A Toolbox for Optimized and Stable Experiments with Ultracold Quantum Gases

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Abstract

Since the successful production of the first Bose-Einstein condensates in dilute alkali vapours in 1995, our understanding of ultracold quantum gases has progressed tremendously. One of today’s principle fields of investigation are the realization of versatile quantum simulators. They overcome the difficult computational problem of simulating an uncontrollable quantum system by replacing it with another one of high controllability. In our experimental apparatus, the combination of ultracold $^{87}$Rb atoms in optical lattices with single-site resolution imaging, the ability of creating arbitrary optical potentials using different techniques and local Faraday imaging (a dispersive imaging technique) will form the basis for our quantum simulations. The steady progress in the research field, requires more and more versatile and stable experiments. Within the scope of this thesis different stepping stones towards this goal are presented.

In a first set of experiments, Faraday imaging as a benchmark tool is explored and utilized. Possible applications of monitoring atom numbers and temperatures, as well as in situ magnetic field measurements are discussed. It is demonstrated how the predictive power of the measurements can be used to detect and circumvent shot-to-shot atom number fluctuations in high precision measurements. In the following, Faraday imaging is used to directly measure the phase-transition from a thermal cloud of atoms to a Bose-Einstein condensate. The influence of the probe on the transition itself is investigated. Moreover, we demonstrate the implementation of an ultracold atom magnetometer in conjunction with Faraday imaging. Single shot precisions of $2\,\text{nT}$ in a total measurement time of just below $1\,\text{ms}$ are reached.

In a second set of experiments, the procedure of finding optimized solutions of producing a Bose-Einstein condensate in variable trap geometries is investigated. There, we examine and utilize the partial knowledge about the topology of the underlying control landscape. Techniques involving both computer-based optimization and the ‘gamification’ of the problem for exploiting cooperative human solving strategies are investigated. The latter experiments proved to be most successful, and by using in this way optimized evaporation sequences, we could increase our over all yield in Bose-condensed atoms by more than 40%. Bose-Einstein condensates as large as $(2.76 \pm 0.01) \cdot 10^6$ atoms could be produced.
Resumé


I et andet sæt af eksperimenter bliver proceduren for at finde optimale løsninger til produktion af Bose-Einstein-kondensater i varierende fældegeometrier undersøgt. I disse udforskes og udnyttes det delvise kendskab til topologi af det underliggende kontrollandskab. Teknikker der anvender både computerbaseret optimering og 'gamification' af problemet, der udnytter menskikers kooperative løsningsstrategier, undersøges. Disse sidste eksperimenter viste sig at være sædeles frugtbare, og ved på denne måde at optimere fordampningssekvenser har vi kunnet øge det overordnede udbytte af Bose-kondenserede atomer med mere end 40%. Det var muligt at producere Bose-Einstein-kondensater så store som $(2.76 \pm 0.01) \cdot 10^8$ atomer.
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The emergence of quantum mechanics at the beginning of the 20th century revolutionized our understanding of many areas of the natural sciences. Not only did it radically change the fundamental understanding of physics, but also found its way into other fields like biology and chemistry, where the explanation of basic molecular bonds is based on quantum mechanics. It came along with a long list of technological advances that influence everybody’s daily life. Modern electronics would be impossible without semiconductor technology of transistors that intrinsically depend on quantum mechanical effects. However, it is still impossible to calculate precisely the physical properties of a larger number of interacting particles, a problem which is exponentially hard, and we are therefore limited to modelling only a few tens of them. The exact knowledge of the individual particles’ states and their coherences is required for understanding in depth the properties of highly correlated systems, like high-$T_c$ superconductors. A way out was presented about thirty years ago by R. Feynman, who proposed the development of quantum simulators. The idea is to replace the complex quantum system under investigation by another, well-controlled one with the same properties.

Since then, multiple prototypes of quantum simulators in different physical systems have been proposed and realized. Besides ultracold quantum gases, there are trapped ions, photonic systems, or superconducting qubits. More exotic systems like strongly interacting nuclear spins on a diamond surface, controlled via nitrogen-vacancy centres below the diamond surface, have been suggested and investigated as a large scale system for quantum simula-
1. Introduction

The importance of the field of ultracold atoms has gained momentum, ever since three groups independently succeeded in 1995 to produce the first Bose-Einstein condensates (BEC) in dilute alkali vapours \([11, 13]\). Four years later the production of the first quantum degenerate gas of fermions \([14]\) followed. These are macroscopically sized, pure quantum objects of exceptional controllability and purity which makes them ideal model systems. Feshbach resonances allow for the precise tuning of interparticle interaction over several orders of magnitude \([15]\). They make it possible, to map out the different regimes of the BEC-BCS cross-over in fermionic systems (see e.g. \([16, 17]\)).

Another method for gaining control over the atoms is by loading them into an optical lattice formed by counter-propagating laser beams. On one hand, this facilitates the study of lower dimensional system, like the Berezinskii–Kosterlitz–Thouless (BKT) transition in two-dimensional layers \([18]\) (amongst others, the work on its theory was honoured with the Nobel prize 2016 to J. M. Kosterlitz and D. J. Thouless), or the preparation of a Tonks–Girardeau gas in one-dimensional tubes \([19]\). On the other hand, optical lattices also allow the realization of artificial ‘crystals’ made of atoms. By changing the lattice depth, the ratio between the on-site interaction and the tunneling probability of atoms between different lattice sites can be tuned. The dynamics of ultracold bosons in optical lattices is described by the Bose-Hubbard model \([20]\) and at low temperatures predicts a phase transition from a superfluid phase to a Mott insulating phase. The former phase exhibits Poissonian statistics of the occupation number of the lattice sites, whereas exact integer site filling and complete absence of particle coherences characterizes the latter one.

The existence of these phases was successfully demonstrated in a pioneering experiment \([21]\). Back then, the single-site atom number was still inaccessible and the detection was performed in momentum space. This changed with the development of quantum-gas microscopes capable of resolving individual atoms in single lattice sites \([22]\). Now atom number fluctuations and correlations became directly accessible \([23, 24]\). Direct benchmarking against \textit{ab initio} theoretical calculations were performed \([25]\), demonstrating impressively the proof of principle of these types of quantum simulators. This development was soon followed by the ability of manipulating, spin-resolved, individual atoms with a strongly focused addressing beam \([26]\). This achievement forms another milestone and triggered yet another series of exciting experiments investigating, for example, quantum magnetism \([27, 29]\) on the single atom level, or the testing...
of quantum field theories \[30\].

Another branch of quantum simulations using ultracold atoms developed in recent years is called \textit{atomtronics} \[31, \[32\] and focuses on simulating systems resembling elements of electronic circuits, thereby exploiting the superfluid nature of the atoms. Experiments include observations of Josephson effects \[33, \[34\] or hysteresis effects in a quantized current \[35\] in weakly linked BECs, both of which happen to be observed in superconducting circuits. More ‘classical’ electronic parts, close to field-effect transistors have also been realized \[36\]. Here, the creation of arbitrarily shaped potentials, using different techniques, is of importance.

In our experimental apparatus, where we work with \(^87\text{Rb}\), the technique of optical lattices with single-site resolution and the ability of manipulating individual atoms will be combined with the generation of arbitrary potentials at the same level of resolution. Although many technical steps are involved, it is important to regard the production of BECs as a starting point of our experiments. Already at this point, high stability and optimality of the system is required. Within the scope of this thesis, different methods and tools were developed to pave the way for finding and providing optimal starting conditions of future quantum simulation experiments.

One field of research is the development and exploitation of techniques to find optimized solutions to problems with complex parameter dependencies. Based on the theoretical example of optimizing the one-dimensional transport of single atoms in a tightly confining potential, novel optimization methods are developed. They are transferred to the experimental problem of BEC production in variable trap geometries. Here, the investigation is extended to the application of the well-established optimization algorithm dCRAB \[37\] and an unprecedented involvement of ‘citizen scientists’, by turning the problem into an engaging collaborative computer game. Human problem solving by the ‘crowd’ is very efficient and a detailed understanding of how people solve problems of this sort, is of extremely high value for the development of versatile optimization processes.

Our setup is complemented with non-destructive Faraday imaging \[38\] which, in contrast to destructive absorption imaging, allows one to probe the same atomic cloud multiple times. In addition, it is sensitive to the spin orientation of the atoms which can be utilized, for instance, for in situ magnetic field measurements. Due to the non-destructive character of the measurements, they form the ideal benchmark tool for atom number, temperature and magnetic
field measurements. This forms the basis for another set of experiments presented in this thesis, in which the different methods are investigated and possible future applications are given. Moreover, the influence of Faraday probing itself is characterized in the case of the phase-transition to a BEC.

Structure of the thesis

- **Chapter 2:** An overview of the experimental apparatus including the optical setup and the different vacuum chambers is given.

- **Chapter 3:** The optical dipole trap, whose construction was one of my main contributions to the experimental setup during the first year of my PhD, is presented in slightly more detail. The different trap configurations and methods of producing BECs are presented.

- **Chapter 4:** This chapter describes our findings on utilizing, among others, optimization landscapes for finding optimized solutions for different kind of problems. The first part discusses the theory case of optimizing the single-atom transport in a tightly confining tweezer potential. I would like to emphasize that the simulations and major parts of the data analysis were conducted by the theory division in our group. I was involved in the presentation and discussion of the data.

  In the second part, parts of the methodology are transferred to our experimental problem of condensing ultracold atomic clouds to a BEC. It is supplemented by further techniques for finding optimized solutions. This involves both computer-based optimization and the ‘gamification’ of the problem for exploiting cooperative human solving strategies.

- **Chapter 5:** Non-destructive Faraday imaging and the corresponding optical setups are introduced and discussed. This sets the basis for the following two chapters.

- **Chapter 6:** In this chapter, our results on probing the BEC phase-transition by using Faraday imaging are presented. Firstly, the use of in situ Faraday probes as a benchmark tool for monitoring atom number and temperature is discussed. Secondly, the *single-shot* imaging of the

\[\text{Apply game elements to problems without a direct game context.}\]
BEC phase-transition and the influence of the probe on it is investigated. Hereby, the transition is crossed both through evaporative cooling and in a conservative, repeatable way by applying a tightly confining dimple potential.

- **Chapter 7**: Two different implementations of an ultracold atom magnetometer by utilizing Faraday imaging are shown. Both are well applicable for future experiments where in situ magnetic field measurements are necessary.
This chapter serves as an overview of the HiRes experiment. As already pointed out in the introduction to this thesis, the final experiment will be at the forefront of state-of-the-art ultracold atoms experiments and combine multiple different experimental techniques.

The planning and building of such an experiment can easily take up to five years and longer without conducting any relevant experiment. Therefore, we decided to take a different path and instead of planning and building the main apparatus at once, the experiment was set up in multiple smaller expansion stages. By this we were not only able to run experiments which were more than pure characterization or calibration, but we could also use the learning experience from these early experiments for the planning of the remaining setup.

The first half of my PhD time was nonetheless mainly spent on building and setting up relevant parts of the setup. However, most of the setup and its technicalities are already described in great detail in [39]. Therefore, this chapter mainly focusses on the changes to the setup since then and presents the major parts of the experiment which are relevant for understanding the sequences of cooling the atoms.

The whole experimental setup extends basically over two optical tables. Section 2.1 reviews the laser setup for preparing all our different laser beams for magneto-optical trapping, pre-cooling and imaging of our atoms, which is based on a single optical table. The other optical table bears the different vacuum chambers in which our experiments are conducted. They are described further
2. Overview of the HiRes Experiment

in section 2.2. Due to the complexity of our science chamber and its versatility in possible experiments, an own section is dedicated to it and presented in 2.3. Section 2.4 concludes the chapter with a description of a typical experimental sequence of how atoms are prepared, cooled and finally detected.

2.1. The laser setup

We are working with $^{87}$Rb atoms which are laser cooled on the D2 transition. Details to spectroscopic data and other properties of $^{87}$Rb can be found in [40]. An overview of the relevant frequencies in our experiment can be found in appendix A.1. In order to obtain a closed cooling cycle, besides a master laser a repump laser is needed pumping atoms which fall into a dark state back into the cooling cycle. The setup of these lasers is briefly described in subsection 2.1.1. For cooling and later for optical pumping and absorption imaging, the light has to be shifted to the according frequencies, mixed and distributed to various fibre ports for guiding the light to the different places on the experimental table. Polarization maintaining fibres guide the light to our experiment. This mixing board is described in subsection 2.1.2. The generation of the light for non-destructive Faraday imaging is a separate setup and will be briefly reviewed at the end of this section.

2.1.1. Master & repump laser

The main cooling transition we are addressing with our master laser is $F = 2 \rightarrow F' = 3$. The setup was recently rebuilt and differs slightly from the one presented in [39]. It is depicted in figure 2.1. The biggest change involves the laser, which is now an external cavity diode laser (ECDL) TOPTICA DL PRO. The in house locking electronics was replaced by the all digital control system TOPTICA DLC PRO. Using this all-in-one system we expect much better long term stability. In addition, the new laser has a higher direct output power which will allow us to implement additional beams for in-situ molasses cooling and imaging for future lattice experiments.

Similarly to the old setup, the light is split into two paths after passing an optical isolator (components remained the same as in [39]) preventing back reflections from other optical elements into the laser. The light of the first
2.1. The laser setup

Figure 2.1.: The optical setup of the master laser. The setup was rebuilt and modified compared to the one in [39]. See text for details.

path is guided through a homebuilt tapered amplifier (TA) for amplification and then coupled into an optical fibre and guided to the mixing board.

A small portion of light in the second beam path is used for locking the master laser by means of frequency modulation (FM) spectroscopy [41] on the $F = 2 \rightarrow F' = (2, 3)$ crossover transition. Before passing the spectroscopy cell, the light passes a double-pass acousto-optic modulator (AOM) configuration which results in an effective red detuning of 300 MHz compared to the $F = 2 \rightarrow F' = 3$ transition. The remaining part of the light is coupled into a different fibre for future use, as described above. At the moment we use this light as a reference for our Faraday laser (cf. subsection 2.1.3 below).

It can happen that atoms fall out of the cooling cycle and end up in the $F = 1$ ground state manifold. Therefore, a repump laser is applied. The setup is quite similar to the one of the master laser and can be found in [39]. In brief, the laser (ECDL, TOPTICA DL 100) is lock by FM spectroscopy on the $F = 1 \rightarrow F' = (1, 2)$ crossover transition. Again, the spectroscopy light is shifted by a double-pass AOM before passing the vapour cell. This time the shift amounts to $-193$ MHz, leading to the laser being effectively on resonance with the $F = 1 \rightarrow F' = 2$ transition. Through an optical fibre the laser light is sent to the mixing board.
2.1.2. The mixing board

The mixing board consists of mainly four sections and the light coming from the master laser is split up accordingly. For more details about the exact optical setups and technicalities, see [39]. Two sections are nearly identical and prepare the light for the 2D and 3D magneto-optical trap (MOT). Both need light which is red detuned from the $F = 2 \rightarrow F' = 3$ transition. In each section, a double-pass AOM allows for appropriate and independent frequency adjustment. In the following, the light is overlapped with the repump light on a polarizing beam splitter cube (PBS) and sent through a TA, before the light is coupled into a single optical fibre in the case of the 3D MOT and two fibres in the case of the 2D MOT. Small portions of the light after the TAs of both sections are sent into a single Fabry-Perot cavity which allows to monitor the overall quality of the laser locks and the power ratio between repump and master laser.

The third section of the mixing board prepares the light for the imaging and push beam. A double-pass AOM shifts the light close to (red detuned) or on resonance to the $F = 2 \rightarrow F' = 3$ transition (depending on the operation for preparing the push or imaging beam, respectively) and allows for fast switching of the light, which is essential for imaging. Afterwards, the light is coupled into three fibres. One contains the push beam, the second one the light for imaging along the longitudinal direction (for definition of directions, see figure 2.2). The third fibre guides the light to a separate optical setup, which allows to distribute the light to four different fibre ports, one of which is shared with the Faraday imaging light (see below).

The fourth and last part of the mixing board is used for providing the optical pumping beam which is used to transfer the atoms to the magnetically trap-pable $|F = 2, m_F = 2\rangle$ state at the end of the molasses phase (cf. section 2.4). Therefore, it needs to be resonant on the $F = 2 \rightarrow F' = 2$ transition. Again a double-pass AOM is used to adjust the frequency and allow for fast pulsing of the beam. Additionally, for ensuring effective optical pumping, the beam is overlapped with a small portion of the repump beam before coupled into a fibre.

All previously mentioned fibre ports are supplied with individual mechanical shutters which allow for complete, physical blocking of the different beams.
2.2. The experimental chambers

2.1.3. The Faraday laser

On a separate breadboard, we prepare the light for Faraday imaging. Another ECDL (again TOPTICA DL 100) is offset beatlocked to the master laser \([42]\) (details to the setup are given in \([39, 43]\)). This provides an easy way to quickly change the frequency of the Faraday laser in a range of \(0.6 – 1.5\ \text{GHz}\) for both, blue and red detuning with respect to the imaging transition. A part of the beat signal is sent to a frequency counter (MICRUCITS, UFC-6000) for online monitoring of the exact offset frequency. The light is overlapped with one of the absorption imaging beams and sent to an optical fibre (cf. previous subsection).

2.2. The experimental chambers

In this section, the vacuum setup with the different chambers is described. It consists in total of four chambers: the 2D MOT and the 3D MOT chamber, which is the starting point of all our experiments, and the cube and science chamber where the actual experiments are conducted. The overall setup is depicted in figure 2.2. As previously mentioned, we decided to build our vacuum setup in multiple steps and to start out in the beginning only with having the intermediate cube chamber attached. This division turned out to be very beneficial. Not only we gained optical access through stretching the whole experiment by the intermediate chamber, but we were also able to perform experiments at a quite early point in time. As before, technical details about single components are omitted in the following and it is referred to \([39]\).

For achieving fast loading rates of our MOT, we make use of a so-called 2D MOT. The 2D MOT chamber is a glass cell and is equipped with two sets of anti-Helmholtz coils for creating a quadrupolar magnetic field along its long side. On top of these coils there are additional offset coils allowing for adjustment of the magnetic field zero with respect to the chamber centre. In conjunction with two pairs of elliptically shaped MOT beams which are concentric to the coils, a 2D MOT can be formed resulting in a cigar shaped atom cloud. The source of the atoms are a set of dispensers inside the glass cell. Electrical connection is ensured via vacuum feedthroughs at the flange which is attaching the 2D MOT cell to the 3D MOT chamber. This flange is equipped with a nozzle of small diameter which establishes a connection between the 2D
MOT and 3D MOT chamber. The atoms are pushed through the nozzle with the help of the push beam which was already previously mentioned in the description of the laser setup. The nozzle fulfils two additional functionalities: First, it is serving as a differential pumping stage to ensure an UHV environment of below $10^{-10}$ mbar in the 3D MOT chamber and second, the nozzle has a 45° angled polished surface, which allows to reflect a retarding laser beam counter-propagating to the push beam establishing the operation of a so-called 2D+-MOT \cite{44}.

The atoms are collected in a 3D MOT in the 3D MOT chamber. This octagonally shaped chamber has in total eight CF40 flanges in the horizontal plane and two CF100 flanges from top and bottom. The latter ones plus the four diagonally situated CF40 flanges are used to shine in the three counter-propagating beam pairs for forming the MOT. The MOT is completed by a set of anti-Helmholtz coils which can produce field gradients of up to 150 G/cm which is sufficient for pure magnetic trapping of the atoms. As the coils are relatively far away from the atoms, large currents are needed and hence their heat load is quite high. They are therefore wound on water-cooled holders. These holders are in turn attached to a motorized transport stage enabling us
2.3. The setup of the science chamber

to magnetically transport the atoms from the 3D MOT to the cube chamber.

These two chambers are connected through a vacuum tube and a flexible bellow. They serve as another differential pumping stage and reduces the background pressure to below $10^{-11}$ mbar in the cube and science chamber which is sufficient for our current experiments. The cube chamber is equipped with a fixed set of high current coils for microwave evaporation as it turned out that the cooling of our transport coils is insufficient for a continuous operation of the experiment. These coils are made of a hollow-core wire which allows for direct, efficient water-cooling. For cancelling residual background magnetic fields or creating an offset field in a certain direction, the chamber is equipped with offset coils.

The cube chamber hosts one of our crossed dipole traps which can be transformed into a hybrid trap. The setup and related experiments are described in greater detail in chapter 3.3 and 4.2, respectively. Furthermore, the chamber has two re-entrant viewports in the horizontal plane. In the early stages of our experiment, an objective with a focal length of 22 mm was attached to one of the viewports which was used for our first experiments with in-situ non-destructive imaging and manipulation of thermal and condensed atom clouds. The results to these experiments are mainly discussed in chapter 6.

Finally, the cube chamber is connected to the science chamber. Details about its specific setup and experimental capabilities are described in the next section. The atoms are transferred to this chamber via optical transport using the dipole beam along the longitudinal direction (for more details on this see sections 2.4 and 3.3.3).

2.3. The setup of the science chamber

Our science chamber is planned for performing various kinds of experiments. First of all, it will host the optical (super-)lattices in combination with single-site imaging of the atoms. Besides manipulation and addressing of single atoms in the lattice by applying optical tweezers, we want to project also arbitrary (static and dynamic) light patterns through the high resolution objective onto the atoms. These patterns serve either for addressing single or groups of atoms, or for creating high resolution arbitrary potentials on the length scale as small as a lattice potential period. In another category of experiments, we will combine the creation and trapping of atoms in micro potentials (dimple traps) with
2. Overview of the HiRes Experiment

![Diagram of the HiRes Experiment chamber with viewports labeled a) and b).](image)

**Figure 2.3.** The final science chamber depicted from the top (left-hand side) and from the side (right-hand side). In total four re-entrant viewports are attached to the chamber. Two of the viewports (marked with a)) are used with the medium resolution objectives. One of the viewports (b) is used as a retroreflector for one axis of the optical (super)lattice. The fourth re-entrant viewport (visible in the side view) hosts the high resolution objective for single-site resolution imaging. It serves as well as a mirror for the optical lattice in the vertical direction. Adapted from [39].

Local dispersive probing using Faraday imaging. This allows to probe a larger system spatially selectively which gives multiple application possibilities: e.g. perform local magnetometry as it will be presented chapter 7.

All these proposals require multiple different experimental techniques which are reviewed in the following. The setup of the vacuum chamber is explained in section 2.3.1, followed by a description of the setup for creating microtraps with an acousto-optic deflector (AOD) in section 2.3.2. The setup for local Faraday probing with a digital micromirror device (DMD) is shown in section 2.3.3. The section is concluded by a brief description of our future high resolution objective in section 2.3.4. As before, more technical details can be found in [39].

### 2.3.1. The vacuum chamber

As we will be working with lattices of multiple different wavelengths and moreover want to manipulate and probe our atoms from different directions, the chamber is equipped with a series of different viewports. The chamber is
2.3. The setup of the science chamber

sketched in figure 2.3. In total there are six viewports in the horizontal plane and two for optical access in the vertical direction.

**Viewports in the vertical direction**

The viewport on the bottom of the chamber is constructed as a re-entrant viewport and will be used for high resolution imaging with our custom made high resolution objective (details below in subsection 2.3.4). It will be used for probing, addressing, and the creation of arbitrary potentials at high resolutions. The viewport serves at the same time as a mirror for the vertical optical lattice which will be shone in from the top of the chamber. To fulfil all these requirements, the window of the viewport has a special coating which is transmissive at $780–790\text{ nm}$ (probing & addressing) and $940\text{ nm}$ (arbitrary potentials). At the same time, it is highly reflective for the planned lattice wavelengths at $912\text{ nm}$ and $1064\text{ nm}$. For creating diffraction limited structures, a high NA is necessary. This requires additionally a large range of angles of incidence between $0–45^\circ$ for the transmission window which has to be considered in the design of the coating.

**Viewports in the horizontal plane**

The four viewports on the diagonals of the chamber are used to shine in two of the three lattice beams as well as resonant light for probing and molasses cooling. In order to circumvent overlapping beams due to internal reflections in the windows, the glass substrates where the lattice beams enter the chamber are angled at $15^\circ$. The lattice beams will be retroreflected to create the standing wave potential. In one case for saving the space for the necessary optics and for enhancing mechanical stability, the beam is retroreflected within the chamber from the opposite viewport. In order to minimize the waists at the position of the atoms and thus maximize the overall intensity for creating an as deep optical potential as possible, the beam will be focused on the reflecting viewport. It is additionally designed as a re-entrant viewport in order to further minimize the distances and attainable waists. The second lattice beam will be retroreflected outside of the chamber in a cat’s eye configuration. The re-entrant viewport is equipped with a customized coating (LaserOptik) which is highly reflective for $\lambda = 790–1064\text{ nm}$, but transmittive for the probing light at $780\text{ nm}$. 
2. Overview of the HiRes Experiment

The remaining two viewports in this plane are also constructed as re-entrant viewports (cf. figure 2.3) and are hosting each a custom made medium resolution objective. The combination of the two allow e.g. to project an arbitrary pattern of probing light onto the atoms with relatively high resolution. The light can be collimated by the other objective and imaged on a camera for detection. The objectives are designed to have a NA of 0.27 at a working distance of $f = 36.8 \, \text{mm}$ for wavelengths of 780 nm. The performance was confirmed in a test setup and allowed to create there diffraction limited spots with a waist of $1.3 \, \mu\text{m}$ as expected. In turn, imaging should perform equally well and one should be able to resolve structures on the same order. The objectives are mounted on a custom made mount allowing for 5-axes adjustment (x, y and z translation, pitch and yaw). The screws adjusting the objective’s position along the optical axis, are equipped with piezo steppers. This allows for computer controlled focusing of the objectives.

For building the objectives into the final setup two reference beams from both sides were used for defining a common optical axis. Afterwards, the objectives are set into the beam paths making sure that tilts are avoided as good as possible from the beginning. An iterative alignment procedure with two reference points on each side was used to align the objectives with respect to the optical axis. Using another $f = 500 \, \text{mm}$ lens, one beam was imaged onto a CCD camera for monitoring the spot size that is created by the complete imaging system. The spot size was minimized by optimizing the distance and removing residual tilts between the two objectives. The correct relative placement of the objectives was confirmed in an independent shear plate measurement of the collimation of an initially collimated beam after passing the two objectives.

During the alignment procedure, it turned out that parts of the beams are cut by the high resolution re-entrant viewport. The beam sizes of the incoming beams therefore have to be limited to a diameter of 9 mm which corresponds to a drastically reduced NA of 0.11. Measurements with above mentioned CCD indicate that we are able to produce spots with a waist of 2.4 $\mu\text{m}$ which should be still good enough for our purposes.

2.3.2. The dimple setup

In the following the optical setup (depicted in figure 2.4) for the preparation and control of single or multiple dimple traps is described. It is placed directly next to the science chamber. A laser beam with wavelength 912 nm is used
2.3. The setup of the science chamber

Figure 2.4.: The optical setup for the dimple beam (simplified scheme). Besides the beam path of the 912 nm laser for creating the dimple (depicted in orange), also the transversal dipole trap beam (red) and the imaging beam (green) is shown (both counter-propagating). The different optical elements are given in figure 3.3. For more details see text.

for creating the optical potential. Central part of the setup is the two-axis acousto-optic deflector (AOD) (INTRACTION DTD-274HA6) which gives us the flexibility to steer the beam and create a large variety of different beam patterns by time-averaging them. A profound characterization of the AOD showing its functionality and versatility is given in [45].

The dimple beam is sent through the backside of one of the medium resolution objectives described above. The AOD has an open aperture of 4 mm. However, for a faster response and shorter beam shifting time, we send a beam with only a diameter of 1.25 mm in the AOD. With this beam we should be able to create up to 32 resolvable spots within the total RF bandwidth of the AOD (16 MHz).

The following optical system is designed to illuminate the whole clear aperture of the objectives and thus to make use of their full nominal NA (as described above the usable NA however is smaller). After the AOD a $f = 40$ mm
2. Overview of the HiRes Experiment

lens focuses the (+1, +1)-order which is recollimated by a $f = 750\,\text{mm}$ achromat. An iris blocks all other orders. The lenses are placed such that the deflection of the AOD results in nearly pure parallel displacement of the beam with respect to the optical axis at the position of the atoms. However, the chromatic shift of the objective which is focused for $\lambda = 780\,\text{nm}$ light has to be taken into account and corrected with the position of the last lens such that previous requirement cannot be met perfectly. On two dichroic mirrors, the beam is overlapped with the dipole trap beam and the imaging beam (cf. figure 2.4).

A flip mirror can be used to image the pattern created by the AOD on a separate CCD camera. For monitoring the beam’s intensity, a part of the beam is split off by a beam sampler and guided on a photo diode after the fibre. The intensity of the beam is regulated via an AOM in conjunction with a servo loop. The AOM is placed before the fibre in the setup of the laser supplying the AOD setup with light (not depicted). The laser used is another Toptica DL Pro at a wavelength of 912 nm. Its output is amplified by a homebuilt TA leading to beam powers of up to 900 mW after the fibre.

As RF source for the AOD, we use a PulseBlasterDDS-II-300-AWG board by SpinCore Technologies. It is equipped with two independent outputs which can deliver RF waveforms with frequencies of up to 100 MHz (the AOD’s center frequency is 27 MHz). A sequence is programmed as a set of instructions, each defining a RF pulse with certain length, intensity and frequency. An individual instruction can be as short as 13.3 ns which is more than sufficient for creating time-averaged potentials. In our case, the system is programmed via a LabView interface. Due to the flexible programming possibilities in terms of single instructions, e. g. a feed forward intensity stabilization of the beam after AOD correcting for frequency dependent deflection efficiencies can be easily developed and is about to be implemented. Home-built RF amplifiers amplify the signals to maximally 1 W for driving the two AOD axis.

The performance of the setup was shown in a test setup [39]. Condensed clouds of atoms were successfully trapped in up to 6 individual traps. We might encounter problems of parametric heating of the atom cloud as soon as the individual frequencies for time averaging become comparable due to interferences of the sound waves within the AOD crystal. In the test setup we discovered a modulation of the beam’s intensity with the frequency difference between following frequency steps which decays within 100 – 300 µs [39]. This effect seems to be a common problem for AODs and was seen also elsewhere [46].

