Transfer reaction study of neutron rich beryllium isotopes.

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# Contents

## Table of Contents

### 1 Introduction
- 1.1 Hundred years of nuclear physics ........................................ 1
- 1.2 The shell model ............................................................. 2
- 1.3 Deformation of nuclei ....................................................... 5
- 1.4 Clustering in nuclei ........................................................ 5
- 1.5 Halo nuclei ........................................................................ 6

### 2 Neutron rich beryllium isotopes
- 2.1 Beryllium isotopes ............................................................ 9
- 2.2 $^{11}$Be ................................................................. 10
- 2.3 $^{12}$Be ................................................................. 11
- 2.4 $^{10}$Be ........................................................................ 13

### 3 Transfer reaction theory
- 3.1 Transfer reactions ............................................................ 17
- 3.2 Experimental differential cross sections ............................... 18
- 3.3 Theoretical differential cross sections ................................. 20
  - 3.3.1 Optical model .......................................................... 20
  - 3.3.2 Coupled channel calculation ......................................... 22
    - 3.3.2.1 The deformed potential ........................................ 23
    - 3.3.2.2 Cluster model ................................................... 23
  - 3.3.3 Distorted wave born approximation ................................ 24
- 3.4 Spectroscopic factors ......................................................... 26

### 4 The experiment
- 4.1 Performing a transfer reaction experiment ......................... 29
- 4.2 ISOLDE ................................................................. 30
- 4.3 REX-ISOLDE .......................................................... 31
- 4.4 Targets ......................................................................... 32
- 4.5 The detector setup ........................................................... 33
4.5.1 Silicon detectors ................................................. 35
4.5.2 Germanium detectors ......................................... 36
4.6 Simulation .......................................................... 37

5 Calibration .............................................................. 39
  5.1 Signals from the detectors .......................................... 39
  5.2 The laboratory frame ............................................. 40
  5.3 Annular Detectors (AD) ........................................... 40
  5.4 Position Sensitive Detectors (PSD) ............................. 42
  5.5 Pad detectors (E-detectors) ....................................... 43
  5.6 Germanium detectors (MB) ....................................... 44
    5.6.1 Energy calibration .......................................... 45
    5.6.2 Position calibration ......................................... 45
  5.7 MINIBALL efficiency ............................................ 46

6 Beam diagnostic ....................................................... 49
  6.1 Coincidence events ............................................... 49
  6.2 Beam characterisation .......................................... 50
    6.2.1 Beam width and offset .................................... 51
    6.2.2 Divergence and direction of the beam ................... 54
    6.2.3 Combining the offset and the angle measurements .... 56
    6.2.4 Beam energy ................................................ 58
  6.3 Experimental data ................................................. 60
  6.4 Beam intensity .................................................. 63
  6.5 Background from contaminants in the target ............... 65

7 Identifying the various reaction channels ...................... 71
  7.1 How to identify a reaction ...................................... 71
  7.2 Excitation energy spectra from high energy particles .... 71
    7.2.1 Particle identification .................................... 72
    7.2.2 Excitation energy spectra ................................ 74
    7.2.3 Gamma gated spectra ...................................... 76
      7.2.3.1 $^{11}$Be .............................................. 76
      7.2.3.2 $^{12}$Be ............................................ 77
      7.2.3.3 $^{10}$Be ............................................ 80
Introduction

This chapter will give a short introduction to the field of fundamental nuclear physics, starting with the discovery of the nucleus a hundred years ago. The most important parts of the field for this thesis will be described.

1.1 Hundred years of nuclear physics

2011 marked the hundredth year for the discovery of the nucleus by E. Rutherford. During these hundred years a lot has happened in the field of physics including quantum mechanics, Bohr’s atom model that leads to a description of the electron orbits around the nucleus. The building blocks of the nucleus (nucleons) have also been identified as the proton (Z) and neutron (N), with the discovery of the latter in 1932 by J. Chadwick. Later the discovery of quarks opened up for an understanding of the deeper structures of the nucleons. Today we have the standard model, for which indications of the final piece (the Higgs boson) have been seen at the LHC within the last year.

Even though the building blocks of the nucleus have been known for 80 years now, a complete and accurate theory to describe the structure of all nuclei is still to be derived. One of the mayor challenges arises from the large variation in the number of nucleons
from one to more than two hundred. Thus neither ab initio nor many body calculations can be used to describe all nuclei. Ab initio calculations are limited by today’s computer power, hence only the lightest and simplest nuclei and reactions can be described using ab initio calculations. The limit today is around $A = 12$. Simplifications and generalisations have been used to go higher in mass, and various theories have been developed.

The theories are challenged by the constant discovery and production of more and more exotic nuclei. For years, only the natural isotopes were known, and most of the theories have been developed from natural occurring isotopes. It was not until the development of the first radioactive ion beam facility (RIB) in ?? that radioactive nuclei were studied. Later the development of RIB facilities has lead to an increase in the number of known nuclei. Fig. ?? shows the development of the nuclear chart from the early days with only natural occurring isotopes to today, where the drip line is reached for many elements, and heavy nuclei, beyond $Z = 110$, have been produced and studied in laboratories. The study of the more exotic nuclei have shown strong effects caused by the imbalance between neutrons and protons. The binding energies, masses, states and shapes of the nuclei are all affected by the number of the neutrons and protons and the ratio between the two. Theories, that are developed for stable nuclei, breaks down when going into the more exotic regions of the nuclear chart. The theories, therefore have to be modified to describe the more and more exotic nuclei discovered.

### 1.2 The shell model

One of the most successful theories in nuclear physics is the shell model. The principle is taken from atomic physics, where electrons are orbiting a core (the nucleus) in different shells, corresponding to different energies and quantum numbers. The problem in nuclear physics is the lack of a central potential. In atomic physics the main force acting on the electrons is from the nucleus, which is much heavier than the electrons, and can be assumed stationary. The electrons movement can then be described from a central potential caused by the nucleus, and other forces enter as perturbations. In a nucleus the forces acting on nucleons are from other nucleons, with similar masses and nuclear
1.2. The shell model

Charges, hence a central potential description is not as useful as for the atom. Instead the potential used in the calculation has to be a combination of the potentials between the individual nucleons (ab initio). Such calculations are too complicated for most nuclei and simplifications have to be made. In the shell model, a nucleus is divided into core and valence nucleons. The valence nucleons are then filled into the shells arising from a core potential. The spin and parity of the energy levels in the nucleus are thus given by the valence nucleons.

In atomic physics, some number of electrons are more stable than others, corresponding to closed shells. The same effect is seen in the shell model, known as the magic numbers: \( N = 8, 20, 28, 50, 82 \) and \( 126 \). The magic numbers arise from large energy gaps between certain shells. The energies of the different shells are determined by the structure of the core potential. The normal core potential is given by a wood-saxon potential with a spin-orbit term:

\[
V(r) = -\frac{V_0}{1 + e^{-(r-R)/a}} + V_{ls}(r)L \cdot S. \tag{1.1}
\]

This form will lead to large energy gaps above the magic numbers. This theory was already developed in 1949 by Mayer and Jensen. The study of more exotic nuclei has shown that the magic numbers break down when going away from stability and approaching the drip lines. This is caused by the core potentials dependency of the nucleons. The potential given in eq. (1.1) is derived from stable nuclei, but the potential is altered when going away from stability, leading to a movement of the levels, that further leads to a closing and opening of energy gaps breaking the magic numbers, like \( N=20 \) in the island of inversion around \( ^{32}\text{Mg} \) and \( N=8 \) in \( ^{12}\text{Be} \), or the appearance of new magic numbers like \( N=16 \) in \( ^{24}\text{O} \). The determination and understanding of the breaking and emerging of magic numbers is an important part of modern nuclear physics, as it can be used to describe the movement of energy levels across the nuclear chart.
Figure 1.2: A plot of the energy levels dependency on the deformation of the nuclei.
The plot is derived using the model by S. G. Nilsson.
1.3 Deformation of nuclei

One of the effects, that causes the movement of the shells, is the deformation of the nuclei. The potential in eqn. 1.1 is derived for a spherical nucleus, but many nuclei have been shown to have a deformed shapes even in the ground state. Two types of deformations can occur, the prolate and the oblate deformation, fig. 1.3. The deformation of isotopes when going away from stability have been studied for many elements. Especially in the Hg isotopes is the deformation prominent with an alternation between spherical and deformed ground states, see Heyde and Wood [] for more details.

![Figure 1.3: The three shapes, prolate, spherical and oblate, of nuclei.](image)

The effect from the deformation on the shells was derived by S. G. Nilsson in 1955 [ ]. Fig. 1.2 shows the energy levels as a function of the deformation. The changes in energy gaps are clearly seen, leading to changes in the magic numbers.

1.4 Clustering in nuclei

Another important aspect in fundamental nuclear physics is the clustering of nucleons inside a nucleus.

Some nuclei are more energetic favorable and stable than others. The best
known is the α-particle (\(^4\)He), which is much stronger bound than the neighboring nuclei. Theoretical calculations are currently trying to establish, when clustering occurs in nuclei [].

The \(\alpha\)-clustering is quite prominent in the \(N\alpha\)-nuclei (\(^4\)He, \(^8\)Be, \(^{12}\)C etc.). All these nuclei shows clear indications of being clusters of \(\alpha\)-particles. \(^8\)Be is an unbound nucleus, that \(\alpha\)-decays, and the triple \(\alpha\)-process leading to the production of \(^{12}\)C shows the great importance of clustering in nuclear astrophysics. \(^{12}\)C is an example of clusters forming a stable nucleus as well. A nucleus can also contain clusters of different particles like \(^7\)Li (\(^4\)He+\(^3\)H) or \(^9\)Be (\(^4\)He+\(^4\)He+n).

The clustering within a nucleus will lead to a deformation of the nucleus, as the clusters will be separated. Taking \(^8\)Be as an example, the two \(\alpha\)-particles will be moving around each other making a deformed shape, rather than forming a spherical nucleus with 4N and 4Z. Furthermore the clustering can lead to new types of excited states due to the excitation of a cluster rather than a single valence nucleon or collective motions.

### 1.5 Halo nuclei

A special type of cluster nuclei is the halo nuclei. Halo nuclei appear close to the drip lines and are characterised by low binding energies and a large matter radii. Furthermore the nucleus left after the removal of one (or two) nucleon(s) is strongly bound. Three types of halo nuclei have been seen, one-proton halos (\(^2\)H and \(^8\)B), one-neutron halos (\(^2\)H, \(^{11}\)Be, \(^{19}\)C and \(^{31}\)Ne) and two neutron halos (\(^6\)He, \(^{11}\)Li, \(^{14}\)Be and \(^{22}\)C). The low binding energy and the large radius in halo nuclei are interpreted as a clustering containing a core (the strongly bound nucleus) and one or two nucleons orbiting the core at a large distance. The orbiting nucleons are called halo nucleons and the large distance leads to the large matter radius. The large distance also makes it easy to separate the halo nucleon from the core, hence the low binding energy.

Interestingly the known two neutron halo nuclei are all Boronenu nuclei. The total system are bound, but two of the three together are unbound, like \(^{11}\)Li, which is bound, but \(n+n\) and \(^9\)Li+n (\(^{10}\)Li) is both unbound.

Halo structures can also occur in excited states of nuclei, even if the ground
state is not a halo state. An example is the excited states in $^{10}\text{Be}$ described in section 2.4.
Neutron rich beryllium isotopes

The neutron rich beryllium isotopes studied in this thesis will be described. The focus will be on the nuclear structure that will be studied in the experiment.

2.1 Beryllium isotopes

This thesis describes an experimental study of the neutron rich beryllium isotopes. Beryllium (Z=4) is in the low mass region (A ≤ 14 for bound isotopes) of the nuclear chart. \(^{9}\)Be is the only stable beryllium isotope, in this thesis neutron rich beryllium isotopes means \(^{10,11,12}\)Be. The low mass makes ab initio calculations possible for most beryllium isotopes. Furthermore beryllium isotopes plays important roles in both the breaking of magic numbers, clustering and halo structures, described in the previous chapter.

One of the most studied beryllium isotopes is the unstable \(^{8}\)Be which is of great importance in astrophysics. \(^{8}\)Be is a two \(α\) cluster. The ground state is a resonance with a width of \(\Gamma = 5.57\) eV \([\text{I}]\). Hence \(^{8}\)Be plays an important role as an intermediate state in the triple alpha process. The two \(α\)-structure of \(^{8}\)Be is conserved in the more neutron rich nuclei like the stable \(^{9}\)Be and \(^{10}\)Be, which in AMD (Antisymmetrised Molecular Dynamics) calculations have been treated as three and four particle clusters \([\text{I}]\). Going further into the neutron rich side of the beryllium isotopes, the separation of the two \(α\)-particles starts to disappear. Nonetheless, the \(α\)+neutrons clustering is seen both in \(^{11}\)Be and \(^{12}\)Be \([\text{I}]\). The clustering in the beryllium isotopes makes the nuclei deformed, especially \(^{8}\)Be is highly deformed. The deformation of \(^{10}\)Be and \(^{11}\)Be is believed to play an important role in the experiment, section 8.3.2.

