological property of the wave functions. Robust!

Brillouin zone = Torus (closed surface)



Phase winding of wave function along closed path:

Two different gauges

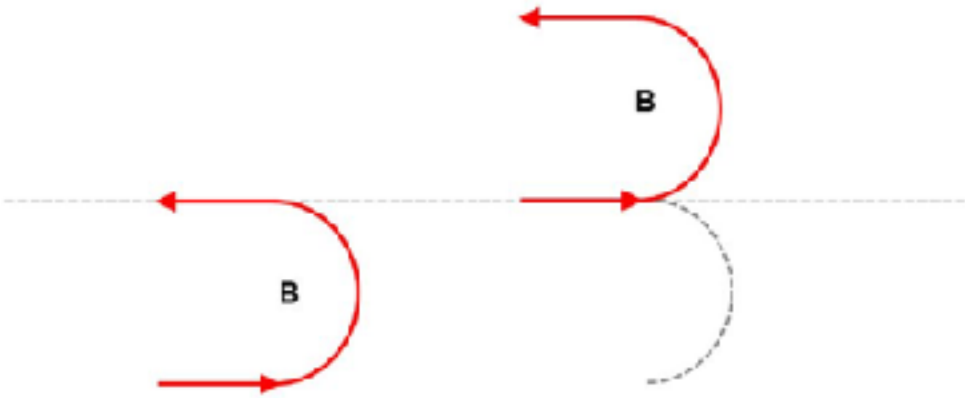
$$\gamma_m(\mathcal{C}) = \begin{cases} \oint_{\mathcal{C}} \mathbf{A}_m(\mathbf{k}) \cdot d\mathbf{k} \\ \oint_{\mathcal{C}} \tilde{\mathbf{A}}_m(\mathbf{k}) \cdot d\mathbf{k} = \oint_{\mathcal{C}} \mathbf{A}_m(\mathbf{k}) \cdot d\mathbf{k} + 2\pi \cdot k \end{cases}$$

Stokes theorem $-\int_{S_2} \mathbf{B}_m(\mathbf{k}) \cdot d^2\mathbf{k} = \int_{S_1} \mathbf{B}_m(\mathbf{k}) \cdot d^2\mathbf{k} + 2\pi k$

$$\boxed{C_n = \frac{1}{2\pi}\int_{\mathrm{BZ}} \mathbf{B}_n(\mathbf{k}) \cdot d^2\mathbf{k} = \text{integer}}$$

Haldane: Key is to *break time-reversal invariance*

Electron in B-field



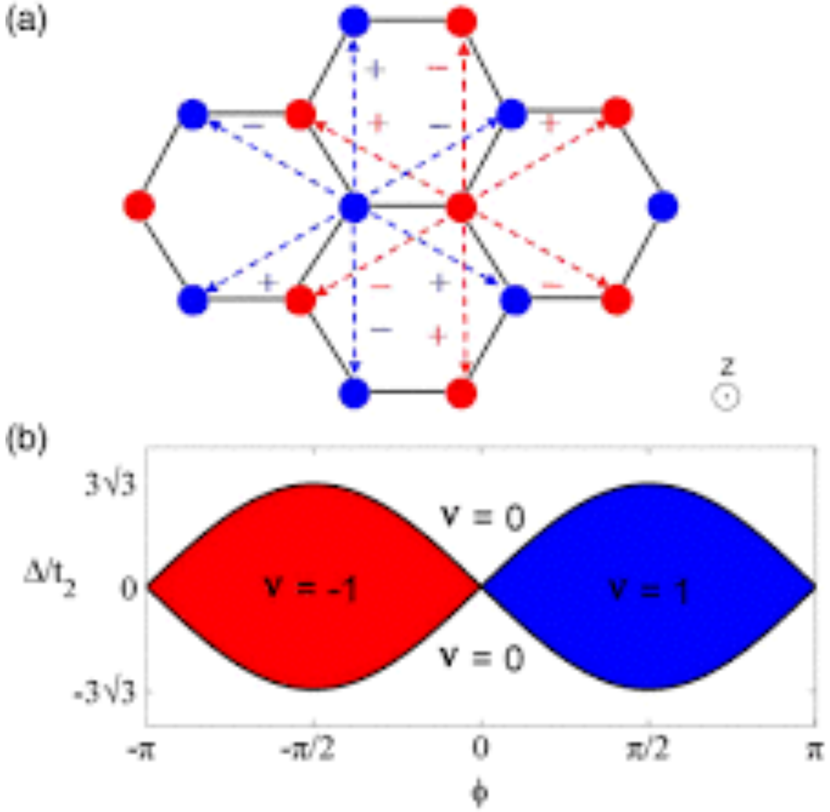
Haldane model (Chern insulator)