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2.3. The setup of the science chamber

So far we were not limited by this problem in our experiments with atoms.

2.3.3. DMD setup for local probing of atom clouds

For local probing we make use of a digital micromirror device (DMD), which in our case is a DLP LightCrafter 6500 from Texas Instruments with $1920 \times 1200$ micro mirrors which can be individually switched to redirect the light into the remaining imaging setup and by that creating arbitrary binary structures. It is set up in direct imaging which means the DMD is placed in the object plane of the imaging system (as opposed to the Fourier plane described in the next subsection) and the image created on the DMD is directly projected onto the atoms. In order to allow for grey scaling, $7 \times 7$ pixels are combined to form a superpixel. The fast switching speed of 10 MHz of the DMD offers also the possibility of applying dynamical patterns. The optical setup is shown in figure 2.5.

The imaging beam delivered by an optical fibre is collimated by a $f = 150\,\text{mm}$ achromat to a size sufficient to illuminate the whole DMD chip, a beamsampler in conjunction with a photo diode directly after the fibre is used to monitor the beam’s intensity. The image emerging from the DMD is collimated by a lens with $f = 120\,\text{mm}$. The following two lenses form a telescope to
2. Overview of the HiRes Experiment

magnify the beam for using the full aperture of the final imaging objective. An iris is used to limit the resolution in the intermediate imaging plane of the system such that a superpixel appears as a point source and individual DMD pixels for grey scaling are blurred.

A $f = 200\text{ mm}$ lens before the DMD which is focusing the beam on the surface of the collimation lens after the DMD is used to reduce the effect of non-isoplanatism \cite{47, 48}. Due to the spherical shape of the object and imaging planes of thin lenses, imaging of large planar objects with coherent light can lead to interference effects. This is corrected by the inserted lens which introduces a spherical phase factor leading to effectively isoplanatic imaging. The improvement for our case is described and characterized in \cite{49}.

Before entering the chamber the polarization of the imaging beam is cleaned by a PBS with a high extinction ration of ($10^4 : 1$, Linos) necessary for Faraday imaging. The beam is overlapped with the dipole beam and the previously described dimple beam on a dichroic mirror.

A more detailed description of the setup for detecting the imaging light after probing the atoms and its performance is presented in chapter 5.2.2.

2.3.4. The high resolution setup

The setup for high resolution imaging and manipulation of atoms is only very briefly described here. Its characterization and description is a central part in \cite{39}. The high resolution objective is custom made (ASE) and designed to have a NA of 0.7 with a working distance of 12.95 mm and an effective focal length of 5 mm. It is corrected to infinity and can operate diffraction limited between 780 – 790 nm without additional correction optics. This means the theoretical resolution limit according to the Rayleigh criterion is $r_{\text{min}} = 679 \text{ nm}$ which should be sufficient to image and resolve single atoms in a lattice with spacing 532 nm.

Very crucial is the flatness of all optical elements in the beam path towards the imaging camera. Overall wavefront distortions $> \lambda/4$ would decrease the performance of the whole imaging system. The effect of the optical coating of the viewport has to be taken into account, but adds only a quadratic relative phase as a function of angle of incidence onto the wavefront. This means it can be either compensated by refocusing the objective or alternatively adjusting the thickness of the viewport window accordingly. Not to run into spatial constrictions, we did the latter one in our case.
2.4. A typical experimental sequence

The system was tested and characterized in a test setup by imaging a test target which was equipped with randomly distributed holes with diameters smaller than the imaging wavelength placed on a substrate that resembles the one used as a viewport window. These holes can be treated as ideal point sources and can be therefore used to extract the point-spread function of the imaging system. According to these measurements the resolution limit $r_{\text{min}}$ (Rayleigh) is between $668 \text{ nm}$ and $688 \text{ nm}$. Therefore, the resolution limit in agreement with the expected limit of $679 \text{ nm}$ and is operating diffraction limited.

Care has to be taken when the system is built into the experimental setup. The additional stress due to the evacuated chamber and welding of the window into the final viewport can lead to additional non-spherical wavefront distortions. They can be at least partially compensated by tightening the viewport screws accordingly. During this process the flatness of the window has to be monitored interferometrically. For this purpose we use a commercial interferometer (TRIOPTICS, µPHASE 1000), which was already used to characterize the flatness of the other optical elements. This process was already simulated successfully in another test setup.

Besides the high resolution imaging, we plan also to shine resolution limited, close-resonant addressing beams at $\lambda = 787.5 \text{ nm}$ and arbitrary potentials at $\lambda = 940 \text{ nm}$ through the objective onto the atoms. The beam patterns can be shaped by using DMDs in a direct imaging setup as described previously (2.3.3). Additionally, we plan separate DMD setups operating in the Fourier plane. Making use of binary holograms both phase and intensity of the light waves can be altered. This technique allows to create arbitrary light patterns and even to correct for wavefront aberrations introduced in the imaging system. The procedure for calculating the holograms and realizing arbitrary trapping potentials with it is described in great detail in [50]. To account for the chromatic shift of the $940 \text{ nm}$ light in the objective, custom made correction optics (ASE) is placed into the respective setup.
2. Overview of the HiRes Experiment

Figure 2.6.: Main stages in our typical experimental sequence. For the description, see text.

2.4. A typical experimental sequence

In this section, a quite detailed description of a typical sequence and the main parameters will be given. In brief, the sequence can be split up in five stages as depicted in figure 2.6: Loading of the MOT, magnetic transport to the cube chamber, microwave evaporation in the cube chamber, loading into the dipole trap and forced evaporation. For experiments in the science chamber the atoms are optically transported. The sequence in more detail:

We start by loading our 3D MOT within 5 s from a 2D MOT. The Rubidium atoms are emerging from dispensers installed in the 2D MOT glass cell (cf. figure 2.2 and section 2.2 below). The MOT beams have a total power of 200 mW and are red-detuned by \( \approx 4.5\Gamma \), where \( \Gamma \) is the natural line width of the D2 line. After a 8 ms long molasses phase, where the magnetic fields are switched off and the detuning is increased linearly from initially \( 7.1\Gamma \) to \( 13.5\Gamma \), the atoms are optically pumped into the magnetically trappable state \( |F = 2, m_F = 2\rangle \) and trapped in a magnetic trap. The optical pumping is initiated by pulsing vertical compensation coils attached to the 3D MOT chamber to define a quantisation axis in the vertical direction. At the same time the optical pump pulse is applied for a total duration of 1.5 ms. For the magnetic trap the field gradient of the coils is ramped up to 150 G/cm within 150 ms. The atoms are transported to the cube chamber within 2 s by moving the coils.

There, the atoms are transferred to a magnetic trap with a field gradient of 130 G/cm created by the fixed set of high current coils. This is followed by microwave evaporation, which transfers atoms from \( |F = 2, m_F = 2\rangle \) to the magnetically untrapped state \( |F = 1, m_F = 1\rangle \). This is done by ramping the frequency of the microwave in three linear ramps from initially 6.93 GHz to a

\footnote{Spherical phase factors can be always corrected by refocusing.}
2.4. A typical experimental sequence

A typical experimental sequence starts with a final frequency of 6.84 GHz over a total duration of 15 s. This results in a cloud of about $5 \cdot 10^8$ atoms at a temperature of $\sim 30 \mu$K.

Subsequently, the atoms are transferred to a crossed dipole trap. The dipole beams are first ramped within 10 ms to their respective loading power. Directly following, the magnetic quadrupole field is lowered in three linear ramps to the final value used later for the evaporation. Both, initial laser powers and magnetic fields, can vary depending on which kind of trap configuration is used. In case the final experiment is conducted in the cube chamber, forced evaporation in the dipole trap is performed. Details about the different trap configurations and respective sequences can be found in chapter 3.3.3. For experiments in the science chamber, the atoms are transported to it by means of optical transport. For that, the magnetic field of the quadrupole coils is lowered to zero while the transport beam along the longitudinal direction is kept at 4 W. A motorized stage changes the effective beam path length of the dipole beam and thereby shifts its focus position. In a s-shaped velocity profile with constant acceleration and deceleration of 112 mm/s$^2$ and a maximal velocity of 250 mm/s, the transport stage is moved within 2.2 s to its final position. Thereby the focus of the dipole beam is shifted over a total distance of 215.8 mm. Due to the specific optical setup of the dipole trap (cf. chapter 3.3.2) the focus moves effectively at twice the speed and double the acceleration/deceleration of the translation stage. During this transport the transverse dipole beam in the science chamber is already ramped up to 7 W. Afterwards, a crossed dipole trap is loaded by symmetrizing the trap in terms of the individual beam’s trap depth. As the waist of the longitudinal beam is about half the size of the transverse beam the power of the longitudinal beam has to be lowered therefore to 1.5 W. More details and the final sequence for producing BECs is described at the end of chapter 3.3.3.

Detection of the atoms

Finally at the end of each sequence, the atoms are detected with time-of flight (TOF) absorption imaging. They are imaged on the $F = 2 \rightarrow F' = 3$ transition (cf. level scheme in the appendix A.1). In the cube chamber, absorption images can be taken either along the longitudinal direction as well as the vertical direction. In the latter case, we are able to reach TOFs of up to 86 ms. In this way, even very large and dense condensate clouds can be recorded. For enhanced precision because of non-linear saturation effects and an unknown exact
2. Overview of the HiRes Experiment

coupling of the probe light to the atoms (due to e.g. imperfect polarization or ambiguous definition of the quantization axis), we performed a so-called \textit{alpha calibration} \cite{51} for the imaging along that direction.

Imaging of the atoms in this chamber is currently implemented in three different directions. Firstly, there is absorption imaging along one of the horizontal lattice axis, which allows for monitoring of the position and beam overlap for the crossed dipole trap as well as partially for the positioning of a dimple beam through the double-objective system. Secondly, there is absorption imaging through the objectives possible. The camera is the same one that was already previously mentioned for the alignment procedure of the objectives. Finally, there is the local Faraday probing, described further down this section.
The Production of $^{87}$Rb Bose-Einstein Condensates

This chapter gives an overview of how Bose-Einstein condensates (BECs) are produced in our experiment. As one of my main tasks during the first half of the PhD was the development and construction of our dipole trap, this separate chapter is dedicated to it. After some introduction to the theoretical background of BECs and how to produce them experimentally, the setup of the dipole trap is described in detail. The chapter is concluded by introducing the main dipole trap geometries and the corresponding evaporation sequences we use. Further experimental investigation of the control landscape of trap geometries is left for chapter 4.

3.1. Bose-Einstein condensates

3.1.1. Bose-Einstein condensation of a non-interacting Bose gas in a harmonic trap

The following description gives a brief introduction on how to derive transition temperatures and critical phase space densities for Bose-Einstein condensation in the case of a non-interacting Bose gas trapped in a harmonic trap. A good overview of the theory behind BECs, which gives more details than presented here, can be found in [52], and is the basis of following section.

At high temperatures, a dilute gas of trapped atoms is well described by Boltzmann statistics. For lower temperatures the quantum nature of atoms
3. The Production of $^{87}$Rb Bose-Einstein Condensates

can not be neglected and, in the case of identical, non-interacting bosons, the mean number of atoms in a certain energy level $\epsilon_\nu$ follows the Bose-Einstein distribution:

$$f_{\text{BE}}(\epsilon_\nu) = \frac{1}{e^{(\epsilon_\nu - \mu)/k_BT} - 1}, \quad (3.1)$$

where $\mu$ denotes the chemical potential. When approaching the transition to a BEC, $\mu$ rises but never exceeds the value of the ground state of the system $\epsilon_0$, which would mean negative and thus unphysical occupation numbers. The transition temperature to a BEC $T_c$ is reached when $\mu \to \epsilon_0$ and is defined as the point when all atoms are just occupying the excited states. Lowering the temperature further leads to a macroscopic occupation of the lowest energy level and a BEC emerges. For large atom numbers, $N \gg 1$, the zero-point motion energy can be neglected and hence $\epsilon_0$ set to zero. The above condition can then be written in the quasi-classical approximation which replaces a sum over the individual states by an integral [52]:

$$N = N_{\text{th}}(T_c; \mu = 0) = \int_0^\infty d\epsilon g(\epsilon)f_{\text{BE}}(\epsilon), \quad (3.2)$$

where $g(\epsilon)$ describes the density of states which depends on the shape of the trap and the dimensionality of the system. In the case of a 3D-harmonic trap integration and solving for the transition temperature yields

$$k_BT_c = \frac{\hbar \bar{\omega} N^{1/3}}{\zeta(3)^{1/3}} \approx 0.94\hbar \bar{\omega} N^{1/3}. \quad (3.3)$$

$\zeta$ denotes the Riemann zeta function and $\bar{\omega} = \sqrt[3]{\omega_x \omega_y \omega_z}$ is the geometric mean of the different trap frequencies. The temperature dependence of the condensate fraction $N_0/N$ below $T_c$ reads as

$$\frac{N_0}{N} = 1 - \left(\frac{T}{T_c}\right)^3. \quad (3.4)$$

Treating the distribution function (3.1) using the local density approximation and performing a direct integration over momentum space gives the density distribution of the thermal, non-condensed atoms:

$$n_{\text{th}}(r) = \frac{1}{N_T^3/2} g_{3/2} \left( e^{(\mu - V_{\text{ext}}(r))/k_BT} \right), \quad (3.5)$$
where $V_{\text{ext}}(r)$ is the external trapping potential and $\lambda_T = \sqrt{2\pi\hbar^2/mk_BT}$ is the thermal de Broglie wavelength. The Bose function $g_{3/2}(z)$ is, in general, defined as $g_p(z) = \sum_{l=1}^{\infty} z^l / l^p$.

A measure for the degeneracy of an atom cloud is the peak phase space density, generally defined as

$$\rho = n_{\text{th}}(0) \lambda_T^3 = g_{3/2}(e^{(\mu - V_{\text{ext}}(r))/k_BT}), \quad (3.6)$$

in accordance with equation (3.5). The critical phase space density $\rho_c$ for Bose-Einstein condensation of a non-interacting Bose gas does not depend on the trap shape and can be obtained by letting $\mu \to 0$, which yields

$$\rho_c = \zeta(3/2) \approx 2.612. \quad (3.7)$$

For temperatures well above the transition temperature $T > T_c$ the peak phase space density in a harmonic trap can be calculated by using Boltzmann statistics. It is given by

$$\rho_{\text{harm}} = N \left( \frac{\hbar \bar{\omega}}{k_BT} \right)^3. \quad (3.8)$$

### 3.1.2. Introducing weak interactions

For a more accurate description of a BEC, interactions have to be taken into account. The many-body Hamiltonian in second quantization describing interacting bosons with mass $m$ trapped in an external potential $V_{\text{ext}}$ is given by

$$\hat{H} = \int \text{d}r \hat{\Psi}^\dagger(r) \left[ -\frac{\hbar^2 \nabla^2}{2m} + V_{\text{ext}}(r) \right] \hat{\Psi}(r)$$

$$+ \frac{1}{2} \int \text{d}r \text{d}r' \hat{\Psi}^\dagger(r) \hat{\Psi}^\dagger(r') V(r' - r) \hat{\Psi}(r') \hat{\Psi}(r). \quad (3.9)$$

The $\hat{\Psi}^\dagger(r)$ and $\hat{\Psi}(r)$ are the quantum field operators for creating and annihilating a bosonic particle at position $r$, respectively. The first line describes the one-body energy of a particle in an external potential, whereas the second term represents pair-wise interaction through the potential $V(r' - r)$. In the weakly
interacting regime of ultra cold temperatures, the collisional properties of the gas can be fully described by s-wave scattering and the corresponding s-wave scattering length $a$. The two-body interaction potential in equation (3.9) can be replaced by the effective contact interaction

$$V(r' - r) = g\delta(r' - r),$$  

(3.10)

with coupling constant $g = \frac{4\pi\hbar^2a}{m}$.

As already described in the last section, Bose-Einstein condensation is characterized by a macroscopic occupation of the ground state of the system. In this limit the field operators can thus be decomposed to allow for perturbations, as described by Bogoliubov [54]:

$$\hat{\Psi}(r, t) = \Phi(r, t) + \hat{\Psi}'(r, t),$$  

(3.11)

where $\Phi(r, t) \equiv \langle \hat{\Psi}(r, t) \rangle$ is identical to the expectation value of the field operator such that $n_c(r, t) = |\Phi(r, t)|^2$ and is referred to as the order parameter, where $n_c(r, t)$ is the density distribution of the condensate. $\hat{\Psi}'(r, t)$ represents the excitations of the system and can be treated as a small perturbation for temperatures much lower than the critical temperature of the phase transition $T \ll T_c$. For $T \to 0$, it can be neglected and by applying the Heisenberg equations of motion one ends up at the Gross-Pitaevskii equation (GPE):

$$i\hbar \frac{\partial}{\partial t} \Phi(r, t) = \left(-\frac{\hbar^2\nabla^2}{2m} + V_{\text{ext}}(r) + gn_c(r, t)\right)\Phi(r, t).$$  

(3.12)

In the case of temperatures above zero, the excitations of the condensate $\hat{\Psi}'$ have to be taken into account. Through the static Popov approximation [55, 56], a solution as an extension to the GPE can be found:

$$i\hbar \frac{\partial}{\partial t} \Psi(r, t) = \left(-\frac{\hbar^2\nabla^2}{2m} + V_{\text{ext}}(r) + 2gn_{\text{th}}(r) + gn_c(r, t)\right)\Phi(r, t).$$  

(3.13)

The two additional terms have a simple interpretation, they can be seen as a correction to the external potential in the form of an additional mean field generated by the thermal and condensed parts. Static solutions to equation (3.13) can be found using the Thomas-Fermi approximation by neglecting the kinetic
energy part of (3.13) which yields an expression for the density distribution of the condensed part of the cloud [56]:

\[ n_c(r) = \frac{1}{g} \left( \mu - V_{\text{ext}}(r) - 2g n_{\text{th}}(r) \right). \]  

(3.14)

The calculation of the thermal part as given in equation (3.5), is now also extended by terms accounting for the interactions:

\[ n_{\text{th}}(r) = \frac{1}{\lambda^3} \frac{3}{2} \exp \left( \frac{[\mu - V_{\text{eff}}(r) - 2g(n_c(r) + n_{\text{th}}(r))]/k_B T}{3/2} \right). \]  

(3.15)

Equations (3.14) and (3.15) form a set of coupled equations which have to be solved self-consistently. They become especially important when one wants to calculate in-situ density distributions of trapped atom clouds [57]. However, simplifications, e.g. in the form of the so-called semi-ideal model [57–60], can be made which will be elaborated on in chapter 6.2.1.

### 3.2. Creating Bose-Einstein condensates in the experiment

In the following two methods for enhancing the phase-space density in order to cross the transition to a BEC will be discussed. The first one, presented in section 3.2.1, involves evaporative cooling of an atomic cloud, which is of a dissipative nature as atoms are lost during the process. Secondly, a way to conservatively cross the phase transition is the application of a strongly focused beam denoted as dimple and will be topic of section 3.2.2.

#### 3.2.1. Forced evaporative cooling

In a simple model of evaporative cooling, atoms with an average temperature \( T \) are trapped in a harmonic potential of depth \( U \). Only atoms which have a certain thermal energy can leave the trap. This leads to a truncation of the high energy tail of the thermal distribution. In this context, one can define the truncation parameter \( \eta = U/k_B T \). Elastic collisions at the rate \( \Gamma_{\text{el}} = n \sigma v \), with \( n \) the atomic density, \( \sigma \) the elastic scattering cross section and \( v \) the relative average velocity of the atoms, lead to rethermalization of the cloud
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ends up at a slightly lower average energy. The probability that an atom with temperature $k_B T > U$ escapes the trap can be estimated by the Boltzmann factor $\propto \exp(-\eta)$. This sets, together with $\Gamma_{\text{el}}$, an overall time scale to the evaporation process. Lowering the potential gradually retains a finite $\eta$ and a constant, ongoing evaporation. The existence of background gas collisions set an upper limit for the time scale of the evaporation process. Therefore, high elastic scattering rates are favoured as otherwise evaporation efficiency might drop significantly.

Scaling laws can be derived \cite{61} which establish simple relations between different relevant parameters such as atom number $N$ and trap depth $U$. They are briefly reviewed, while losses due to background gas collisions are neglected. It will be concluded with a suggested time dependence of the trap depth $U(t)$ for evaporation at a fixed $\eta$.

The total energy loss rate during evaporation is given by

$$\dot{E}_{\text{tot}} = \dot{N}(U + \alpha k_B T) + \frac{\dot{U}}{U} \frac{\dot{E}_{\text{tot}}}{E_{\text{tot}}},$$

(3.16)

where the first term describes the loss due to the escape of an atom with average energy $U + \alpha k_B T$. In general it can be assumed that $0 \leq \alpha \leq 1$. For atoms in a harmonic potential with pure $s$-wave scattering, one can show that $\alpha = (\eta - 5)/(\eta - 4)$ \cite{62}. The second term of equation (3.16) accounts for the change of total energy due to changes in the potential depth. At the same time the energy of a classical gas is given by $E_{\text{tot}} = 3 N k_B T$ which leads to the relation $\dot{E}_{\text{tot}} = 3 k_B (\dot{N} T + N \dot{T})$.

Assuming a fixed $\eta$, equation (3.16) can be now integrated, yielding

$$\frac{N}{N_i} = \left( \frac{U}{U_i} \right)^{3/[2(\eta' - 3)]},$$

(3.17)

where $\eta' = \eta + \alpha = \eta + (\eta - 5)/(\eta - 4)$. $U_i$ and $N_i$ are the initial trap depth and atom number, respectively. Considering an energy independent elastic scattering cross section, one finds $\Gamma_{\text{el}} \propto N \tilde{\omega}^3 / k_B T$ and can derive a similar relation for the elastic scattering rates:

$$\frac{\Gamma_{\text{el}}}{\Gamma_{\text{el},i}} = \left( \frac{U}{U_i} \right)^{\eta'/[2(\eta' - 3)]}.$$

(3.18)
3.2. Creating Bose-Einstein condensates in the experiment

As mentioned above, the time-scale for the evaporation is determined by $\Gamma_{el}$ and $\eta$. In the scope of the $s$-wave Boltzmann equation [62] one finds an expression for the evaporation rate to lowest order of $\exp(-\eta)$ [61]:

$$\dot{N} = -2(\eta - 4) \exp(-\eta) \Gamma_{el} N.$$  \hspace{1cm} (3.19)

Together with equations (3.17) and (3.18), this allows one to derive a closed form for the time dependence of the trap depth which can be directly related to laser powers (cf. section 3.3.1):

$$\frac{U(t)}{U_i} = \left(1 + \frac{t}{\tau}\right)^{-\beta} = \frac{P(t)}{P_i},$$  \hspace{1cm} (3.20)

where $t = [t, t_f]$ defined through $U(t_f) = U_f$ the final potential depth at the end of the ramp. $\beta$ and $\tau$ are given as

$$\beta = 2(\eta' - 3)/\eta',$$  \hspace{1cm} (3.21)

$$\frac{1}{\tau} = \frac{2}{3} \eta' (\eta - 4) \exp(-\eta) \Gamma_{el,i}.$$  \hspace{1cm} (3.22)

As can be seen, the initial collision rate $\Gamma_{el,i}$ determines the time scale of the evaporation ramp defined by $\tau$. Equation (3.20) is used to describe the ramps in our experiment, as will be seen later in this chapter. Thereby, $\beta$ and $\tau$ will be treated as separate parameters.

3.2.2. The dimple trick

By shining a strongly focused, far detuned beam (dimple) onto a trapped cloud of atoms, the potential that the atoms experience is deformed. As long as the additional beam deforms the overall potential only locally, this leads to an increase of the atomic density in its vicinity (see figure 3.1). If the effective trapping volume of the additional beam is much smaller than the one of the surrounding trap, only a small fraction of the atoms will populate the dimple potential. The major part of the cloud acts as a reservoir which is in thermal contact with the atoms in the dimple. By adding the dimple adiabatically, the overall temperature can be assumed to stay constant. This in turn will leave the thermal de Broglie wavelength unchanged (cf. section 3.1.1) and hence lead to a local increase of the phase space density, whereby the overall entropy of the gas does not change. In this way the phase transition to a BEC can be induced.
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**Figure 3.1.:** Illustration of the dimple trick. Top line: (a) The red solid line shows the reservoir potential created by the CDT before shining in the dimple. The atoms (depicted in blue) are evaporatively pre-cooled close to the transition to a BEC. Adding the potential of the dimple (b), locally increases the phase space density which is enough to cross the phase transition. Bottom line: corresponding experimental absorption images for the respective situations. Taken from [39].
3.3. A versatile optical dipole trap for cooling and transporting $^{87}$Rb

This particular technique is generally known as *dimple trick* [63, 64]. In an equivalent picture, the chemical potential $\mu$ of the cloud remains unchanged while deforming the trap with the dimple as depicted in figure 3.1. However, due to the deformation the lowest energy state comes closer to the level of the chemical potential, consequently leading to a macroscopic occupation of that level (cf. the Bose-Einstein distribution given in equation (3.1)).

For estimating the increase in phase space density for a given dimple and certain reservoir conditions, one can follow a relatively simple model presented in [65]. Here, it is assumed that the entropies before and after the applied change of the trapping potential by the dimple have to be the same. For a non-interacting ideal gas, the entropy of this system can be calculated with the Helmholtz free energy $F$, via the thermodynamical relation $S = -\partial F/\partial T$. $F$ in general is given by

$$ F = -k_B T \ln(Z) \approx N k_B T \left( \ln(N) - \ln(Z_1) - 1 \right) = N k_B T \left( \ln(\rho) - 1 \right), $$ (3.23)

where $Z$ and $Z_1$ are the full and the single particle partition functions, respectively. Hereby, the relation $Z = Z_1^N/N!$ which holds for identical particles and Stirling’s formula $\ln N! \approx N \ln N - N$ was used for the simplification. $\rho = N/Z_1$ denotes the peak phase space density as given in equation (3.6) on page 27. Assuming Boltzmann statistics $Z_1$ becomes

$$ Z_1 = \lambda_T^{-3} \int dV \exp \left( -\frac{U(r) - U(0)}{k_B T} \right). $$ (3.24)

Using these relations the initial and final entropies can be calculated and due to the implied adiabaticity of the process equated: $S_i(T_i, N, U_i) = S_f(T_f, N, U_f)$. This system of equations can be solved numerically for $T_f$ and hence a value for the peak phase space density of the deformed trap can be obtained. A more detailed treatment including numerical results is shown in [39].

3.3. A versatile optical dipole trap for cooling and transporting $^{87}$Rb

This section describes the design of our dipole trap and the different configurations we use. After a short summary of how optical potentials are formed
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in section 3.3.1, the technical details of our specific setup are presented in section 3.3.2. Finally, typical trap geometries and corresponding evaporation sequences are discussed in 3.3.3.

### 3.3.1. Optical potentials for trapping ultracold atoms

The ability to trap neutral atoms with a focused and far detuned laser beam relies on the interaction between the beam’s oscillating electric field and the dipole moment of the atom. Due to the inversion symmetry of the electron wave function, neutral atoms do not possess a permanent dipole moment. However, it can be induced by the electric field of the laser beam itself.

For the typical intensities used in a dipole trap, the relation between the electric field amplitude $\hat{E}$ and the induced dipole moment $\hat{p}$ is linear and is described by the dynamic polarizability $\alpha(\omega)$. Following [66] it can be written as

$$\hat{p} = \alpha(\omega) \hat{E}, \quad (3.25)$$

where $\omega$ is the angular frequency of the driving field. Note that in this notation all quantities are of complex nature. The potential resulting from this light-atom interaction reads

$$U_{\text{dip}}(r) = \frac{1}{2 \epsilon_0 c} \Re(\alpha(\omega)) I(r), \quad (3.26)$$

with $I = 2 \epsilon_0 c |\hat{E}|^2$ denoting the intensity of the dipole beam. The strength of the potential is determined by the real part of the polarizability $\Re(\alpha(\omega))$. As shown in [66], for sufficiently large detunings from an atomic transition (i.e., $|\Delta| \gg \Gamma$), an expression for the polarizability can be found using a semi-classical description. This yields the potential

$$U_{\text{dip}}(r) = \frac{3\pi e^2}{\omega_0^2} \frac{\Gamma}{\Delta(2\omega_0 + \Delta)} I(r), \quad (3.27)$$

where $\omega_0$ is the resonance transition frequency, $\Gamma$ is the natural linewidth and $\Delta = \omega - \omega_0$ the detuning from resonance. This potential is attractive if the dipole beam is red detuned ($\Delta < 0$) and repulsive for blue detuning ($\Delta > 0$).

In a similar approach, an expression for the off-resonant photon scattering rate $\Gamma_{\text{sc}}$ can be derived. It turns out that it is proportional to the imaginary
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part of the dynamic polarizability $\Im(\alpha(\omega)).$ One obtains:

$$\Gamma_{sc}(r) = \frac{1}{\hbar c_0 c} \Im(\alpha(\omega)) I(r)$$

$$= \frac{6\pi c^2}{\hbar \omega_0} \left( \frac{\omega}{\omega_0} \right)^3 \frac{\Gamma^2}{\Delta^2(2\omega_0 + \Delta)^2} I(r).$$

(3.28) (3.29)

In the case of not too large detunings compared to the atomic transition frequency ($|\Delta| \ll \omega_0$), one can further simplify above expressions for the potential depth and scattering rate, respectively. This leads to a simple relation between the two:

$$\Gamma_{sc} = \frac{U_{dip}}{\hbar} \frac{\Gamma}{\Delta}. \quad (3.30)$$

That is, for a low heating rate of the atoms in a red-detuned trap due to off-resonant scattering of photons, large detunings are favourable. In order to compensate for the resulting reduced potential depth, high intensities are required.