The neutron rich beryllium isotopes also shows interesting behavior from a shell model point of view. \(^{11}\)Be is known for the inversion of the \(1/2^+\) and \(1/2^-\) states indicating a mixing of the \(1s0d\) and the \(0p\) shell. A mixing that
leads to the breaking of the N=8 magic number in $^{12}$Be.

$^{11}$Be is the starting point of the experiment and is studied through scattering experiments. $^{10,12}$Be are studied through one neutron transfers from or to $^{11}$Be using a deuterium target.

The three nuclei will be described in more detail in the next sections.

2.2 $^{11}$Be

$^{11}$Be is one of the few known one neutron halo nuclei, section 2.2. The neutron separation energy is only 501 keV and the matter radius is much larger than for the neighboring nuclei ($r_m = 2.91$ fm in $^{11}$Be compared to $r_m = 2.09$ fm for $^{11}$B). The charge radius on the other hand is comparable ($r_c = 2.47$ fm in $^{11}$Be compared to $r_c = 2.48$ fm for $^{11}$B). This is all the characteristics of a halo nucleus. $^{11}$Be is interpreted as a core of $^{10}$Be and a neutron orbiting the core at a relative long distance, fig. 2.1. $^{11}$Be is considered a two body system ($^{10}$Be+n) in this thesis and the detailed description of $^{10}$Be is omitted. The halo structure and the deformation of $^{11}$Be is strongly affecting reactions involving $^{11}$Be. Elastic scattering of $^{11}$Be on heavier nuclei close to the coulomb barrier have been studied and potentials that work for nuclei close to $^{11}$Be fail to reproduce the experimental scattering cross sections of $^{11}$Be. Recent DWBA calculations on a $^{10}$Be(d,p)$^{11}$Be experiment also showed a discrepancy between theory and experiment. An attempt to determine the reaction potentials for halo nuclei is made by A. Bonnacorso et al. They conclude, that a neutron halo leads to a large diffuseness of the imaginary part of the potential.

The mixing of the 1s0d and the 0p shell is another interesting effect seen in $^{11}$Be. Despite the low neutron separation energy, $^{11}$Be have two bound
states a 1/2\(^{+}\) ground state and a 1/2\(^{-}\)-state with an excitation energy of \(E^* = 0.320\) MeV. Furthermore a low lying resonance at \(E^* = 1.78\) MeV is known, fig. 2.2. The particle structure in the two body model of the three states/resonance are:

\[
|^{11}\text{Be}; 1/2^+\rangle = \alpha_+ |^{10}\text{Be}; 0^+_1\rangle |n; s_1/2\rangle + \beta_+ |^{10}\text{Be}; 2^+_1\rangle |n; d_5/2\rangle.
\]

(2.1)

\[
|^{11}\text{Be}; 1/2^-\rangle = \alpha_- |^{10}\text{Be}; 0^+_1\rangle |n; p_{1/2}\rangle + \beta_- |^{10}\text{Be}; 2^+_1\rangle |n; p_{3/2}\rangle.
\]

(2.2)

\[
|^{11}\text{Be}; 5/2^+\rangle = |^{10}\text{Be}; 0^+_1\rangle |n; d_{5/2}\rangle.
\]

(2.3)

Only the simplest configuration is taken for the resonance, but small amounts of other terms is expected as well.

The small energy required to excite the neutron from the s-shell to the p-shell and the fact that the p-shell configuration is placed between the s- and the d-shell configurations indicates a strong mixing of the two shells. The 0\(^p\) and the 1s0d shells are strongly separated close to stability (\(E^*_{1/2^+} - E^*_{1/2^-} = 3.09\) MeV for \(^{13}\text{C}\) and 5.18 MeV for \(^{15}\text{O}\)). The mixing is an example of the movement of the shells when going away from stability. The strong mixing and the low separation energy affect the scattering and transfer reaction as well.

\(^{11}\text{Be}\) is easily excited and a strong coupling between the two bound states is expected, as well as a coupling to the low lying resonance, which has to be taken into account when interpreting the data, section 8.4

### 2.3 \(^{12}\text{Be}\)

The mixing of the 1s0d and the 0\(^p\) shell is proven to appear in \(^{12}\text{Be}\) as well, breaking the N=8 magic number. The mixing was already suggested by F. Barker et al in 1976 [], who studied the beta decay of \(^{12}\text{Be}\). The \(\beta\)-decay
strength was too weak for the ground state to be purely described by a $p^2$ configuration of the two valence neutrons. The theory was supported by the low lying $2^+_1$ excited state, and was finally confirmed in 2003 with the discovery of the low lying $0^+_2$-state ($E^* = 2.2$ MeV) by Shimoura et al [], fig. 2.3. The mixing of the shells have been studied in several break up reactions [] as well as a few transfer reactions []. The first transfer experiment populating $^{12}\text{Be}$ ($^{10}\text{Be}(t,p)^{12}\text{Be}$) by Fortune et al [] were able to populate the ground state, the $2^+_1$- and the $1^-_1$-state. No spectroscopic factors were determined and the spin assignment for the $1^-_1$-state was wrongly suggested to be $0^+$. Furthermore low lying resonances were studied. The second transfer reaction, a low energy $^{11}\text{Be}(d,p)^{12}\text{Be}$ transfer like the one described in this thesis, was also unable to identify the $0^+_2$-state, but spectroscopic factors were determined for the other three bound states []. The experimental spectroscopic factors were not consistent with theoretical ones, and new calculations have caused doubt on the results from the transfer reaction experiment []. Hence the amount of shell mixing in $^{12}\text{Be}$ is still to be determined, and the aim of this experiment is to provide new spectroscopic factors for all bound states including the $0^+_2$-state from a (d,p) reaction.

Furthermore, three body calculations have been performed by C. Romero-
Redondo et al in 2007 [1] successfully describing the bound states of $^{12}$Be. The model predicts a bound $0^-$-state [1], which has never been seen. New improved calculations of the three body model have recently been performed, in order to predict the spin and parity of the low lying resonances [1]. The $0^-$-state is moved up in excitation energy in the calculation, and a $0^-$-resonance just above the 1n-threshold is suggested.

### 2.4 $^{10}$Be

The very long lifetime of $^{10}$Be ($1 \times 10^6$ y) and the possibility of using stable beams to study $^{10}$Be have made the study of $^{10}$Be accessible for many years, and the structure of $^{10}$Be is well known. The $^{11}$Be(d,t)$^{10}$Be channel of the experiment has proven very interesting, both from an experimental and technical point of view and for an understanding of the structure of $^{11}$Be and $^{10}$Be. The ground state of $^{11}$Be has two components, eqn. 2.1, and removing the halo neutron in a transfer would leave $^{10}$Be in either the ground or the first excited state, seen in fig. 2.4. The high lying $1^-_1$ and $2^-_1$-states can be populated by knocking out one of the core neutrons, leaving the halo neutron orbiting the new $^9$Be core. This is consistent with the interpretation of the two states being one-neutron halo states [1]. The last two states require multistep reactions to populate, and the reaction cross sections are expected to be small. Especially the $0^+_2$-state, which requires both a knockout of a core neutron and an excitation of another core neutron from the $0p_{3/2}$-shell to the $1s_{1/2}$ shell. The $2^+_2$-state

![Figure 2.4: The bound states and the lowest resonances of $^{10}$Be. The neutron separation energy and the Q-value of the $\beta$-decay is indicating as well. The picture is taken from the TUNL datagroup [1].](image-url)
requires a core neutron knockout but only an excitation of the halo neutron from the $1s_{1/2}$ shell to the $0p_{1/2}$ shell, which is much more likely.

The four highly excited states in $^{10}$Be are very close in excitation energy, fig. 2.4, making them hard to distinguish in a transfer experiment if only charged particles are detected. Two of the states are successfully identified and a third tentatively seen in this experiment, shown in chapter 7, proving how strong a setup with both charged particle and gamma detection is. The high resolution (100 keV) for germanium detectors used for gamma detection is a necessity to separate events from the four states.

The gamma energy from the decay of all excited states in $^{10,11,12}$Be are shown in table 2.1 along with the branching ratios. The values are taken from C. M. Matoon et al.[] for $^{10,11}$Be and S. Shimoura et al.[] and H. Iwasaki et al.[] for $^{12}$Be. The only states that might produce gammas within 100 keV of each other is the four excited states in $^{10}$Be, the rest should be easily identified in a gamma spectrum, chapter 7. All four states decays to the $2^+_2$-state generating four gamma lines, which is pairwise separated with less than 100 keV. Only the $2^+_2$- and the $2^+_1$-states decays completely (or almost) to the $2^+_1$-state. The $0^+_2$ and the $1^-_1$ have two other strong decays, a 219.2 keV ($0^+_2 \rightarrow 1^-_1$) and a 5959 keV ($1^-_1 \rightarrow 0^+_1$), which can be used to determine the population of the $0^+_2$ and the $1^-_1$ states respectively, section 7.2.3.
Table 2.1: known gamma decays in $^{10,11,12}$Be. The decays are taken from []

<table>
<thead>
<tr>
<th>Nucleus</th>
<th>Decay</th>
<th>$E_{\gamma}$ [keV]</th>
<th>BR [%]</th>
</tr>
</thead>
<tbody>
<tr>
<td>$^{10}$Be</td>
<td>$2^+_1 \rightarrow 0^+_1$</td>
<td>3367</td>
<td>100</td>
</tr>
<tr>
<td></td>
<td>$2^+_2 \rightarrow 2^+_1$</td>
<td>2590</td>
<td>91</td>
</tr>
<tr>
<td></td>
<td>$1^-_1 \rightarrow 0^+_1$</td>
<td>5959</td>
<td>66</td>
</tr>
<tr>
<td></td>
<td>$1^-_1 \rightarrow 2^+_1$</td>
<td>2593</td>
<td>34</td>
</tr>
<tr>
<td></td>
<td>$0^+_2 \rightarrow 2^+_1$</td>
<td>2812</td>
<td>65.7</td>
</tr>
<tr>
<td></td>
<td>$0^+_2 \rightarrow 1^-_1$</td>
<td>219.2</td>
<td>34.3</td>
</tr>
<tr>
<td></td>
<td>$2^-_1 \rightarrow 2^+_1$</td>
<td>2896</td>
<td>100</td>
</tr>
<tr>
<td>$^{11}$Be</td>
<td>$1/2^- \rightarrow 1/2^+$</td>
<td>320.0</td>
<td>100</td>
</tr>
<tr>
<td>$^{12}$Be</td>
<td>$2^+_1 \rightarrow 0^+_1$</td>
<td>2107</td>
<td>100</td>
</tr>
<tr>
<td></td>
<td>$0^+_2 \rightarrow 0^+_1$</td>
<td>$511^1$</td>
<td>83</td>
</tr>
<tr>
<td></td>
<td>$0^+_2 \rightarrow 2^+_1$</td>
<td>144</td>
<td>17</td>
</tr>
<tr>
<td></td>
<td>$1^-_1 \rightarrow 0^+_1$</td>
<td>2680</td>
<td>100</td>
</tr>
</tbody>
</table>

$^1$pair production
This chapter will give a short motivation for using transfer reactions to study the neutron rich beryllium isotopes. Furthermore the procedure for calculating both experimental and theoretical differential cross sections will be described.

3.1 Transfer reactions

Transfer reactions are a powerful tool to probe single particle excitations in a nucleus. This makes transfer reactions very useful when studying single particle behaviours of states. A particle, or cluster of particles, is transferred from one nucleus to another in transfer reactions, generally described like:

\[ A + a \rightarrow B + b. \]  \hspace{1cm} (3.1)

\[ B = A + v. \]  \hspace{1cm} (3.2)

\[ b = a - v. \]  \hspace{1cm} (3.3)

\( v \) is the transferred particle and \( B \) is the particle to be studied. The final states of the final nuclei can be described by taking the ground states of the initial nuclei and add or remove one particle (cluster). In the simplest picture, the two cores (the part of the two nuclei, that is not transferred) are assumed not to be affected by the transfer, hence the final states are completely determined by, which state the transferred particles was in, and which state ends up in. Any core excitations or multistep transfers are considered to be higher order contributions. Eventhough all four elements in eqn 3.1 is nuclei \( b \) will be refered to as particles in this thesis.

In this thesis, I study one neutron transfers (\( v = n \) in eqn 3.1), both adding a neutron to the \(^{11}\text{Be} \) nucleus ((d,p)) and removal of one ((d,t)), leading to single neutron excitations of \(^{12}\text{Be} \) and \(^{10}\text{Be} \) respectively. It has already been stated in chapter 2 that both \(^{11}\text{Be} \) and \(^{12}\text{Be} \) to a large extent can be described
as a $^{10}$Be core and one or two neutrons respectively. This description works especially very well for $^{11}$Be with its halo structure.