The first topological insulator

Electrons hopping honeycomb lattice:

Seen in 2013 in $(Bi,Sb)_2Te_3$

Gave rise to topological band theory

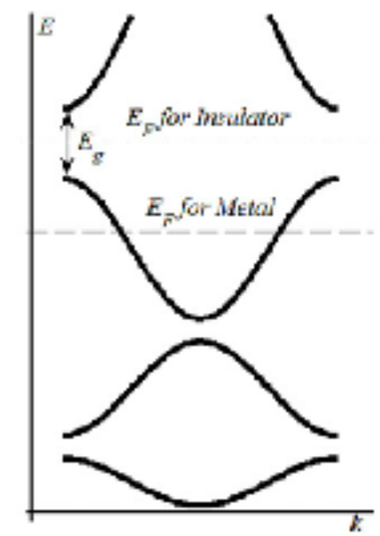


Topological band theory

Bloch: two kinds of materials:

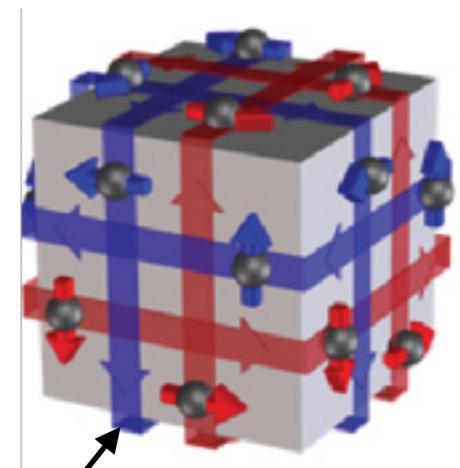
❶ Metals with partly filled band

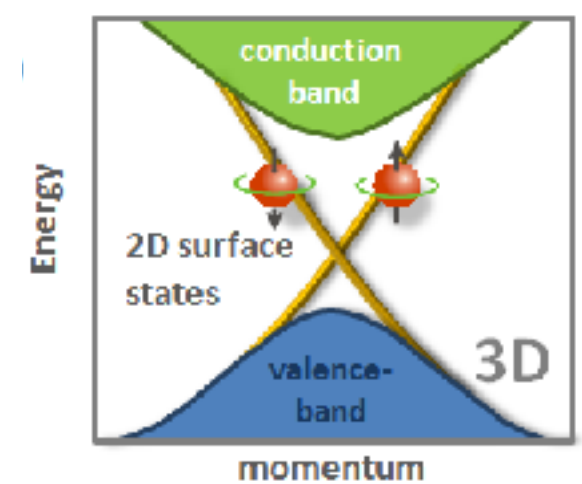
❷ Insulators with filled bands



After more than 90 years:

❸ Topological insulators:
Insulating in the bulk,
conducting on the edges





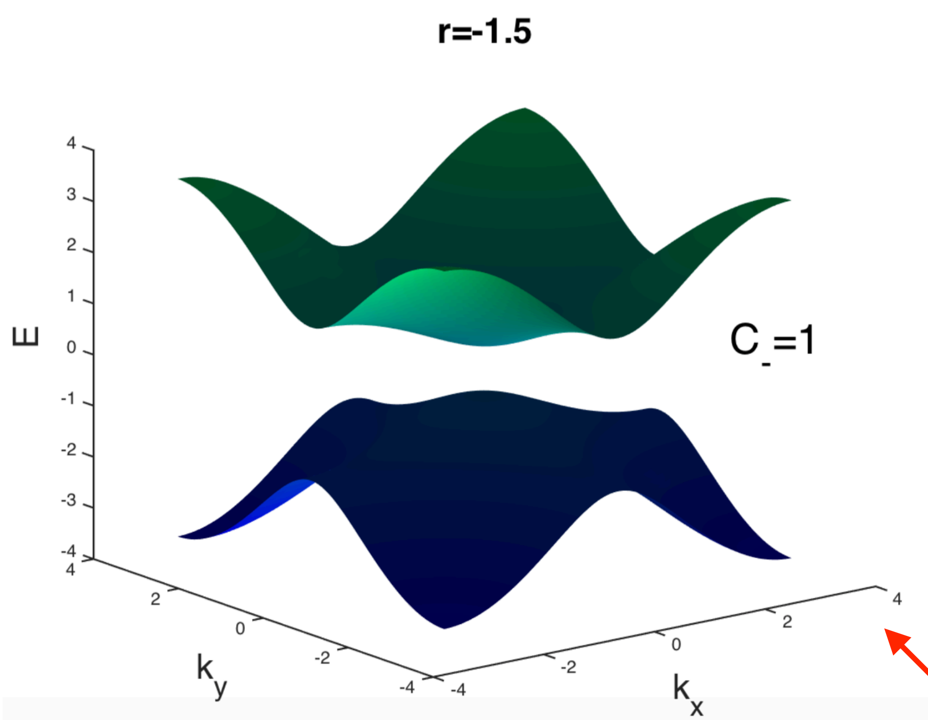
Topologically protected surface states

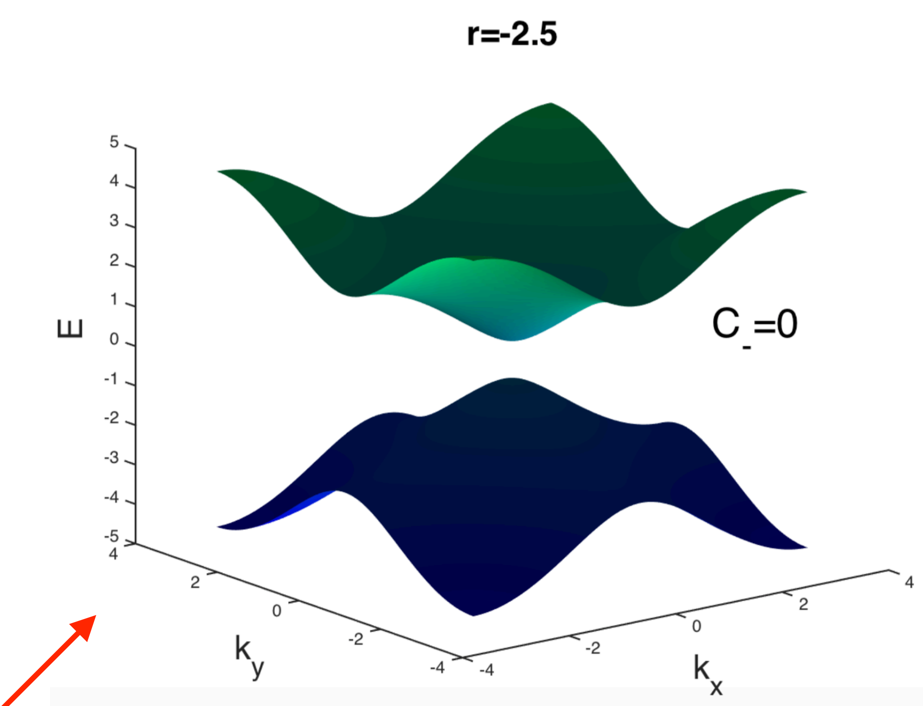
Example: Chern insulator

$$\mathcal{H}(\mathbf{k}) = \mathbf{h}(\mathbf{k}) \cdot \vec{\sigma} = \begin{bmatrix} r + \cos k_x + \cos k_y & \sin k_x - i \sin k_y \\ \sin k_x + i \sin k_y & -(r + \cos k_x + \cos k_y) \end{bmatrix}$$