For a more precise calculation of the trapping potential and the scattering rates, the fine structure splitting into D1 and D2 line has to be taken into account and their individual contributions have to be considered. Therefore equations (3.27) and (3.29) are replaced by:

$$U_{dip}(r) = \pi c^2 \left( \frac{\Gamma_{D1}}{\omega_{D1}^2 \Delta_{D1}(2\omega_{D1} + \Delta_{D1})} + \frac{2\Gamma_{D2}}{\omega_{D2}^2 \Delta_{D2}(2\omega_{D2} + \Delta_{D2})} \right) I(r) \quad (3.31)$$

$$\Gamma_{sc}(r) = \frac{\pi c^2}{2\hbar} \left( \frac{\Gamma_{D1}^2}{\omega_{D1}^2 \Delta_{D1}^2(2\omega_{D1} + \Delta_{D1})^2} + \frac{2\Gamma_{D2}^2}{\omega_{D2}^2 \Delta_{D2}^2(2\omega_{D2} + \Delta_{D2})^2} \right) I(r), \quad (3.32)$$

where the subscripts D1 and D2 of $\omega$, $\Delta$ and $\Gamma$, refer to the resonance transition frequency, the detuning and the natural line width of the respective transition. The easiest way to form optical potentials for trapping neutral atoms is by the use of red-detuned Gaussian beams. The intensity distribution $I_{sb}$ of a single beam with power $P$, wavelength $\lambda$, waist $w_0$ and Rayleigh range $z_R = \pi w_0^2/\lambda$ propagating in the z-direction is given by:

$$I_{sb}(r, z) = \frac{2P}{\pi w^2(z)} \exp \left( -\frac{2r^2}{w^2(z)} \right) \quad \text{with} \quad w(z) = w_0 \sqrt{1 + \left( \frac{z}{z_R} \right)^2}. \quad (3.33)$$
A harmonic approximation of the resulting potential can be obtained by performing a Taylor expansion up to the second order. This allows one to define trap frequencies for atoms with mass $m$, which are given by $\omega_{\text{rad}} = \sqrt{4U_0/mw_0^2}$ for the radial direction and $\omega_{\text{ax}} = \sqrt{2U_0/mz_R^2}$ for the axial direction, respectively. $U_0 = |U_{\text{dip}}(0)|$ is the potential depth at the centre of the beam. By superimposing multiple beams different trap geometries can be realized. Retroreflecting a single beam while retaining its polarization, leads to an intensity modulation along the beam’s propagation direction due to interference. An optical lattice with wave number $k$ and intensity distribution

$$I_{\text{latt}}(r, z) = 4I_{\text{sb}}(r, z) \cos^2(kz)$$

is formed. Note that the peak intensity compared to the single beam is enhanced by a factor of 4. More complex lattices are possible by combining multiple beams which leads to multidimensional lattices [67]. In our future experiments we will superimpose beams with multiple wavelengths. This allows to construct superlattices resulting in fish-like potentials [68, 69].

In many cases, a single, non-reflected beam doesn’t provide enough confinement along the axial direction to allow for sufficient evaporation to produce a BEC. The thermalization rate which is related to the trap frequencies is too small and drops especially for low powers (cf. section 3.2.1). To overcome this, one can cross two beams in order to form a crossed dipole trap (CDT) [66].

Another method for creating a stronger confinement is to use a combination of a single dipole beam and a magnetic trap, called a hybrid trap [70]. Generally, the drawback for the use of pure, static magnetic traps are Majorana spin-flips of atoms near the magnetic field zero to magnetically untrapped states [71]. As this happens predominantly for the coldest atoms in the cloud, which are on average closest to the centre of the trap, this leads to a heating effect which gets progressively worse with the evaporation, eventually preventing the cloud from condensing to form a BEC. In a hybrid trap this is circumvented by placing the beam underneath the centre of the magnetic trap (cf. figure 3.2). In a typical experimental sequence, atoms are first evaporatively pre-cooled via microwave or radio frequency (RF) evaporation in a pure magnetic trap until Majorana spin-flips start to dominate. Then, the atoms are transferred to the single beam by lowering the magnetic field gradient of the quadrupole trap to a final value slightly below the levitation gradient of the atoms against gravity. For $^{87}$Rb atoms in the $|F = 2, m_F = 2\rangle$ as in our case, the levitation gradient
Figure 3.2.: Illustration of the loading process from a magnetic trap to a hybrid trap. (a) The dipole beam is displaced by the distance $z_0$ underneath the magnetic trap. In our case the magnetic field gradient is lowered from an initial value of $B' = 150 \text{ G/cm}$ to a final value of 15.3 G/cm during the transfer from the magnetic to the hybrid trap. This corresponds to the levitation field against gravity for the state $|F = 2, m_F = 2\rangle$ of $^{87}\text{Rb}$. The change of the potential as a function of the vertical position during the transfer is shown in (b).

is 15.3 G/cm. In the ideal case, this leads to an almost complete transfer of the atom cloud into the dipole trap. The remaining magnetic field ensures an axial confinement high enough for further forced evaporative cooling, which is achieved by lowering the dipole beam power. Additionally, the adiabatic change of the shape of the trap from a linear to a harmonic geometry already leads to an increase in phase space density during the transfer process without any further evaporation.

The trapping potential in this hybrid configuration including gravity can be written as:

$$U_{\text{hybr}}(x, y, z) = U_{\text{mag}}(x, y, z) + U_{\text{dip}}(x, y, z - z_0) + U_{\text{grav}}(z)$$

$$= m_F g_F \mu_B B' \sqrt{x^2/4 + y^2/4 + z^2} + U_{\text{dip}} + mgz,$$

where $U_{\text{dip}}$ is given by equation (3.31), $m_F$ is the magnetically trapped hyperfine state, $g_F$ the according Landé g-factor, $\mu_B$ the Bohr magneton and $B'$ the magnetic field gradient of the quadrupole field. The corresponding trap frequency to the axial confinement in a hybrid trap can be approximated by

$$\omega_{\text{ax}} = \sqrt{\frac{m_F g_F \mu_B B'}{4m} \frac{B'}{z_0}}.$$

As can be seen, it is independent of the dipole beam power, which illustrates the advantage of the hybrid trap.
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The setup of the dipole trap consists of four parts:

- A setup preparing the longitudinal dipole beam.
- Two separate setups for the transversal beams of the cube and science chamber (for the definition of the directions longitudinal and transversal cf. figure 2.2 and 3.4).
- A setup distributing the output beam of the fibre amplifier to multiple high power fibres supplying the setups above.

The setup of the longitudinal beam includes a motorized translation stage which enables one to shift the focus of the beam in axial direction. This allows one to optically transport atoms between the cube and science chamber or to change the effective trapping volume of the CDT. In the following, the individual setups are described briefly.

**Setup for distributing the beams**

Our initial beam is provided by a NUfern NuAMP fibre amplifier working at a nominal wavelength of 1064 nm with a maximal output power of 50 W. It
is seeded by an Innolight Mephisto laser. The output beam is collimated with an aspheric lens \((f = 18.54 \text{ mm})\) resulting in a beam of \(\approx 1 \text{ mm}\) diameter which is ultimately split up into five separate beams. Three of which supply the three different dipole beams and two which will be further split up for preparing the optical lattices in the future. All PBS cubes used for dividing the beams are air-spaced (Altechna) suitable for high power applications. The setup is presented in figure 3.3.

In a first step, the initial beam is split in two parts. Each of them is sent through a double stage optical isolator (Thorlabs IOT-5-1064-VHP) which prevents back-reflections from following optical elements into the fibre amplifier. In an earlier version only one isolator was used and the beam split up afterwards. However, due to the high power densities thermal lensing effects were induced which severely limited the achievable coupling efficiencies into the optical fibres. The beam passing through the upper isolator in figure 3.3 provides the power for the two transversal beams. A motorized half wave plate (Thorlabs PRM1Z8) allows one to guide the beam either towards the cube chamber or the science chamber and thus allows one to have a CDT in either of the two chambers. An AOM (Crystal Tech 3110-197) is used to stabilize the power in conjunction with a PID module developed by the institute’s internal electronic workshop. The respective photo diodes for monitoring are placed in the setups for the preparation of the dipole beams after the high power fibres (see below). The zeroth order of the AOM is recycled and directed through another AOM stabilizing and supplying one of the lattice beams and in this way allows one to switch between the dipole trap and the optical lattice.

The other beam which passes through the lower isolator is used for the longitudinal dipole beam. As before, an AOM is used either to redirect and power stabilize the beam for the dipole trap operation or to provide the light for the remaining two lattice axes. All non-used beams are eventually dumped in beam blocks suitable for high beam powers (Vision Lasertechnik). The longitudinal and transversal dipole beam for the cube chamber are coupled into photonic crystal fibres NKT Photonics LMA-PM-15. The alignment procedure could be simplified significantly by the use of specially designed fibre couplers (Schäfter + Kirchhoff 60FC-SMA) with \(f = 15 \text{ mm}\) focal length. The transverse beam for the science chamber is coupled into a regular polarization maintaining fibre (Schäfter + Kirchhoff) which is equipped with an end cap for an enhanced effective mode field diameter at the fibre tip.

Special care has to be taken while coupling into the fibres in order not to
overheat or burn the fibre tips. As a precaution each fibre is coupled “cold” by chopping the beam with the help of the AOM resulting in a low duty cycle of \( \sim 20\% \) at a frequency of \( \sim 50–100\) Hz while incrementally increasing the output power of the fibre amplifier. Additionally in this way the system of AOM and fibre is optimized at cold working conditions. This procedure is reasonable, as we require the highest available powers right at the beginning of a laser ramp. Using this alignment procedure we achieve coupling efficiencies of about 70–80\% resulting in beam powers of up to 7 W for the transverse beams and 11 W for the longitudinal beam after their corresponding fibres.

**Longitudinal dipole beam including a translation stage for optical transport**

The longitudinal dipole trap beam can be used for multiple purposes:

- It can be set up to work in a hybrid trap configuration.
- It forms together with the transversal beams of the cube and the science chamber a CDT.
- It is used for optically transporting the atoms between the cube and the science chamber.

A central part of the setup is a motorized translation stage (THORLABS DDS220/M) equipped with two mirrors which allows one to shift the position of the focus.
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in the axial direction. In addition to the possibility for optical transport, this enables us to easily change the effective waist of this beam in the CDT which could – besides creating different static trap volumes – also be used to realize a dynamically changing trap, synchronized to the evaporation in a similar fashion as it was proposed to experiments performed in the group of D. Weiss [72]. The waist was planned to be around $40\mu m$. The relatively small Rayleigh range gives enough axial confinement, in order to be able to act as optical tweezers, as was already successfully demonstrated in the group of I. Bloch [73, 74]. The setup is depicted in figure 3.4.

After the fibre coupler, the beam is collimated to a beam diameter of approximately 1.6 mm. Therefore, the same type of special couplers as for the incoupling are used. Subsequently, the beam’s polarization is cleaned by a Brewster plate. It is expanded by a first telescope consisting of a $-75\text{mm}$ and $400\text{mm}$ lens to a beam diameter of 8.8 mm. A focusing lens ($f = 500\text{mm}$) produces an intermediate focus which can be shifted by the translation change. A 1:1 telescope formed by two $f = 500\text{mm}$ lenses images the focus into the experimental chamber.

Measurements of beam profiles showed constant waist sizes of around $w \approx 45\mu m$ for different translation stage positions along the transport distance. They are supported by independent trap frequency measurements yielding $w = (47 \pm 5)\mu m$ [75]. The last mirror before the chamber is dichroic (THORLABS DMSP805) which overlaps the longitudinal imaging beam with the dipole beam. Additionally, the mirror mount is equipped with a piezo driven motor (NEWPORT PICOMOTOR) which allows one to adjust it remotely and could be used to correct the beam pointing during optical transport. At the moment this feature is not in use.

For intensity stabilization, in conjunction with an AOM and a PID circuit (cf. previous section), a photo diode (THORLABS PDA20CS-EC) is placed behind one of the mirrors. It records a small portion of the dipole light which is transmitted through the mirror due to imperfect reflectance.

Transversal dipole beam in the cube chamber

The setup for the transversal beam in the cube chamber closely resembles the one of the longitudinal, apart from missing an intermediate focus and the motorized translation stage. Therefore no additional drawing is given.

We decided to aim for a focus of $80\mu m$ in order to be able to create larger
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trap volumes. Similarly, as in the longitudinal setup, a telescope made of plano-concave/convex lenses with $f = -75 \text{ mm}$ and $f = 200 \text{ mm}$, respectively expands the beam to a diameter of $\approx 5.4 \text{ mm}$. The final focus of the beam is created by a $f = 500 \text{ mm}$ lens which is placed on a manual translation stage. The waist was measured to be $w_x = (83.5 \pm 2.0) \mu\text{m}$ and $w_y = (86.4 \pm 0.4) \mu\text{m}$ for the horizontal and vertical direction in the plane of the CCD camera (IDS UI-1240SE-M-GL) used, respectively. Trap frequency measurements yield a waist of about $89 \mu\text{m}$.

**Crossed dipole trap in the science chamber**

The medium resolution objective of the science chamber, requires more careful consideration of the beam path. Because of the small focal length of the objective a small incoming beam is needed to produce the relatively large focus of $\approx 80 \mu\text{m}$. The often used approximation for the calculation of the waist $w_0$ of a collimated beam with $1/e^2$ diameter $D$ that is focused by a lens

$$w_0 \approx \frac{2\lambda f}{\pi D}, \quad (3.37)$$

breaks down as soon as the Rayleigh range of the incoming beam is comparable or shorter than the lens’ focal length $f$. An exact calculation of the Gaussian beam propagation is therefore needed. In addition, we were facing space limitations due to the optics that will be needed for the optical lattice beam preparation in future. We decided for a setup with a single lens with $f = 350 \text{ mm}$ after the outcoupler of the dipole beam. It creates a focus with a waist of $w_{in} = 150 \mu\text{m}$ about $3 \text{ cm}$ before the principle plane of the objective. As the double objectives were aligned with high precision with respect to the atoms with the help of imaging light, the dipole beam’s focus position was adjusted by placing the $f = 350 \text{ mm}$ lens on a translation stage and optimizing its position.

**3.3.3. The four main trap configurations**

Historically, our first trap for producing BECs in the experiment was the hybrid trap. Its simplicity in the setup and potential for producing large condensates were the main arguments for using it. However, it soon turned out that the overall stability of our system was very low; amongst others we were struggling
3.3. A versatile optical dipole trap for cooling and transporting $^{87}$Rb

![Figure 3.5.](image)

**Figure 3.5.:** Illustration of the capabilities of the combined crossed dipole trap with the magnetic quadrupole field. The green arrows depict the possibility of translating the focus of one of the dipole beams and hence varying the trapping volume.

with instabilities in our MOT and drifting magnetic offset fields which both were directly influencing the results obtained in the hybrid trap. Especially the zeroing of the compensation fields was difficult and laborious back then. We therefore turned to a CDT which is intrinsically less affected by residual magnetic fields. This enabled us to work with mainly two configurations producing more stable BECs: a *narrow crossed dipole trap* (NCDT) where the foci of transversal and longitudinal beam were overlapped and a *wide crossed dipole trap* (WCDT) featuring a larger volume and a nearly spherically symmetric trap. The latter is achieved by displacing the focus of the longitudinal beam about 7 mm away from the crossing point. The results presented in chapter 6 were obtained in the latter configuration.

The NCDT seems to show a strict upper limit for the achievable number of atoms in the BEC of $(0.56 \pm 0.01) \cdot 10^6$ atoms. Density calculations suggest that this is the result of the emergence of density limiting three-body losses during the loading of the trap. However, especially for experiments in the science chamber this effect is undesired. Overall more atoms are required due to the parametric heating and additional losses during the optical transport. Consequently efforts were made to be able to return to a single beam trap by improving the stability of the MOT. Knowledge was gained about how to stably load and transport atoms in the magnetic trap and transfer them to a hybrid trap. Magnetizable parts that get influenced by the moving transport coils caused our previous problems. Consistent operation of these and optimization
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Figure 3.6.: Absorption images for the transition from a thermal cloud of atoms to an almost pure BEC. The images were taken along the vertical direction at different points of the last two evaporation ramps in the CDT after a time-of-flight of 30 ms.

During the course of these optimizations, we also found the possibility to operate a wide hybrid trap by displacing the focus, as in the CDT case, with the magnetic field as axial confinement instead. The magnetic part of the trap was sufficiently deep to hold the atoms against being dragged towards the focus of the beam. However, this trap configuration was the least stable in terms of long-term stability, probably due to the comparably weak confinement of the magnetic field instead of a transversal beam. An illustrative sketch of the variability in trap geometries is given in figure 3.5.

Due to limitations in our experimental control, initially the evaporation ramps were implemented in form of four linear ramps which were optimized one by one. One of these sequences used in the BEC transition in our original WCDT as discussed in chapter 5 is shown in figure 3.6. A series of absorption images which was taken during the last two evaporation ramps shows the transition to an almost pure BEC. To get a measure for the evaporation efficiency, we calculated the phase space density (cf. equation (3.8)) at different points during the evaporation by evaluating the temperature and number of atoms with similar images. A result for these kind of measurements taken during the third evaporation ramp can be seen in figure 3.7 which shows the phase space density as a function of the atom number in a double-logarithmic plot. The efficiency of the evaporation, which is defined as $\gamma = -\frac{d \log(\rho)}{d \log(N)}$ can then be easily obtained via an allometric fit to the data. In this case, we obtain $\gamma = 3.3 \pm 0.4$. 
3.3. A versatile optical dipole trap for cooling and transporting $^{87}$Rb

Figure 3.7.: Measurement of the phase space density vs. the number of thermal atoms at different points along the third evaporation ramp in the CDT. The solid red line shows an allometric fit to the data and yields an evaporation efficiency of $\gamma = 3.3 \pm 0.4$.

which is a good value for evaporation in a purely optical trap [78]. One can already see the start of the formation of a BEC at the end of this ramp. The phase space density that was calculated only with parameters obtained from the thermal atoms is saturating and therefore excluded from the fit.

A later update of our experimental control program allowed us to define arbitrary functions for any analogue output channel [79]. We implemented from then on all our evaporation ramps with a parametrization according to the ideal ramps following equation (3.20) presented before in section 3.2.1. In this way, the sequence for producing a BEC can be defined using only a few parameters: The magnetic quadrupole field during evaporation defined through the current through the coils $I_{QP,\text{evap}}$, the position of the translation stage $x_{\text{trans}}$, the initial and final powers $P_{i,k}$ and $P_{f,k}$ defined separately for the two dipole beams (replacing subscript $k$ by $l$ for longitudinal or $t$ for transversal accordingly), $\tau$ and $\beta$ defined in equation (3.20). The combination of $\tau$, $\beta$ and the ratio between initial and final power defines the duration of the ramps. For simplicity, they are chosen to be the same for the two beams, hence the same combinations of parameter values are assigned. Additionally, the current through the verti-
3. The Production of $^{87}$Rb Bose-Einstein Condensates

cal offset field coils can be set which affects the effective distance between the magnetic field zero and the dipole beam, denoted as $z_0$ in equation (3.35). The general experimental sequence until the loading of the dipole trap is described in chapter 2.4. As briefly pointed out there, the loading of a given trap configuration is initiated by ramping the dipole beams to the desired initial power and setting the vertical magnetic offset field. Afterwards, the quadrupole field is lowered in three linear ramps to the value of the according trap. A first relatively steep ramp relaxes the trap in 0.5 s from initially 133.0 G/cm to 29.6 G/cm. The second ramp lowers, within 1 s, the gradient to the levitation gradient of 15.3 G/cm and constitutes the effective transfer of the atoms to the dipole trap. In the last, again steeper ramp, the final gradient for the evaporation is reached within 0.3 s. The optimized values for the evaporation parameters of the four traps – NCDT, WCDT, hybrid and wide hybrid trap – as well as typical yields in condensed atoms and their uncertainties are given in table 3.1. It can be concluded that the NCDT is absolutely seen the most stable trap, most probably due to above discussed density limitations that are reached during the loading phase.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>NCDT</th>
<th>WCDT</th>
<th>Hybrid trap</th>
<th>Wide hybrid trap</th>
</tr>
</thead>
<tbody>
<tr>
<td>$I_{QP,\text{evap}}$ (A)</td>
<td>0</td>
<td>0</td>
<td>15</td>
<td>19.4</td>
</tr>
<tr>
<td>$I_{\text{off}}$ (mA)</td>
<td>100</td>
<td>0</td>
<td>375</td>
<td>0</td>
</tr>
<tr>
<td>$P_{i,t}$ (W)</td>
<td>1.25</td>
<td>7.8</td>
<td>4</td>
<td>10</td>
</tr>
<tr>
<td>$P_{i,t}$ (mW)</td>
<td>5.55</td>
<td>5.2</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>$P_i/P_f$</td>
<td>24</td>
<td>13.4</td>
<td>160</td>
<td>19</td>
</tr>
<tr>
<td>$\tau$ (ms)</td>
<td>120</td>
<td>150</td>
<td>500</td>
<td>575</td>
</tr>
<tr>
<td>$\beta$</td>
<td>1.6</td>
<td>1.25</td>
<td>2.4</td>
<td>1.33</td>
</tr>
<tr>
<td>$x_{\text{trans}}$ (mm)</td>
<td>85.6</td>
<td>93</td>
<td>85.6</td>
<td>93</td>
</tr>
</tbody>
</table>

| BEC yield ($\times 10^6$ atoms) | 0.56(1) | 1.1(1) | 1.9(2) | 1.6(3) |

Table 3.1.: Experimental parameters used to describe the final evaporation sequence for creating a BEC.
Creating a BEC in the science chamber

In the science chamber we have multiple ways of creating a BEC. Besides performing evaporative cooling in the CDT of the science chamber, we also have the possibility of shining in a dimple beam which allows us to cross the phase transition in a conservative way as discussed in section 3.2.2. The two paths are briefly sketched out in the following.

Both start with the transfer of the atoms to the longitudinal dipole beam followed by the optical transport as described above and in chapter 2.4. In the last step of the transfer process the magnetic quadrupole field is ramped completely to zero. During the optical transport the transverse beam in the science chamber is ramped to 5 W. The final value of the translation stage position of the optical transport is set to \( x_{\text{trans}} = 120 \text{ mm} \) which allows one to form effectively a NCDT in the science chamber. For loading the crossed dipole trap, it is matched with respect to the individual beam’s effective trap depth. Therefore, the beam power of the narrower longitudinal beam is lowered in a linear ramp within 1 s to 1.07 W, which sets the starting point for the evaporation. In the case of a BEC in the CDT, the same parametrization as before is used. The following values turned out to be optimal: \( P_{f,l} = 599 \text{ mW}, \) \( P_{f,t} = 599 \text{ mW} \) (the initial values are given above), \( \tau = 120 \text{ ms} \) and \( \beta = 1.6. \) Using this method, BECs as large as \( 6 \cdot 10^5 \) atoms can be produced.

Alternatively, the evaporation in the CDT can be stopped earlier and the transition to a BEC crossed by shining in a dimple beam which is ramped up within 80 ms to various powers. The CDT serves thereby as the reservoir (cf. section 3.2.2). A thorough investigation of different combinations of powers and the resulting yields of condensed atoms was performed. For only detecting the atoms trapped in the dimple, the reservoir was discarded before the atoms in the dimple were measured 20 ms later in TOF absorption images. The results are presented in figure 3.8. For higher dimple powers, there is a band visible between 80 and 600 mW in which the loading into the dimple is enhanced but only thermal atoms are accumulated. One has to go to lower dimple powers where the loading of thermal atoms already decreases to observe the emergence of a BEC. This can be qualitatively understood within the framework of the theory behind the dimple trick, where the dimple potential is only a local disturbance without influencing the reservoir. The experiments presented in chapter 7 rather work with thermal atoms.
Figure 3.8.: Loading of the dimple as a function of final dipole trap power $P_{f,l}$ and dimple power $P_{dimple}$. The top graph shows the number of thermal atoms, whereas the lower graph depicts the number of atoms in the condensate measured in TOF absorption imaging.
Complex Optimization Landscapes

The quest for novel, previously unknown solutions for different kinds of optimization problems is the main object of investigation in the following chapter. Our approach is to examine and exploit the topology of the underlying control landscape. Parts of the following discussion will serve as a manuscript for a publication which is currently in preparation [80].

The abstract concept of landscapes for describing complex correlations of multiple variables leading to a certain output was introduced in the early 1930s by S. Wright for a biological study [81]. Here, it was used to facilitate a deeper understanding of the influence of low-level genetic properties, representing ‘the input variables’, on the high-level evolutionary dynamics as the ‘fitness’ or ‘height’ of the landscape as illustrated in figure 4.1. This concept has proven to be a very powerful investigation tool across different disciplines: e. g. social science [82], computer science [83] and physics [84].

In the case of evolutionary biology, the structure of the mapping of a certain genetic configuration (genotype) to the fitness of an organism is still not understood and hence modelled as random [85, 86] (however, there are recent advances in studies on empirical fitness landscapes, see e. g. [87]). This leads to rugged landscapes which are rich in local suboptimal extrema, making it difficult to find the global optimum with local optimization algorithms. In contrast, in the case of quantum optimal control problems under the assumption of a fully controllable system without any constraints imposed on the controls, the
landscape is trap free \cite{84,88}. That means, in this regime, it is sufficient to use a local search algorithm for finding optimized solutions. This powerful statement is no longer strictly valid when energy and time resources are finite since absolute controllability breaks down. This introduces local suboptimal traps and the application of purely local search algorithms might fail. In this context the quantum speed limit (QSL) is defined as the shortest process duration, where an optimal solution can be achieved \cite{89}. As these examples show, the properties and understanding of the landscapes is essential for efficient optimization \cite{84}.

In this chapter we will focus on two different problems, both exhibiting an optimization landscape with complex topology. The first one is a theoretical study on fast single atom transport, which will be topic of section 4.1. We will examine the case where controllability is extremely broken \cite{90} leading to a very rugged control landscape. Therefore, more sophisticated optimization techniques are needed. This could either be achieved by looking for inspiration in computer science, which is currently pursued in our group. Here in this chapter, however, we aim at developing a novel search methodology relying more directly on the topology of the control landscape. In this context, we are especially interested in the distribution of the solutions in the control landscape and the clustering of them into ‘established’ solution strategies. In physics we often have particular ways of solving problems. Any given problem may have a number of conventional strategies that can be employed. The question we address is whether these established canonical strategies are unique or if many other strategies exist that are potentially more efficient. In this context of the single atom transport problem, different solution strategies were identified in previous work \cite{90}. Here we go beyond this work, and the question of uniqueness of strategies is addressed by forming convex combinations of them and thereby investigating the parameter space in between. If the strategies are truly discrete, then one would expect always to observe a ‘valley’, i.e. a region of low quality solutions. It might turn out, however, that some kind of connections exist. This unexplored region of the optimization landscape is illustrated in figure 4.1 by the ‘foggy’, grey patch with the question mark. As will be explained below in section 4.1, we do indeed identify a continuum of solutions connecting the strategies which were previously thought to be

\footnote{Full control means, for a given \(n\)-dimensional Hilbert space \(\mathcal{H}\), every possible unitary operator has to be able to be realized with the given control (cf. Eq. (4.1)).}
Figure 4.1.: Illustration of a partially unknown control landscape with some controls represented in two dimensions. The third dimension shows the fitness of the landscape. There are multiple known suboptimal extrema and two clusters of seemingly well performing solutions. However, the extrema themselves and the region between are unexplored and unknown which is depicted by the grey, ‘foggy’ patch. In the optimization problems of this chapter, we want to especially focus on these areas.

discrete. Thus, a much richer landscape topology emerges. In a next step, we exploit the assumption that in physics solution strategies vary slowly under small perturbations, which allows us to introduce a new optimization scheme jumping from ‘peak to peak’ in the landscape.

The gained insight will be applied to the problem of BEC production, presented in section 4.2. In analogy with the single atom transport example, we form convex combinations of existing strategies and explore the space between them in order to test whether they are indeed distinct strategies or just low dimensional manifestations of a more complex high dimensional landscape. Twenty years of experimentation with BECs has resulted in the crystallization of a discrete number of conventional approaches to create one. In recent years attempts have been made to explore a wider range of solutions using closed loop optimization [91–95], where the output of the system under investigation is fed back as an input to the algorithm. However, although this has provided improved solutions it has neither yielded insight into the structure of the optimization landscape nor given any information of global optimality.
In addition, other avenues for finding optimized ways of producing BECs are pursued. In section 4.3, the local search algorithm, CRAB [96, 97], and its extension to global search, dCRAB [37], are explained as well as results from the experimental implementation in our problem. In section 4.4, the remote control interface established for the dCRAB helped us to develop a simple client that permits live control over certain parts of the experimental sequence. These experiments served as a stepping stone towards the development of a computer game resulting in an unprecedented ‘gamification’ of an experimental problem in real-time. First results are presented in section 4.5. The chapter is concluded with an outlook in 4.6.

4.1. Single atom transport at the edge of the Quantum Speed Limit

The following section describes results for optimizing the problem of single atom transport at the quantum speed limit (QSL) in a given potential in a theoretical framework. Based on a novel perception of the underlying optimization landscape, new methods for finding optimized solutions to the problem are established. The presented work is an extended investigation of findings discussed in [90] and is based on data that was originally obtained in the Bring Home Water challenge (BHW challenge) of the citizen science computer game Quantum Moves. In subsection 4.1.1, the concept of optimization problems and landscapes is formally introduced, followed by a description of the BHW challenge and the development of new methods for treating the optimization problem in 4.1.2. A list of formal definitions that were introduced within these two subsections in conjunction with optimization landscapes is provided in subsection 4.1.3.

4.1.1. Theoretical background

Mathematical formulation of the optimization problem

The Hamiltonian describing the system can be in general separated into two parts. A drift Hamiltonian $H_0$ and a control Hamiltonian $H_c(u(t))$

$$H(t) = H_0 + H_c(u(t)),$$  (4.1)
4.1. Single atom transport at the edge of the Quantum Speed Limit

where \( u(t) \) is the control field, that is used to steer the Hamiltonian. In general, the control field \( u(t) \) is represented by a multi component vector. For simplicity, the vector nature and the time dependency is omitted in the notation throughout this section where not explicitly needed. The unitary transformation that drives the time evolution is given by the differential equation

\[
i \hbar \frac{\partial}{\partial t} U(u, t) = H(t) U(u, t),
\]

(4.2)

and transfers a certain initial state \( |\psi_i\rangle \) to a final state

\[
|\psi_f\rangle = U(u, t) |\psi_i\rangle.
\]

(4.3)

The objective of the optimization is to find a control field \( u^* \) that allows for a transition from the initial state \( |\psi_i\rangle \) to a given target state \( |\psi_t\rangle \). Therefore, a functional \( J(u) \) is defined as a cost function which allows one to assign a fitness value to a given control field \( u \). In our case of a state transfer problem, it is convenient to use the fidelity \( F \), defined through the overlap between the final state after time evolution (equation (4.3)) and the target state \( |\psi_t\rangle \), as such a functional:

\[
J(u) = F = |\langle \psi_t | \psi_f \rangle|^2.
\]

(4.4)

The graph of \( J(u) \) is often referred to as optimization landscape. The objective of the optimization is to find a control field \( u^* \) such that \( F \) is maximized.