The one neutron removal experiment ($(d,t)$) could be used to investigate the ground state structure of $^{11}$Be, which is known to be a linear combination of an s-neutron and a ground state $^{10}$Be and a p-neutron and an excited $^{10}$Be:

$$|^{11}\text{Be}\rangle = |^{10}\text{Be}; 0^+\rangle |n; s_{1/2}\rangle + |^{10}\text{Be}; 2^+\rangle |n; d_{5/2}\rangle .$$

The loosely bound halo neutron should be easily transfered leaving the $^{10}$Be core in either the ground state or the excited $2^+_1$-state. This structure has been studied in various $(d,p)$ and $(p,d)$ reactions, for instance [ ]. As it will be shown in chapter 7 also the higher lying states of $^{10}$Be will be populated in the $(d,t)$ transfer. This is done by knocking out one of the core neutrons from $^{11}$Be leaving $^{10}$Be in a single neutron excited state. For the four high lying states, the halo neutron will still be lightly bound ($< 900$ keV) and in two cases at a large distance making it possible to study the suggested halo structure of the $1^-_1$- and $2^-_1$-states.

A one neutron transfer reaction is also a very good tool for studying the mixing of the sd and the p shell, which occurs in $^{12}$Be as described in section ???. All bound states in $^{12}$Be contains some amount of a $^{11}$Be$_{gs}$ and one neutron in an s-, p- or d-shell. Especially the $|^{10}\text{Be}; 0^+_1\rangle |n; s_{1/2}\rangle |n; s_{1/2}\rangle$ component in the two $0^+$-states is interesting in the study of the mixing of the states. This component should be strongly populated in a $^{11}$Be$(d,p)^{12}$Be reaction. The main aim of the experiment is to determine the amount of this component in the two $0^+$-states, which will be done by determining the spectroscopic factors, which will be described in section 3.4. First differential cross sections will be described, starting with experimental ones.

### 3.2 Experimental differential cross sections

The differential cross section from an experimental point of view is a measure of the probability for a particle being emitted in a given solid angle. The differential cross section is given by:

$$\frac{d\sigma}{d\Omega}(\theta, \phi) = \frac{N_b(\theta, \phi)}{l_A \cdot n_t} \frac{1}{d\Omega} .$$

(3.4)
3.2. Experimental differential cross sections

$N_b$ is the number of reactions per unit time, $I_A$ is the beam intensity, $n_t$ is the particle density in the target and $d\Omega$ is the solid angle. This can be rewritten as:

$$\frac{d\sigma}{d\Omega}(\theta, \phi) = \frac{N_b(\theta, \phi)}{N_A \cdot n_t} \frac{1}{d\Omega}.$$  \hspace{1cm} (3.5)

Here $\frac{N_b}{d\Omega}$ is the number of outgoing particles for a solid angle and $N_A$ is the total number of beam particles in the experiment.

The detection efficiency of the setup has to be taken into account in order to convert the number of detected particles in a solid angle to the total number of outgoing particles in the solid angle. Instead of calculating the detection efficiency for all solid angles, simulations of the experiment is used to take the detection efficiency into account. The simulation will be described in detail in section ??, but the basic idea is to generate $N_{A}^{\text{sim}}$ events uniformly distributed over $4\pi$. Only events hitting the detectors will be registered. This way the cross section can be described as:

$$\frac{d\sigma}{d\Omega}(\theta, \phi) = \frac{1}{n_t} \frac{N_{A}^{\text{exp}}}{N_{A}^{\text{det}}} \frac{N_{A}^{\text{sim}}}{N_{A}^{\text{det}}}.$$  \hspace{1cm} (3.6)

Here sim stands for simulated data while exp stands for experimental data.

In order to calculate the cross section the five values in eq. (??) need to be determined. The easiest is $n_t$. It is given by:

$$n_t = \frac{\rho dt N_d}{M} = \frac{1 \cdot 10^{-3} \text{g/cm}^2 \cdot 6.02 \cdot 10^{23} \text{mol}}{16.1 \text{g/mol}} \cdot 2 = 7.47 \cdot 10^{19} \text{cm}^2. \hspace{1cm} (3.7)$$

Here $\rho$ is the density of the target and $dt$ is the thickness of the target, these two are combined and are given in table 4.1. $M$ is the molar mass of the target and $N_d$ is the number of reaction particles per molecule (two deuterons per ethylene molecule).

The number of beam particles for the experimental data ($N_{A}^{\text{exp}}$) is determined by rutherford scattering on $^{107}\text{Ag}$, and is described in section 6.4. The number of beam particles for the simulation is simply the number of simulated particles, and is given in section 4.6.

It has proven easier to determine $\frac{N_{A}^{\text{exp}}}{N_{A}^{\text{sim}}}(\theta, \phi)$ rather than the two numbers individually. This is done by fitting excitation spectra from simulated data to
the experimental ones:

\[ N_{\text{det}}^{\text{exp}} = a N_{\text{det}}^{\text{sim}} \]  
\[ a = N_{\text{det}}^{\text{exp}} / N_{\text{det}}^{\text{sim}}. \]  

(3.8)

This way the statistical uncertainties are taken into account by the fit. An example of such a fit is given in section 8.2.

### 3.3 Theoretical differential cross sections

The theoretic differential cross sections are calculated using the program FRESCO []. A short description of the theory behind the calculations will be given in this section. The theory has been taken from []. The section starts with scattering theory before treating DWBA calculations for transfer reactions. Real potentials will in the following sections be noted with a \( V \), while complex potential will be noted with a \( U \).

#### 3.3.1 Optical model

The simplest calculation is the optical model (OM). OM is used to determine elastic scattering cross sections like \( (p,p) \) and \( (d,d) \). Only the elastic channel is calculated, all other reaction channels are treated as loss of flux. The scattering process is described by the wavefunction of the particle in a potential given by the interaction between the target and the particle, hence the job is to solve the Schrödinger equation:

\[
[\hat{T} + U(R) - E]\Psi(\vec{k}, \vec{R}) = 0
\]  
(3.9)

It is assumed that the potential is spherical symmetric making it possible to separate the angular and radial part of the wavefunction, and the wavefunction can be written as an expansion in polynomials of \( \cos(\theta) \):

\[
\Psi(\vec{k}, \vec{R}) = \frac{1}{kR}\sum_{L=0}^{\infty}(2L + 1)^{1/2} \chi_L(k, R)P_L(\cos(\theta))
\]  
(3.10)

The radial functions \( \chi_L(K, R) \) can then be calculated for all \( L \), and the solution can be written in terms of Hankel functions and the S-matrix \( S_L \).
In order to determine the cross section, the solution needs to be matched with another description of the particle wavefunction. The incoming particle is assumed to be described by a plane wave traveling in the z-direction:

$$\Psi_{\text{inc}}(\vec{k}, \vec{R}) = e^{i k z}.$$ (3.11)

The wavefunction will be distorted by the potential, and the outgoing wavefunction is assumed to be of the form:

$$\Psi_{\text{out}}(\vec{k}, \vec{R}) = e^{i k z} + f(\theta, \phi) e^{i k R}.$$ (3.12)

The probability of finding the particle in a given angle, and hence the cross section is then given by the square of \(f\):

$$\frac{d\sigma}{d\Omega}(\theta, \phi) = |f(\theta, \phi)|^2.$$ (3.13)

Normally there will not be any \(\phi\)-dependence of \(f\). If the reaction involves a charged particle and target, the potential will consists of a coulomb part and a nuclear part, and \(f\) can be seperated into two:

$$f(\theta) = f_c(\theta) + f'(\theta),$$ (3.14)

where \(|f_c(\theta)|^2\) is the Rutherford cross section and \(f'(\theta)\) is the nuclear \(f\)-value. By comparing the two descriptions of the wavefunction it can be shown, that the nuclear \(f\)-value is given by:

$$f'(\theta) = \frac{1}{2ik} \Sigma L (2L + 1)(S_L - 1)P_L(\cos(\theta)),$$ (3.15)

hence \(S_L\) needs to be determined in order to get the cross section. If \(S_L = 1\) it won’t affect the \(f\)-value. This happens if the nuclear potential does not affect the radial wavefunction. For each potential there exists a \(L_1\)-value for which \(S_{L \geq L_1} = 1\), hence in calculation of the cross section it is not necessary to go to \(L = \infty\).

For a real potential the norm of \(S_L\) is unitary: \(|S_L| = 1\), but if the potential is complex the norm will be less than one, indicating a loss of flux. This is what is used in the optical model. An imaginary part is added to the nuclear potential to take into account the loss of flux to all the other reaction channels. The real part of the potential describes the elastic scattering, while the imaginary part removes the flux going to other reaction channels.
3.3.2 Coupled channel calculation

Coupled channel calculations can be used to determine the cross section of inelastic scattering. The $1/2^-$-state in $^{11}$Be is populated through inelastic scattering in the experiment described in this thesis, which will be shown in chapter 7.

Like in the OM calculation, the idea is to solve the Schrödinger equation to determine a scattering amplitude. The differential cross section will then be the norm squared of the $f$-value. If the cross section is calculated in the center of mass frame, it doesn’t matter whether $^{11}$Be or the deuteron is considered the incoming particle. $^{11}$Be is used as the beam particle for the CC calculation. The wave function of $^{11}$Be will be given by:

$$
\Psi_{^{11}\text{Be}}(\xi, \vec{R}) = \phi_1(\xi)\chi_1(\vec{R}) + \phi_2(\xi)\chi_2(\vec{R}). \quad (3.16)
$$

The indices 1 and 2 refer to the two different states (1 being the ground state), $\vec{R}$ is the relative coordinates between the two nuclei and $\xi$ is the internal coordinates in $^{11}$Be. The internal wavefunctions for $^{11}$Be ($\phi(\xi)$) are eigenfunctions for the internal hamiltonian:

$$
H_{^{11}\text{Be}}\phi_1(\xi) = \epsilon_1 \phi_1(\xi) \quad (3.17)
$$

$$
H_{^{11}\text{Be}}\phi_2(\xi) = \epsilon_2 \phi_2(\xi). \quad (3.18)
$$

The total hamiltonian is given by:

$$
H = H_{^{11}\text{Be}} + K + U(\xi, \vec{R}). \quad (3.19)
$$

Here $K$ is the kinetic energy operator and $U$ is the interaction between the target and the projectile. $\Psi_{^{11}\text{Be}}$ will then obey the Schrödinger equation:

$$
(E - H)\Psi = 0. \quad (3.20)
$$

Making the projections onto the ground state gives:

$$
0 = \phi_1^*(\xi)(E - (H_{^{11}\text{Be}} + K + V(\xi, \vec{R})))\Psi(\xi, \vec{R}) \quad (3.21)
$$

$$
= (E - \epsilon_1 - K - U_{11}(\vec{R}))\chi_1(\vec{R}) - U_{12}(\vec{R})\chi_2(\vec{R}),
$$

with:

$$
U_{nm}(\vec{R}) = \int \phi_n^*(\xi)U(\xi, \vec{R})\phi_m(\xi)d\xi. \quad (3.22)
$$
Doing a projection on the excited state as well gives two coupled equations to be solved:

\[
(E - \varepsilon_1 - K - U_{11}(\vec{R}))\chi_1(\vec{R}) = U_{12}(\vec{R})\chi_2(\vec{R}).
\]  
\[\text{(3.23)}\]

\[
(E - \varepsilon_2 - K - U_{22}(\vec{R}))\chi_2(\vec{R}) = U_{21}(\vec{R})\chi_1(\vec{R}).
\]  
\[\text{(3.24)}\]

The matrix elements of the coupling between the two states need to be determined before the coupled equations can be solved. Remark: If \(U_{12} = 0\) the first equation reduces to the optical model Schrödinger equation. The matrix elements can be calculated in two different ways, using a deformed potential or describing the nucleus by a cluster model.

### 3.3.2.1 The deformed potential

A spherical potential like the one used in the OM calculation can only couple two states with the same spin and parity. If instead the radius of the nucleus is dependent on the angle (\(R(\theta')\)), it is possible to couple states with different spins and parity. A deformed potential can be expanded on spherical harmonics:

\[
U(\xi, \vec{R}) = \sum_{\lambda} V_{\lambda}(R)Y_{\lambda 0}(\theta', 0).
\]  
\[\text{(3.25)}\]

\(\lambda\) is the multipole number with \(\lambda = 1\) being the dipole transition. The states that can be coupled by a given \(\lambda\) obey:

\[
|J_i - J_f| \leq \lambda \leq J_i + J_f
\]  
\[\text{(3.26)}\]

and

\[
\pi_f = (-1)^\lambda \pi_i.
\]  
\[\text{(3.27)}\]

\(V_{\lambda}\) is the projection of a spherical potential onto a given spherical harmonics:

\[
V_{\lambda}(R) = \int V(\vec{R} + \delta_{\lambda})Y_{\lambda 0}(\theta', \phi')d\Omega.
\]  
\[\text{(3.28)}\]

\(\delta_{\lambda}\) is called the deformation length and is a measure of the deformation of the potential.