Free parameter

<u>Two energy bands</u>



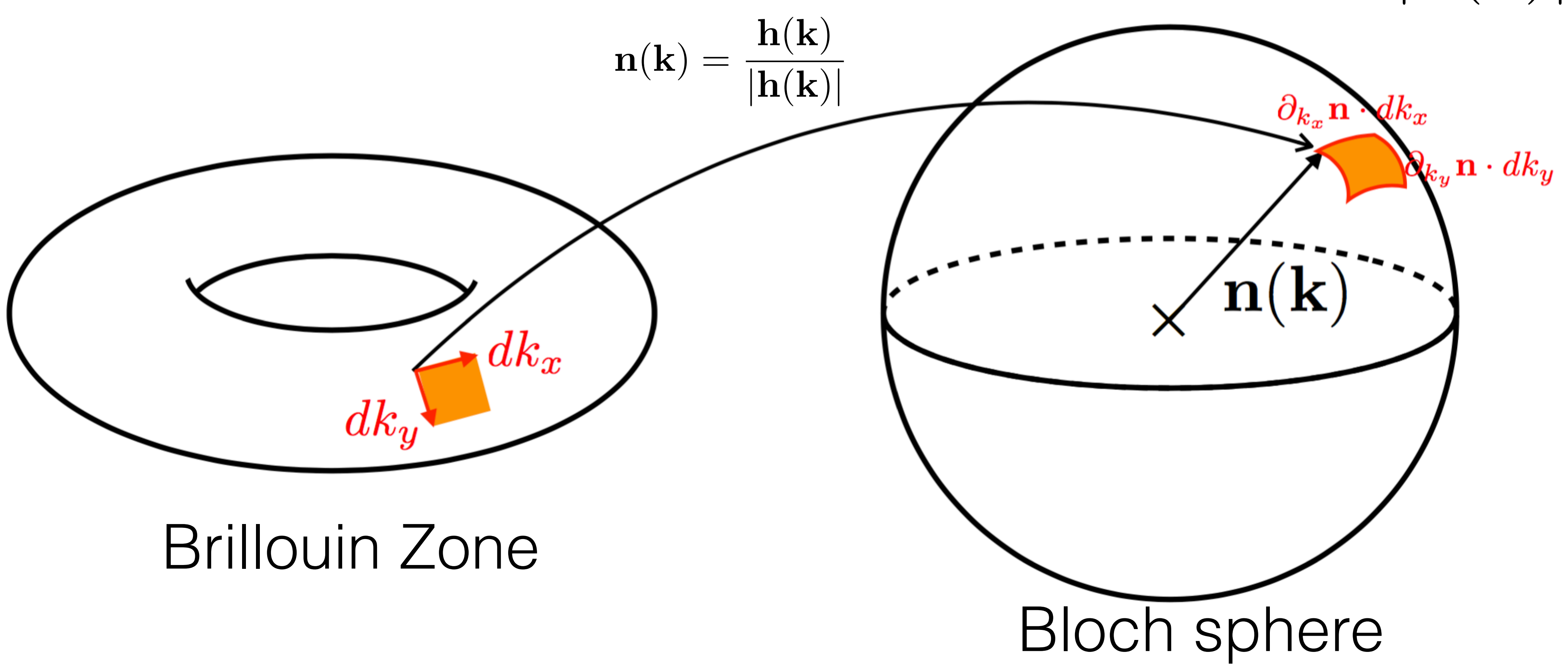


Topologically distinct!

$$\mathcal{H}(\mathbf{k}) = \mathbf{h}(\mathbf{k}) \cdot \vec{\sigma} = \begin{bmatrix} r + \cos k_x + \cos k_y & \sin k_x - i \sin k_y \\ \sin k_x + i \sin k_y & -(r + \cos k_x + \cos k_y) \end{bmatrix}$$

Eigenvalues: $\epsilon_k = \pm|\mathbf{h}(\mathbf{k})|$

Define unit vector on Bloch sphere $\mathbf{n}(\mathbf{k}) = \dfrac{\mathbf{h}(\mathbf{k})}{|\mathbf{h}(\mathbf{k})|}$



Chern number: # times **n**(**k**) paints the Bloch sphere

Zero band gap ⇒ **h**(**k**)=0 ⇒ Chern number ill defined!