In our numerical treatment, \( u(t) \) has to be discretized in finite time steps \( \Delta t \). This allows one to assign a dimensionality to the optimization problem through the number of time steps, thereby assuming that the individual time steps are independent of each other.

**Representation of multi dimensional optimization landscapes**

The dimensionality of the optimization problem can be reduced significantly by introducing a parametrization \( h(\bar{c}) \) with parameters \( \bar{c} \), which maps from a low dimensional space to the full high dimensional control space. This can

\footnote{Note, that \( H(t) \) is explicitly time-dependent. Therefore, the integration of (4.2) is not trivial.}
for instance be the expansion of $u$ into a certain functional basis $f_j(t)$ with coefficients $c_j$

$$u(t) = \sum_j c_j f_j(t), \quad (4.5)$$

where the $c_j$ form a set of parameters $\vec{c}$. A certain parametrization we call also a representation of the problem. Based on $h(\vec{c})$ a new optimization landscape can be defined through the set $\{\vec{c}, J(h(\vec{c}))\}$. It is clear that the topology of the landscape crucially depends on the parametrization that is used. This dimensionality reduction might introduce local traps in the landscape, since it imposes constrictions to the control fields that can be realized. Hence, the conditions for a trap-free landscape given in \[84, 88\] do not hold any more.

The dimensionality of a certain parametrization can be still large. For visualization purposes a further dimensionality reduction can be beneficial. Several methods have been investigated over the past year in our group \[98\] in order to find a three-dimensional representation of the full optimization landscape, which can be easily plotted. Among them are Principal Components Analysis (PCA) \[99\], Sammon mapping \[100\], Stochastic Neighbor Embedding (SNE) \[101\], Locally Linear Embedding (LLE) \[102\] and t-Distributed Stochastic Neighbor Embedding (tSNE) \[103, 104\].

tSNE has been shown to be both efficient and reliable for visualization in our case and will be used in several cases in the following analysis in this chapter. As it is not a very common tool in our community, its working principle is briefly outlined here. The method relies on comparison of probability distributions. In a first step, the probability of similarity between two objects $x_i$ and $x_j$ in the high dimensional space is calculated according to

$$p_{ij} = \frac{\exp(-||x_i - x_j||^2/2\sigma_i^2)}{\sum_{k \neq l} \exp(-||x_k - x_l||^2/2\sigma_i^2)}, \quad (4.6)$$

where $\sigma_i$ is a variable that is found through binary search to estimate the number of neighbours of object $x_i$. This probability is equated to the probability of similarity of the objects in the low dimensional space $y_i$ and $y_j$, represented by a Student’s t-distribution

$$q_{ij} = \frac{(1 + ||y_i - y_j||^2)^{-1}}{\sum_{k \neq l}(1 + ||y_k - y_l||^2)^{-1}}. \quad (4.7)$$
4.1. Single atom transport at the edge of the Quantum Speed Limit

Now, $y_i$ and $y_j$ is found by minimizing the difference between the two probability distributions given by the Kullback-Leibler divergence $E$ between $q_{ij}$ and $p_{ij}$:}

$$E = \sum_i \sum_j p_{ij} \log \left( \frac{p_{ij}}{q_{ij}} \right).$$

(4.8)

4.1.2. Investigating solutions of the Bring Home Water challenge

The BHW challenge is a level in the citizen science game Quantum moves and ‘gamifies’ the single atom transport problem [90]. A screenshot is shown in figure 4.2. The task is to move an atom in one dimension from a starting point into a designated target area. Initially, the atom is held in a static Gaussian shaped potential. The atom is picked up and transported by using a movable optical tweezer of variable potential depth. The objective is to reach the target area as quickly as possible (promoted by the introduction of a time penalty in the game). At the same time, it is required to arrive with as high fidelity as possible with respect to the motional ground state of the tweezer potential. This represents a non-trivial optimization problem. Solutions to it are for example valuable for the realization of a large scale quantum computer of ultracold atoms in optical lattices [105].

Purely numerical optimization methods to these kind of problems use tailored search algorithms like the Krotov algorithm [106]. The Krotov is a local optimization technique exploiting the structure of the Schrödinger equation to obtain information about the gradient of the cost function, unlike less sophisticated local search algorithms like the Nelder-Mead simplex method [107] (short simplex, its principle is briefly described in section 4.2.2). This ensures monotonic increase of the cost function for each iteration step. It was also used in our case and due to its locality implemented with multiple start overs using sinusoidal seed functions [90]. It was shown, that the optimization of player solutions from the BHW challenge outperforms such a purely numerical approach for transport durations close to the QSL. A powerful optimization method of using player solutions as seeds for the Krotov algorithm called CHOP was developed [90]. In order to compare the different solutions obtained with CHOP, a distance measure was introduced. Hereby, a distance map $D_{ij}$ is defined, which compares the overlap between two wave functions $|\psi_i(x, t)\rangle$ and
The BHW challenge shown in four screenshots. Starting from the initial situation 1), one has to utilize the movable tweezer to pick up the atom, which is illustrated by the square of its wave function and held in a static potential tweezer potential. Two strategies, 2a) and 2b), were identified of how to most efficiently pick up the atom and transport it to the designated target area 3).

\[ |\psi_j(x, t)\rangle \]

at each time step \( t \) for a given total transport time \( T \):

\[ D_{ij} = \frac{1}{T} \int_0^T \langle f_{ij}|f_{ij}\rangle dt, \quad (4.9) \]

where \( |f_{ij}(x, t)\rangle = |\psi_i(x, t)\rangle - |\psi_j(x, t)\rangle \) is the difference between the wave functions at each position \( x \). A clustering analysis performed on \( D_{ij} \) revealed that solutions fall into two distinct groups denoted as ‘clans’ [98]. The solutions forming a distinct clan all follow a similar strategy for the atomic transport, to which one can assign a physical interpretation. One of the clusters exploits quantum tunnelling, while the members of the other clan use a shovelling strategy (cf. figure 4.2). Following the tunnelling strategy, the optical tweezer is moved to a certain position at the left-hand side of static potential where the tunnelling rate is maximized. In the shovelling strategy, the atom is directly picked up by forming a deep combined potential of optical tweezer and static potential. A more detailed description of the distinct solution strategies can be found in the supplementary material of [90].
4.1. Single atom transport at the edge of the Quantum Speed Limit

Figure 4.3.: Visualization of the construction of a bridge between the two clans of solutions. The colours denote the type of each solution. (a) The result of using convex combinations of solutions and local perturbations of them to establish a connection between the two clans is shown. (b) Local Krotov optimization was applied and close to optimal solutions are attained.

The superlandscape

The question arises, how distinct and local these solutions are in the context of the dimensionality of the full control space. Do they really represent physically distinct strategies? If so, what is the topology of the border region between the two? Is the transition smooth or abrupt? Given the high dimensionality of the problem, an exhaustive exploration of the whole space is impossible to realize with a reasonable amount of time and resources. Instead, we investigated the topology of the landscape spanned by convex combinations of the two clans and local random perturbations to these solutions \[98\]. As depicted in figure 4.3(a), where a 2D visualization of the landscape is shown, the decline in fidelity of the interpolated points and the multitude of points yielding zero fidelity suggest, that the clans can be seen as extremely small regions of good, nearly optimal solutions in the underlying optimization landscape.

However, if the perturbed solutions are used as seeds for above mentioned Krotov algorithm, new solutions are found, reaching a fidelity that resembles that of the original optimized solutions (see figure 4.3(b)) which resulted in the appearance of a bridge. We formally define a bridge as a high fidelity
curve with respect to local parameter variations that connects two different strategies in the optimization landscape in a monotonical manner. It has to be pointed out that the displacement of the solutions from the initial seed in the course of the Krotov optimization process is negligible. Additionally, each initial seed converged to a different optimum, i.e. new, distinct solutions have been found. This suggests that as long as we search in the correct volume of the optimization landscape, it is possible to optimize any initial seed to a good fidelity by using the Krotov algorithm. That is, the landscape is locally very rugged, but rich in optima.

We therefore define a new meta landscape, denoted superlandscape, which only consists of local optima obtained via some local optimization such as the Krotov method. The local optimizer $O$ introduces a mapping from some initial seed of the control $u$ to an optimized solution $u^*$. The superlandscape is then formed by the set

$$\{(u, J(u^*))\}, \text{ where } O : u \rightarrow u^*. \quad (4.10)$$

An illustration of the superlandscape is shown in figure 4.4(a). We see it as a smooth envelope function spanned on top of the optima found by the local optimization algorithm in the underlying rugged landscape.

With this understanding, we propose a method of performing optimization in this new landscape which we call Krotov Algorithm with Simplex (KRASI). It utilizes the fast Krotov optimization to identify a local optimum and combines it with simplex search \[107\]. Thereby, the simplex is applied directly to the superlandscape as defined in equation (4.10). The method of optimization is as follows:

1. Choose a seed (solution).

2. Initiate a simplex search, that allows the control variables to change.

3. Optimize the seed with the Krotov algorithm.

4. Evaluate the optimized solution from step 3 with the simplex algorithm and if convergence is attained stop. Otherwise go back to step 3 with different control parameters.

Choosing eleven of the interpolated solutions including the two clan solutions (cf. figure 4.3(b)) and optimizing them according to KRASI, resulted in
4.1. Single atom transport at the edge of the Quantum Speed Limit

Figure 4.4.: The concept of the superlandscape. (a) shows an illustration of an optimization landscape with densely lying local optima which is spanned by an overlying superlandscape. (b) Starting from the established bridge of figure 4.3(b) (here the infidelity is plotted), different points were optimized using the KRASI scheme. Significantly improved solutions as shown in figure 4.4(b). Here, the infidelity, $I = 1 - F$, is shown in the same 2D visualization as above. A total of 2000 iterations were used for one KRASI optimization run. The black lines show the trace of each KRASI iteration step leading to improvement.

The best KRASI optimized solution reached the fidelity $F = 0.998$. The fidelities of tunnelling and shovelling clan for a total transport time $T \approx 0.182$ below the QSL amount to $F_{\text{tunnel}} = 0.928$ and $F_{\text{shovel}} = 0.864$. With a mean distance of 2.8 and 7.1 to the tunnelling and shovelling solutions as defined by equation (4.9), the best KRASI solution does not resemble any of the solutions in the two clans already found. The path of the best solution can be seen in figure 4.5. As expected by the distance measure, the best solution lies in between the tunnel and shovel solutions. Comparing the absolute positions and amplitudes, the KRASI solution can be interpreted as a variant of the tunnelling strategy as the moveable tweezer stops at a similar position and does not reach the position of the static potential.

The success of this method indicates that the superlandscape is locally smooth. This makes sense from a physical point of view, where the assumption of continuity of physics requires that two closely related optimal solutions (in terms of physical parameters) yield a very similar quality. Hence, this leads to a smooth landscape for solutions that are closely related in their physical
4. Complex Optimization Landscapes

![Figure 4.5](image)

Figure 4.5: Plot of the new found optimum using the KRASI optimization scheme. For comparison, the paths of the shovel and tunnel strategy are shown, respectively. The dashed lines mark the parameters of the static potential, where the atom is located in the beginning.

content. Therefore, this method should find applications within all areas of physics where a high dimensional parameter space is required to define a set of solutions, but at the same time small perturbations to individual dimensions dramatically affect the outcome. One very important requirement of KRASI is the ability to very efficiently locate the optima which is provided by the Krotov.

In conclusion, we found a new powerful method of searching the complex control landscape in our implementation of the single atom transport problem. Moreover, new concepts of how to perceive and investigate such landscapes were developed. This approach will be adjusted and employed to the experimental problem of BEC production in different trap geometries in the following section.

4.1.3. Overview of introduced formal definitions

Previously introduced terms with respect to landscapes and their representation are collected in following list:

- **Optimization landscape**: The graph made from a set of control variables $\vec{u}$ and their fitness/cost function $J(\vec{u})$ (such as fidelity $F$): $\{(\vec{u}, J(\vec{u})\}$. 

60
4.1. Single atom transport at the edge of the Quantum Speed Limit

- **Parameterization**: A function that maps from a low dimensional space to the full high dimensional control space. Note, that there is different optimization landscape for different parametrizations. For a 1D parametrization we have \( \{(z, f(h(z)))\} \) as an optimization landscape for the parametrization \( h : h(z) = \vec{u}. \)

- **Representation**: Equivalently used as the term of parametrization. Note, that there exists also a *visual representation* of a landscape, usually referred to as *visualization*.

- **Solution strategy**: A dense cluster of high fitness solutions using some metric. In this sense, both the shovelling and tunnelling clans are (solution) strategies. A clan is synonymous with a strategy. The trap geometries introduced in chapter 3.3.3 and that will be readopted in the next section, are also strategies. We saw that we can usually ascribe a physical interpretation to strategies. Note, that these solution strategies have nothing to do with how an optimization problem is solved. Within this definition we may also speak of the best solution within a strategy.

- **Search strategy**: Methods for finding new, possibly better solutions to a problem. In that sense, local or global optimization algorithms are search strategies.

- **Bridge**: A high fitness curve with respect to local variations connecting two different strategies in the optimization landscape. The connection is monotonical in terms of fitness.

- **Connection**: A curve connecting two different strategies in the optimization landscape with respect to local variations in its parameters. Monotony is not required.

- **Superlandscape**: The optimization landscape formed by the control variable and their value after local optimization (such as (d)CRAB or Krotov). If \( O : \vec{u} \rightarrow \vec{u}^* \) describes some local optimization algorithm, the superlandscape is given by the set \( \{ (\vec{u}, J(\vec{u}^*)) \} \).
4. Bose-Einstein condensates in different trap geometries

Inspired by the mathematically and theoretically well defined problem of the Bring Home Water challenge, the implications that were drawn from the analysis of its solution landscape are applied to the experimental challenge of producing a BEC. As introduced in chapter 3, we have access to an extremely versatile trap geometry which allows to switch between a crossed dipole trap and a hybrid trap, both featuring a variable trap volume by being able to change the effective waist of one of the beams. Given the long cycle time of the experiment of $\approx 35$ s and the presence of noise in the experimental results, this analysis will necessarily be more qualitative and incomplete than the theoretical analysis above. Nevertheless, as we shall see the methodology can be used to obtain new insights into the landscape topology of this complex problem. In subsection 4.2.1 a primary landscape is established based on the principal BEC production strategies currently applied in many different experiments. The topology of the landscape is investigated using the methodology of the previous section by trying to find connections between seemingly local clusters of solutions. We emphasize that these methods in themselves are of local nature. Subsection 4.2.2 demonstrates the application of a Nelder-Mead simplex algorithm for optimizing in the landscape of a given parametrization.

Keeping the landscape aspect of the problem always in mind, the search for new, previously unknown solutions is extended in the following section, thereby using different resources:

- The application of the local optimization algorithm dCRAB which is executed remotely by theorists at Ulm University (section 4.3).

- The use of a proprietary functional based remote control client and testing it with the help of people with a physics background (section 4.4). Both parties, the theorists implementing the algorithm and the people using the remote control client, had only rudimentary knowledge about the optimization landscape. Their solutions are compared to the ones we already found. Besides finding new solutions, these experiments demonstrate successfully the capability of remotely controlling our experiment using different interfaces.
4.2. Bose-Einstein condensates in different trap geometries

- The combination of citizen science with the real time control of an experimental apparatus (section 4.5). The response and solution finding process of different groups of people without explicit physics background, knowing even less about the experimental details and the physics behind, is investigated. Their decisions on developing new attempts are driven and steered by being able to compare older solutions and its performance to the ones of other participants. This resembles a mixture between a particle swarm and an evolutionary algorithm in which the individual agents are represented by actual human beings.

All the measurements presented in this and the following sections were obtained in the time span of multiple months. Therefore, overall experimental drifts could not be excluded. Fixed benchmark sequences were used to monitor the performance of the experimental apparatus on a daily basis and results were renormalized assuming a linear correlation. In all our discussions, apart from the results discussed in section 4.5, the effect of introducing further constraints, or adjusting the figure of merit to e. g. take into account the time of the whole evaporation process are omitted. Obviously, this would lead to a change of the optimization landscape under investigation.

4.2.1. Investigating the topology of the BEC creation landscape

Before investigating the topology of the landscape for creating a BEC, a reasonable parametrization of the problem had to be found. In the case of our trap geometry we used initially in total 9 parameters which were already briefly described in chapter 3.3.3. As a reminder, they are the initial and final powers of the two beams, \( \tau \) and \( \beta \) as defined in equation (3.20) to shape the evaporation ramps, the currents through the vertical magnetic offset coils and the quadrupole coils \( I_{\text{off}} \) and \( I_{\text{QP, evap}} \), respectively and the position of the translation stage \( x_{\text{trans}} \) for changing the effective volume of the trap.

Here, it is useful to point out the subtleties of our choice of parametrization and its influence on the optimization landscape. In general, a distinction has to be made between solution strategies featuring different combinations of external control variables like laser powers or magnetic field gradients and strategies leading to inherently different physical traps and evaporation sequences. Different sets of external control variables can lead to the same physical trap,
characterized by the trap depth and a set of trap frequencies. In an optimization landscape represented by external control variables, these strategies would be regarded as different. In our discussions, we primarily consider the external control variables, as we would like to distinguish also between similar trap geometries realized through different combinations of lasers and magnetic fields. The potential energy surfaces of each strategy can subsequently be analysed to determine if they represent distinct physical strategies.

There are two main trap configurations that can be realized and used to create a BEC with the parameter space available: A crossed dipole trap and a hybrid trap. In both cases, the major design criteria are to firstly feature a deep enough initial potential for being able to trap many atoms after the microwave evaporation and secondly to retain a high collision rate for efficient evaporation throughout the whole evaporation process (i.e., having high enough trap frequencies, while lowering the potential, cf. chapter 3.2.1). In the case of the crossed dipole trap, the necessary confinement is provided by the second crossing beam, whereas the superimposed quadrupolar magnetic field takes that part in the hybrid trap.

A second characterizing feature for trap configurations can be the effective trapping volume. We can directly influence it by translating the focus of the longitudinal beam, leading to an enhanced effective waist of this beam in the region of the trap and thus a larger volume. It turned out that although only one beam is used for the hybrid trap, it is still possible to change the effective volume by this method. With an optimized loading sequence, a large volume trap exhibits rather a more spatially mode-matched type of loading with many atoms at a rather low phase space density. On the contrary, a small volume trap shows a more dimple type loading $[63–65]$, where the evaporation is started with lower atom numbers but a higher initial phase space density. This is of course a quite crude classification, because the overall combination of loading and evaporation with different trap volumes is much more complex. Nonetheless, this inspired us to define four initial ‘conventional’ trap configurations: a small volume crossed dipole trap (narrow crossed dipole trap, or from now on $NCDT$), a large volume counterpart (wide crossed dipole trap, $WCDT$) and similarly a hybrid and wide hybrid trap (cf. chapter 3.3.3). Similar as in the previous section, we can define these as physically inspired solution strategies to the problem.

As usually done in these kind of experiments, these strategies are optimized in one-dimensional (1D) line scans in an iterative procedure with respect to
4.2. Bose-Einstein condensates in different trap geometries

Figure 4.6.: (a) Example of line scans used to find optimized parameters for the hybrid trap. The range in which the parameters were scanned was normalized. (b) 2D scan between NCDT and WCDT showing a connecting bridge between the two. The solid red line is a simple theoretical estimate of where to find the bridge within the given parameter space. The inset shows a diagonal cut through the landscape and illustrates, a simple convex combination of all variables fails to find a bridge. For more details, see text.

every parameter mentioned above. That is, each control parameter is scanned individually, whereas the others are kept fixed at a certain value. The value with the best figure of merit is chosen (in our case the number of condensed atoms, $N_{\text{BEC}}$) and one continues with the next parameter. We therefore call these strategies locally optimal. An exemplary set of these scans for some of our parameters is shown in figure 4.6(a) for the optimization for the hybrid trap. The optimized sets of parameter values are the ones given already earlier in table 3.1 on page 46.

Having established locally optimal strategies, we would like to search for bridges connecting different ones like it was done previously in the Bring Home Water problem. As before, a bridge is defined as a connection of different strategies, with a monotonically de- or increasing figure of merit of the system under investigation. A seemingly intuitive connection should exist for traps of the same type, like the NCDT and WCDT, which are basically crossed dipole traps but with different effective volumes, which is only affected by the change of one beam. However, a simple connection made of convex combinations of all
4. Complex Optimization Landscapes

parameters fails and gives merely a connection instead of a bridge according to the definitions presented before (cf. inset in figure 4.6(b)). This result can be understood by considering e.g. the depth of the dipole potential as a function of the displacement of the focus. Equations (3.27) and (3.33) in chapter 3.3.1 show that these correlate quadratically for a single beam, whereas the power is affecting the potential depth only linearly. This suggests to treat the translation stage position $x_{\text{trans}}$ representing the displacement of the focus separately, which yields a 2D scan of parameters while still maintaining the strategy of forming convex combinations for the individual dimensions. The result is shown in figure 4.6(b) and demonstrates that changing the representation of a problem yields a bridge and disproves the local character of the solution strategies involved. It has to be pointed out that no further optimization is involved to come to the above result. The solid red line marks the position within the given parameter space, which yields the same potential depth at the beginning of the final evaporation ramp. That is, we are following the same physical reasoning which lead to the 2D scan itself. Even though it only considers the initial situation of the final evaporation and not the evaporation itself, this estimate resolves remarkably well the position of the bridge.

Efforts were made to find also bridges for other trap combinations especially between NCDT and hybrid trap. Therefore, different representations were chosen resulting in various scans of different parameter combinations. However, none of them turned out to be successful. At this point it is not clear if bridges exist at all within the range of our parameter space. These scans rather illustrate the complexity of the underlying landscape. As an example, the extension of the scan space in the third dimension is presented in figure 4.7(a)–(e) for parameters between the NCDT and hybrid trap. As in the case of going to two dimensions in the above example, the third dimension is obtained by separating a single parameter from the others (in this case $\beta$). For the sake of completeness, the other dimensions are: magnetic fields and powers of the longitudinal beam on the x-axis, and the powers of the transversal beam and $\tau$ on the y-axis, respectively. The effect of this change is more subtle than for the appearance of the bridge, but the result is nonetheless intriguing as it reveals a moving mountain peak which features in one case even better BEC atom numbers as the pure hybrid trap, our so far best performing trap configuration.

All these scans exemplify the existence of a higher lying landscape which is describing the full complexity of the problem. A tSNE plot is created to
4.2. Bose-Einstein condensates in different trap geometries

Figure 4.7.: Illustration of a scan in three dimensions from the NCDT to the hybrid trap. Each frame (a)–(e) represents a scan point in the third dimension (denoted $z$). The description of the different parameters $x$, $y$, and $z$ is given in the text.
4. Complex Optimization Landscapes

Figure 4.8.: Two dimensional tSNE representation of the landscape showing the variety of different trap configurations that are accessible in our experiment. The plot contains the four main configurations which were scanned and optimized by 1D line scans. The connecting bridges were found either directly, by performing multidimensional, interpolated scans between different trap configurations, or by identifying and using intermediate trap configurations which can’t be ascribed to any of the strategies. For more details see text.

illustrate the relative position of the four initial strategies, which is shown in figure 4.8. Here, 1D line scans that were also used for the local optimization are rastering each conventional trap configuration. In addition, the attempts of bridge construction are added to the plot. As the colour coding indicates, in all other trap combinations but the connection between the WCDT and NCDT which was discussed above, $N_{\text{BEC}}$ is not monotonically de- or increasing, i.e. they are not forming a bridge in our terms. The bigger patches of data points are 2D scan whereas the linear traces represent multiple line scans. For this purpose, also intermediate trap configurations which can’t be ascribed to any of the strategies were identified and used to establish a connection via multiple stopovers. For the sake of clarity, the 3D scan of figure 4.7 is omitted in this plot.
4.2. Bose-Einstein condensates in different trap geometries

Figure 4.9.: Illustration of a 2D simplex optimization. The depicted landscape is spanned by $x$ and $y$, which represent each a certain set consisting of parameters from table 3.1. The solid lines show the traces of the optimizer for three executions with different starting points.

4.2.2. Local simplex optimization

As an attempt to find more connecting bridges between different trap configurations, we implemented a search of the parameter space for optimal solutions using the Nelder-Mead method [107] (short simplex algorithm) in our experimental control. As its name says it searches the landscape by forming simplices. The cost function value at the corners of a simplex is interrogated. For the next iteration the simplex is deformed depending on the cost function values. An exemplary application is shown in figure 4.9, where the simplex was applied on a 2D landscape spanned by $x$ and $y$, which are sets of aggregated original parameters describing our trap. In this representation, there appear two local optima which are in the given parameter space close to the NCDT and the hybrid trap, respectively. Starting with three seeds at different locations, the trace of the optimizer is illustrated by the differently coloured data points. In all cases it reaches to the local optimum. Applying our implementation of the simplex to optimizations in higher dimensions, however, turned out to be very noise driven and therefore this method was not pursued any longer.
4. Complex Optimization Landscapes

With respect to the theory case of section 4.1 it can be concluded, that the rather long cycle times of about 30 s prohibit us to perform an as extensive exploration of the landscape as in the theory case of section 4.1. We don’t have a fast locally optimizing algorithm like the Krotov at hand and a similar combination of different local search algorithms is therefore not feasible. Yet, as the investigation the in previous subsection showed, a multitude of new, in one experiment even better solutions could be revealed by applying a similar method as before, which contained the mixing of established solutions in the search for connecting bridges. It was confirmed that the locality of a solution strategy strongly depends on its problem representation and the way the landscape is searched. The question still remains, if there are completely new and even better solutions that cannot be trivially found in the available set of representations given by our choice of parameters (cf. table 3.1). Therefore, alternative approaches for optimizing the whole evaporation process in an even higher dimensional space are investigated, which will be the topic of the following sections.

4.3. Remote optimization with dCRAB

The first approach in finding new solutions to our problem of BEC creation is to make use of a more sophisticated computer-assisted optimization. Previous experiments used global closed-loop optimization strategies like genetic algorithms [91–94] or machine learning approaches [95] to optimize experimental sequences. However, similarly as in our own implementation of a simplex algorithm, they were limited to a fixed small amount of previously introduced parameters which forbids arbitrary ramp shapes for laser powers or magnetic field gradients as a function of time. As we can’t model the complex problem of an evaporation process in detail, we need an algorithm which is capable of efficient optimization in a high dimensional space without detailed knowledge of the underlying system (a ‘black box’ optimizer). Similar problems can be found in quantum optimal control theory (QOCT) [108], which aims at finding an optimized way of coherently transferring a quantum system from an initial state $|\psi_i\rangle$ to a certain target state $|\psi_f\rangle$ by applying a control pulse acting on

\footnote{It has to be pointed out that for these types of problems which are not solvable analytically this question will remain.}
the Hamiltonian of the system. In fact, the optimization problem introduced in section 4.1.1 is a typical process treated in the framework of QOCT.

Ab initio calculations require an exact treatment of the system, which is often not possible due to multiple reasons, e.g. a restricted knowledge of the system or limited numerical capacities. Due to the multitude of dimensions, a direct search in the full space of control pulses is not efficient. For the results of the first section the problem was gamified. Another approach is the use of sophisticated algorithms. The so called chopped random-basis (CRAB) quantum optimization [96, 97] later extended to the dressed CRAB (dCRAB) algorithm [37] were developed and are tailored for such kind of problems. Both were already successfully applied to ultracold atom experiments [109]. The latter algorithm is used in our case to optimize the BEC creation process. It is not implemented directly in our experiment, but is executed remotely by the people from Ulm University developing it.

In the following subsections, first the working principle of (d)CRAB will be explained (4.3.1), followed by a short description of the implementation of the ability to externally remotely control our experiment (4.3.2). Finally, experimental results are discussed and set into relations with the landscape interpretation of the problem (4.3.3).

4.3.1. The working principle of (d)CRAB

The CRAB algorithm rephrases the problem of optimizing a set of $N_u$ different control pulses or driving fields $u_i(t)$, $i = 1, \ldots, N_u$ controlling the dynamics of the system. Therefore, an initial guess $u_i^0(t)$ is corrected by multiplying it with a real valued function [96, 97]

$$u_i(t) = u_i^0(t) f_i(t).$$

(4.11)

For simplicity, it is assumed that the set of control pulses only consists of one element in the following discussion, i.e. $u_i(t) = u(t)$. The corresponding correcting function $f(t)$ is expanded in a functional basis which is truncated after the $N_c^{th}$ element. Typically, as well as in our case, this expansion is performed in terms of Fourier harmonics and takes the form

$$f(t) = 1 + \sum_{k=1}^{N_c} A_k \sin(\omega_k t) + B_k \cos(\omega_k t).$$

(4.12)

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4Special thanks to Jonathan Zoller and his help!
4. Complex Optimization Landscapes

Here, $\lambda(t)$ is a shape function setting the boundary conditions appropriately. The frequencies $\omega_k$ are chosen randomly around the harmonics,

$$\omega_k = \frac{2\pi}{T}(k + r_k),$$

(4.13)

where $T$ is the total duration time of the process. The $r_k$ are picked randomly from a uniform distribution between $-0.5$ and $0.5$. This leads to a non-orthogonal basis expansion, which turned out to speed up the convergence time of the algorithm because of the coverage of a larger functional space compared to a pure orthogonal basis \[97\].

The performance of a specific $f(t)$ for QOCT problems is given by a cost function which can be defined in different terms, e.g. the final system energy or the fidelity as the state overlap between the final state (obtained by applying the control pulse corrected by $f(t)$) and the target state $F = |\langle \psi_f | \psi_f \rangle|^2$ (cf. equation \[4.4\]). As before, the figure of merit is the number of condensed atoms in our case. Together with the $A_k$ and $B_k$ this defines a multivariable optimization problem which can be solved by any numerical method suited. Here, a variant of the aforementioned mentioned simplex algorithm is used \[107\].