### 3.3.2.2 Cluster model

The theory behind the cluster model is similar to the one used in transfer 3.1.
reactions described in section ?? . The cluster model can be used when the excitation is a one nucleon excitation like the $1/2^-$-state in $^{11}$Be. The reaction will be described in a three body model with the nucleus ($^{11}$Be) as a core ($^{10}$Be) and a valence nucleon (n), and a target particle (d), shown in fig. ?? . The target particle will react with both the valence nucleon and the core, and the reaction potential can be described by two independent potentials:

$$U(\vec{r}, \vec{R}) = V_{n+d}(\vec{R}_1) + U_{^{10}\text{Be}+d}(\vec{R}_2) - U_{^{11}\text{Be}+n}(\vec{R}).$$

(3.29)

The internal wavefunctions can be calculated from eq. (3.17) and (3.18) using the hamiltonian:

$$H = K + V_{^{10}\text{Be}+n},$$

(3.30)

where $V_{^{10}\text{Be}+n}$ is the potential from the core seen by the neutron. Knowing the potential and the wavefunction of the internal states, the matrix elements can be calculated from eq. (3.22).

3.3.3 Distorted wave born approximation

The calculation of transfer reactions is very similar to the cluster model of the coupled channel calculation. The basis is still a three-body model with a core ($a$), a valence particle ($v$) and a reaction particle ($b$). The difference comes in the transfer of the valence particle instead of an excitation. Fig. 3.2 shows the initial ($\alpha$) and final ($\beta$) situation of the transfer reaction. The reaction particle now acts as a core for the final configuration. The cross section is then
3.3. Theoretical differential cross sections

proportional to the square of the transition amplitude.

\[
\frac{d\sigma}{d\Omega} = \frac{\mu_\alpha\mu_\beta}{2\pi\hbar^2 k_\alpha} |T(k_\alpha, k_\beta)|^2, \tag{3.31}
\]

with the transition amplitude given by either:

\[
T_{\text{prior}} = \langle \chi^{(-)}_\beta \phi_\beta \phi_\beta | V_{3a} + U_{AB} | \chi^{(+)}_\alpha \phi_\alpha \phi_\alpha >, \tag{3.32}
\]

called the prior representation, or:

\[
T_{\text{post}} = \langle \chi^{(-)}_\beta \phi_\beta \phi_\beta | V_{3b} + U_{AB} | \chi^{(+)}_\alpha \phi_\alpha \phi_\alpha >, \tag{3.33}
\]

called post. Post and prior representation give the same result in DWBA. The transition amplitude can be written in integral form:

\[
T = \int d\vec{R}_\alpha d\vec{R}_\beta \chi^{(-)}_\beta (\vec{R}_\beta) I_{\beta\alpha}(\vec{R}_\alpha, \vec{R}_\beta) \chi^{(+)}_\alpha (\vec{R}_\alpha), \tag{3.34}
\]

with:

\[
I_{\beta\alpha}(\vec{R}_\alpha, \vec{R}_\beta) = \langle \phi_\beta \phi_\beta | V_{3a} + U_{AB} | \phi_\alpha \phi_\alpha >. \tag{3.35}
\]

\( I_{\alpha\beta}, \chi_\alpha \) and \( \chi_\beta \) are needed to determine the transition amplitude. The \( \chi \)'s can be determined by a set of coupled channel equations like eq. (3.23) and (3.24).

\[
(E_\alpha - K_\alpha - U_{aa}(\vec{R}_\alpha))\chi_\alpha(\vec{R}_\alpha) = \int U_{\alpha\beta}(\vec{R}_\alpha, \vec{R}_\beta) \chi_\beta(\vec{R}_\beta) d\vec{R}_\beta, \tag{3.36}
\]

\[
(E_\beta - K_\beta - U_{\beta\beta}(\vec{R}_\beta))\chi_\beta(\vec{R}_\beta) = \int U_{\beta\alpha}(\vec{R}_\alpha, \vec{R}_\beta) \chi_\alpha(\vec{R}_\alpha) d\vec{R}_\alpha. \tag{3.37}
\]
The integral \((I_{\alpha\beta})\) is done by first calculating the overlap functions \((\phi_A \text{ and } \phi_B)\) from the Schrödinger equation like in the cluster model.

### 3.4 Spectroscopic factors

An important parameter in transfer reaction experiments and the study of single particle structures of nuclei is the spectroscopic factor. The aim of the \(^{11}\text{Be}(d,p)^{12}\text{Be}\) experiment described in this thesis is to determine the spectroscopic factor for each state populated in the reactions.

For many years spectroscopic factors have been interpreted as a measure of the amount of a given particle configuration of a state. As an example take the ground state of \(^{12}\text{Be}\), which is known as a linear combination of three different particle configuration:

\[
|^{12}\text{Be}; 0^+_1\rangle = \alpha |^{10}\text{Be}; 0^+_1\rangle |2n; s_1^2\rangle + \beta |^{10}\text{Be}; 0^+_1\rangle |2n; p_1^2\rangle + \gamma |^{10}\text{Be}; 0^+_1\rangle |2n; d_5^2\rangle. \tag{3.38}\]

The factors can be determined by probing the different particle configuration like the \(\alpha |^{10}\text{Be}; 0^+_1\rangle |2n; s_1^2\rangle\) in a \(^{11}\text{Be}(d,p)\) reaction. The spectroscopic factor determined by this experiment is then interpreted as the value \(\alpha^2\). The validity of this interpretation has been questioned in recent years. The spectroscopic factor is a quantity describing effects on the total volume of a nucleus, while direct reactions happens at the surface. Hence modifications can occur especially when going away from stability \([\ldots]\). It has been proposed to use the absolute normalization constant (ANC) instead, which is a surface quantity and may describe reactions better, especially for exotic nuclei, \([\ldots]\) and references therein. The validity of spectroscopic factors for exotic nuclei has been tested experimentally for various types of experiments, and while a strong effect is seen in nucleon knock-out reactions, no changes have been seen in one nucleon transfer reactions, \([\ldots]\). The debate is still ongoing, but it will not be covered in this thesis, and only spectroscopic factors will be determined.

The spectroscopic factor is defined as a scaling factor between an experimental determined differential cross section and a theoretical one:

\[
S := \left| \frac{\frac{d\sigma}{d\Omega}}{\frac{d\sigma}{d\Omega}} \right|^2. \tag{3.39}\]

From this definition and the description of the differential cross sections given in the two previous sections it is clear why the spectroscopic factor is equal to the norm squared for a given particle configuration.
Both the theoretical and the experimental determined differential cross section is calculated for a given configuration of a state, like the

$$|^{10}\text{Be}; 0^+_1 \rangle |2n; s^2_{1/2} \rangle$$

configuration of the $0^+$-states in $^{12}\text{Be}$. Only one particle configuration for each state can be probed by a given transfer reaction. This is easily seen by adding a neutron to the ground state of $^{11}\text{Be}$, the only $0^+$-configuration possible is the $s^2_{1/2}$, hence the differential cross section is proportional to:

$$\frac{d\sigma}{d\Omega}_{\text{experiment}} \propto |\langle 2n; s^2_{1/2} | (^{10}\text{Be}; 0^+) |^{12}\text{Be}; 0^+ \rangle|^2 = |\alpha|^2$$ (3.40)

with $|^{12}\text{Be}; 0^+ \rangle$ taken from eqn. 3.38.

For the theoretical calculation it is a choice made in the parameters. The theoretical cross section is calculated with the assumption, that the state is completely described by a given particle configuration corresponding to

$$\frac{d\sigma}{d\Omega}_{\text{theory}} \propto |\langle 2n; s^2_{1/2} | (^{10}\text{Be}; 0^+) |^{12}\text{Be}; 0^+ \rangle|_{\text{theory}}|^2 = 1.$$ (3.41)

The proportionality constant is the same, hence from the definition of the spectroscopic factor:

$$S = |\alpha|^2.$$ (3.42)

This is valid if the reaction is a single step process, if the transfer is done in a two step process, for instance where the initial nucleus is excited, new configurations can be populated. Taking again the $0^+$-states of $^{12}\text{Be}$. If $^{11}\text{Be}$ is excited to the $1/2^-$ state, the single particle configuration populated is the:

$$|^{10}\text{Be}; 0^+_1 \rangle |2n; p^2_{1/2} \rangle$$

configuration. These effects requires a deeper theoretical analysis and are not included in this thesis.
A description of the experimental procedure will be given in this chapter. Including the experimental setup, the detectors and the production of our RIB. Furthermore a short description of the simulation will be given.

4.1 Performing a transfer reaction experiment

The motivation for studying neutron rich beryllium isotopes, especially $^{12}$Be, in a transfer reaction should be clear from the last two chapters. Performing a transfer reaction involving a radioactive isotope is a complicated task though. The short lifetime of $^{11}$Be ($\tau = 19.9$ s) makes it impossible to use it as a target. Instead the reaction is done in inverse kinematics using a radioactive ion beam (RIB) of $^{11}$Be and a deuteron target. This requires both a RIB facility, to produce a $^{11}$Be beam and a foil containing deuterons. Deuterons are almost stable and foils containing deuterons is easily accessible. A plastic foil, where the protons are exchanged with deuterons, are used in this experiment. Several RIB facilities, that can produce low energy $^{11}$Be beams, exists today, the ISOLDE facility at CERN, Switzerland, was used in this experiment. Furthermore, detectors to detect at least the outgoing charged particles in order to determine a differential cross section. Gamma detectors are also used, in this experiment, to separate the individual states, section 2.4. All the components will be described individually in this chapter. The experiment is performed by directing the low-energy $^{11}$Be beam onto a target containing deuterons. Two (or more) particles goes out of the target. The light particles (p, d or t) are detected while the heavy fragments are sent to a beam dump.

The experiment is performed as a series of small runs. A run ranges from minutes to a few hours in time. A data file containing the signals from the detectors are created for each run. During the experiment small runs with other targets, needed for the analysis, are performed, section 4.4.
4.2 ISOLDE

The radioactive $^{11}$Be beam was produced at ISOLDE, CERN []. ISOLDE was one of the first facilities at CERN, and one of the first low energy RIB facilities in the world. The ISOLDE facility is shown in fig. 4.1. The beams are produced at one of the two primary target stations, one connected to the General Purpose Separator (GPS), which was used in the experiment, and one connected to the High Resolution Separator (HRS). The beam production is done through fragmentation of heavy ions. A Tantalum (Ta) target was used for the $^{11}$Be production. High energy protons ($\sim 1.4\text{GeV}$) from the PS booster hits the primary Ta target creating an unstable nucleus which decays through emission of several light nuclei, including $^{11}$Be. The target is heated to temperatures between 700°C and 1400°C making the fragments evaporating from the target and diffusing onto the ion source. The created nuclei are laser ionized creating a positive ion, which can be accelerated through high voltage. The ions are accelerated up to 60 keV and sent to one of the two separators (GPS or HRS). The separators are a series of bending magnets, one for GPS and two for HRS (fig. 4.1), which separate the accelerated beam particles according to their masses. Only the low-energy $^{11}$Be particles goes through the separator and into the ISOLDE experimental hall. The rest of the emission fragments are sent to a beam dump.
4.3 REX-ISOLDE

The $^{11}\text{Be}$ beam from the GPS has to be accelerated further, in order to get a beam energy high enough to induce nuclear reactions. This is done at the REX-ISOLDE postaccelerator \cite{1}. Fig. 4.2 shows a diagram of the postaccelerator. The idea of the REX is to create highly charged nuclei, which can be accelerated through a short linear accelerator. The ability to use a short LINAC, combined with the placement outside of the highly radioactive target area of ISOLDE, is the strength of REX-ISOLDE.

The acceleration is done in three steps. First the ions are trapped, bunched and cooled in a penning trap (REXTRAP). The cooled ions are afterwards sent to an electron beam ion source (REXBIS). The EBIS creates highly charged ions using a magnetically compressed electron beam \cite{2}. $^{11}\text{Be}$ is totally stripped creating $^{11}\text{Be}^{4+}$. The REXTRAP uses a noble gas to cool the beam and therefore the bunched beam needs
to be separated once again, which is done through two bending magnets. The noble gas used is neon, and the bending magnets are only able to separate particles with different \( q/A \), hence the separator cannot distinguish \( ^{11}\text{Be}^{+4} \) and for instance \( ^{22}\text{Ne}^{+8} \). A neon gas without \( ^{22}\text{Ne} \) had to be used in the REXTRAP. A purified \( ^{20}\text{Ne} \) gas is used as a cooler gas reducing the \( ^{22}\text{Ne} \) contamination to less than 1%. Last step is the acceleration of the beam. The acceleration is done in a LINAC, that is able to accelerate the beam up to 3 MeV/u. A beam energy of 2.85 MeV/u was used in the experiment. After the reacceleration the beam is directed, through a bending magnet, onto the reaction target placed at the MINIBALL station.

![Diagram of the REX post accelerator](image)

**Figure 4.2:** Diagram of the REX post accelerator. The low-energy beam from ISOLDE enters the penning trap, where it is bunched and sent to the EBIS. The EBIS strips the beam particles and send them through a \( q/A \) separator to the LINAC. The LINAC consists of four different types of cavities, that can accelerate the beam up to 3 MeV/u.