However, the use of fixed $\omega_k$ defines again only a limited search space which might be governed by local false traps for the optimization algorithm. To circumvent this, dCRAB was developed as an extension to the CRAB algorithm. It starts out exactly as the CRAB optimization. As soon as no improvement in the cost function is detected any more, or after a fixed amount of iterations, a new CRAB run (denoted as superiteration) is started ‘dressing’ the current best pulse with a new correcting function and a new random basis \[37\]. For the $j$th superiteration this can be written as

$$u^j(t) = u^{j-1}(t) f^j(t),$$

(4.14)

where $u^{j-1}(t)$ represents the best pulse from the previous superiteration. For $j = 1$, this corresponds to the initial pulse guess in equation \[4.11\]. By the change of basis, the problem is reformulated, the topology of the current optimization landscape changes and false local traps are resolved \[37\].

4.3.2. Realization of the remote control

Within our experimental control program Alice, the experimental sequences are organized in blocks \[79\]. Each block can either consist of other blocks or
4.3. Remote optimization with dCRAB

![Diagram of remote control interface between dCRAB and Alice via a web connection.]

**Figure 4.10.** Scheme for the remote control interface between an external client and our experimental control program Alice via a web connection. Dressed crab illustration: by kind permission of Jonathan Zoller, Ulm University.

represent a physical channel controlling some hardware among others analogue and digital output channels. The blocks cannot only be placed in series but can also be executed in parallel or in loops. This allows for the programming of versatile experimental sequences [110].

We have realized the possibility to load a set of parameters which are given in a simple text file format into a sequence. This enables us not only to overwrite constant variables in a block, but also to read in whole time-resolved waveforms on the fly, which then can be output on an analogue channel. This feature is used to build a closed-loop optimization scheme with a remote client. In particular, the remote client represented by the dCRAB algorithm which is implemented on a local machine at Ulm University writes such a text file and sends it via a web connection to our computer hosting Alice. The experimental sequence is compiled and executed with the given pulses. At the end of the sequence, an absorption picture of the cold atom cloud is taken and the BEC atom number $N_{\text{BEC}}$ extracted on our analysis computer. This value is handed to Alice which closes the optimization loop by writing the result into another text file which is sent back to the computer hosting the optimizer via the same web connection. The scheme is depicted in figure 4.10.

### 4.3.3. Results

Different experiments were performed in order to investigate the advantages of the dCRAB optimization in a high-dimensional space. Starting from an
initial guess for the dipole beam intensity ramps according to the parameters of Table 3.1, the pulses were modified by the dCRAB algorithm. During the individual experiments, the trap volume i.e. the position of the translation stage $x_{\text{trans}}$ and the current for the magnetic offset field $I_{\text{off}}$ were kept at a fixed value.

In the specific implementation of dCRAB, the pulse is modified by adding a correcting function centred around zero and expanded in the Fourier basis onto the pulse:

$$u_i(t) = u_i^0(t) + f_i(t).$$

This should be contrasted to the original CRAB scheme of equation (4.11) where the correction is applied by a multiplication. In our case, the expansion only contains the first harmonics with a frequency randomly picked at the beginning of every superiteration. The only free parameters for modifying the pulse are the amplitude and the phase.

To overcome shot-to-shot fluctuations and thus resulting in an optimization driven and influenced by noise, an adaptive averaging scheme is applied with a stepwise increasing number of averages for higher yields in $N_{\text{BEC}}$. Outliers to high atom numbers are in this way re-evaluated. However, we still keep the number of time-consuming evaluations low at early stages of the optimization which decreases the overall convergence time. This would be an ideal application of Faraday pulses as benchmark tools as described later in chapter 6.1. This tool however is unfortunately not available for these experiments.

In a first experiment the quite well understood parameter space around the WCDT was investigated. Here, only the laser ramps were modified, as exclusively pure crossed dipole trap configurations were considered. The experiment’s starting point was intentionally chosen to yield a smaller BEC, in order to give the algorithm enough room for improvement. It could be shown that the dCRAB algorithm found steadily better solutions, but overall the numbers of condensed atoms were converging to the ones obtained after the manual optimization of the WCDT at $\approx 1.1 \cdot 10^6 – 1.2 \cdot 10^6$ atoms. Apart from the artificially restricted search space, the maximally achievable laser powers might have been a limiting factor for gaining higher $N_{\text{BEC}}$. In the course of the different experiments, it was also tried to find better ramps for the loading of the dipole trap. Here, an alternating optimization scheme between improving the loading and the evaporation was used. However, there was no evidence on
4.3. Remote optimization with dCRAB

![Graph](image)

**Figure 4.11.** (a) Optimization with dCRAB from an intermediate point between hybrid trap and NCDT. The number of condensed atoms obtained in the hybrid trap could be exceeded by 20%. (b) Another optimization run from a different starting point, yielding a similar \( N_{\text{BEC}} \). The inset shows a 1D scan between hybrid trap and NCDT. The starting point for the optimization runs in (a) and (b) corresponds to the trap parameters at 0.4 and 0.2, respectively and are marked with green circles.

significant improvement. For this reason, the loading was fixed to the sequence as described in chapter 2.4 in all following optimization runs.

We now focus on experiments performed in the region between the hybrid trap and the NCDT. For these, besides the shape and the duration of the laser ramps, the value of the magnetic field gradient during the evaporation (being constant in time) is controlled by the optimization algorithm. All in all, this adds up to in total six parameters which have to be optimized by the underlying simplex algorithm.

The starting point for the optimizations is chosen on the basis of a 1D scan along the convex combination of pulse parameters for the hybrid trap and NCDT as defined in table 3.1. For clarity, the scan is depicted in the inset of figure 4.11(b). As before, we want to give the algorithm enough room for improvement and started the first optimization at a point yielding clearly non-optimized values. In terms of normalized units of the scan shown in the inset, the starting point for this optimization corresponds to position 0.4 which seems to be connected to the hybrid trap but not to the NCDT as the lack of atoms in
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the region between 0.5 and 0.9 indicates. The result of this first optimization is shown in figure 4.11(a). The plotted BEC atom numbers are averaged values. The number of repetitions is changing according to the adaptive averaging scheme mentioned above. In this sense, the iteration number given in the plots is an effective iteration number counting the finished evaluation of an averaged value. After a strong improvement in the first 20 effective iterations, the rise in terms of BEC atom numbers flattened, in order to accelerate again towards the end of the run. We surpass the old record by more than 20% ending at a new optimum of about $2.35 \cdot 10^6$ atoms. Although it seems that the experiment was aborted prematurely, following runs starting at the new optimum indicated that not much further improvement can be expected.

One of our objectives also in this set of experiments is to show the connectedness of different solutions through bridges. As we seem to be able to establish such a bridge between the hybrid trap and the point 0.2 of the above mentioned parameter scan (cf. inset of figure 4.11(b)), we start another optimization run from there. We then want to bridge the newly found optimum via this starting point to the hybrid trap. The results of the optimization are shown in the main plot of figure 4.11(b). The increase in atom numbers as a function of effective iterations is not as high as in the previous optimization run starting from 0.4, but the final value of convergence is comparable.

In order to find a well resolved bridge between the optimization’s starting point and the new optimum, we perform as a first approach stepwise linear interpolations between the pulses belonging to the individual improvement steps the dCRAB algorithm found on its path to the new optimum (i.e. following the steps of the red solid line in figure 4.11(b)). The result is presented in figure 4.12(a). The individual stepping stones of the optimization are marked by the vertical red lines, whereas the first step represents the transition from the hybrid trap to the starting point of the optimization. As indicated by the rising and falling atom numbers along the sequence, this method reveals only a connection in our terminology. In contrast to this approach, shown in figure 4.12(b), a direct linear interpolation between the hybrid trap and the new optimum is performed. Surprisingly, this quite simple technique proves to be successful in this case and shows monotonically increasing atom numbers. Taking into account the high dimensionality of the whole space available for optimization this result cannot be expected. The best counter example is the stepwise interpolation.

One can imagine the situation as illustrated in figure 4.13. The optimum lies
4.3. Remote optimization with dCRAB

<table>
<thead>
<tr>
<th>BEC atom number (×10^6)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Data</td>
</tr>
<tr>
<td>Optimization step</td>
</tr>
</tbody>
</table>

![Graph](a)

![Graph](b)

**Figure 4.12.** (a) Linear interpolation from the hybrid trap through the starting point of the optimization in figure 4.11(b) to its optimum. Thereby the pulses between each optimization step (marked by vertical dashed lines) is interpolated. (b) Direct linear interpolation between the pulses for the hybrid trap and the found optimum in figure 4.11(b). The error bars represent the standard error for five repetitions.

in a part of the landscape that was previously not covered by our parametrization. Using the dCRAB, new parameters (here depicted by the two axes x1 and x2) were introduced. The red circles depict the path of the direct linear interpolation (a) leading to a steady increase in the atom numbers (b). The piecewise linear interpolation path (blue triangles) follows the steps that were found by the dCRAB algorithm (orange triangles). We believe that in our case some of the steps, especially in the beginning when the number of averages per iteration step was low, were noise driven and therefore atom numbers were measured systematically too high, as indicated by the error bars. This does not affect the overall convergence of the quite robust dCRAB algorithm but reveals when performing the repeated interpolation of the pulses.

The fact that the first step towards the starting point in figure 4.12(a) doesn’t show monotony could be explained by the difference in the parametrization used for the interpolation compared to the one of the scan in the inset of figure 4.11(b) (here we observe monotony between these two points). In this case, not the shapes of the ramps were interpolated but its describing parameters
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Figure 4.13.: (a) Visualization of the path of the direct linear interpolation (red circles) versus the piecewise interpolation following the trace of the dCRAB algorithm (blue triangles) towards the optimum in the optimization landscape spanned by the parametrization x1 and x2. The bigger orange triangles depict the steps found by the optimizer. (b) Number of atoms in the BEC as a function of iteration steps along the path. The linear path exhibits constantly increasing atom numbers, whereas following the optimizer leads to a more rugged outcome and a decrease in atom numbers in one step caused by a noise driven ‘detour’.

from table 3.1, which is not equivalent.

In order to get a better insight into the subtle differences of the solutions, a distance analysis is performed. As a measure we use a weighted Manhattan distance definition $d_{i,j}$ between pulse sets $i$ and $j$:

$$d_{i,j} = \sum_k \alpha_k \left( \int_0^1 |u^i_k(\tau) - u^j_k(\tau)| \, d\tau \right) + \beta |T_i - T_j|, \quad (4.16)$$

where the different $k = 1, 2, 3$ correspond to the rescaled pulses of magnetic field gradient and the individual beams’ powers of unit length weighted by $\alpha_k$, respectively. The overall difference in duration of the ramps $|T_i - T_j|$ is taken into account as an additional addend with weighting factor $\beta$. For giving a comparable weight to changes in the laser ramps at the beginning and at the end, where the differences in power can be as large as two orders of magnitude, the amplitudes are logarithmized before being further processed. The result of the distance analysis of the stepwise interpolation, as well as the direct interpolation is plotted in figure 4.14. The distance is measured with respect
4.3. Remote optimization with dCRAB

Figure 4.14.: Distance analysis with respect to three different reference points for the two different optimization methods presented in figure 4.12. In (a) the path of the stepwise interpolation is shown, where the red vertical lines mark the optimization steps of the dCRAB algorithm. (b) represents the direct interpolation path.

to three reference points: the hybrid trap, NCDT and the starting point of the dCRAB optimization, corresponding to the red, blue and black curves, respectively.

In both depicted cases, the distance towards the hybrid trap increases rapidly in the beginning, due to the fact that switching on the transverse beam which is not present in the hybrid trap is a quite distinct feature. As before, the original optimization steps are visible as discontinuities in the distance traces and are marked by the red vertical lines in figure 4.14(a). It can be seen that the dCRAB algorithm first recedes from the starting point, for coming intermediately closer again and finally converging to a distance of around 0.025. A similar, but much smaller change in distance towards the hybrid trap can be observed. The optimum is therefore relatively close to the starting point and nearly on the same hypersphere with radius 0.1 around the hybrid trap. The alternating distance illustrates, that the optimizer took a quite complex path ‘crawling’ around the optimum in order to reach it finally. The small increments in the atom number at the end of the original optimization in figure 4.11(b) are pointing towards similarly small changes in distance. Obviously, this analysis is quite coarse with respect to the high dimensionality of the total parameter space. Moreover, absolute distances depend heavily on the chosen metric.
Therefore, this discussion should be only seen as a form of interpretation.

4.3.4. Conclusion

We have shown the successful application of a black box optimizer having access to the full parameter space. Additionally, we gained valuable knowledge for future quantum experiments about the implementation and performance of these kind of algorithms with our experimental apparatus. Although only working locally, the chopped basis set lead to an efficient, fast converging optimization, whereas the randomization and repeated start-over of the process circumvented local traps. In this way, a new optimum could be found. The path of the optimizer in the high dimensional landscape between the hybrid trap and the new optimal solution was reconstructed by performing a distance analysis on the different solutions. This was compared to the path of a direct interpolation. Surprisingly, the latter, simpler approach proved to be successful to establish a bridge between the solutions. The failure of following the optimization path was accounted to possible noise influencing the algorithm. Nonetheless, the overall convergence of the optimizer was robust against these disturbances. An even more refined averaging and re-evaluation scheme could lead to more robustness and further enhanced behaviour.

4.4. Human remote optimization

After the successful application of the dCRAB algorithm, we involve other people to control directly our experiment. We developed a simple client which allows to create ramps for magnetic fields and the two laser beams based on piecewise defined functions. The interface between the client and the experimental control is the same as for the realization of the dCRAB control in previous section (cf. figure 4.10). A screenshot of the program can be seen in figure 4.15. As a test of the interface, we gave this program to two different people with physics background and knowledge about evaporative cooling of ultracold atoms: an undergraduate physics student doing an internship in our lab and a postdoc who was working with us at an earlier stage of the experiment, who is now affiliated to the University of Nottingham. Both were only

\[5\text{Special thanks to Lærke Lyhne and Mark Bason!}\]
4.4. Human remote optimization

Figure 4.15.: Screenshot of the initial human remote optimization tool used in a first trial of remote controlling the experiment. It is a LabView based stand-alone client and allows one to define piecewise defined variable based functions for describing the ramps of the two dipole beams and the magnetic quadrupole field. It enables to perform 1D scans of set of variables and gives immediate feedback on the obtained atom numbers.

partially familiar with our current experimental setup and had only limited information about working solutions. However, they were e.g. acquainted with the different trap geometries we were able to realize and had knowledge about our own preferred parametrization of table 3.1, without knowing exact numbers. They were provided with a working, non-optimized solution which was the same as the starting point of the computer assisted optimization presented in figure 4.11(a) of previous section. Both were having control over the ramps not only for the evaporation but also for the loading of the dipole trap, i.e. right after the end of the microwave evaporation (cf. chapter 2.4).

This experiment highlights once more the remote control features that have been developed. The individual traces of the optimization are depicted in figure 4.16, where the upper panels show the distances defined by equation 4.16 with respect to the hybrid trap, NCDT, the starting point of the optimization and the optimum found by the dCRAB algorithm in figure 4.11(b). The lower panel shows the achieved number of atoms in the BEC. Although having only limited time of about half a day each, both parties proved to be quite successful and could each more than double the number of BEC atoms at the end of the sequence. Slight drifts in the apparatus might have influenced the individual
Figure 4.16: Optimization traces by two physicists. The upper panels depict the distance measures according to equation (4.16) with respect to the hybrid trap, NCDT, the respective starting points and the optimum found by the dCRAB algorithm (cf. figure 4.11(b)). The lower panels show the number of condensed atoms that were achieved in the individual steps. The traces in (a) were produced by an undergraduate student, (b) by a former postdoc of the experiment.

results, as no benchmark pulses were applied during the optimization. Due to this and the extremely limited number of participants, this is more an exemplary experiment with nonetheless prominent results. It can be seen as a stepping stone towards the experiments presented in the subsequent section.

The individual strategies seem to differ. The student spent considerably long time in exploring and scanning traps with a lot of CDT properties as indicated by the relative closeness to the NCDT, before turning after approximately 300 iterations more towards traps with hybrid trap character, which reflects also in an increase in atom numbers. One interpretation here is that the types of CDT that were tried out were density limited and thus no larger BECs could achieved. The experienced experimentalist immediately works with a geometry close to the hybrid trap achieving high number of atoms from the beginning on, but fails to improve the results in the later course of the experiment which might have been also induced by apparatus drifts. The best atom number yields are around \((2.1 - 2.2) \times 10^6\), which is slightly higher than in our pure hybrid trap. The successful feature in the sequence at the end of the student’s experimental time seems to be the introduction of a slow magnetic quadrupole field ramp towards
lower fields during the forced evaporation in the laser beams. Whereas the other person used constant magnetic fields for this part of the sequence throughout the whole optimization, similar as we do it in our standard sequences. Using this new strategy BECs with up to $2.4 \cdot 10^6$ atoms can be produced.

4.5. Crowd-sourced search for novel solutions

Motivated by the quite successful outcome of the previous experiment, we decided to go a step further and thought about possibilities to involve and engage people without any physics background in participating in the search for novel solutions of creating BECs in our trap geometry. We saw already in the Bring Home Water challenge \cite{90} that the gamification of our types of problems proved to be eminently useful similarly as in other citizen science game projects, where e. g. the human sense for creativity pursuing seemingly non-optimal paths was used to investigate the complexity of protein folding \cite{111} in the game Foldit or the structure of RNA \cite{112} in EteRNA. In other challenges, the humans’ exceptional visual perception was used to categorize galaxies \cite{113} or mark neurons and synapses in electron microscope pictures of the retina \cite{114}.

4.5.1. The Alice game

Our project is one of the first real-time implementations of an open laboratory in physics. To our knowledge, there exists only the IBM Quantum Experience \cite{115} and the project Quantum in the Cloud \cite{116}. Both allow realizing a quantum computing sequence on a real system. The former uses five superconducting qubits, whereas the latter is a system realized on a photonic chip. However, these projects are addressing rather ‘experts’, whereas we believe in the gamification of the problem supported with a didactics strategy to engage the public. In our case, we face the challenge of turning the programming of laser and magnetic field ramps into an engaging game. It is clear that a functional based approach at the level of complexity that is needed in our case, is for many people too abstract and would therefore quickly lead to frustration. Likewise, the unassisted, blind search for solutions would probably prove unsuccessful for the most of the participants. The real-time operation of the laboratory implicates additional difficulties. Each submitted solution has to be processed which leads to a time delay of at least 30s per solution for run-
4. Complex Optimization Landscapes

Figure 4.17.: Screenshots of the *Alice challenge* \[^{117}\]. In (a) a frame from the promotional video \[^{118}\] is shown. Players can control the magnetic quadrupole field depicted in yellow and the two dipole beams in red and blue. The control happens in a client (b) programmed with the cross-platform engine \([\text{Unity}]^{119}\) and features a spline editor for shaping the ramps where the same colour coding was used.

We therefore developed a client in form of a game using the cross-platform engine \([\text{Unity}]^{119}\) and promoted through our online community \([\text{ScienceAtHome}]^{116}\). As depicted in figure 4.17, the ramps are implemented by differently coloured spline curves providing an intuitive way of shaping them. In order to account for the high powers/magnetic fields during the loading and the low ones at the very end of the evaporation sequences, the displayed ramps represent logarithmized and normalized versions of the real valued ones. The lower bounds for the dipole beams were set to 1 mW which is below powers where we are able to trap atoms. The minimal magnetic fields are set to \(\approx 0.5 \text{ G/cm}\), values where we don’t observe any measurable influence on existing trap configurations. In cases where these values are set, the respective outputs in the experiment are pulled to zero. The duration of the ramps, before the number of atoms are detected in a TOF measurement, are fixed to a certain value and can’t be influenced by the players. As in the previous example of section 4.4, the user generated ramps incorporate the loading of the dipole trap.

The general setting of the game is of collaborative fashion. The participants can see the solutions and corresponding results of other players and have the possibility to copy them completely or only a certain spline curve. This allows the people to reproduce working solutions easily and try to improve them further. The yield in condensed atoms of a solution is converted into a score
4.5. Crowd-sourced search for novel solutions

in order to form a high score list. In this way, best working solutions are easily accessible and people are driven through the motivation of beating the high score. A video produced by us \[118\] (figure 4.17(a)) introduces people into the idea behind magnetic traps, optical dipole traps and evaporative cooling. A small tutorial gives an introduction in how to handle the control of the game. As experimental noise and imperfection easily can lead to confusion and a strange game experience, the participants were warned explicitly beforehand that we are running a real experiment which is subjected to fluctuations and problems can occur.

Using this client, two different game modes were realized:

- A round based version of the game. Here, access was restricted to 142 participants who signed up beforehand. Small teams of five players each were formed and only one team at once was playing. This kept waiting times for an individual player for results at a minimum. In each round, every player was asked to hand in one solution. After the solutions from all players were collected, they were realized in the experiment and the results given to the players. Each team was allowed to play 15 of these rounds.

Due to the restricted number of players and time, we didn’t expect novelties of good solutions, and rather the players’ game behaviour was the object under investigation. For example, some of the teams were given the additional direct information how often a certain solution was copied from the previous round. The outcome of these experiments is currently being analysed and will not be part of this thesis. The maximum number of condensed atoms that was achieved by a team was \(1.8 \cdot 10^6\).

- A ‘swarm’ version of the game. In this mode, the game was open to everyone and there were basically no limit in the number of participants. The submitted solutions were placed in a queue, and, depending on the length of the queue, an estimated process time was displayed. In this way, players could join, submit one or a set of solutions and come back at a later time to review the results.

In the following section first results of the latter game mode will be highlighted.
4. Complex Optimization Landscapes

4.5.2. Results

Investigating the best working solutions

In the ‘swarm’ version of the Alice game (short *swarm game*), we had about 700 individual participants. The game was open for participation one week, 24 hours per day, with brief interruptions to fix system problems, e.g. to relock lasers. Directly after the game was launched and made accessible to the public, we realized that people performed extremely well and reached soon a saturation level in the results corresponding to the numbers of our largest BECs. It was therefore decided to restart the game on a regular basis two to three times per day. As experienced players knew the good solutions relatively quickly, we additionally changed the length of the ramps at each restart. In a total of 15 sessions, we covered a range for the ramp duration from 1.75 s to 8 s. Counting all sessions, 7577 solutions were submitted. Figure 4.18 shows the distribution of the attained numbers of condensed atoms, \( N_{\text{BEC}} \), taking all solutions into account. More than 73\% yielded a BEC. Although filters were set in the fitting algorithm for excluding bad fits of strangely shaped atomic clouds, there might be still a small fraction of falsely assigned condensate sizes.

In a first step, we investigate the best solutions that were found in the individual sessions. For each session, we therefore extract \( N_{\text{BEC}} \) of the 20 largest BECs and their corresponding ramps. The results as a function of total ramp

![Figure 4.18: Histogram for the achieved number of condensed atoms, \( N_{\text{BEC}} \), for all submitted solutions in the swarm game.](image)
4.5. Crowd-sourced search for novel solutions

Figure 4.19.: (a) Best BEC yields for the individual game sessions versus the respective ramp duration. One data point represents the average of the 20 best solutions and the error bars depict the standard deviations. The corresponding laser and magnetic field ramps of the coloured data points were extracted and are investigated in (b) using the same colour code. Here, from top to bottom, the magnetic field gradient, the power of the longitudinal dipole trap (LDT) and the power of the transversal beam (TDT) are plotted on a logarithmic scale, similarly as the players saw the curves in the game client. The solid lines represent the mean of all ramps under investigation, whereas the standard deviation is marked by the shaded areas.

The time set in the different sessions is shown in figure 4.19(a). The data points represent thereby the mean value, whereas the error bars are the standard deviation of the individual data sets which holds also for the other data sets presented in this section if not stated otherwise. We observe that the largest BECs are basically independent for ramp durations $\geq 3$ s yielding $(2.42 \pm 0.06) \times 10^6$ atoms, where the red marked point was excluded. This number is in the same range as our student achieved previously (cf. section 4.4) and also in agreement with the results obtained with the dCRAB algorithm (section 4.3.3). Although we observed drifts in benchmark pulses that were run in parallel every 10$^{th}$ iteration of the experiment, the players seemed to adapt to changing conditions leading to the observed plateau. Below 3 s, $N_{\text{BEC}}$ decreases drastically and no solution was found for ramp durations below 1.75 s, although attempts were made for a settings as low as 1.25 s.

The sets of best solutions of three sessions with ramp durations of 2, 4 and 7 s corresponding to the coloured data points in figure 4.19(a) are investigated
more carefully to highlight differences in the strategies. Figure 4.19(b) depicts with solid lines the mean of the respective ramps, where the same colour coding was applied. The shaded area represents the standard deviation. Clear differences especially when coming to short ramps are visible, where the transverse beam power is, compared on a relative time scale, partially two orders of magnitude more powerful than in the other two examined strategies. A simple physical argument for this result can be given. Faster ramps usually require higher thermalization rates for an equally efficient evaporation, which is provided in this case by the transverse beam through higher trap frequencies in the longitudinal direction. For longer ramp times such high thermalization rates are not needed and it is more beneficial to support a larger trap volume through a lower power of the transverse beam. This conjecture is supported by differently shaped clouds after time-of-flight. In the case of short ramps the aspect ratio is close to 1, whereas the cloud is usually elongated for high \( N_{\text{BEC}} \) and longer ramps.

Surprisingly large BECs of \((2.76 \pm 0.01) \cdot 10^6\) atoms were achieved for a ramp duration of 4 s in one of the sessions. The corresponding ramps are represented by the data coloured in red in figure 4.19. The reason, or the distinct feature in the ramps allowing for such large numbers is not entirely clear and currently under investigation. The extent of the bounds on the ramps of the transversal beam indicates that its effect could be of less importance, and, in the case of this specific session, some exceptional good conditions were induced by uncontrollable drifts in the apparatus. However, there are regions where the band narrows. Moreover, there were no anomalies visible in the benchmark pulses. The exact same ramp combination for both lasers and magnetic field gradients were not realized in any other session. Despite the fact that there might have been undetected drifts, the participating players were able to adjust themselves to these new conditions and find the combination leading to the remarkable results.

**The evolution of a solution**

In the following, we are looking at the development of strategies within a single game session. As people are likely to tend to copy or mimic solutions, there is an evolutionary process expected. Following procedure is therefore used to filter and identify common main strategies out of the up to 1000 individual solutions that are submitted in a single run: The data set is divided into
4.5. Crowd-sourced search for novel solutions

Figure 4.20.: Evolution of strategies over the course of a single game session. From top to bottom the ramps for the magnetic field gradient, as well as the longitudinal and the transversal beam power are plotted on a logarithmic scale. Four evolutionary steps are shown as an example. The corresponding $N_{\text{BEC}}$ is denoted through the colour code, where the black, solid line represents the seed solution and the red, dotted line the best solution at the end of the session.

A careful placement of the bins allows one to follow the main solution strategy in the course of a session.

Figure 4.20 exemplifies the procedure on a data set. Four colour coded steps with steadily increasing $N_{\text{BEC}}$ were chosen to show the most prominent changes. The whole data set of 490 individual solutions was subdivided into 19 bins. The presented data shows average and standard deviation of the 11 best solutions in the four chosen bins. Remarkably here is the relatively big change from the initial seed, represented by the data in black, to the second step (green), which happened within a few single iterations and lead to a drastic increase of $N_{\text{BEC}}$. The later changes were of smaller nature. The final step to the ramps represented by the red data points happened only at the very end. It entails lowering the power of the longitudinal beam at the very beginning of the ramp. Instead, the transversal beam’s power was increased, which is however followed by a stronger initial decrease of the power, in order to end up at the same final
power as in the previous step.

A clustering analysis similarly as was performed in section 4.1.2 could simplify the procedure of identifying distinct strategies but depends heavily on the chosen metric. In this specific case, it might be feasible to perform a combined analysis taking also the resulting trap properties like trap frequencies and potential depth into account.

**Performance of selected solutions for different ramp durations**

Finally, we investigate the performance of selected solutions when implementing them using different total ramp duration times, $T_{\text{ramp}}$, while preserving the shape of the ramps. This points towards the question of how universal a given solution is, or if they are optimized purely for the time that was set during the respective optimization experiment. Three ramp shapes were chosen for this ramp duration sweep: The optimum obtained in the dCRAB optimization (cf. section 4.3.3), the ramps that yielded the largest BEC in the Alice game with an original $T_{\text{ramp}}$ of 4 s and the best performing ramps for very short ramp durations of 1.75 s (cf. figure 4.19).

The results are presented in figure 4.21. As described in previous subsections, slow long term drifts in the apparatus lead to changes in the optimality of solutions. Therefore for comparison, the atom numbers in this figure were renormalized to the values that were obtained in the original optimization experiments. The data represents the average over five repetitions. All solutions are stable for long ramp times (i.e. the yielded $N_{\text{BEC}}$ is independent of $T_{\text{ramp}}$) and seem to be only limited by the finite life time due to background gas collisions or parametric heating by noise in the dipole beams or magnetic fields. This changes for short ramp durations. The dCRAB solution is performing worse for ramps shorter than 5 s, where the original total ramp duration was 4.92 s. A similar behaviour is observed for the best solution of the Alice game. Here, $N_{\text{BEC}}$ starts to degrade for $T_{\text{ramp}} < 4$ s, which corresponds to the same time it was originally optimized for.

Interestingly, the solution optimized for very short $T_{\text{ramp}}$ seems to be performing best exclusively for these durations. Here, there is even a decrease in $N_{\text{BEC}}$ visible for $T_{\text{ramp}} > 2.5$ s, before numbers are stable again as described.