### 4.4 Targets

A foil of deuterated polyethylene (CD\(_2\)) was used as the primary target in the experiment, but five different targets were used in total. Table 4.1 shows the five targets along with a short description of the purpose of the target. The deuterated polyethylene was used to produce the \( ^{11}\text{Be}+d \) reactions investigated in this thesis. Unfortunately the target contained a significant amount of carbon and a small fraction of protons. It is not possible to create a pure CD\(_2\) target and a few percent contamination of CH\(_2\) is expected. \( ^{11}\text{Be} \) can react with both the carbon and the protons in the target producing protons, deuterons and fusion products, which will act as a background on the real \( ^{11}\text{Be}+d \) events. To determine and remove the background a few runs on a pure carbon target as well as on a regular polyethylene (CH\(_2\)) target were performed throughout the experiment. The effect of the background is described in section ???. The reactions from the two background targets could be used to investigate the reactions of \( ^{11}\text{Be} \) on both carbon and protons as well, and indeed the \((p,p)\) and \((p,d)\)
reactions are studied in chapter 9.

The total number of $^{11}\text{Be}$ particles in the experiment is needed to determine the differential cross section, eqn. 3.6. This is done by determining the beam intensity through a reaction with a known cross section, like Coulomb scattering. The low Z-value of the two nuclei (Z = 4 and 1) makes the (d,d) scattering mainly nuclear and the channel cannot be used to determine the beam intensity. Instead a silver target was used. Short beam intensity measurements were performed regularly during the experiment. The beam intensity will fluctuate during the days of an experiment, and this has to be taken into account to get the right scaling of the angular distributions. By using a silver target the beam intensity is only measured at an instant time, hence regular measurements are needed. The beam measurement and the calculation of the total number of $^{11}\text{Be}$ in the experiment will be described in section 6.4.

The last target was a stopper foil made of a thick aluminum foil. The target was used to create a $^{11}\text{Be}$ gamma source. The $^{11}\text{Be}$ particles will be stopped in the target and afterwards $\beta$-decay to excited states in $^{11}\text{B}$. The excited $^{11}\text{B}$ will then gamma decay producing gammas with energies up to 8 MeV. This $^{11}\text{Be}$ source is used to determine the high energy detection efficiency of the MINIBALL, described in section 5.7.

<table>
<thead>
<tr>
<th>Target</th>
<th>Thickness</th>
<th>Purpose</th>
</tr>
</thead>
<tbody>
<tr>
<td>CD$_2$</td>
<td>1. mg/cm$^2$</td>
<td>Primary target. $^{11}\text{Be} + \text{d}$ reactions.</td>
</tr>
<tr>
<td>$^{107}\text{Ag}$</td>
<td>1.9 mg/cm$^2$</td>
<td>Beam intensity measurement through Coulomb scattering.</td>
</tr>
<tr>
<td>CH$_2$</td>
<td>1.1 mg/cm$^2$</td>
<td>Used to determine the background from reactions on protons.</td>
</tr>
<tr>
<td>C</td>
<td>1.5 mg/cm$^2$</td>
<td>Used to determine background from fusion products of $^{12}\text{C} + ^{11}\text{Be}$.</td>
</tr>
<tr>
<td>Al</td>
<td>$\sim$ 200 $\mu$m</td>
<td>Energy and efficiency calibration of MINIBALL.</td>
</tr>
</tbody>
</table>

Table 4.1: The five different targets used in the experiment.

4.5 The detector setup

Two types of detectors were used in the experiment, silicon detectors for charged particles and germanium detectors for gammas. A standard setup for transfer reaction experiments at ISOLDE has been developed combining the MINIBALL [?] and the
T-REX []. A picture of the setup is seen in fig. 4.3.

The MINIBALL is an array of 24 germanium detectors placed in eight clusters. A Ge-cluster is seen in fig. 4.5. The clusters are placed on a movable frame, making it possible to adjust the position of the individual clusters to optimize the angular coverage for each experiment. The MINIBALL has a resolution down to 100 keV, which is a factor of 10 higher than the resolution of the charged particle detectors.

The T-REX setup used for charged particle detection consists of 16 silicon detectors placed in a barrel like configuration, fig. 4.4. The barrel has four sides and two end caps. The four sides are made from eight square detectors two on each side. The barrel is designed to have two end caps made from four annular detectors (AD) each. Each AD covers almost a quarter of the end cap. The end cap covering the very backward angles was missing in the experiment. The end caps have a hole in the middle to let the beam and heavy fragments slip through. The T-REX covers nearly all angles from $8^\circ$ to $152^\circ$ in the laboratory system [?]. The target is placed in the middle of the setup (red dot in fig. 4.4) using a target ladder. The four detectors making up the forward side of the barrel are shielded with a 11.57 µm thick mylar foil in order to protect the detectors from heavy fragments. Light particles pass through the foil while heavy particles like beryllium isotopes are stopped. The mylar foil lowers the resolution of the charged particle data, due to straggling in the foil. Furthermore it increases the
lower limit in energy for charged particles to be detected.

Figure 4.4: A drawing of the T-REX setup including the laboratory frame described in section 5.2. The target is represented by a red dot. Half of the detectors on the top and left right side is omitted to give a view inside the detector. The beam is assumed to follow the z-direction.

4.5.1 Silicon detectors

The silicon detectors in the T-REX is telescope detectors with a thin detector (ΔE-detector) in front of a thick one (E-detector). The advantage of two detectors in telescope is the ability to make particle identification, which will be described in section 7.2.1. Ideally the particles go through the front detector and stop in the back one, hence the thinner a front detector the better. The combined energy deposited in the two detectors corresponds to the total energy of the incoming particle.

The momentum vectors of the particles are required to calculate excitation energy spectra, chapter 7. Hence the position within the detectors of the particles have to be determined as well. This is done by using segmented detectors as ΔE-detectors. The thick E-detectors is only used for measuring the energy and consists of only one large pad (the E-detectors are sometimes called pad-detectors). The thickness of the ΔE- and E-detectors can be seen in table 4.2.

The T-REX setup consists of two types of ΔE-detectors with different segmentations. The eight detectors making up the side of T-REX are square detectors with an active area of 50x50 mm$^2$, fig. 4.4. The detectors are position sensitive detectors (PSD). The front side of a PSD is divided into 16 strips while the backside is one big
pad. The PSD has one readout for each strip on the frontside and one for the backside. The backside readout gives the total energy deposited in the detector ($\Delta E$) while the frontside readouts give a signal indicating the position in the corresponding strip. The front side of the detectors has a resistant layer and a readout in one end of each strip. If a particle hits a strip close to the readout (one end) the current lost in the resistant layer is small and the front and back side readout is the same. If a particle hits a strip in the end away from the readout all current is lost in the resistant layer. Hence the signal from the front side ranges from 0 in one end to $\Delta E$ in the other, providing the position, section 5.4.

The annular $\Delta E$-detectors are divided into 24 strips and 16 rings. The position is determined by the strip and ring number, section 5.2. The energy is determined from either the energy signal in the ring or the strip. An energy difference between the two signals of less than 500 keV is required for an event to be accepted.

<table>
<thead>
<tr>
<th>Place in T-REX</th>
<th>Detector</th>
<th>Thickness [µm]</th>
</tr>
</thead>
<tbody>
<tr>
<td>Barrel side</td>
<td>PSD</td>
<td>140</td>
</tr>
<tr>
<td></td>
<td>Pad</td>
<td>1000</td>
</tr>
<tr>
<td>End cap</td>
<td>AD</td>
<td>500</td>
</tr>
<tr>
<td></td>
<td>Pad</td>
<td>500</td>
</tr>
</tbody>
</table>

Table 4.2: Detector types and thickness for the silicon detectors in T-REX

### 4.5.2 Germanium detectors

The MINIBALL setup consists of 8 clusters, each cluster contains three germanium detectors (fig. 4.5), and each detector is divided into six segments giving the MINIBALL a high angular resolution. The high angular resolution is needed in order to make doppler correction of the gammas.

Each germanium detector has seven readouts, one for each of the six segments and one for a common core. The core gives the total energy deposited in the detector while the segment readouts give the energy deposited in each segment. As the gamma ray are scattered in the detector, the energy of the gamma can be divided between several segments, hence the core signal is used

![Figure 4.5: A picture of a MINIBALL cluster with the three segmented germanium detectors. The detectors are the three hexagons to the right. Each detector has six triangular segments. The part to the left is electronics and a container for liquid nitrogen.](image)
4.6 Simulation

The lack of $4\pi$-coverage and the non spherical structure of the T-REX makes the detection efficiency for a given solid angle dependent on the angle. Rather than calculating the detection efficiency for all angles a simulation of the experiment is made. The simulation code is written by V. Bildstein [1] using Geant4 [4] and the g4miniball [2] package. The simulation can simulate both transfer and scattering reactions. $N$ reactions are produced with a flat distribution in the center of mass frame. The direction in the laboratory frame of the outgoing particles are determined from kinematics and root-file, containing information on the energy, detector and strip number for each detected particle, is generated. The simulated data can afterwards be analysed using the same program as the experimental data.

The flat distribution ensures only the detection efficiency causes variations in the number of detected particles in a given angle. The angular distribution of a reaction can be determined by comparing the number of experimental detected particles with the number of detected particles in the simulation, section 8.2.

The detailed structure of the beam can be set in the simulation, including the spread in energy, the reaction depth in the target, the shape and offset of the beam, and the angle and divergence of the beam. This is used to test and determine the detailed structure of the experimental beam in chapter 6 and to improve the simulations used to determine the angular distributions.

The simulations is also used to determine the width of the lowest lying resonance known in $^{12}$Be, section 10.3.
In this chapter, the calibration of the silicon and germanium detectors will be described. A calibration of both the energy and the position signals are performed.

5.1 Signals from the detectors

The momentum vector for one of the outgoing particles is required to determine the reaction type of each event. Four parameters \((x, y, z, E)\) can be measured as described in the last chapter. Details on the electronics will not be given in this thesis, for more information see V. Bildstein et al \cite{1}. In short terms the signals from the detectors are sent through a preamp, an amplifier and an analog to digital converter (ADC). The ADC number, the outlet number and the value of the signal (channel number) are afterwards stored in data files. The ADC and outlet number provide information on the detector number and signal type (position or energy), while the channel number of the signals provide the value of either the position or the energy. The channel numbers have to be converted to either position or energy, which requires a calibration.

The energy and position calibration for the silicon detectors are done using an \(\alpha\)-source, described in section 5.4 and 5.3. The \(\alpha\)-particles are stopped in the \(\Delta E\)-detectors and the light particles from the reactions of \(^{11}\text{Be}\) on deuteron are used to calibrate the pad-detectors (\(E\)-detectors) shown in section 5.5. An \(^{152}\text{Eu}\) source and the \(^{11}\text{Be}\) beam are used for the energy calibration of the germanium detectors. The positions of the germanium detectors are determined using a stable beam of \(^{22}\text{Ne}\) on a deuteron target, all this is described in section 5.6.

Furthermore the detection efficiency of the MINIBALL has to be determined. The efficiency has to be taken into account when producing gamma gated spectra, section 7.2.3. The MINIBALL efficiency is determined by K. Wimmer \cite{2} and will only briefly be described at the end of this chapter.
5.2 The laboratory frame

The position of an event should be given relative to the reaction point. The laboratory frame is defined as shown in fig. 4.4. Origin is placed in the center of the target, which is also the center of the barrel created by the PSDs in the T-REX. The z-axis is placed parallel to the PSDs and perpendicular to the ADs. The y-axis is the horizontal axis and the x-axis the vertical one perpendicular to the z-axis.

The beam is assumed to be moving along the z-axis and all reactions are assumed to happen in origin. This assumption will be investigated in chapter 6 where the structure of the beam will be determined. With this assumption the direction (angle and azimuthal angle) of the outgoing particle can be determined using the coordinates of the event.

The position of an event in the lab frame is determined in two steps. First the position in the detector \((pos)\) is calculated, using the formulas described in the next sections. Afterwards the position is transformed from the detector frame to the laboratory frame. The transformation requires knowledge of the individual detectors position in the laboratory frame. The positions of the silicon detectors are determined from the construction of the T-REX and the transformations are given by:

\[
\begin{align*}
    x_{PSD} & = 50 \text{ mm}(pos - 0.5) \cos\left(\frac{\pi}{2} N_{detector}\right) + 29 \text{ mm} \sin\left(\frac{\pi}{2} N_{detector}\right) \\
    y_{PSD} & = 50 \text{ mm}(pos - 0.5) \sin\left(\frac{\pi}{2} N_{detector}\right) + 29 \text{ mm} \cos\left(\frac{\pi}{2} N_{detector}\right) \\
    z_{PSD} & = 8 \text{ mm} + 3.125 \text{ mm} N_{strip} \\
    \theta_{AD} & = \arctan\left(\frac{9 \text{ mm} + 2 \text{ mm} N_{ring}}{63 \text{ mm}}\right) \\
    \phi_{AD} & = -59.3411 \text{ mrad}N_{strip} + 712.09 \text{ mrad} + \frac{\pi}{2}(1 - N_{detector}).
\end{align*}
\]

The detector numbers \((N_{detector})\) are counted clockwise looking towards the beam starting with 0 for the top detector. A transformation from \((x, y, z)\) to \((\theta, \phi)\) is easily done.

The positions of the germanium detectors have to be determined individually, which will be done in section 5.6.2.