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6In the original experiment, the dCRAB algorithm was only optimizing the evaporation ramp. The loading was fixed to our standard loading procedure (cf. chapter 3.3.3). Here for this experiment, the loading and the evaporation was taken as ‘the ramp shape’.
4.6. Outlook

In conclusion, we demonstrated how the detailed understanding of a control landscape can help to facilitate the search for novel, previously unknown solutions to different kind of optimization problems. We think of solution strategies as a new way to investigate a higher dimensional landscape. Once a working strategy is found, one should not see it as an end point but rather as a perfect starting point for further and deeper investigations. On the example of the single atom transport problem, we introduced the concept of the superlandscape and developed the search method KRASI. It is composed of a Krotov algorithm which establishes a link to the superlandscape. This is combined above. By using this sweep technique, we even yield a small BEC for the extreme short time of $T_{\text{ramp}} = 1.5\,\text{s}$. These results demonstrate that both, the computer algorithm and the Alice game players adapt to given constraints and conditions and find best working solutions in the given settings.
4. Complex Optimization Landscapes

with a simplex search which is then effectively only scanning the superland-
scape. The successful application promises transferability to other quantum
optimal control problems of similar kind. There are in principle no limits set
on which algorithms to use for the optimization in the superlandscape and the
exploratory benefits of a genetic optimizer were for example recently investi-
gated [120].

The methodology of investigating the landscape topology in the transport
problem was in the following transferred to the production of BECs in different
trap geometries. As in the theoretical case, a basic understanding could be
developed, but as deep and systematic investigations were prohibited by the
rather long cycle times of the experiment. Therefore, alternative methods of
finding solutions were pursued. The application of the dCRAB algorithm and
the introduction and testing of a simple remote control client proved to be
successful. Future collaborations through the conduction of remote controlled
experiments seem feasible. In both cases, algorithm and remote control client,
it was given control over the shape of the ramps describing dipole beam powers
and magnetic field gradients. New evaporation sequences for creating BECs
were found with an improvement in condensate atom numbers of more than
20\% compared to our previous optimum until then. A distance analysis in the
given parameter space could ascribe both the solutions high similarities to a
hybrid trap.

Finally, we presented first results from the Alice game, the gamification of
the optimization problem of the evaporation process. They underline once more
– in accordance to previous studies – that the human intuition is valuable in
many aspects and enables to quickly adapt to changing conditions. Similar
condensate numbers as in the two previous cases were reached. In one case,
however, they surpassed our old record by far and novel solutions were found,
yielding $(2.76 \pm 0.01) \cdot 10^6$ atoms. Moreover, the players were also able to op-
timize the BEC production in the extreme conditions of very short ramps. A
sweep scan of the ramp duration showed, that these ramps were exclusively
optimal under the original optimization conditions.

The results show that a deeper understanding of the human problem solving
process is at the heart of improving and developing versatile computer algo-
rithms that are not only able to cope with rugged landscapes like in the case
of the BHW challenge [90], but also to optimize in landscapes that are sub-
jected to experimental noise and changing conditions. Our data obtained in
the team-based version of the Alice game might be able to contribute here.
In many ultracold atom experiments resonant light is used to image atoms. The sample is destroyed during the imaging process. However, the ability of taking multiple images of the same atomic cloud is needed in certain situations, e.g. for the measurement of correlated processes. There are several techniques available to perform non-destructive measurements of atomic samples\(^1\). Among others, like partial-transfer absorption imaging \([121]\), there are imaging techniques which make use of dispersive light-matter interactions. Phase contrast imaging (PCI) was used to record the first non-destructive observations of BECs \([122, 123]\). This technique is complemented by polarization phase contrast imaging \([124]\), commonly known as Faraday imaging \([38]\). The working principle is comparable to the classical Faraday effect, where the polarization axis of linearly polarised light is rotated proportional to the magnetic field component parallel to the propagation direction of light. However, the underlying physics is different.

This chapter gives an introduction into Faraday imaging as it is used in our experiment. Section 5.1 describes the principles of light-matter interactions which lead to the Faraday rotation we are making use of. We established two different implementations of Faraday imaging: Dark field Faraday imaging (DFFI) and dual port Faraday imaging (DPFI). The corresponding setups are reviewed in section 5.2. The performance of DPFI, which is currently employed

\(^1\)As it will be discussed later, there are no perfectly non-destructive measurements. Less destructiveness will be always accompanied by a smaller signal.
5. Faraday Imaging

in the experiment, is investigated in more detail in section 5.3.

5.1. Dispersive light-matter interactions

The interaction between a single atom and a single light mode is described under the dipole approximation by the interaction Hamiltonian:

\[ \hat{H}_I = -\hat{d} \hat{E}, \]  

(5.1)

with the complex electric field and dipole operators \( \hat{E} \) and \( \hat{d} \), respectively. In the off-resonant, dispersive regime, the polarizability tensor \( \hat{\alpha} \) can be introduced to describe the light-matter interactions. Hereby, the excited state populations are neglected and adiabatically eliminated. As shown in detail in [125], \( \hat{\alpha} \) can be expanded into a series of irreducible spherical tensors \( \hat{\alpha}_{f,f'} = \hat{\alpha}^{(0)} \oplus \hat{\alpha}^{(1)} \oplus \hat{\alpha}^{(2)} \). This leads to an effective interaction Hamiltonian consisting of three terms, a scalar (rank 0), a vectorial (rank 1) and a tensorial (rank 2) term:

\[ \hat{H}_{I,\text{eff}} = \hat{H}_I^{(\text{scal})} + \hat{H}_I^{(\text{vec})} + \hat{H}_I^{(\text{ten})}. \]  

(5.2)

The polarizabilities \( \alpha^{(k)} \), which are related to the strength of the interaction, depend on the coupling strengths \( \alpha^{f',(k)}_f \) and effective detunings \( \Delta_{f,f'} \) between the different involved transitions \( f, f' \), where the same sign convention is used as in chapter 3.3.1. They have the general form:

\[ \alpha^{(k)} = \alpha_0 \sum_{f,f'} \alpha^{f',(k)}_f \frac{\Delta_{f,f'}}{\Delta_{f,f'}^2 + \Gamma^2 / 4}, \]  

(5.3)

with the classical polarizability \( \alpha_0 = 3\epsilon_0 \hbar \Gamma \lambda^3 / 8\pi^2 \) for the general transition with wavelength \( \lambda \) and natural linewidth \( \Gamma \). The geometry and state dependent physics is contained in the individual coupling factors \( \alpha^{f',(k)}_f \) which are described by Wigner 6-\( j \) symbols [125, 126].

It turns out, that the influence of the tensorial part of the interaction \( \hat{H}_I^{(\text{ten})} \) can be neglected as the corresponding \( \alpha^{(2)} \to 0 \) tends to zero for the large (blue) detunings with respect to the \( F = 2 \to F' = 3 \) transition we are using in our experiments. The remaining scalar and vectorial contributions in equation (5.2)
5.1. Dispersive light-matter interactions

can be written as

\[ \hat{H}_I^{(\text{scal})} = \frac{1}{3} g \alpha^{(0)} \hat{N}_{\text{at}} \hat{N}_{\text{ph}}, \quad (5.4) \]

\[ \hat{H}_I^{(\text{vec})} = \frac{1}{2} g \alpha^{(1)} \hat{F}_z \left( \hat{N}_+ - \hat{N}_- \right), \quad (5.5) \]

where the number operators for atoms \( \hat{N}_{\text{at}} \), photons \( \hat{N}_{\text{ph}} \) and for right/left circular polarized photons \( \hat{N}_+/\hat{N}_- \) are introduced. \( \hat{F}_z \) is the collective atomic angular momentum operator for the \( z \) component defined by the propagation direction of the light and \( g = \omega/2\epsilon_0 V \) is the field factor connected to the interaction volume \( V \). In our case of \( ^{87}\text{Rb} \) atoms in the \( F = 2 \) ground state manifold, the polarizabilities for the D2 line take the form:

\[ \alpha^{(0)} = \alpha_0 \left( -\frac{1}{10} D_{21} - \frac{1}{2} D_{22} - \frac{7}{5} D_{23} \right) = \alpha_0 \chi^{(0)}(\Delta), \quad (5.6) \]

\[ \alpha^{(1)} = \alpha_0 \left( \frac{1}{20} D_{21} + \frac{1}{12} D_{22} - \frac{7}{15} D_{23} \right) = \alpha_0 \chi^{(1)}(\Delta). \quad (5.7) \]

The effective detunings for the different involved transitions is contained in the \( D_{f,f'} = \Delta_{f,f'} / \Delta_{f,f'}^{1/4} \) which simplifies to \( D_{f,f'} \approx \frac{1}{\Delta_{f,f'}^{1/4}} \) in the case of large detunings, which is the well-known scaling. The dependency of \( \chi^{(0)} \) and \( \chi^{(1)} \) on the detuning is indicated by a general detuning \( \Delta \).

In the limit of large atom numbers and for spin-polarized atoms in the same internal state with individual average \( z \)-projection \( \langle \hat{f}_z \rangle \), the collective angular momentum can be treated classically which yields \( \hat{F}_z = N_{\text{at}} \langle \hat{f}_z \rangle \). This allows one to introduce the spatial density \( n(r) = N_{\text{at}} / V \) in equations (5.4) and (5.5). Now the circular polarization photon states form an eigenbasis for both Hamiltonians and the corresponding eigenenergies \( E_+/E_- \) can be directly read off. This enables one to calculate the introduced phase-shift on the light mediated through the light-matter interaction.

For linearly polarized light which can be written as an equal superposition of right and left circular polarization, this leads to an overall scalar phase shift of \( \theta_S = \int 1/2(E_+ + E_-)dt / \hbar \), due to the scalar part of the interaction in (5.4) which is commonly detected in PCI. The vectorial part in (5.5) introduces a differential phase shift between the two circular components \( \theta_F = \int 1/2(E_+ - E_-)dt / \hbar \), which effectively leads to a rotation of the polarization plane of the light about the angle \( \theta_F \), the Faraday angle. The integration over the time
can be substituted by a spatial integration along the propagation direction of the light, which yields, together with equations (5.5) and (5.7), the detectable Faraday angle

$$\theta_F = \frac{3\Gamma \lambda^2}{16\pi} \langle \hat{f}_z \rangle \chi^{(1)}(\Delta) \int n(r)dz = c(\Delta) \int n(r)dz.$$

For simplicity all constants and detuning dependent factors are contained in $c(\Delta)$. The integral on the right-hand side in equation (5.8) represents the column density of the probed atom cloud. Given a fixed projection of the atomic spins $\langle \hat{f}_z \rangle$ (in our case of atoms prepared in $|F = 2, m_F = 2\rangle$, $\langle \hat{f}_z \rangle = 2$), $\theta_F$ is directly proportional to the column density. Due to the dependency on the spin projection, Faraday imaging does not only allow one to probe density distributions, but can also be used to monitor and map out magnetic fields, as will be shown later in chapter 7.

5.2. Experimental realization

In the following section, the optical setups and methods used to extract Faraday rotation angles from the probe light beam are reviewed. Section 5.2.1 deals with our original DFFI setup used to record the data presented in chapter 6. For current and future experiments, a new setup was developed which allows for homodyne detection of the probe light in a dual port scheme. It is presented in section 5.2.2. A short comparison of the two methods is drawn in 5.2.3.

5.2.1. Dark field Faraday imaging

The advantage of this imaging technique lays in the very simple setup which is depicted in figure 5.1. In our case, linearly polarized light, which is blue detuned by 1–2 GHz with respect to the $F = 2 \rightarrow F' = 3$ imaging transition, illuminates a cloud of cold $^{87}$Rb atoms. The atoms are spin-polarized along the propagation direction of the imaging light with the help of a small magnetic guidance field. For analysing the polarization rotation of the light induced by the dispersive light-matter interaction as described in previous section, only a polarizing beamsplitter cube (PBS) is needed. For DFFI, the initial polarization axis is adjusted such, that all the non-rotated light is reflected by the
5.2. Experimental realization

Figure 5.1.: Principle of non-destructive Faraday imaging. Light-atom interactions lead to a turn of the polarization axis of the linearly polarized light. The turning angle is proportional to the column density of the atom cloud. A PBS which reflects the non-turned part of the light separates the two polarization components. The turned part is recorded with a detector. Taken from [38].

PBS. Commonly the imaging technique is therefore often referred to as dark field Faraday imaging (DFFI). The rotated part of the light transmitted by the PBS therefore contains all the information about the atom cloud.

This light is collimated by an objective with effective NA = 0.2 and a focal length of $f = 22$ mm. After the PBS, an image is created with a 500 mm achromat and recorded with an ANDOR iXON ULTRA camera. This camera is equipped with an electron multiplying charge-coupled device (EMCCD), which allows images to be taken at high gain factors, effectively suppressing read-out noise, and yet supporting high frame rates in the kHz range. The whole imaging system has a total magnification of around 23 and an effective resolution of 3–4 µm. The setup is described in more detail in [39].

The signal $I(\theta_F)$ that is acquired in DFFI, scales as $\sin^2 \theta_F$. However, due to the imperfection of the PBS the finite cube suppression $CS := I(0)/I(\pi/2)$, defined as the ratio between minimal and maximal transmission at $\theta_F = 0$ and $\pi/2$ has to be taken into account. For the measured signal this yields

$$I(\theta_F) = I_0 \frac{\sin^2 \theta_F + CS \cos^2 \theta_F}{1 + CS},$$

(5.9)

where $I_0$ is the initial light intensity incident on the atoms. For our setup, the CS is spatially dependent and ranges from $1 \cdot 10^{-3}–1 \cdot 10^{-4}$ and is therefore mapped out with the camera. For determining the initial $I_0$, one can make
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Figure 5.2.: The DPFI setup (a) and some sample images of probe beam patterns created by the DMD (b)-(d). More details are given in the text.

use of the finite CS and take background images without atomic signal corresponding to $I(0)$. This allows one to eliminate $I_0$ in equation 5.9 and one can extract the Faraday rotation angle:

$$S_{DFFI}(\theta_F) = \sin^2 \theta_F = \left( \frac{I(\theta_F)}{I(0)} - 1 \right) \left( \frac{\text{CS}}{1 - \text{CS}} \right).$$  \hspace{0.5cm} (5.10)

Due to quite a high CS, our signals for $I(0)$ are usually close to the electronic noise of the camera and hence too low to obtain a proper calibration of the incident intensity. We therefore guided an attenuated portion of the non-rotated light onto our camera which could be used to rescale reference pictures accordingly.

5.2.2. Dual port Faraday imaging

The basic setup for DPFI is very similar to the one of DFFI. The only difference is that the second port of the PBS in figure 5.1 is now also recorded by the camera albeit in a different region of the CCD. In order to maximize the sensitivity, the initial polarization of the probing light is rotated by 45° such that non-rotated light is split into equal amounts and a balanced detection is achieved. The disadvantage of this scheme is that the two independent arms after the PBS towards the camera have to be set up in exactly the same way
5.2. Experimental realization

including identical imaging lenses placed at the same relative distances. This is necessary for achieving the same magnifications for the images of the two ports on the camera and hence being able to precisely balance the detection.

To circumvent these problems, we use a Wollaston prism as the polarization sensitive element (FOCTEK, custom made). The two distinct polarization components (horizontal and vertical) leave the prism with a small separation angle of \( \approx 0.3^\circ \) for \( \lambda = 780 \text{ nm} \) in our case. This enables us to use the same optical elements for both ports which simplifies the alignment process tremendously. Additionally, a common-mode rejection of the effect of vibrating optical elements is achieved. A simplified sketch of the setup is shown in figure 5.2 (a). The setup for preparing the probe light which includes a DMD for shaping the beam’s spatial intensity profile, is presented in chapter 2.3.3. The collimated probe beam from the imaging objective is rotated by a half wave plate into the 45° measurement basis which is mounted in a precision rotation mount (THORLABS PRM1/M). The Wollaston prism splits the probe beam into its horizontal (H) and vertical (V) polarization components. A first \( f = 300 \text{ mm} \) achromat creates an image in an intermediate plane. Here, parts of the beams can be masked with a razor blade, which can also be used to effectively protect parts of the camera’s CCD from stray light. For our applications so far, the non-illuminated areas defined through the DMD were dark enough and no stray light could be detected with the camera. A pair of achromats with \( f = 125 \text{ mm} \) and 400 mm projects the intermediate image onto the same camera as that was used for DFFI. The overall magnification of the imaging system is \( M = 29 \) which was obtained through a cross-calibration with absorption imaging. One is referred to \([39]\) and chapter 2.3.1 for a more detailed characterization of the objectives’ performance.

In order to simplify the realization of multi-image sequences of variable length, each of the camera exposures is triggering a micro controller board (ARDUINO UNO) which then sends synchronized pulses of variable length to the probe light controlling AOM \([43]\).

The intensity of the two signals in the two ports recorded by the camera are

\[
I_{\text{DPFI}}^{(H,V)}(\theta_F) = I_0 \frac{1 \pm \sin(2\theta_F)}{2}. \tag{5.11}
\]

In contrast to DFFI, \( I_0 \) can be simply obtained by forming the sum of the two components \( I_{\text{DPFI}}^{(H,V)} \), which yields for the calculation of the balanced Faraday
5. Faraday Imaging

As another advantage compared to DFFI, we also gain information about the sign of the rotation angle. The calculation is in principle done pixel per pixel. Depending on the level of spatial structure information needed, the images can be also divided into grids of larger cells containing multiple pixels before processing. If this is known in advance, the subgrouping should already be implemented as hardware binning on the camera due to a better signal to noise performance [43]. Two general conditions have to be met for optimum DPFI imaging: Firstly, the intensities of the two beams \( I^{(H,V)} \) has to be well balanced. In our case, the balancing is adjusted with the half wave plate in the precision mount in calibration runs. Small imbalances can be corrected in the post processing of the data by taking reference pictures without atoms. Secondly, the images of the different ports have to be well centred with respect to each other, before further processing according to equation (5.12) is done. To facilitate the correct centring, we can make use of the DMD by creating some small scale light patterns. Some samples of patterns are shown in figure 5.2 (b)-(d) (only the image of a single port is depicted). They can be used to numerically optimize the relative shift of the patches by minimizing the standard deviation of the balanced signal. Both calibration procedures – the centring and balancing – are discussed in more detail in [43].

The final procedure for obtaining Faraday images is shown in figure 5.3. Although carefully balanced and centred, there is still a residual pattern visible in the processed atom pictures. The origin is not entirely clear, but as it arises after subtracting the contributions of the two ports from each other, this indicates a non-uniform splitting of the two polarization components, be it due to imperfections in the Wollaston prism or already in the initial beam. For the moment, we therefore subtract an averaged reference picture to get a flat background (see figure 5.3 (e)).

### 5.2.3. Comparison and limitations of both methods

At first sight, the sensitivity of DPFI for small signals seems to be enhanced compared to DFFI, as the signal scales in the former case linearly (cf. equation (5.10)) whereas in the latter case it scales only quadratically with the angle signal

\[
S_{DPFI}(\theta_F) = \sin(2\theta_F) = \frac{I_{DPFI}^{(H)} - I_{DPFI}^{(V)}}{I_{DPFI}^{(H)} + I_{DPFI}^{(V)}}. \tag{5.12}
\]
5.2. Experimental realization

Figure 5.3.: Procedure for extracting the Faraday angle in DPFI. The raw images (a) and (b) are treated according to equation (5.12). Due to a residual pattern in the images additional reference images (c) are recorded and subtracted from the images with atoms (d) resulting in the final Faraday image (e).

(cf. (5.12)). However, the contributing shot-noise in DFFI only depends on the signal, whereas in DPFI due to the balanced illumination a constant shot-noise level is present which has to be taken into account even for no Faraday rotation. A more detailed noise analysis for the different dispersive imaging techniques is given in [38], with the result that apart from the influence of different polarizabilities involved in the formation of the signal, all imaging methods show the same signal-to-noise ratio (SNR) for small angles.

As is the case for the dispersive interaction forming a dipole potential, both methods are subjected to off-resonant scattering which either leads to losses from the trap due to heating or to Raman type scattering into different internal states less sensitive or dark to the probe. Both effects result in a decrease of the Faraday signal. It turns out that both the achievable SNR and the off-resonant scattering rate $\Gamma_{sc}$ scale similarly $\propto I_0/\Delta^2$. All measurements have therefore a finite destructivity and one has to always consider the trade-off between destructivity and SNR.

Off-resonant scattering of imaging light in the case of DFFI is discussed in [39]. A detailed calculation about light scattering induced atom losses due to transfers to untrapped states for $^{87}$Rb atoms in a magnetic trap is presented
in [28]. The influence of the probing onto the BEC transition and a more detailed discussion about the precision of DFFI in our experiment can be found in chapter 6.2.

Due to the scalar part of the polarizability there can be common mode diffraction effects leading to a distortion of the acquired image. As they are common mode (i.e. independent of the polarization), they can be cancelled by the differential imaging scheme of DPFI [27]. However, we neither observed the effect in DPFI nor in DFFI. Probably in our regime of detunings and the use of the D2 line instead of the D1 line for our measurements, the magnitudes of the polarizabilities are in general small enough, so that the effect is negligible. Similarly, we did not observe the even smaller differential diffraction effects which would show up in the balanced images of DPFI.

5.3. Performance of DPFI

In the following section, the performance of our DPFI imaging setup is characterized. Section 5.3.1 discusses the noise response of the system and the limitations coming with it, whereas in 5.3.2 the locality of the probe beam generated by the DMD setup is investigated. The section is concluded with some comments on the destructivity of the measurements. This characterization is still under investigation. For the measurements presented later in chapter it is, however, sufficient.

5.3.1. The detection limit of the setup

We use for Faraday imaging an ANDOR iXON 897, which is a CCD camera with an optional electron multiplying (EM) register enabling two general modes of how photons are converted to digital counts. In both modes, the number of photons \( N_{\text{ph}} \) impinging on the CCD chip are converted through the photo electric effect to a certain amount of photo electrons \( N_e \). The conversion factor is given by the quantum efficiency

\[
\eta = \frac{N_e}{N_{\text{ph}}}. \tag{5.13}
\]

\( N_{\text{ph}} \) and \( N_e \) obey the same statistics, i.e. for a coherent state of light they are Poisson distributed and \( \text{Var}(N_e) = \eta \text{Var}(N_{\text{ph}}) \). In the first operation mode, the conventional mode, these photo electrons are then converted pixel by pixel
5.3. Performance of DPFI

through an analog to digital (A/D) converter to digital counts. The gain $G$ relates the number of digital counts $N_c$ to $N_e$ and through equation (5.13) to $N_{ph}$:

$$N_c = G N_e = G \eta N_{ph}. \quad (5.14)$$

In many data sheets instead of the gain, the sensitivity $g = 1/G$ is often given. In the second mode, the EM mode, before digitization the electrons are sent through the EM register where high pixel shift voltages generate a controlled avalanche of secondary electrons to amplify low signals by a fixed factor, the EM gain. For $N_c$ one therefore yields

$$N_c = EM G \eta N_{ph}. \quad (5.15)$$

The relative advantages for both modes become apparent when taking technical noise sources into account and looking at the SNR. The main technical noise sources for an (EM)CCD camera are typically the following:

- **Read-out noise $\delta_{\text{read-out}}$:** it occurs in every read-out process in the A/D converter and is therefore dependent on the gain settings of the converter.

- **Noise due to dark current $\delta_{\text{dark}}$:** thermally induced electrons that accumulate over time. Can be decreased by cooling the camera. For short exposure times its contribution is negligible, however it is amplified for high EM gains.

- **Clock induced charges $\delta_{\text{CIC}}$:** through pixel shift induced charges. As above they are usually negligible, but might be amplified through EM gain.

- **Due to the stochasticity of the avalanche process in the EM register, the noise through it is enhanced by a factor $F = \sqrt{2}$, when using EM gain.**

Considering all these independent noise sources with the variance of the measured photo electron signal $\text{Var}(N_e)$, the variance in camera counts using the EM register is:

$$\text{Var}(N_c) = \delta_{\text{read-out}}^2 + F^2 EM^2 G^2 (\delta_{\text{dark}}^2 + \delta_{\text{CIC}}^2 + \text{Var}(N_e))$$
$$= \delta_{\text{read-out}}^2 + F^2 EM^2 G^2 (\delta_{\text{dark}}^2 + \delta_{\text{CIC}}^2 + \eta N_{ph}), \quad (5.16)$$
where in the second line an illumination with a coherent state of light with Poissonian statistics is assumed. The factor $F^2 = EM^2 = 1$, if the conventional gain register is used. The SNR := $N_c/\sqrt{\text{Var}(N_c)}$ therefore reads:

$$\text{SNR} = \frac{\eta N_{ph}}{\sqrt{(\delta_{\text{dark}}/EM)^2 + F^2 G^2 (\delta_{\text{read-out}}^2 + \delta_{\text{CIC}}^2 + \eta N_{ph})}}.$$  \hspace{1cm} (5.17)

As can be seen from (5.17), the use of EM gain effectively lowers the contribution of read-out noise. For signals $\gg \delta_{\text{read-out}}$, all technical noise terms can be neglected and the best achievable SNR is related to the shot noise limit, yielding $\sqrt{\eta N_{ph}}$ and $\sqrt{\eta N_{ph}}/2$ for the conventional mode and EM mode, respectively. For intense signals, the conventional mode is therefore advantageous, as it does not suffer from the additional stochastic noise factor $\sqrt{2}$. In the limit of low photon numbers the EM mode is preferred.

An investigation of the noise characteristics and the detection limits of our camera in DPFI was performed. Therefore, images with no atoms were recorded at multiple different illumination levels choosing an illuminated area that resembles the size of a typical atom cloud. Two cases are analysed: First evaluating the unnormalized balanced signal $I_{\text{bal}} = I^{(H)} - I^{(V)}$ in units of photon numbers with $I^{(H,V)}$ given in equation (5.11) (see figure 5.4(a)) and second calculating the Faraday angles according to equation (5.12) (figure 5.4(a)). In both cases the variances are investigated as a function of total incident photon number per pixel $I^{(H)} + I^{(V)}$.

The variances and SNR of equations (5.16) and (5.17) and the corresponding limits hold when looking at the unnormalized balanced signal. Applying error propagation to (5.12) in the limit of small angles, the shot noise limited variances in the Faraday signal turn out to be

$$\text{Var}(S_{\text{DPFI}}(\theta_F))_{\text{SN, conv}} = \frac{1}{\eta N_{ph}} \quad \text{and} \quad \text{Var}(S_{\text{DPFI}}(\theta_F))_{\text{SN, EM}} = \frac{2}{\eta N_{ph}}, \hspace{1cm} (5.18)$$

for the conventional and the EM mode.

For the analysis, a preferably imaging artefacts free region of interest (ROI) is chosen. In this ROI, each pixel is treated individually and statistics are obtained over the whole data set of one illumination level. The corresponding shot noise limits are plotted as black dashed and dotted lines. Three cases are investigated: using the conventional gain register, applying an EM gain of 5 and
5.3. Performance of DPFI

![Graphs showing performance](image)

**Figure 5.4.** Detection limits for DPFI using different camera settings. (a) shows the variance of the balanced signal in units of photons as a function of the mean incident photon number per pixel. The solid lines are quadratic fits to the data weighted by the variance. (b) is a zoom into the region above 100 photons/px and displaying the detection limit converted to angles. The black dashed and dotted lines show the bounds for shot-noise limited performance.

EM gain of 20. In figure 5.4(a), polynomial fits to second order are applied. It can be seen that we are close to being detection limited when using the conventional gain register above 600 photons/px. For photon numbers above 1 \cdot 10^5, a quadratic contribution due to technical noise is observed. We believe that this additional noise arises from imaging artefacts due to the imperfect beam. For the data set with EM = 5, a close to diffraction limited probe light spot showing almost no artefacts instead of the larger illuminated area was chosen with the DMD. As a consequence, the scatter of the data is reduced. In all cases the investigated ROIs are comparable.

Using the EM register, the enhanced noise due to the stochasticity in the process of generating secondary electrons is visible. In general, we are between a factor of 1.25 and 1.5 above the expected theoretical limit. The use of low EM gains $EM < 10$ seem not to be beneficial at all, as the noise level is always above the one of the conventional gain register. Figure 5.4(b) shows the detection limit in terms of angles and is a zoom into the shot noise limited regime.

For detecting low signals the data can be binned across multiple pixels.
However, special care has to be taken for the analysis. The response of different pixels across the CCD chip varies slightly which leads to an additional artificial noise term referred to as flat field noise. It scales quadratically with the number of counts $N_c$ but has always the same spatial profile. If binning is inevitable, it therefore can be circumvented by careful characterization of the effect or by subtracting two images at the same illumination from each other before the analysis. In these cases the binning should be additionally implemented as hardware binning, as charges on pixels which are chosen to be binned are collected before the read-out process. In this way the contribution from read-out noise is only acquired once. So far, these effects were negligible for our experiments, but have to be taken into account in the future when going to experiments where we are interested in the noise as variable itself.

5.3.2. Locality of the probe and destructivity

In another experiment, the locality of the probe beam created and shaped by the DMD was investigated. As the beam is intended to be used to probe small clouds, for example in an array of micro traps, where only some of them are probed, such as monitoring the local magnetic field, it is necessary that this happens only in a spatially confined area without disturbing or destroying neighbouring clouds. In order to check the locality, a cold cloud of atoms is loaded from the CDT in the science chamber into the dimple (for the general sequence cf. chapter 3.3.3). Afterwards, the CDT is switched off and following a short hold time, two time-wise separated, local DPFI probes which are diffraction limited in size are applied. The position of the first probe with respect to the dimple position is scanned in one dimension from experimental realization to experimental realization, whereas the second probe is always applied at the position of the dimple and the signal sum of the Faraday angle evaluated. The presence of the first probe leads to a partial atom loss from the dimple and hence can be observed as a decrease of signal in the second probe. In a second comparing run, no first probe was applied. The result of both scans is presented in figure 5.5.

The presence of the first probe pulse leads to a decrease of the signal of about 30%. A Gaussian fit was applied to the loss signal yielding a $1/e^2$ diameter of $(7.8 \pm 0.6) \mu m$, which is in agreement with other independent measurements of the obtainable spot size of the double objective system. The result can rather be seen as an upper bound as the signal is probably represented by a
Figure 5.5.: Investigation of the locality of the probe. Two DPFI images are taken in a row, where in the first one the position of probe is scanned. In the second picture, the probe is applied at the position of the atoms and the measured sum of the signal of the second image is plotted. The solid line represents a Gaussian fit to the drop in signal. The dashed lines are guide to the eyes.