5.3 Annular Detectors (AD)

First the four segmented \(\Delta E\)-detectors that make up the end cap of the T-REX barrel is calibrated. The calibration is performed by placing an \(\alpha\)-source at the target position. The \(\alpha\)-source consists of four emitters \((^{148}\text{Gd}, ^{239}\text{Pu}, ^{241}\text{Am} \text{ and } ^{244}\text{Cm})\) with \(\alpha\)-energies 3182.7 keV, 5156.6 keV, 5485.6 keV and 5804.8 keV.
The signal from the AD’s are merged together two and two in the ADC’s to reduce the number of outlets. The top and left AD, and the bottom and right share the same ADC and outlet numbers. Each pair has a total of four outlets, an address and an energy signal for the front (strips) and for the back side (rings). The energy signal provides the energy ($\Delta E$) and the address provides the strip/ring number.

The address signal contains 32 peaks divided into two groups, fig 5.1. Each peak represents a strip or ring, and each group corresponds to a detector. Channels from 400 to 700 represents hits in either the top or bottom detector (dependent on the ADC number) and channels 1000 to 1300 represents hits in the left or right detector. The mean value of each peak ($C_N$) is determined for each peak and a gate is made to determine the strip/ring number:

$$N = [C_N - 6, C_N + 6].$$  \hfill (5.6)

$N$ is the strip/ring number.

![Figure 5.1: The histogram showing the address from an ADC. Each peak represents a strip in the top (400-700) or left AD (1000-1400).](image)

The energy calibration is done with a simple linear relation:

$$\Delta E = a_{\Delta E} (C_{\Delta E} - b_{\Delta E}).$$  \hfill (5.7)

$C$ being the channel number. Fig. 5.2 shows a linear fit using the four known energies from the $\alpha$-source. The energy calibration is performed for each strip and ring in each detector. A channel number spectrum is generated for each strip/ring by using the address gates just determined. The channel numbers are determined using a gaussian fit to the four peaks in a channel number spectrum.
5.4 Position Sensitive Detectors (PSD)

The position sensitive detectors, that make up the four sides of the T-REX barrel is described in section 4.5.1. The PSD’s have 17 readout each, one for each of the 16 strips providing the position in the strip, and one from the rear side providing the energy.

The resistive front layer provides a signal ranging from 0 to $\Delta E$ dependent on the position in the strip. To generate a common scale for the position of each event the position signal is divided by the total energy, making the range 0 to 1. The calibration of the position is then:

$$P_{OS} = \frac{a_{pos}(C_{pos} - b_{pos})}{\Delta E}.$$  \hspace{1cm} (5.8)

It is clear that the position signal is dependent on the energy signal. Unfortunately the energy signal is also dependent on the position signal as shown in fig. 5.3. The detected energy is reduced for particles dependent on the position in the strip, due to the resistivity in the detector. The relation between the channel number and the energy is no longer linear but given by:

$$\Delta E = a_{AE} \left( \frac{C_{AE} - b_{tilt}}{a_{tilt}(1 - P_{OS}) + 1} - b_{AE} \right)$$  \hspace{1cm} (5.9)

The strong dependency between the energy and the position signal makes an iterative process necessary to calibrate the PSD detectors.

The calibration is further complicated in the four forward PSD’s due to the Mylar foil in front of them. The energy loss of the $\alpha$-particles in the Mylar is dependent on
the incoming angle giving a further dependency between the energy and the position, fig. 5.3B. The three peaks are bend, rather than straight like in the backward PSD’s. Furthermore, only three peaks are detected, the α with the lowest energy is stopped in the Mylar foil.

5.5 Pad detectors (E-detectors)

The pad detectors only provides an energy, hence no position calibration is done.

The energy calibration is inspired by the ΔE – E plots used to distinguish protons from deuterons etc., section 7.2.1. A clear relation between the energy deposited in the ΔE- and the E-detector exits. The relation is dependent on the particle type, the energy of the particle and the angle between the particle and the norm of the detector (θdet). The energies of the α’s from the α-source used to calibrate the energy of the ΔE-detectors are not strong enough to enter the pad detectors. Instead particles from the reactions of 11Be on deuterons are used. The reaction produces both protons, deuterons and tritons. Spectra of the channel numbers are made gated on various intervals in θdet and ΔE. The spectra contain two or three peaks corresponding to protons, deuterons and tritons, or one or two of them. Fig. 5.4A shows two pad
energy spectra taken from the top detector strip 0 with the following gates:

\begin{align*}
1.95 \text{ MeV} & \leq \Delta E \leq 2.05 \text{ MeV} \quad (5.10) \\
2.95 \text{ MeV} & \leq \Delta E \leq 3.05 \text{ MeV} \quad (5.11) \\
\end{align*}

A strong peak at 0.5 MeV (p) and a weak one at 1 MeV (d) is seen for the spectrum with a $\Delta E = 2$ MeV gate. Three peaks are seen in the other representing the three particles. The shift in energy of the proton and deuteron peak between the two spectra is clearly seen. A third gate around $\Delta E = 4$ MeV is made as well. Energy spectra is generated for the three gates in all strips in each detector and as many peaks as possible is determined for each spectra.

The corresponding energies have been determined using SRIM [1], and a linear fit is made using peaks from all gates and strips in a detector, fig. 5.4B. The large uncertainty in the channel number for each peak in, fig. 5.4A, is compensated by the large amount of points, and a linear relation is clear.

![Figure 5.4: (A) Two histograms of channel numbers made with two gates on the $\Delta E$-signal; Red: $\Delta E \approx 3$ MeV, Blue: $\Delta E \approx 2$ MeV. All three particles are seen at 3 MeV, only the two lightest are seen at 2 MeV. (B): A linear fit to all the identified peaks in one of the four forward PSD’s.](image)

### 5.6 Germanium detectors (MB)

The 24 germanium detectors in the MINIBALL all have six segments and a core. The core gives the total energy in the detector, and the segments provides a better determination of the position of the detected gamma. Pulse shape analysis could be
5.6. Germanium detectors (MB)

used to determine the position of the gamma in the segments [?]. This is not done in this analysis, the angular resolution given by using the center of the segments is sufficient to make a doppler correction of the gammas, chapter 7.

5.6.1 Energy calibration

The gammas from the experiment ranges in energy from 300keV to 6MeV, requiring a long energy range for the germanium detectors. The energy calibration is therefore made with two sources. A low energy source (\(^{152}\)Eu) providing gammas from 121keV to 1400keV. The \(^{152}\)Eu source was placed in the target holder. The high energy source was \(^{11}\)Be produced by the REX-ISOLDE and stopped in an aluminum target, section 4.4. \(^{11}\)Be provides gammas from 2100keV to 6800keV [?]. An energy calibration, using the gamma energies given in table 5.1, is made for each segment and core using a linear relation between the channel number and the energy.

![Figure 5.5: Two gamma energy spectra made from the \(d(^{22}\text{Ne},p)^{23}\text{Ne}\) reaction. A peak close to 1016 keV, but slightly shifted, is seen in both spectra. The spectra are made from a detector placed before (red) and after (blue) the target.](image)

5.6.2 Position calibration

The position of each segment relative to the target has to be determined. This is done using gammas from a \(d(^{22}\text{Ne},p)^{23}\text{Ne}\) reaction. A proton gated gamma spectrum is made for each segment and the gammas from the strongest line is identified, fig. 5.6. The strongest line is known to be the \(1/2^+ \rightarrow 5/2^+\) transition in \(^{23}\)Ne with a gamma energy of \(E_{^{23}\text{Ne}} = 1016.85\text{keV}\). The angle of the segments can then be determined from
Table 5.1: $E_\gamma$ for the gamma sources used to calibrate the germanium detectors. All values are in keV.

<table>
<thead>
<tr>
<th>$^{152}\text{Eu}$</th>
<th>$^{11}\text{Be}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>121.781</td>
<td>2124.47</td>
</tr>
<tr>
<td>244.698</td>
<td>4154.90</td>
</tr>
<tr>
<td>344.279</td>
<td>4665.90</td>
</tr>
<tr>
<td>778.904</td>
<td>5340.47</td>
</tr>
<tr>
<td>964.079</td>
<td>5851.47</td>
</tr>
<tr>
<td>1085.87</td>
<td>6278.81</td>
</tr>
<tr>
<td>1408.01</td>
<td>6789.81</td>
</tr>
</tbody>
</table>

The doppler shift:

$$\theta = \arccos\left(\frac{1}{\beta}\left(1 - \frac{E_{23}\text{Ne}}{\gamma E'}\right)\right). \quad (5.13)$$

$E'$ is the doppler shifted gamma energy, corresponding to the mean value of the peak and $\beta$ and $\gamma$ the relativistic factors determined from the energy and angle of the proton:

$$\gamma = \frac{T_A + Q - T_b - T_B + m_B c^2}{m_B c^2} \quad (5.14)$$

$$\beta = \sqrt{1 - \frac{1}{\gamma^2}}. \quad (5.15)$$

$A, b$ and $B$ refer to eqn. 3.1 and are here $^{22}\text{Ne}$, p and $^{23}\text{Ne}$ respectively. $T_B$ is calculated from momentum conservation (eqn. 7.5).

### 5.7 MINIBALL efficiency

The efficiency calibration of the MINIBALL is done with four different gamma sources, $^{152}\text{Eu}$, $^{60}\text{Co}$, $^{207}\text{Bi}$ and $^{11}\text{Be}$. The activity is known for the first two sources and the efficiency can be determined from the number of detected gammas and the run time. The activity of the latter two are unknown, hence only the relative intensity between individual gamma lines in the two sources can be determined. The relative intensities are then scaled to overlap with the efficiency from the first two sources. A fit to the following function taken from RadWare [?] is made:

$$\epsilon_{\text{MB}} = \exp\left(\left[(A + Bx + Cx^2)^{-G} + (D + Ey + Fy^2)^{-G}\right]^{-1/G}\right) \quad (5.16)$$
with:

\[ x = \log\left(\frac{E}{100\text{keV}}\right) \]
\[ y = \log\left(\frac{E}{1000\text{keV}}\right). \]

The parameters will not be given here, but the determined efficiencies for the four sources along with the fit is shown in fig. 5.7. The MINIBALL efficiency for each of the detected gammas in the transfer experiment is determined. The efficiencies are shown in table 5.2.
Figure 5.7: The detection efficiency of the MINIBALL as a function of the gamma energy. The measured efficiencies are marked for the individual sources. The dotted line represents the best fit.

<table>
<thead>
<tr>
<th>Nuclei</th>
<th>$\gamma$ [keV]</th>
<th>$\varepsilon_{MB}$ [%]</th>
</tr>
</thead>
<tbody>
<tr>
<td>$^{12}$Be</td>
<td>511</td>
<td>8.2</td>
</tr>
<tr>
<td></td>
<td>2100</td>
<td>3.5</td>
</tr>
<tr>
<td></td>
<td>2700</td>
<td>3.0</td>
</tr>
<tr>
<td>$^{11}$Be</td>
<td>320</td>
<td>12</td>
</tr>
<tr>
<td>$^{10}$Be</td>
<td>219.2</td>
<td>14</td>
</tr>
<tr>
<td></td>
<td>2600</td>
<td>3.2</td>
</tr>
<tr>
<td></td>
<td>2900</td>
<td>2.8</td>
</tr>
<tr>
<td></td>
<td>3300</td>
<td>2.5</td>
</tr>
<tr>
<td></td>
<td>6100</td>
<td>1.5</td>
</tr>
</tbody>
</table>

Table 5.2: The MINIBALL efficiency for each of the gamma lines in $^{10,11,12}$Be.
In this chapter it will be demonstrated how coincidence events from direct reactions can be used to determine the structure of a radioactive beam. The technique will be used on the $^{11}$Be beam in order to improve the transfer reaction analysis.

### 6.1 Coincidence events

The setup is designed to detect the light particles (p, d and t) and let the heavy fragments slip through to a beam dump, but in some cases it is possible for the heavy nucleus to be detected in the annular detectors, who covers angles down to 8 degrees. Fig. 6.1 shows the relation between the angles of the two outgoing particles for the three main reactions. Only the reactions populating the ground state of the final nucleus is shown, as they provides the largest outgoing angles. The angles covered by the T-REX is marked as grey areas in the plot. The plot shows that coincidence of both $^{11}$Be+d and $^{10}$Be+t is very likely. Only the d($^{11}$Be,t)$^{10}$Be events are used in this chapter to simplify the analysis as much as possible, especially when determining the beam energy, section 6.2.4. The statistic is still plentiful.

The angular range of the light particles is limited in coincidence events compared to single particle events due to the requirement of a minimum eight degree angle for the nucleus. The statistic of these coincidence events is much lower than in single particle events (one third for the (d,t) reactions), which makes them less suitable for determining angular distributions and differential cross sections. Instead the coincidence events can be used to determine the shape and structure of the beam. There is a complete knowledge about the total energy and momentum of the final state in a coincidence event. The complete kinematic enables a study of the energy and momentum vector of any of the in- or outgoing particles. In this chapter the incoming particle ($^{11}$Be) will be studied. The information about the beam gained from coincidence events can then be used to improve the analysis of the single particle events and the simulation of the reaction.
Figure 6.1: The outgoing angle of the heavy fragment ($\theta_B$) as a function of the outgoing angle of the light particle ($\theta_b$). The three lines in the plot represents: Blue: (d,t), Red: (d,d) and Black: (d,p). The grey area represent the area covered by the T-REX, the dotted line indicate the distinction between the AD’s and the PSD’s.