The general destructivity of the probe in terms of atom loss in a thermal cloud of atoms held in the CDT was investigated for different detunings and probe pulse lengths which is detailed in [43]. We observed the expected behaviour that the loss is related to the off-resonant scattering rate $\Gamma_{sc} \propto I_0/\Delta^2$. For our typical detuning of 1.1 GHz to the blue of the $F = 2 \rightarrow F' = 3$ transition, we lose half of the Faraday signal within 14 pulses using durations of 2 $\mu$s length. Increasing the duration to 20 $\mu$s, only about 6 pulses can be applied before half the signal is lost. The non-linearity in the two results illustrates the complexity of the processes involved like rethermalization, possible Raman type scatterings to different internal atomic states and an increased scattering convolution of dimple size and probe beam size. More importantly, when the first probe is applied far away from the dimple position, no difference can be observed compared to the case when no first probe beam is applied at all. It can therefore be concluded that the probe is truly local at the diffraction limit and no other stray light effects are present.
5. Faraday Imaging

rate due to density effects close to condensation \cite{129}, which requires further characterization under more controlled conditions, resolving for example also the temperature of the probed cloud.

It has to be noted that the number of DPFI pulses that can be applied before the signal is lost, depends strongly on the chosen experimental conditions and decreases drastically if only atoms trapped in the dimple are probed. Although it can be related, the destructivity in terms of atom losses must not be confused with the destructivity of a certain quantum state in the language of non-demolition measurements.
The following chapter presents an extensive study of different applications of DFFI, introduced in chapter 5. It is based to great extent on a manuscript which is currently in the peer-review process. A preprint version of the paper is available [130]. Certain sections are identical.

As already pointed out in the introduction to this thesis, the field of quantum simulations has faced tremendous progress [3, 6], since it was first suggested by R. Feynman in 1982 [4]. The constant advances led to the need for more and more accurate experiments. Many of the state-of-the art quantum simulators aim to pin down the boundaries between discrete phases of many-body systems, such as Ising spin transition points [27, 131] or magnetic phases [132], with the highest possible accuracy. Current simulations focus on providing tight experimental bounds for benchmarking theoretical models, such as finite temperature bosonic superfluids in optical lattices [25]. In section 6.1, a first experiment demonstrates, how dispersive Faraday probes that were applied during the evaporation process of an atomic cloud to a BEC can be used as a benchmark tool in order to bin the subsequently acquired data. In our specific case, this additional information allows for high-resolution mapping of the non-linear parameter dependence on the critical point in the transition from a thermal cloud to a BEC, although there are significant fluctuations of the experiment from sequence to sequence.

In section 6.2 the dispersive probe itself is used to extract condensate fractions of condensed clouds in-situ. The procedure is described and a detailed comparison with results obtained from equivalent absorption images in time-of-
Figure 6.1.: (a) Experimental setup showing crossed dipole trap (red) at a wavelength of 1064 nm and the imaging lens. Detuned light (blue) is used to dispersively probe the BEC phase transition. A dimple potential (orange arrow) at a wavelength of 912 nm allows for repeated phase transition crossings. (b) Evaporation scheme illustrating the dispersive “benchmark” pulse at \( P_0 \) (yellow) and the probing around the phase transition (red) realized either with destructive absorption imaging or with multiple dispersive probe pulses. The inset schematically shows the critical curve describing the power, \( P_c \), at which the transition to a BEC occurs, as a function of the dispersively measured phase space density, \( \text{PSD}_0 \), measured at \( P_0 \). The arrows illustrate the paths of individual experimental realizations for different \( \text{PSD}_0 \).

flight is presented. This allows for the single-shot determination of the critical point using multiple dispersive probe pulses as well as the quantification of the deterministic shift of the phase transition due to probe induced heating.

Finally in section 6.3, several consecutive phase transitions are probed using the periodic addition of a focused ‘dimple’ potential in a single experimental realization. The chapter is summarized with an outlook.

### 6.1. Faraday imaging as dispersive benchmark probe

The first experiments presented in the following took place in the cube chamber (cf. chapter 2.2). The setup was slightly different compared to today. A simplified version is shown in figure 6.1(a). The atoms are prepared and loaded into
the dipole trap as described in chapter 2.4. For these experiments the WCDT configuration was used, where the focus of the longitudinal beam was displaced such that the effective waists of both beams were roughly 70 µm at the crossing point of the beams and hence a symmetric trap was obtained (see also chapter 3.3.3). Forced evaporative cooling was performed by lowering the power in each beam from an initial power 5.5 W in four linear ramps to final values between 900 and 600 mW, at which stage partially condensed clouds with condensate fractions \( \frac{N_0}{N} \approx 0–0.6 \) and total atom numbers \( N \approx 2.1–0.8 \times 10^6 \) were produced.

The cloud is probed non-destructively, by using our DFFI setup (cf. chapter 5.2.1). Here, the probing light was blue detuned by \( \Delta = 1.5 \) GHz from the \( |F = 2 \rangle \rightarrow |F' = 3 \rangle \) transition and propagated along the same axis as the transverse dipole beam. Intensities of 10 mW/cm\(^2\) are used, corresponding to around 400 photons/(µm\(^2\)µs). A small bias magnetic field of around 1 G along the light propagation direction preserved the atomic sample’s spin polarization. For these experiments, a single objective (effective NA 0.2, focal length \( f = 22 \) mm, infinitely corrected for imaging at 780 nm, cf. chapter 5.2.1 and see [39] for details) was attached to the cube chamber and used for high resolved, non-destructive imaging of the atom cloud.

In each realization of the experiment, a 20 µs dispersive benchmark (DB) light pulse probes the thermal cloud at a trapping power of 1.1 W (see figure 6.1). The total atom number and temperature is extracted from this in-situ image: typical values are around \( 3 \times 10^6 \) atoms at 1 µK giving a reference phase space density \( \text{PSD}_o \approx 0.25 \). The influence of this pulse is at the 1% level in terms of heating and even lower in terms of atom losses. This minimally destructive probing was chosen to allow for the exploration of the maximal possible range of condensate fractions.

Thus, a reference PSD is known for each run of the experiment before the phase transition occurs. Assuming that the subsequent evaporation efficiency is deterministic, the critical dipole trap power, \( P_c \), at which the phase transition is crossed can be predicted. A high PSD\(_o\) will lead to efficient evaporation and thereby a phase transition at high optical power. If PSD\(_o\) is reduced, \( P_c \) reduces until a BEC is no longer obtained below a certain PSD\(_o\). The dependence of \( P_c \) on PSD\(_o\) defines a critical curve illustrated in the inset of figure 6.1(b). In the following we use this example to demonstrate the benefits of dispersive probing for the determination of phase diagrams in the presence of environmentally induced fluctuations.
Figure 6.2.: BEC growth curves from absorption imaging data for different PSD bins. The inset shows the corresponding histogram of PSDs round 1.1 W. The red and yellow shaded areas depict the two selected data groups for the respective growth curves.

In the absence of the DB probe, PSD\(_{o}\) is not known in a particular realization. The critical curve can, however, be determined approximately by assuming slowly varying experimental conditions such that stability can be assumed about at least two consecutive realizations of the experiment. In that case, two sequences of measurements are repeatedly interchanged. In one of them the evaporation sequence is interrupted early to determine PSD\(_{o}\), the other one is the normal evaporative cooling sequence to a BEC. Nonetheless, the fact that the particular PSD\(_{o}\) is not known for the latter sequence introduces fundamental limitations for obtaining low error bars by averaging over several realizations as shown below.

In this first set of experiments, absorption images were acquired at nine different points during the last linear evaporation ramp between 900 and 600 mW (cf. figure 6.1). The predictive power of our DB method is quantified by binning different experimental runs in which PSD\(_{o}\) is approximately equal. To limit the systematic error due to inaccurate analysis of absorption images at small condensate fractions, we redefine the critical power, \(P_c\), as the point at which the BEC fraction reaches 10%. In order to determine \(P_c\) in each bin,
6.1. Faraday imaging as dispersive benchmark probe

Figure 6.3.: Experimental determination of the critical curve corresponding to a section of the inset of figure 6.1(b). Each blue point is obtained from 4 individual evaporation series. The horizontal error bars correspond to the standard error of the PSD in each bin. The yellow point is the result of evaluating the whole data set without binning. The dashed red box is ±2 standard deviations of the whole dataset and highlights the skewedness of the data. The inset highlights (PSD₀ = 0.256–0.26, P𝑐 = 750.4–753.3 mW) the area around the point representing the whole dataset and shows its systematic shift compared to a quadratic fit to the binned data.

the condensate fraction is fitted with the heuristically motivated function

\[ \frac{N_0}{N} = \frac{\alpha (P - \beta)}{1 - \exp(\gamma (P - \beta))}, \]  

(6.1)

where \( P \) denotes the dipole beam powers at which the cloud is released for absorption imaging. \( \alpha, \beta \) and \( \gamma \) are fitting parameters, with \( \beta \) being related to the transition point. For illustration, we first apply this procedure to two groups of data points (see inset of figure 6.2). As expected, a clear systematic shift between the two groups is seen in figure 6.2.

In the following we examine more systematically how much information about the shape of the critical curve can be extracted. Therefore, the data is divided into 21 bins corresponding to 21 different PSD₀. For each bin the following analysis is performed: Full data sets are selected randomly and fits
with the BEC growth function are performed. This is repeated until all available data of the bin under investigation was used. Out of the obtained set of fit results, the mean value and the standard error of $P_c$ can be extracted.

The result as a function of $\text{PSD}_o$ is displayed in figure 6.3 and a fit with a quadratic model $^1$ is applied. From the confidence bounds of the fit (1σ), we obtain the standard error on the estimated $P_c$. Comparing the confidence bounds of the quadratic fit with the result of averaging all the data (orange point in the inset of figure 6.3), both methods give similar uncertainties at this particular point. This demonstrates the first main result: the DB-method gives a significant increase in the obtained information about the critical curve without a noticeable decrease in precision.

Moreover, indiscriminately averaging all data points assumes a linear response over the entire range. This is not valid as shown by the fit. The inset of figure 6.3 demonstrates clearly that the error bounds from simple averaging do not represent the true bounds. This illustrates another key point of these findings: in the case of even slightly non-linear dependence on the control variable the precision does not improve with averaging of more data samples as is conventionally assumed. On the contrary, it leads to inaccuracy and misleading error bounds.

This effect can be quantified and understood based on the following argument, which is valid for any measurement with a noisy control parameter. Assuming a control variable $x$ that is fluctuating around its mean by some amount $x'$, each realization of an experimental outcome following some function $f(x)$ will be measured at slightly different values $x = \langle x \rangle \pm x'$. The mean measurement result $f_{\text{meas}} = \langle f(x) \rangle$ will simply be the convolution of the true response function with the distribution of the control variable. In the case of a normally distributed fluctuating control variable with variance $\sigma_x^2$ and assuming that the fluctuations are sufficiently small such that $f(x)$ can be approximated by its Taylor expansion to second order, one finds a relatively simple differential equation showing the introduced systematic shift, $f_{\text{meas}} = f(x) + \frac{1}{2}f''(x)\sigma_x^2$.

A more general calculation leads to the same result even for non-Gaussian distributions of control values $^2$. Since we assume a quadratic fit to our measured data the offset to the ‘true’ result can easily be found. The whole curve

$^1$Since the variation in $\text{PSD}_o$ is primarily due to fluctuations in atom number, rather than temperature, the finite curvature is due to the decrease in evaporation efficiency for samples with few atoms and thus a slower collision and re-thermalization rate.

$^2$Klaus Mølmer, private communication
is shifted by $\Delta = -1/2f''_{\text{meas}} \sigma_x^2$. In the case of averaging all data this shift is $\Delta = 0.25 \text{mW}$, while the binning results in a negligible shift of $5.1 \mu\text{W}$ for the whole curve.

In general, if there is no \textit{a priori} knowledge of the shape of the critical curve, a conservative estimate would have to employ the standard deviation of the full data set illustrated by the dashed box in figure 6.3. This highlights the usefulness of the benchmark pulses for the sensitivity-enhancement of general quantum simulation experiments. In this particular case, the benchmark method yields a prediction precision improvement of a factor of 10, when comparing the confidence bounds of the fit to the binned data to the otherwise necessary conservative estimate.

Given the predictive power of the benchmarking technique, it is interesting to explore the limits of this approach. Therefore, we compare the variance of absorption measurements in the vicinity of the BEC transition to the conditional variance given the knowledge of the benchmarking probe. This is achieved by evaluating the correlations between the benchmark pulse and absorption measurements taken at various final trap depths. A measurement of peak angle during a benchmark pulse and the total atom number inferred from absorption imaging is show in figure 6.4(a). We define the noise reduction factor (NRF) for the variables $x, y$ as the ratio between the original and reduced variances:

$$NRF = \frac{\sigma_y^2}{\sigma_{y,\text{red}}^2}$$

$$\sigma_{y,\text{red}}^2 = \frac{1}{N-1} \sum_{i=1}^{N} [y_i - \bar{y}(x_i)]^2,$$

where $\bar{y}(x)$ is determined by simple linear regression. Stronger correlations result in higher values of NRF.

The associated NRFs are shown in figure 6.4(b). Slightly before the transition point, we obtain a reduction in variance by a factor 14 indicating a strong correlation between the two variables. After the transition point, however, the reduction decreases significantly and ends at roughly 2 for the data points at the lowest final evaporation power, 600 mW. This drastic drop in predictability is attributed to a combination of small classical fluctuations of the trap bottom and quantum noise arising due to the stochastic nature of the condensate formation \[133\]. The drop demonstrates that \textit{single} benchmark measurements cannot give detailed information on the stochastic dynamics in...
6. Measurement-enhanced determination of BEC phase diagrams

Figure 6.4.: (a) Correlation plots between the peak rotation angle from the dispersive benchmark probing and the total atom number from absorption imaging for various final trap depths (from top to bottom: higher to lower final evaporation power). (b) The extracted NRFs are shown for the different final evaporation powers.

the region around the BEC transition. Nonetheless, the simplicity of this technique, makes it a powerful tool.

6.2. Non-destructive probing of the BEC transition

Having established a first useful application of dispersive probing as a benchmark tool supporting another measurement, we now will use the probe itself for the actual measurement. As before, the transition of a thermal cloud to a BEC is investigated. In contrast to absorption imaging in time-of-flight, the non-destructive nature of the dispersive probe allows us to take multiple images of the same atom cloud across the transition. That requires to probe the cloud in-situ. In section 6.2.1, first the procedure of extracting BEC fractions of Faraday images which recorded the cloud in-situ is described together with a discussion about the quality of the outcomes by comparing single Faraday probes with equivalent absorption images. Afterwards, the experimental results are dis-
6.2. Non-destructive probing of the BEC transition
cussed in section 6.2.2, detailing the influence of the probe on the measurement
result itself. The presented experiment is similar to a setting demonstrated in
[123], where the phase transition to a BEC was non-destructively measured
using phase contrast imaging. Here, the transition was induced by applying a
sudden quench, whereas we follow it during continuous evaporation.

6.2.1. Fitting technique

In-situ modelling of the cloud

In contrast to time-of-flight absorption imaging where densities are low and
the mean-field interaction between the atoms is negligible, the interaction’s
influence on the cloud’s density profile has to be considered in the case of in-trap
images. Instead of taking the complete influence of mean-field interactions on
the condensed and thermal part in a self-consistent way into account, we employ
the so-called semi-ideal model [57–60], which only includes the interactions of
the condensed part on the thermal part. This leads to a modified effective
trapping potential for the thermal part (cf. chapter 3.1.1):

$$V_{\text{eff}}(r) = V_{\text{ext}}(r) + 2gn_c(r).$$

In the Thomas-Fermi approximation, the condensed and thermal cloud’s den-
sity distributions are then given by

$$n_c(r) = \frac{1}{g}(\mu - V_{\text{ext}}(r)) \quad \text{and}$$
$$n_{\text{th}}(r) = \frac{1}{\lambda_T^3}g^{3/2}\left(e^{\beta(\mu-V_{\text{eff}}(r))}\right).$$

where $\lambda_T$ is the thermal de-Broglie wavelength. Usually, the chemical poten-
tial $\mu$ and the number of condensed atoms $N_0$ have to be determined self-
consistently. We circumvent this by applying a further simplification and ex-
panding the condensed fraction into a series depending on the reduced chemical
potential $\tilde{\mu} = \frac{\mu}{k_BT}$ [59]. Truncation after the first non-trivial order and solving
6. Measurement-enhanced determination of BEC phase diagrams

for the condensate fraction yields:

\[
\frac{N_0}{N} = 1 - \left( \frac{T}{T_c} \right)^3 - \eta \frac{\zeta(2)}{\zeta(3)} \left( \frac{T}{T_c} \right)^2 \left[ 1 - \left( \frac{T}{T_c} \right)^3 \right]^{2/5}, \quad \text{with} \quad (6.7)
\]

\[
T_c = \frac{\hbar \omega N^{1/3}}{k_B \zeta(3)^{1/3}}, \quad (6.8)
\]

the critical temperature for the onset of condensation for a non-interacting, ideal gas. \( \zeta \) is the Riemann zeta function and \( \eta \) a dimensionless scaling parameter described by \( \eta = \frac{\mu T}{k_B T} = 0 \).

Given a cloud with atom number \( N \) at temperature \( T \), we are now able to calculate the expected condensate fraction and the density distributions for thermal and condensed part, respectively.

To extract information about atom number, temperature and BEC fraction from the in-situ Faraday probe, each image is first converted from EMCCD counts into a rotation angle (cf. chapter 5.2.1). The resulting image is then azimuthally averaged around the point with the highest rotation angle (see figure 6.5(a)). This radial profile is then compared to multiple profiles obtained for realizations of the ‘semi-ideal model’ with atom clouds of different \( N \) and \( T \) and a given trap geometry. As the calculation of a model profile was time consuming, profiles were generated once for a grid of parameter values. The best agreeing parameters were found by minimizing the resulting \( \chi^2 \) in this grid as illustrated in figure 6.5(b). The best agreeing pair of \( N \) and \( T \) is found. The obtained peak densities are within 20% of those extracted from images.

A problem arises for clouds which are just above the condensation threshold. Note that due to interactions this corresponds to temperatures \( T < T_c \). Hence, there is a range of temperatures, where a BEC is predicted for a non-interacting gas, but not according to our implementation of the semi-ideal model. In order to close this gap, we linearly interpolate the density distributions between the cases of just having a small interacting BEC and a non-interacting thermal cloud in this temperature range.

The 1\( \sigma \) confidence bounds to \( N \) and \( T \) are found by looking for the ellipsoid in the landscape of figure 6.5(b), where \( \chi^2 \) takes the value \( \chi^2_{\text{min}} + 1 \) [134]. Using this method, the errors for the condensate fraction for a single fit are on the level of 1%. Nonetheless, we observe a large systematic deviation if comparing these results to equivalent ones obtained through absorption images in the same experimental realizations. These deviations cannot be explained only
6.2. Non-destructive probing of the BEC transition

Figure 6.5.: Illustration of the fitting procedure. (a) The raw image (shown in the inset) is azimuthally averaged around the point with the highest rotation angle resulting in the blue data points. The error bars depict the standard error obtained in the averaging process. A least-squares fit to the semi-ideal model was performed. The red dashed line shows the thermal atom distribution of the best fitting realization, the blue dashed line the corresponding sum of thermal and condensed atom distribution. (b) Contour plot of the value of $\chi^2$ as a function of atom number $N$ and temperature $T$, the effective parameters of the semi-ideal model for the same data shown in (a). The minimal value of $\chi^2$ corresponds to the best fit. A star indicates that point. The point marked by a circle is the corresponding absorption imaging result of the same experimental realization.

by a mismatch of peak densities. Therefore, we performed a more careful comparison of Faraday and absorption imaging results.

Accuracy of Faraday images

For this comparison, we use a single non-destructive pulse of varying duration. A range of potential depths across the BEC transition are chosen and the cloud profiles recorded. BEC fractions from the fitted profiles are then compared to those measured for the same potential with absorption imaging. This analysis (see figure 6.6(a)) shows a systematic shift towards higher BEC fractions in the in-situ images and best agreement for long durations.
6. Measurement-enhanced determination of BEC phase diagrams

![Figure 6.6:](image)

**Figure 6.6:** (a) Single Faraday probing of the BEC transition for different imaging pulse durations (colored symbols) compared to results of conventional absorption imaging (black squares). The systematic overestimation of BEC fractions obtained in Faraday imaging is visible. (b) Simulation of the influence of shot noise on the fit results in Faraday imaging as a function of mean number of detected photons. For the modelling, realistic pairs of total atom number $N$ and temperature $T$ were chosen corresponding to three different BEC fractions (from top to bottom $\frac{N_0}{N} = 0.50$, 0.35 and 0.23). The lines are guides to the eyes. A clear systematic shift of the BEC fraction is observed for photon numbers below $10^3$. The yellow bar indicates the photon numbers used in our experiments.

To understand the origin of this trend we model the effect of finite photon numbers on the determination of BEC fractions. For a given photon flux and cloud profile we generate sample images which include the Poissonian statistics of the imaging light and apply the fitting procedure described previously. The mean and standard deviation in BEC fraction are then extracted from the series of images and repeated for different photon fluxes. The results shown in figure 6.6(b) demonstrate a clear, systematic shift consistent with observations. This effect persists for all the three simulated condensate fractions. The range of photon fluxes used in the experiment (yellow bar) indicate that agreement between in-situ and absorption imaging cannot be expected for higher condensate fractions.
6.2. Non-destructive probing of the BEC transition

Figure 6.7: (a) Single-shot probing of the BEC formation: For each of the three pulse durations 8 repetitions are shown. The pink line, derived from absorption images, indicates where 10% BEC fraction is crossed (thickness indicates 2σ of confidence bounds). (b) BEC fraction measured via absorption imaging after multiple pulses of Faraday probing have interacted with the sample. Data and corresponding fits are shown for different durations. In the inset the 10% transition point is plotted vs. the Faraday pulse duration. It corresponds to the same line that is marked in (a).

6.2.2. Experimental results

Based on the detailed understanding of the individual in-situ images presented in previous section, we repeatedly probe the sample in a single experimental run. The ultimate goal is accurate probing of the transition at the quantum limit. This is at present limited by classical fluctuations in the probing light and the trap geometry as well as insufficient imaging resolution, which will be topics of future research. Here we address an important first step: the modification of the phase transition due to the heating caused by the probing. This is important for all future studies due to the intrinsic trade-off between destructivity and signal-to-noise ratio.

During the final stages of the evaporation process up to 6 pulses are taken at 220 ms intervals for various pulse durations. After the final pulse in the sequence, an absorption image is taken and the condensed fraction extracted. The results of this in-situ probing of the condensation process using three different probe durations are shown in figure 6.7(a). A clear shift of the BEC transition point towards lower dipole powers and shallower traps is observed for increasing pulse durations due to heating.
6. Measurement-enhanced determination of BEC phase diagrams

This shift is confirmed independently by the absorption images taken at the end of the sequence. The extracted BEC fractions from these are shown in figure 6.7(b). The data corresponding to the same Faraday probing pulse duration are fitted with the same heuristically motivated function (6.1) introduced already earlier in this chapter. Note that even with the high signal-to-noise of absorption imaging there are still difficulties associated with correctly measuring small BEC fractions. This is illustrated by the fact that the dataset corresponding to the highest probe duration is not pulled as much towards zero around the transition as the other series. Therefore as in section 6.1, we use the point where 10% condensate fraction is reached as the critical power. The errors are defined from the 1σ confidence bounds. The resulting BEC transition points together with the corresponding pulse durations are fitted with a linear function (see inset of figure 6.7(b)). This fit function and its 2σ confidence bounds are displayed as the red curve in figure 6.7(a) which shows good agreement with the Faraday data.

The observed heating rate is somewhat higher than simple theoretical estimates would suggest, which we attribute both to classical variations in probing conditions and to multiple scattering events due to the high spatial density. Our results reinforce the need to consider heating effects in future studies of the quantum effects of dispersive probing [129, 135].

6.3. Multiple crossing of the BEC phase transition

Finally, we show steps towards the mapping of the entire BEC phase transition diagram in a single run, using the periodic addition of a focused ‘dimple’ potential [63]. The dimple allows for the repeated conservative crossing between thermal cloud and BEC as explained in more detail in chapter 3.2.2. Studies to date have only employed this effect in combination with individual, destructive absorption images: BEC production has been characterized [65, 136, 137] and in one pioneering experiment multiple crossings have been explored [64]. In our experiments the evaporation is stopped before condensation is reached. At this point the cloud serves as a reservoir in which a 912 nm wavelength laser beam focused through the imaging objective to a waist of 7 µm produces an additional potential (cf. also figure 6.1). In our experiments this potential is periodically cycled from a depth of 0 to 1.12 µK at a rate of 10 Hz.

We first characterize the cloud evolution with absorption imaging. Maximum
6.3. Multiple crossing of the BEC phase transition

![Graph](image)

**Figure 6.8.** (a) Reservoir atom number (blue squares) and its temperature (orange dots) as measured with absorption imaging at the beginning of the corresponding dimple cycle. The curves are fits to the data: double exponential for the atom number and linear for the temperature. (b) BEC atoms measured via absorption imaging (blue circles) as the dimple trap depth was cycled. Red squares show the peak rotation angle from multiple dispersive images of 2 µs duration during the first two cycles; error bars correspond to 1σ over several repetitions. (c) Phase-diagram of the atoms in the dimple potential. Above the black line a BEC should be observed (blue part of the sinusoidally cycling curve), while below one expects only thermal atoms (orange part). The blue dots indicate absorption measurements containing a BEC, while the orange squares indicate purely thermal clouds.

BEC atom numbers of around $1.5 \cdot 10^5$ atoms, 10% of the total atom number, are measured while after 30 cycles the condensed cloud of $2 \cdot 10^4$ atoms is still distinguishable from the thermal reservoir, see blue data points in figure 6.8(b). The condensation dynamics in the dimple involves three parts of the sample (reservoir, BEC and thermal fractions in the dimple). The total atom number decay and the reservoir temperature increase are used to estimate a critical dimple depth:

After a variable number of full dimple cycles we measure the conditions of the cloud that serves as a reservoir. The extracted number of atoms and temperature are shown in figure 6.8(a). The atom number is well fitted by a double exponential function: the initial decay corresponds to a $1/e$ lifetime of about 0.2 s and is followed by a slower decay with a lifetime of about 1.5 s. The temperature data is fitted with a linear function, resulting in an initial temperature of $(0.37 \pm 0.01)$ µK and a heating rate of $(9 \pm 5)$ nK/s. The uncertainties on the fit results are quite broad, which is mainly due to difficulties in measuring
temperatures accurately on clouds very close to the BEC transition.

From the temperature and the trapping frequencies, the rms radius of the reservoir cloud is calculated to be between 13 and 19 µm. On the contrary, the dimple beam has a waist of only 7 µm. Because of the different sizes we can distinguish the increase in density due to the atoms trapped in the dimple potential using in-situ dispersive Faraday probing. In this way, we can estimate that 50% of the reservoir atoms concentrate in the volume defined by the dimple, when this is at its deepest value of 1.12 µK. We use only this portion of the atoms in the following calculations.

From the fitted atom number and the time-varying trapping frequencies of the cycling dimple potential we calculate the critical temperature for the non-interacting gas, $T_c$ (cf. equation (6.8)), as a function of time; we compare this with the fitted temperature. When interaction is included, equation (6.7) gives non-zero condensate fractions when $T/T_c < 0.88$ for our experimental conditions.

Thus, we can place the experimental results into two groups on a phase diagram, see figure 6.8(c). The cycling dimple potential draws a line, which crosses the critical boundary corresponding to the threshold value for $T/T_c$. We find that even this simple model can predict the presence/absence of a BEC reasonably well during the first 20 cycles. The inaccuracy in the measurement of the reservoir temperature and non-adiabatic dimple loading dynamics reduce the effectiveness of the model to make predictions on the final cycles.

Finally, we perform the first dispersive probing of multiple quantum phase transitions by introducing a varying number of weak Faraday pulses within each dimple cycle at different pulse durations. In figure 6.8(b) red squares indicate the maximal angle of Faraday rotation obtained in each probing pulse. The non-zero signal in the absence of the dimple is caused by the background polarisation rotation of the reservoir. The data clearly demonstrate our ability to track the loading into the dimple over two realizations of the BEC production. As before, we study the destructivity of the probing by interrupting the probing at various points and determine the number of condensed atoms using absorption imaging. For one single weak probe applied repeatedly at maximum dimple power at least 10 dimple cycles can be monitored, whereas only a few transitions can be monitored when seven pulses per cycle are applied. This

\[ \text{At present, imaging resolution does not allow to distinguish BEC and thermal parts in the dimple.} \]
means that both correlation measurements between, as well as high-resolution monitoring of several quantum phase transitions in a single shot should be feasible.

6.4. Outlook

In summary, we have demonstrated the predictive power of non-destructively probing a cold cloud before it crosses the BEC phase transition. This provides enhanced information about the non-linear dependence of a phase transition on a control parameter. Importantly we demonstrate that using conventional methods the precision of a quantum simulation does not necessarily increase with additional repetitions unlike our method. We envision similar techniques to be applicable for any quantum many-body phase transition in which classical fluctuations play a role. The information from a series of benchmark probes analyzed in real-time by a FPGA can be used to give direct feedback in the evaporation process and stabilize the atom numbers down to the shot noise level \[138\]. Simulations in collaboration with National Instruments were initiated to investigate the possibility of real-time extractions of BEC fractions on a FPGA using machine learning approaches.

In addition, we have demonstrated repeated probing of both one and several phase transitions in a single shot. This enables the single-realization mapping of full phase diagrams as well as the investigation of stochastic effects such as increased fluctuations at the phase transition \[133\] and possible non-Markovian dynamics \[139\] for multiple crossings.
In this chapter, our first attempts at using ultracold clouds of $^{87}$Rb as a magnetometer are presented. Measurements are performed using DPFI, introduced in chapter 5. The chapter opens with a short overview on optical magnetometry utilizing light-matter interactions with atom clouds and vapours. Afterwards, two types of magnetometers and their implementation in our experiment are demonstrated: Firstly in section 7.2, a vector magnetometer that infers direction and magnitude of a magnetic field through measuring the macroscopic spin orientation of the atom cloud aligned along the magnetic field. Secondly in section 7.3, a magnetometer, in which we measure the Larmor precession frequency of the atoms, which directly gives a precise, absolute magnetic field value without relying on other calibrations. Both can be used to monitor locally the magnetic fields at the position of the atoms. Suggestions for possible future applications are given in section 7.4.