6.2 Beam characterisation

The characterization of the beam is done in three steps. First the width and offset of the beam is determined from the azimuthal angle of the two outgoing particles. Secondly the incoming angle and divergence of the beam is determined. Finally the beam energy is determined in two ways.

The order of these steps is determined by the inter-dependency of the quantities. The displacement can be determined independently of the incoming angle and the beam energy, while the two others are very dependent on the displacement and width of the beam, as it will be shown in section 6.2.2 and 6.2.4.

The analysis is derived for a general reaction of the type seen in eqn. 3.1 and tested on simulations before applied to the experimental data.

The outgoing angles of the particles are calculated using eqn. 5.1-5.5 and assuming the reaction occurred in origin. If this is not true, the angles derived are wrong. The beam is characterised by looking at the errors caused by the assumptions.
6.2.1 Beam width and offset

The beam width and offset can be determined by calculating the error in the $x$ and $y$ coordinates of the two particles. The principle is to use the azimuthal angles ($\phi$) of the two outgoing particles. The azimuthal angles are given by:

$$\phi = \tan \frac{y}{x}.$$ 

It is clear that $\phi$ is independent on energy, $z$ and the incoming angle, making it ideal to determine the offset of each event. The two outgoing particles will move back to back in the center of mass frame (CM) due to momentum conservation. This leads to the following relation for the azimuthal angles in CM:

$$|\phi_b - \phi_B| = \pi.$$ 

This relation is unaffected by the transformation from CM to the laboratory frame, as the transformation is done along the beam axis, changing only $\theta$'s. The transformation is slightly dependent on the incoming angle. The effect is negligible though, as it will be shown from the simulations. Combining the two equations we get the following relation:

$$y_b x_b = y_B x_B.$$ 

If this relations does not hold for an event, the $x$- and $y$-values are wrong, implying that the reaction did not occur in origin but at a point we will note as $(x_A, y_A)$ to indicate it is the $x$- and $y$-value of the incoming particle. The equation is made into a function of $x_A$ and $y_A$ to determine the offset:

$$y_A - y_{A_0} x_A - x_{A_0} = y_B - y_A x_B - x_A.$$ 

This gives a linear relation between the offset in the $x$- and $y$-direction. The offset is assumed to be as minimal as possible, leading to the following minimization problem:

$$\min(x_A^2 + y_A^2),$$

which can be solved using a Lagrangian multiplier:

$$x_A = \frac{x_B y_B - x_B y_b}{(y_B - y_b)^2 + (x_B - x_b)^2(x_B - x_b)}, \quad (6.1)$$

$$y_A = \frac{x_B y_B - x_B y_b}{(y_B - y_b)^2 + (x_B - x_b)^2(x_B - x_b)}, \quad (6.2)$$

The offset for each coincidence event can now be determined using these two equations. The form of the beam can be illustrated by plotting $x_A$ vs. $y_A$ (xy-plots). Fig. 6.2
shows $xy$-plots for four simulations with parameters shown in table 6.1. The A-D corresponds to the ones in table 6.1. Fig. 6.2A shows the analysis of the simulation with a thin beam. The width of the spot in fig. 6.2A arises from the uncertainty in the $x$- and $y$-values of the AD. The same effect is seen in fig. 6.2B, where the spot size of 6 mm diameter is again 1 mm to large. All this indicates an uncertainty of 1 mm in the diameter coming from the uncertainty in the AD strip sizes. Two things should be noted.

First, the distribution in fig. 6.2B is peaked at (0,0) rather than uniformly distributed, indicating that the assumption of a minimal shift is too strong. This makes it difficult to give a description of the distribution of the beam. Simulations with different distributions have to be performed to see the effect of the distribution. A gaussian distribution with a FWHM width of 5 mm is compared to simulation B in fig. 6.3. The projection on the x-axis ($x_A$) is shown as well. It will be shown in section 6.3, that the experimental data is closest to a uniform distribution, hence the gaussian simulation will not be used more in this chapter.

Secondly, the shape of the peaks are determined partly by the shape of the beam and partly by the setup. The T-REX setup lacks detectors close to $\phi = \pi/4 + N\pi/2$, which causes the missing events at $x = \pm y$ in fig. 6.2. A setup with $4\pi$ angular coverage would produce a perfect circle for a circular beam spot, as used in the simulation.

The total offset can also be determined, illustrated in fig. 6.2C. Comparing fig. 6.2B and C the two spots are identical except the latter is shifted -1 mm in $x$ corresponding to the shift added to simulation C. This shows that a shift in the beam will be detected with this technique. Once again the beam spot is peaked at (0,0) in fig. 6.2C, confirming the insensitivity to the detailed distribution of the beam spot. The final

<table>
<thead>
<tr>
<th>Sim.</th>
<th>$w_A$ (mm)</th>
<th>$s_A(x,y)$ (mm)</th>
<th>$(\theta_A,\phi_A)$ (mrad)</th>
<th>$\Delta_A$ (mrad)</th>
<th>$E^*$ (MeV)</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>0</td>
<td>(0,0)</td>
<td>(0,0)</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>B</td>
<td>5</td>
<td>(0,0)</td>
<td>(0,0)</td>
<td>0</td>
<td>0/3.31</td>
</tr>
<tr>
<td>C</td>
<td>5</td>
<td>(-1,0)</td>
<td>(0,0)</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>D</td>
<td>5</td>
<td>(0,0)</td>
<td>(87,0)</td>
<td>0</td>
<td>0</td>
</tr>
</tbody>
</table>

Table 6.1: The beam structure for the four Geant4 simulation used to check the technique. All simulations are made with a uniform distribution on a disc in the $xy$-plane. $w$ is the diameter, $s$ is the offset, $\theta$ is the incoming angle and $\Delta$ the divergence of the beam. $E^*$ is the excitation of $^{10}\text{Be}$. 
6.2. Beam characterisation

Figure 6.2: A plot of $x_A$ vs. $y_A$ for the four simulations in table 6.1. A-D refers to the simulation numbers. The shape of the peaks are caused by the setup, which does not cover $4\pi$.

In conclusion, the width, shape and offset of the beam can be determined with an uncertainty of $\Delta w = 1$ mm using only the $x$- and $y$-coordinates of the two outgoing particles.
6.2.2 Divergence and direction of the beam

The direction of the beam can be determined from momentum conservation. The momentum vector of the beam is given by adding the momentum vectors of the two outgoing particles:

\[ \vec{p}_A = \vec{p}_b + \vec{p}_B \] (6.3)

with the momentum vectors of the outgoing particles given by:

\[ \vec{p} = \sqrt{\frac{2mE}{x^2 + y^2 + z^2}} (x, y, z). \]
The incoming angle and azimuthal angle is then given by:

\[
\tan \theta_A = \sqrt{\left(\frac{P_x}{p_A}\right)^2 + \left(\frac{P_y}{p_A}\right)^2}
\]

\[
\tan \phi_A = \frac{p_y}{p_x}.
\]

The incoming angle (\(\theta\)) and the azimuthal angle (\(\phi\)) for each beam particle can be determined by combining the four equations. This way the \(x\)- and \(y\)-component of the angle can be determined by \(\theta_x = \theta \cos \phi\) and \(\theta_y = \theta \sin \phi\). The dependency on all eight observables can lead to large uncertainties in the results.

The angle have been calculated for the four simulations from table 6.1. Fig. 6.4 shows a \(\theta_x\) vs. \(\theta_y\) plot for the four simulations. The large effect of the width of the

![Figure 6.4: A plot of \(\theta_A \cos \phi\) vs. \(\theta_A \sin \phi\) for the four simulations in table 6.1. A-D refers to the simulation numbers.](image-url)
beam on the calculated angle is clearly seen. The effects of the energy and z-position can be ignored in the simulations and it will be shown in section ?? that the effect is negligible when applying this technique to the experimental data. Fig. 6.4A shows that the uncertainties in x and y lead to a 35 mrad uncertainty in $\theta_A$. The dependency on x and y is confirmed by simulation B. The width of the beam leads to a significant spread in the calculated incoming angles ($\theta_A = 70$ mrad), fig. 6.4B. The comparison of fig. 6.4C and D shows the difference between an offset in the beam and an incoming angle in the beam. The offset is transformed into an offset in the calculated incoming angle, while an angle will lead to a spread in the calculated angle. The large spread in fig. 6.4D indicates that the framework for the incoming angle breaks down at large angles.

To reduce the influence of the beam width and offset on the calculated divergence and direction of the beam the former should be determined before calculating the angles.

### 6.2.3 Combining the offset and the angle measurements

The strong dependence of the beam width on the incoming angle shown in section ?? can be investigated further and the resolution on the determined incoming angle can be improved. This will be done in this section.

Fig. 6.5 shows a plot of the $x_A$-value determined in section 6.2.1 vs. $\theta_x$ determined in section 6.2.2 for the four simulations. A similar plot can be made for the $y$-components, which will not be shown, as the interpretations are similar to the ones for the $x$-components. Ideally these plots would show the emittance of the beam, but from fig. 6.5A it is clear that experimental resolution effects must be understood first. The beam in fig. 6.5A has 0 mm mrad emittance. The extra width is caused by the same uncertainties seen in fig. 6.2 and 6.4. Instead the plots can be used to determine the offset, width, angle and divergence independently.

Each plot can be interpreted as having two stretched components, a vertical one at $x_A = 0$ mm and a diagonal one ($\theta_x \approx 25x_A$). The two components are centered around (0,0) for simulation A and B, as expected. The diagonal in fig. 6.5C is shifted -1 mm in x compared to fig. 6.5B, indicating that an offset in the beam will lead to a shift in the diagonal component corresponding to the offset of the beam, and only a minor shift in the vertical one. An angle in the beam will on the other hand affect the vertical component, as shown in fig. 6.5D, but not the diagonal one. The vertical component in fig. 6.5D is centered at $\theta_x \approx 80$ mrad, compared to the 87 mrad used in the simulation. Furthermore, a width in the beam will stretch the diagonal component while a divergence in the beam will stretch the vertical component, hence all four parameters (offset, width, angle and divergence) can be determined from a $x_A$.
Figure 6.5: A plot of $x_A$ vs. $\theta_A \cos \phi$ for the four simulations in table 6.1. A-D refers to the simulation numbers.

The two components are an effect of the setup, and with a setup covering $2\pi$ the two components will merge into one. The two components can be separated by gating only on light particles hitting the left or right PSD, or the top and bottom. This is done for simulation C, and can be seen in fig. 6.6. It is clear from fig. 6.5 and 6.6, that the top and bottom PSD’s should be used to determine the offset in the $x$-direction ($x_A$) and the left and right PSD’s should be used to determine the $x$-component of the incoming angle. The opposite is the case for the $y$-components. Ideally only particles detected in the $xz$-plane should be used to determine $\theta_z$ and $y_A$, due to the large sensibility to these two components and the insensibility to $\theta_y$ and $x_A$. Likewise, only particles in the $yz$-plane should be used to determine $\theta_y$ and $x_A$.

Thus an $x_A$ vs. $\theta_z$ and a $y_A$ vs. $\theta_y$ plot will be used to determine the angle and divergence of the incoming beam in section 6.3.
6.2.4 Beam energy

The beam energy is determined in two ways, using first energy and then momentum conservation. In both cases the beam energy is determined at the reaction point, assumed to be in the center of the target. The beam energy before the target can then be calculated by adding the energy lost in the target before the reaction.

Energy conservation gives:

\[ T_A = T_b + T_B + E^* - Q. \]

\( E^* \) is the total excitation energy of the outgoing particles, but normally only B is excited. In this experiment only \(^{10}\text{Be} \) can be excited. The excitation energy cannot be directly measured, hence the beam energy minus the excitation energy is calculated instead, and then a correction for the excitation energy is made afterwards:

\[ E = T_A - E^* = T_b + T_B - Q. \]

The advantage of this method is the independency on the position of the reaction, the only uncertainties arises from the energy of the particles and the energy loss in the target. The disadvantage is the required need of information about the excitation energy.

The second method is calculating the beam energy from:

\[ T_A = \frac{(P_{x_A}^*)^2 + (P_{y_A}^*)^2 + (P_{z_A}^*)^2}{2m_A}. \]
The momentum vector is calculated using eqn. 6.3. This method gives the beam energy, but it is dependent on the position of the reaction, making the uncertainty dependent on the width of the beam.

Fig. 6.7 shows the calculated beam energy from simulation B in table 6.1. In the simulation half of the events populate the ground state of \(^{10}\text{Be}\) and the other half populate the first excited state \((E^* = 3.31 \text{ MeV})\). The beam energy is set to \(T_A = 31.35 \text{ MeV}\) at the reaction point. Fig. 6.7A is the energy calculated using energy conservation. Two peaks are shown at 31.24 and 27.90 MeV with widths 0.150 and 0.157 MeV respectively. The lowest peak is from events populating the excited state in \(^{10}\text{Be}\), and it is lowered with 3.3 MeV corresponding to the excitation energy as expected. Fig. 6.7B shows the energy determined from momentum conservation, only one peak emerges as the method ignores the excitation energy, but the peak is broader than the two in fig. 6.7A due to the uncertainty in the \(xy\)-plane. The energy calculated with the second method is 31.45 MeV with a 0.5 MeV width. The beam energy set in the simulation is within the error of all three reconstructed energies proving the validity of the two equations.