### 7.1. State of the art

This discussion focuses on optical magnetometry, for other similar performing technologies like superconducting quantum interference devices (SQUIDs) one is referred to respective literature [140]. The field of optical magnetometry was born almost 50 years ago, using an optically pumped, heated $^{87}$Rb vapour cell to exploit the ground state Hanle effect for precise measurements of magnetic fields [141, 142]. Simply speaking, in this experiment an initial
unpolarized atom cloud is partially polarized by the optical pumping beam. Under the influence of an external magnetic field, the polarization starts to undergo Larmor precession (see section 7.3 for a more detailed discussion on Larmor precession). This in turn affects the optical response of the atom cloud to the pumping beam which is measured with a photo detector and appears as sharp resonance features in the detected signal as a function of the external magnetic field. Using a modulated probing scheme with lock-in detection, weak DC magnetic fields of $10^{-9} \text{G} = 100 \text{fT}$ could be measured, a remarkable feature for the time. However, integration times of about 10 s were used and the vapour cells had a large volume, i.e. the spatial resolution was very low.

Most optical magnetometers still work according to the above general scheme: A polarized atom cloud is affected by an external magnetic field and the effect on the atoms' polarization is measured by a probe beam. Hereby, the means for achieving an initial polarization can be of different kind as well as the implementation and detection of the probe beam. The maximal achievable single shot precision of an optical magnetometer has two contributions: There is an atomic shot noise term and a photon shot noise term due to the measurement process. We will describe both terms.

Considering a single atom, the minimal magnetic field that can be detected is limited by the finite measurement time $\tau$ leading to a finite resolvable angle in Larmor precession (cf. equation (7.10) below)

$$\delta B_{sa} = \frac{\hbar}{g_F \mu_B \tau},$$

(7.1)

where $g_F$ is the Landé g-factor and $\mu_B$ the Bohr magneton. With a cloud of $N$ atoms obeying the Poisson statistics, one gains a factor of $1/\sqrt{N}$ and repeating the measurement continuously in a total measurement time $T$, enhances the precision by another factor of $\sqrt{T/\tau}$. The final atomic shot noise contribution therefore reads as

$$\delta B_a = \frac{\hbar}{g_F \mu_B} \frac{1}{\sqrt{NT\tau}}.$$  

(7.2)

In some literature this is referred to as single shot sensitivity although it is defined in units of $[\text{T}]$ (cf. Eq. (7.2)). Consequently, it is often not used consistently. We will call the uncertainty of a single magnetic field measurement single shot precision. The measurement time independent quantity given in units $[\text{T}/\sqrt{\text{Hz}}]$, we will call single shot sensitivity instead.
The same arguments can be used for the limitation due to photonic shot noise in the measurement. Using the DPFI scheme, as we do in our magnetic field measurements, and having SNR = 1, the finite resolvable Faraday angle is given by (cf. equation (5.18) in chapter 5.3.1)

\[ \delta \theta_F = \frac{1}{2\sqrt{\eta N_{ph}}} \]  

(7.3)

One yields the most sensitive magnetic field measurement, when probing the atom cloud while the collective spin is pointing perpendicular to the axis of probing. Employing equation (5.8), the resulting Faraday angle integrated over the whole atom cloud for a small displacement \( \delta \phi_L \) of the collective spin reads

\[ \theta_{F,\text{min}} = c(\Delta) N \delta \phi_L. \]  

(7.4)

Setting \( \delta \theta_F = \theta_{F,\text{min}} \) and solving for \( \delta \phi_L \) finally yields for the single shot precision of magnetic field measurements due to photonic shot noise:

\[ \delta B_{\text{ph}} = \frac{\hbar}{g_F \mu_B} \frac{\delta \phi_L}{\sqrt{T\tau}} = \frac{\hbar}{g_F \mu_B} \frac{1}{2c(\Delta)N} \frac{1}{\sqrt{\eta N_{ph} T\tau}}. \]  

(7.5)

Both noise contributions from (7.3) and (7.5) are uncorrelated and have to be summed in quadrature for the overall single shot precision of a magnetometer

\[ \delta B = \sqrt{\delta B_{a}^2 + \delta B_{\text{ph}}^2}. \]  

(7.6)

Usually \( \tau \) is related to the finite spin-coherence time which limits the sensitivity especially in vapour cell magnetometers like the one discussed above, in which the thermal atom cloud is subjected to spin-relaxation due to collisions with the walls or spin-exchanging collisions with other atoms [144]. At high densities the latter effect is the dominating factor for limiting the sensitivity. It is however possible to suppress this effect in a spin-exchange relaxation-free (SERF) magnetometer [146] and yield higher sensitivities of up to 0.54 fT/\( \sqrt{\text{Hz}} \) at a probe volume of only 0.3 cm\(^3\).

Ultracold atoms do not suffer from these problems. Wall collisions are absent and decoherence due to spin-changing collisions can be excluded by preparing
atoms in a single stretched state. Additionally, they can be confined with the help of optical dipole traps in extremely small volumes which allows for high spatial resolution. The finite coherence time is now dominated by differential AC stark shifts and magnetic field inhomogeneities combined with the diffusion of atoms in the trap. Applying optical lattices which yields even tighter spatial confinement allows one to circumvent this effect.

This resulted in a few experiments, involving cold thermal clouds as well as BECs in variable trap configurations. One of the pioneering experiments, using condensed clouds, was performed in the group of D. Stamper-Kurn in 2007 [147]. Here, the BEC was held in a single beam dipole trap and a spatially resolved Larmor precession signal was recorded with phase contrast imaging. The measured magnetic field was artificially induced by another off-resonant, circular polarized beam. By analysing an integrated signal a sensitivity of \( (\delta B \sqrt{T}) = 8.3 \text{ pT/}\sqrt{\text{Hz}} \) could be achieved, where \( T \) is the complete cycle time of the experiment including preparation and detection. This is worse than the sensitivity of the above mentioned SERF magnetometer, but the probe volume was only 200 \( \mu \text{m}^3 \). In other similar performing experiments, an ultracold, non-condensed cloud – again trapped in a single beam – was probed by state-selective absorption imaging in a Ramsey type of experiment [148]. They were able to extract magnetic fields with high temporal and spatial resolution of 120 \( \mu \text{s} \) and 50 \( \mu \text{m} \).

All these measurements are limited by the fundamental uncertainty in the collective spin in the form of projection noise. Spin squeezing can be applied to achieve subprojection noise performance, which was successfully demonstrated in the groups of M. Mitchell [149] and M. Oberthaler [150]. In the former case, squeezing was performed on a cold, but thermal cloud yielding a sensitivity of 54.6 \( \text{pT/}\sqrt{\text{Hz}} \) in a trapping volume of 3.7 \( \cdot 10^6 \mu \text{m}^3 \). In the latter experiment, squeezing was performed on a BEC in an optical lattice. Due to the smaller atom number the sensitivity was only 1.86 \( \text{nT/}\sqrt{\text{Hz}} \), however at a much smaller probe volume of only 90 \( \mu \text{m}^3 \).

The examples of optical magnetometers mentioned represent only a small portion. An exhaustive list, together with an overview of the individual magnetometer’s performances and comparisons of them to other available technologies, is presented in the supplementary material of [150].
7.2. Vector magnetometer

The basic working principle of our vector magnetometer is the measurement of the macroscopic spin orientation of the atom cloud, which, in case of careful preparation, is parallel to the net magnetic field vector. This can be accomplished using Faraday imaging [38], as the Faraday rotation angle $\theta_F$ of the light’s polarization axis is sensitive to the projection of the cloud’s collective total angular momentum onto the propagation direction of the light, defined as the $z$-direction (see equation (5.8) in chapter 5.1). Making use of DPFI enables one to detect the sign of the Faraday angle and hence the projection. To find the exact orientation of the angular momentum vector, external fields have to be scanned and therefore dynamic control over them is necessary.

In the typical operation scheme, the offset field $B_z$ along the $z$-direction is swept while the cloud is probed with Faraday imaging. The transition from positive to negative Faraday angles is observed resulting in a signal of the form

$$S(B_z) = -A \frac{B_z - B_{z,0}}{\sqrt{(B_z - B_{z,0})^2 + B_t^2}} e^{-\kappa B_z}, \quad (7.7)$$

where $A$ is a normalization constant, $B_{z,0}$ and $B_t = \sqrt{B_{x,0}^2 + B_{y,0}^2}$ are the unknown field components in the $z$ and transversal directions respectively. The factor $e^{-\kappa B_z}$ accounts for the loss of signal during the measurement, assuming a constant sweep rate of $B_z$. Consequently, $B_{z,0}$ can be directly read off from the zero crossing, whereas $B_t$ determines the slope of the signal in the vicinity of the zero crossing. In any case, for absolute magnetic field values a precise knowledge of the calibration of the scanning coils is necessary.

The specific setup of the offset field coils in our science chamber was briefly mentioned in chapter 2.3.1. As depicted in figure 7.2, besides coils in the vertical direction, there are offset coils controlling the fields in the horizontal plane denoted as sets A and B. They are mounted on the diagonal viewports of the chamber, i.e. they define a coordinate system which is rotated by 45° with respect to the one defined by the probe beam. Nonetheless, we prefer to use the axis of the probe beam for the description of our measurements. In first approximation, by neglecting effects of inhomogeneity due to imperfect placement of the coils, a field along the $z$-direction is therefore created by an equal weighted sum of fields from coil sets A and B. Positive field directions in the respective coil sets are thereby defined along the diagonal from coil 1 to coil
2. Correspondingly, a field along the \(x\)-direction can be defined through the difference. Therefore a new parametrization for the currents \(I_A\) and \(I_B\) driving the coil sets is introduced:

\[
I_{\text{sum}} = kI_A + I_B \quad \text{and} \quad I_{\text{diff}} = I_B - kI_A \quad (7.8)
\]

\[
\Leftrightarrow \quad I_A = \frac{I_{\text{sum}} - I_{\text{diff}}}{2k} \quad \text{and} \quad I_B = \frac{I_{\text{sum}} + I_{\text{diff}}}{2} \quad (7.9)
\]

In order to account for asymmetries due to different placement of the two coil sets, the factor \(k\) is introduced. Following the operation scheme described above, in a standard magnetometer sweep \(I_{\text{sum}}\) is changed to perform a sweep of the magnetic field along the \(z\)-direction.

To find the right value of \(k\), the fields are zeroed in an iterative procedure: In a first step, the transversal fields are minimized by repeatedly operating the magnetometer each time with a slightly different value for \(I_{\text{diff}}\). The best compensating value corresponding to the steepest slope around the magnetometers signal’s zero crossing is chosen. A similar procedure is done with the current through the vertical coil \(I_{\text{vert}}\). In the next step, the magnetometer is operated by sweeping \(I_{\text{diff}}\) instead and keeping \(I_{\text{sum}}\) at a fixed value. Now the parameter \(k\) is changed from sweep to sweep. In the ideal case for the right \(k\), the resulting magnetometer signal is constant as \(I_{\text{diff}}\) should not induce a magnetic field.
Figure 7.2.: Different signal traces obtained with the vector magnetometer. No magnetic field sweeps were performed in the case of the blue and red data points and $B_z$ was kept at a constant value. For the green and yellow points $B_z$ was swept, where transversal fields were not compensated in the former one but in the latter one as can be seen from the complete loss of signal due to depolarization of the atom cloud. The green solid line is a fit according to equation (7.7).

change in the z-direction. With the new best guesses for $I_{\text{diff}}$, $I_{\text{vert}}$ and $k$, the procedure is repeated.

Prior to a magnetometer measurement, a cloud containing approximately $6 \times 10^6$ atoms close to the BEC transition in the CDT of the science chamber (cf. also chapter 3.3.3) is produced. The magnetic offset fields are then ramped within 200 ms to values aligning the collective spin of the atoms either parallel or anti-parallel to the z-direction. This represents the starting conditions for the magnetometer sweep. Thereafter, typically 40 DPFI shots of 2 µs length at a rate of 50 Hz and at our typical detuning are applied (cf. chapter 5.3.2) while $I_{\text{sum}}$ is swept. The integrated signal in a ROI over the atom cloud is evaluated. It turned out that the magnetometer is sensitive to the phase of the mains supply and lead to a jitter effect of up to 5 mG. Therefore, a synchronization of the start of the magnetometer sweep with the mains phase was implemented [43]. The chosen imaging rate of 50 Hz ensures, that probing happens always at the same mains phase.

Four typical magnetometer signal traces are presented in figure 7.2. In case
of the red (blue) data points, \( I_{\text{sum}} \) was not swept and kept at the fixed upper (lower) sweep range corresponding to (anti-)parallel alignment of the collective spin. Therefore, only the decay of the signal during probing is observed. In the other two traces, the magnetometer was operated normally and \( I_{\text{sum}} \) was swept in a range corresponding to the magnetic field in the \( z \)-direction specified on the abscissa (absolute calibration was obtained through cross-calibration with the vertical coil which was calibrated using Larmor frequency measurements as described in the next section). Thereby, the green data points represent a trace with a non-compensated transverse field component. A fit according to equation (7.7) is applied which is displayed as green, solid line yielding a residual transverse field of \((2.60 \pm 0.15) \text{ mG}\). The overlap with the non-swept data points indicates that no loss of atoms through the sweep of the magnetic field was induced. In contrast, the yellow data points show the magnetometer operated at compensated transversal fields. The sweep of the \( z \)-component guides the magnetic field zero through the atom cloud resulting in its depolarization and a complete loss of the Faraday signal.

The fit results presented above show that we are able to reach a single shot precision of \(150 \mu \text{G} = 15 \text{ nT}\) with this relatively simple type of magnetometer. Comparing the time scale of the relative magnetic field change \( \left| \frac{dB}{dt} \right| \) during the sweep to the time scale given by the Larmor frequency \( \omega_L/B = 700 \text{ kHz}/\text{G} \) for \(^{87}\text{Rb}\) in the \( F = 2 \) ground state, cf. next section) and assuming that depolarization occurs when the two time scales are comparable, the loss of polarization corresponds to a compensation on the order of \(0.1 \text{ mG}\) which is in agreement with the fit uncertainties.

Using this magnetometer, stability scans of our magnetic fields can be performed. Ideally the transversal fields are set to non-compensating values in order to be able to record and fit complete magnetometer traces for the best possible precision. Figure 7.3 shows a stability scan over more than an hour, where a single sequence from preparing the atoms until obtaining a result from the magnetometer is about \(30 \text{ s}\) long. The changes of the magnetic fields with respect to the start of the measurement for the \( z \) and the transversal directions are displayed in blue and red, respectively. The shaded areas represent the \( 1 \sigma \) confidence bounds of the fits and the black solid lines are running averages over 10 sequences, i.e. about \(5 \text{ min}\). The overall precision seems to be on the same order of magnitude as in the above example. It can be seen that the stability of \( B_z \) seems to be on average better than \(1 \text{ mG}\), whereas \( B_t \) shows a sudden
7.3. Larmor precession magnetometer

In the Larmor precession magnetometer the precession of an atom cloud’s collective spin about the net magnetic field axis is measured. In a semi-classical picture, the movement of an individual alkali atom’s single outer electron with total angular momentum $\mathbf{F}$ induces a magnetic dipole moment $\mu = \gamma \mathbf{F}$, with the gyromagnetic ration $\gamma = g_F \mu_B / \hbar$ as a proportionality constant, where $g_F$ is the Landé g-factor and $\mu_B$ the Bohr magneton. In an external magnetic field a torque $\mathbf{\tau} = \mathbf{\mu} \times \mathbf{B}$ acts on the atom which leads to a precession of the total angular momentum with the Larmor frequency

$$\omega_L = \frac{g_F \mu_B}{\hbar} B.$$  \hfill (7.10)
For $^{87}\text{Rb}$ atoms in the $F = 2$ ground state, $\omega_L/B$ amounts to 700 kHz/G. A precise measurement of the frequency therefore yields a precise measurement of the magnetic field.

Similar to the vector magnetometer, the collective spin projection along the $z$-direction is measured with DPFI and the precession is observed as a modulation of the light’s polarization (cf. figure 7.1 for the definition of directions). Considering the maximum achievable contrast in cases when the collective spin is pointing (anti-)parallel to the $z$-direction, this method is therefore most sensitive to magnetic fields oriented perpendicular to the imaging direction. For slow magnetic field changes at a relative rate slower than the current Larmor frequency $\left| \frac{dB}{dt} \cdot \frac{1}{B} \right| > \omega_L$, the collective spin can follow the magnetic field and we have the situation of a sweep in the vector magnetometer. In order to initiate a macroscopic, measurable Larmor precession, the relative orientation of the spin and the magnetic field have to be changed non-adiabatically. This can be achieved either by an additional optical pumping beam [146, 151, 152], or by redistributing the occupation of the Zeeman sublevels with RF fields [147]. Both have the effect of changing the orientation of the collective spin.

In our experiment, we change the orientation of the magnetic field itself on a fast time scale in order to initiate Larmor precession. Therefore, the vertical compensation coils are driven by a fast current supply (developed at our institute) which also allows high bandwidth stabilization of the magnetic fields for future use. By pulsing the current, field changes of up to $\approx 11.7 \text{G/ms}$ for the first 100 mG can be induced. Afterwards, the field response is exponential with a transient field changing rate of $\approx 2.2 \text{G/ms}$.

The experiments start out by cooling the atoms in the CDT of the science chamber to temperatures just above the transition point to a BEC. Afterwards, a fraction of the atoms is loaded into the dimple of 200 µW power. The CDT remains on and serves as a reservoir which proves to be beneficial for keeping a good signal throughout the measurement. Using a 100 ms slow magnetic field ramps through the offset coils, the collective spin is aligned along the $z$-direction. The magnetic guiding field in the $z$-direction is reduced to a minimal value at which the cloud retains spin polarization. With a phase-lock on the 50 Hz of the mains supply, the actual magnetometer sequence is initiated: The vertical compensation coil is pulsed for 20 µs inducing a sudden field change of about 600 mG, enough to initiate Larmor precession. In the next step, the field along $z$ is decreased to zero within 10 ms. At the same time, the vertical field is brought to the value chosen for investigation which has to correspond
7.3. Larmor precession magnetometer

Figure 7.4.: Measurements using the Larmor frequency magnetometer. In (a) measurements of the Larmor frequency for two different magnetic fields are shown. The lines represent fits with a damped sinusoidal oscillation. The extracted frequency in the upper panel corresponds to a magnetic field of \((14.41 \pm 0.02)\, \text{mG}\), whereas in the lower panel \((6.31 \pm 0.02)\, \text{mG}\) were measured. (b) Multiple measurements at different currents driving the vertical offset coils allow for an absolute calibration. Each data point is averaged over five repetitions, where the error bars denote the standard deviation of the measurement results.

to Larmor frequencies that are compatible with the frame rates of the EMCCD camera detecting the balanced Faraday signal. Here, we work in a partial frame transfer mode, where the Faraday probe is localized only to the extent of the dimple by setting the DMD appropriately. We record 41 images at a rate of \(44.3\, \text{kHz}\). The acquisition sequence is triggered after another waiting time of \(9.98\, \text{ms}\), the total duration thus corresponds to being in phase again with the 50 Hz from the power line. The observable Larmor frequencies are not limited to the maximal frame rate of the camera, as also the aliasing signal can be analysed and the absolute frequency extracted together with the knowledge of the acquisition rate.

Figure 7.4(a) shows two typical Larmor signals. Damped sinusoidal oscillations are fitted to the data in order to extract the Larmor frequency. Due to the direct linkage to the magnetic field through equation \((7.10)\), this provides a measurement of the magnetic field with a single shot precision of \(20\, \mu\text{G} = 2\, \text{nT}\) according to the \(1\sigma\) confidence bounds of the fits about an order of magnitude

\[
\text{Vertical magnetic field } B_{\text{vert}} (\text{mG})
\]

\[
\text{Vertical offset current } I_{\text{vert}} (\text{mA})
\]
better than the vector magnetometer. Taking into account the whole cycle time of the experiment of about 35 s for this sequence, this corresponds to a sensitivity of \( (\delta B \sqrt{T}) = 11.8 \text{ nT}/\sqrt{\text{Hz}} \). Due to the fixed constant image acquisition rate and the finite uncertainty in the resolvable signal, this sensitivity is constant over different applied magnetic fields in the observed range of 10–140 mG. Improvement could be achieved by distributing the acquisition points unevenly and e.g. getting a better resolution of the signal’s zero crossing after some unobserved periods of Larmor precession. Without prior probing, a loss of the Larmor precession signal is observed approximately 100 ms after its initiation. This can be caused by a dephasing of the coherent spin precession along the probing direction which washes out the measured signal. Dephasing can be induced due to magnetic field inhomogeneities or differential AC stark shifts along the longitudinal axis of the dimple. Comparing to other experiments which also directly observe the Larmor precession signal our single shot precision is comparable \[153\]. In \[152\] a single shot precision of 18 pT is reported which was however obtained by averaging over 30 runs. Both experiments used much larger probe volumes on the order of mm³, whereas in our case our dimple volume in a conservative calculation extends about \(3.6 \cdot 10^3 \mu\text{m}^3\).

As presented in figure 7.4(b), we are now able to obtain an absolute calibration of the vertical offset coils at the position of the atoms using this method. Here, 16 different current settings were applied and the resulting magnetic field in the vertical direction was measured through Larmor frequency measurements. This result can be used to cross calibrate the horizontal coils in measurements with the vector magnetometer described in previous section.

### 7.4. Outlook

As shown above, we can successfully demonstrate the implementations of two different magnetometers and single shot precisions of 15 nT and 2 nT could be achieved for the vector magnetometer and the Larmor precession magnetometer, respectively. Applying successive optical pumping in different directions and measuring the Larmor precession signal, it could be implemented as a vector magnetometer gaining information of all three magnetic field components \[154\]. So far, our magnetometers have been working with only a single dimple, but an extension to use a grid of atoms trapped in micro traps is feasible. Combining the information yielded from multiple traps could allow the
Figure 7.5.: In-situ absorption image of atoms in six micro traps. The outer four traps could be probed using our implementation of local Faraday imaging for tracking e.g. the magnetic field, whereas the inner two represent a coupled double-well system.

extraction of magnetic field gradients or curvatures. Together with fast analysis real time feedback could be applied. Figure 7.5 exemplifies such a scenario. Depicted are atoms trapped in a configuration of six micro traps produced by the AOD through time averaging the potential (see chapter 2.3.2). The two in the centre are closely spaced and could represent a weakly coupled double-well system exhibiting Josephson junction dynamics [33]. The outer four traps could be selectively probed by using local Faraday probing with the DMD (cf. chapter 5.3.2). In a different scenario, one of the traps in the centre itself could be weakly probed to e.g. induce spin squeezing and hence influence the dynamics of the overall system. Due to the versatility of local probing patterns with the DMD and generation of arbitrary potentials with the AOD, a broad variety of experiments are possible with this part of the setup.
Conclusion and Outlook

Within this thesis, a versatile toolbox for a stable and optimized experiments for quantum simulation was presented. Here, Faraday imaging plays a central role. Two main applications were shown. In the first one, the monitoring of atom numbers and temperatures at an earlier stage of the experiment was used as a benchmark tool. It was shown, that the predictive power of these measurements provide enhanced information which can be correlated with the actual measurement of the system under investigation. In a second application, Faraday imaging was utilized to implement two types of ultracold atom magnetometers. Single shot precisions of 15 nT and 2 nT for a vector magnetometer and a Larmor precession magnetometer were reached, respectively. Although the latter one was only sensitive to a single magnetic field component, it could be turned into a vector magnetometer by implementing optical pumping along different axes. Moreover, the possibility of using Faraday imaging for probing phase transitions in a single experimental sequence was demonstrated in the case of the BEC transition. The extension to multiple successive phase transitions would allow the mapping of entire phase diagrams in a single sequence. Both, magnetic field monitoring and atom number measurements, could be implemented for direct processing on a FPGA in a closed loop application. The reliable preparation of atomic clouds with atom numbers at the shot noise level using active feedback was successfully demonstrated [138].

In the second field of investigation, tools for the optimization of different kind of problems were investigated. In a first case, the theoretical problem of the fast one-dimensional single atom transport below the quantum speed limit
was investigated. It was shown, how the detailed understanding of the control landscape can be exploited to develop novel search methods. The concept of the superlandscape as a smooth meta landscape on top of the extremely rugged control landscape was introduced and KRASI as a successful optimization strategy in this landscape established. In the second case, the experimental problem of BEC production in different trap geometries was investigated. Similar techniques were applied to develop an understanding of the control landscape in this problem. The search for novel, optimized solutions under the consideration of the control landscape was complemented by the implementation of a computer-based optimization with the dCRAB algorithm. A simple remote control client based on the same interfacing to the experimental control was developed allowing for human remote controlled optimization. In both cases, computer-based and human-based optimization, new, better sequences for creating BECs were found. In a further step, the human remote controlled experiment was extended and the problem was gamified, in order to engage also non-experts. We could demonstrate that human problem solving strategies are of high value especially when the underlying optimization landscape is slowly changing and subjected to noise. The best solutions obtained in these experiments yielded BECs which were as large as \((2.76 \pm 0.01) \cdot 10^6\) atoms, which is an improvement of more than 40\% compared to our best ‘standard’ strategy of the implementation of a pure hybrid trap.

Currently the experiment is being equipped with the single-site resolution microscope \[39\] which will give access to a large variety of possible experiments. Several of the tools and techniques presented within this thesis will be valuable for our future experiments. Some perspectives are given below.

**Quantum simulation**

Most of the quantum simulations in optical lattices performed so far, were realized without any spatial limitations through other optical potentials and are in that sense open systems. The experiments in the direction of quantum magnetism presented in the introduction to this thesis \[28, 29\] are examples. The addressing and initialization of the spin impurities in these systems was also limited to simple geometries. Our unique system of DMDs (cf. chapter 2.3.4) that enable the creation of arbitrary beam patterns at two different wavelengths for addressing as well as for applying far-off resonant optical potentials, provide versatile possibilities. They will allow for the implementation of novel quan-
tum systems. Recently, it was shown how the creation of isolated subsystems whose dynamics is still described by the Bose-Hubbard Hamiltonian [20] can be used to investigate the subtleties of local entropy [155]. This information can be used to understand the dynamics of local thermalization after the application of a quench [156] (i.e. a sudden, non-adiabatic change of the system Hamiltonian). In this experiment a similar DMD setup in the Fourier plane was used to selectively address a group of atoms.

Using our ability for creating arbitrary far-off resonant potentials, we are not restricted solely to lattice systems. Through Fourier synthesis it is easy to create for example Laguerre-Gaussian beams [50, 157] for forming ring-shaped traps. Non-cubic arrays are also possible in a similar fashion as done in other experiments [158].

**Quantum computation**

The ability of addressing multiple atoms at the same time and the application of localized tweezer potentials allows also for the realization of a large scale quantum computer with single- and two-qubit gates as proposed in [105]. First steps in this direction were successful [159], where the entanglement of two transportable atoms in tweezers is shown. The biggest problem so far, was the reliable initialization of such large scale systems. A bottom-up approach for the construction of such kind of arrays in conjunction with imaging and real-time feedback was demonstrated in [46, 160].

Two-qubit operations were demonstrated already in optical lattices for many atoms in parallel [161, 162]. In a different approach it is proposed to use long period superlattices (i.e. superimposing two lattices of similar wavelengths) to realize single- and two-qubit gate operations [69]. These are implemented by using the effects of site dependent AC stark shifts as well as changing the relative phase and amplitude of the contributing beams. These type of lattices are also planned to be implemented in our experiment. Single-qubit operations using this method were successfully realized in an experiment [163].

**Advanced Faraday probing**

As demonstrated in chapter 5.3.2, we are able to apply localized Faraday probing. This can not only be used to implement local magnetic field monitoring as suggested in chapter 7.4 but also to introduce locally measurement-induced
8. Conclusion and Outlook

Figure 8.1.: (a) Illustration of the phase diagram of the non-destructively probed Mott insulator to superfluid transition in an optical lattice. Up to now it is unclear, how such measurements would influence the phase diagram as a function of the probing strength depicted as a third axis. Adapted from [39]. (b) Screenshot of a possible extended version of the Alice experimental control accessible to everyone.

spin squeezing by repeated measurements. More specifically, one can think of BECs in a weakly coupled double-well system, for example generated by our dimple setup described in chapter 2.3.2. A carefully prepared system could be governed by tunnelling dynamics that can be ascribed to the Josephson effect, similarly as in [33]. It is unclear and would be exciting to investigate what dynamics emerges when probing only one of the two wells non-destructively with local Faraday imaging.

Faraday probing could be also used to identify quantum phases in optical lattices by applying a non-uniform probing of the lattice sites and analysing the variance of the measurement signal. Using this technique, the superfluid to Mott insulator transition or the identification of even more complex quantum phases in superlattices seems viable, as proposed in [164, 165]. Again in this case, it is unclear which dynamics is driven by the measurement induced back-action. This is illustrated in figure 8.1(a), where the superfluid to Mott insulator phase diagram is depicted. Adding a weak non-destructive probe, corresponds to adding another axis to the phase diagram with the probing strength as a parameter. This region is so far an uncharted one.
The ‘democratic’ lab

The successful gamification of the optimization problem of the evaporation process showed the advantages of human problem solving strategies. It would be very interesting to continue it in a similar fashion in the future. There could be two approaches. One is to formulate concrete challenges like the optimization of the BEC production. The second approach could follow the ideas of the *IBM Quantum Experience* [115] or *Quantum in the Cloud* [116] and give the possibility to design whole experimental sequences. A screenshot of a possible user interface that we envision is depicted in figure 8.1(b). A prototype was demonstrated at the National Instruments week [166] in August 2016 in Austin, Texas. In this relation, a transport sequence created in *Quantum moves* [90] was implemented for atoms trapped in a dimple potential created by the AOD (cf. chapter 2.3.2) which then was measured with Faraday imaging. A master’s thesis dealing with this topic is currently under preparation [167]. The established interface within the scope of the dCRAB optimization can be easily used for further scientific collaboration already now.
Appendix
A. Appendix

A.1. $^{87}$Rb level scheme

Figure A.1.: The level scheme for the $^{87}$Rb D2 line and implementation of our cooling scheme. Adapted from [39] and [40].
Bibliography


Bibliography


Bibliography