![Figure 6.7](image-url)  
Figure 6.7: Spectra of the incoming energy determined from energy conservation (left) and momentum conservation (right). The spectra are made from simulation B.

Which method to use depends on the reaction, the setup and information known about the final states. The first method requires a clear information about the excitations of the final nuclei and has a stronger energy dependency. The second method is limited due to the beam spot. The methods can also be used together for confirmation as it will be done in section 6.3.

In conclusion the beam energy can be determined within 0.2 MeV with both methods. The spread in the energy determined using momentum conservation is affected by the beam width. A beam width of 5 mm leads to a FWHM energy spread
of $\Delta E = 0.5$ MeV.

### 6.3 Experimental data

The method will now be applied to the $d(^{11}\text{Be},t)^{10}\text{Be}$ data of the experiment.

The first step is determining the beam width, offset and shape. Fig. 6.8 shows a plot of the calculated $x$’s and $y$’s, both 2D and projections. Only the outline of the plot is useful, hence no $z$-color is used. Comparing the projections with fig. 6.3C+D shows indications of a round beam spot with a uniform distribution and a small gaussian tail. The uniform distribution, which makes up the main part of the beam, has a diameter of 6 mm and is shifted $-1.3$ mm in the $x$-plane. The tail arises partly from particles in the beam halo and partly from beam particles scattered on the edge of an 8 mm collimator placed 176 mm before the target.

To determine the incoming angle and the divergence of the beam a plot similar to the ones in fig. 6.5 has to be made, both for the $x$- and $y$-component, fig. 6.9A and B.

The incoming angle is determined first from the vertical components. $\theta_x$ is shifted $20(5)$ mrad compared to simulation C, while $\theta_y$ is centered around zero. This indicates an incoming angle of $\theta_A = 20(5)$ mrad towards the left.

A simulation using the determined value of the beam width ($w_A = 6$ mm), offset ($s_A = -1.3$ mm) and angle ($\theta_A = 20$ mrad) is made to determine the divergence. The $x$ vs. $\theta_x$ and $y$ vs. $\theta_y$ for the simulated data is plotted in fig. 6.9C and D respectively.

The difference between the simulated and the experimental data is caused by either the tail of the beam width (the diagonal component) or the divergence (the vertical component). The majority of the events is in the experimental and the simulated data are placed within the same area, confirming a beam spot of 6 mm shifted $-1.3$ mm in $x$ and with an incoming angle of 20 mrad. This also indicates a very small divergence of the beam, and an upper limit is set to $\Delta \theta \leq 30$ mrad. Small tails in both width and divergence are present, but they drop off very rapidly.

The rms emittance of stable beams accelerated to 2.85 MeV/u at REX has been determined as $\leq 2.8$ mm mrad. To a good approximation, the particle distribution is Gaussian in $(x, x')$ phase space and therefore 95% of the beam is bounded by an emittance a factor 6 times larger than the rms emittance at $\sim 17$ mm mrad. The emittance was measured with a high intensity 1 nA beam, composed mainly of $^{20}\text{Ne}$ leaked into the ion source from an adjacent Penning trap. An emittance meter was used employing both the slit-grid method and, with an upstream quadrupole, the three-gradient method to measure the emittance. It should be stressed that this mode of ion source operation is far from that typically used to charge breed radioactive beams and the increased intensity in this scenario is likely to cause degradation of the emittance from the source; the high intensity is needed to make the measurements. The results
Figure 6.8: A xy-plot of the experimental data from the $d^{(11}\text{Be},t)^{10}\text{Be}$ reaction. The plot should be compared to the plots in fig. 6.2 and fig. 6.3

of the stable beam measurement [?] predict just a few mrad of beam divergence at the target with a beam width of 6 mm, consistent with the analysis and upper limit for the emittance detailed in this paper.

The beam energy has been determined using both methods described in section 6.2.4, the plots of the beam energies can be seen in fig. 6.10. Method one, using energy conservation, produces three peaks as expected. $^{10}\text{Be}$ has six bound states, but the four highest are separated with only 300keV, making them indistinguishable in the analysis, see fig. 2.4. Method two, using momentum conservation, only produces one peak. The mean values and widths of the peaks are determined by a gaussian fit. The calculated energies along with the statistical uncertainties given by the fits can be seen in table 6.2 and the energy spreads in table 6.3. A wide distribution at low beam energies is seen in both methods, this is from reactions on the carbon in the
Figure 6.9: A plot of \( x_A \) vs. \( \theta_A \cos \phi \) (A) and \( y_A \) vs. \( \theta_y \) (B) for the experimental data and for simulation using a beam with \( w_A = 6 \) mm, \( s_A = -1.3 \) mm, \( \theta_A = 20 \) mrad and \( \phi_A = \pi/2 \) rad (C) and (D). In order to avoid saturation in the scatter plot, only one third of the data is plotted.

The width of the beam should lead to a larger width in method two, but the four peaks have comparable widths and all four peaks are much wider than the ones from the simulations (fig. 6.7). The average energy spread from the four peaks is \( \Delta \frac{E}{E} = 5\% \). This is higher than a previous determined beam spread measured using silicon detectors at REX-ISOLDE [7]. The beam energy was measured there for a 300 keV/u stable beam. The beam spread was determined to \( \Delta \frac{E}{E} = 0.5\% \) and an additional spread of \( \Delta \frac{E}{E} = 1.4\% \) was seen, caused by the silicon detectors. An increase in energy to 2.85 MeV/u should not increase the total beam spread from 2% to 5%. This indicates a new dominating effect.

The large energy spread is expected to stem from a large uncertainty in the re-
6.4. Beam intensity

The beam energy from the experimental data, determined from energy (left) and momentum (right) conservation.

Figure 6.10: The beam energy from the experimental data, determined from energy (left) and momentum (right) conservation.

action depth. The beam energy is calculated assuming the reactions occur halfway through the target. If the reaction happens in the beginning (or end) of the target, the energy loss corrections are wrong. The effect of the reaction depth has been investigated by calculating the beam energy using the experimental data assuming a reaction at three depths in the target; in the beginning of the target, in the middle and at the end of the target, fig. 6.11. The mean values are determined with gaussian fits and the energy loss in the target before the reaction is added. The calculation showed, that the uncertainty in the reaction depth leads to an uncertainty of $\Delta E_{\text{in}} = 0.8$ MeV in the reconstructed beam energy. Including this effect along with the uncertainty from the silicon detectors may account for most of the observed energy spread.

The expected beam energy of 31.35 MeV is within the uncertainty of the measured value, though slightly higher. The expected value of 31.35 MeV is calculated by extrapolating a beam energy measurement at 300 keV/u to 2.85 MeV/u. The measurement at 300 keV/u is a precise measurement performed on a stable beam [?]. The extrapolation will lead to an uncertainty in the provided beam energy, which is higher than the uncertainty from the coincidence events. Making the values determined from coincidence events more reliable.

6.4 Beam intensity

The beam intensity is needed to calculate the differential cross section, section 3.2. The elastic scattering cross section is mainly nuclear, section 8.2. Hence the elastic
channel cannot be used to determine the beam intensity. Instead six runs with a silver target have been performed throughout the experiment. These reactions are below the coulomb barrier and the elastic scattering can be used to determine the beam intensity. The beam intensities from the six measurements can be seen in table 6.4.

The beam intensity will fluctuate during the experiment. To see this fluctuation a plot of detected particles (p, d and t) pr. second for each run are plotted as a function of the file number (timescale), fig. 6.12. The plot shows a very intense beam at the beginning of the experiment, but the intensity slowly drops during the experiment. The six measurements of the intensity is also indicated in the plot. The number of detected deuterons pr. incoming $^{11}\text{Be}$ is determined by taking the number of detected deuterons pr. second from a run next to an intensity measurement, and divide by the beam intensity. The values are given in table 6.4. There is a significant fluctuation in the calculated ratios especially in the first (I) measurement. The first measurement is taken with a very intense beam, fig. 6.12, and the low beam intensity value is expected to be caused by deadtime due to the high intensity, hence this measurement is ignored. An average value for the last five is made:

$$\frac{N_d}{N_{^{11}\text{Be}}} = 0.76(17) \cdot 10^{-6}. \quad (6.4)$$

The total number of $^{11}\text{Be}$’s on the deuteron target can then be calculated by taking the total number of detected deuterons ($N_d^{\text{total}} = 842400$) and dividing it with the ratio, giving:

$$N_{^{11}\text{Be}}^{\text{total}} = 1.11(25) \cdot 10^{12}. \quad (6.5)$$
6.5 Background from contaminants in the target

With the detectors calibrated and under control, and the structure and energy of the beam determined, the only part left to understand, before calculating the differential cross sections, is the target.

The primary target is deuterated polyethylene, which contains, beside deuterons,
carbon and protons. $^{11}$Be can react with the protons and the carbon nuclei and produce protons, deuterons and tritons, which will form as a background in the analysis. An example of the background has already been seen in fig. 6.10, but it will be even more eminent when calculating excitation energy spectra, chapter 7. It is crucial to understand the background, in order to get the right normalisation and angular distribution. The coincidence events have proven extremely effective in the understanding of the background from contaminations in the target. While the background and the real events are strongly entangled in the single particle events (fig. 7.3), in coincidence events the two are clearly separated. Fig. 6.10 shows a clear separation between the peaks from (d,t) events and the broad, low energy, distribution from reactions on carbon in the target.

Runs have been performed in this experiment using a target of pure carbon and a regular polyethylene target in order to determine the background, section 4.4. The data from these runs are analysed separately, assuming the events are from $^{11}$Be+d reactions. This will generate distributions similar to the background in the reactions

Figure 6.12: The beam energy from the experimental data, determined from energy (left) and momentum (right) conservation.
### 6.5. Background from contaminants in the target

<table>
<thead>
<tr>
<th>Run</th>
<th>$I_{\text{beam}}$ [$10^6$/s]</th>
<th>$\frac{N_d}{N_{11\text{Be}}}$ [$10^{-6}$]</th>
</tr>
</thead>
<tbody>
<tr>
<td>I</td>
<td>3.93</td>
<td>2.3</td>
</tr>
<tr>
<td>II</td>
<td>6.12</td>
<td>0.86</td>
</tr>
<tr>
<td>III</td>
<td>5.52</td>
<td>1.0</td>
</tr>
<tr>
<td>IV</td>
<td>6.06</td>
<td>0.72</td>
</tr>
<tr>
<td>V</td>
<td>5.23</td>
<td>0.69</td>
</tr>
<tr>
<td>VI</td>
<td>4.46</td>
<td>0.55</td>
</tr>
</tbody>
</table>

Table 6.4: The measured beam intensities from runs with $^{11}\text{Be}$ on Ag, and the calculated ratio between number of incoming $^{11}\text{Be}$ and number of detected deuterons.

The distributions have to be scaled, to take into account the difference in run time and number of nuclei in the different targets. Normally the background distributions are scaled to fit the single particle events, but the entanglement can lead to a large uncertainty. The coincidence events are used in this experiment instead. The beam energy is calculated for all coincidence events with identified deuterons and tritons assuming $d(^{11}\text{Be},d)^{11}\text{Be}$ and $d(^{11}\text{Be},t)^{10}\text{Be}$ reactions. The calculations are done for all three targets (CD$_2$, C and CH$_2$). The deuteron data is used to determine the background from protons in the target, only a limit amount of $(p,t)$ events are seen but a significant amount of $(p,d)$ is detected. Fig. 6.14 shows the calculated beam energies for the three targets. Both energy (left) and momentum (right) conservation are used for $(d,t)$ (top) and $(d,d)$ (bottom).

The broad distribution is clearly from reactions on carbon nuclei in the target, and is used to determine the scaling factor for the carbon target data:

$$\text{run}_d = 7.5\text{run}_C$$

A rough estimate of the ratio between carbon nuclei and deuterons in the target can be given, by taking the scaling factor times the ratio between the total lengths of the runs. A scaling of 7.5% corresponds to a ratio of:

$$\frac{N_d}{N_{12\text{C}}} = 0.53 \text{ (close to 1:2 as expected).}$$

Determining the scaling factor for the proton target data is more complicated. Only the deuteron data can be used. A peak at 31 MeV is seen in the deuterated polyethylene data, which is caused by reactions from either carbon or protons in the target. The carbon data contributes to the peak but cannot reproduce the total peak
with the scaling factor determined from the broad distribution. The rest is either the
tail of the main peak at 29.56 MeV or reactions on protons. The peak from the proton
data (scaled with 0.4 in the figure) produces an energy slightly too high, but this might
be caused by an unknown effect. Only an upper limit can be set:

$$\text{run}_d = 0.2\text{run}_p$$  \hspace{1cm} (6.8)

This corresponds to approximately a 1% proton contamination in the deuterated
polyethylene.

The strong separation between the real and background data shows, that full
kinematic events can be used to effectively select true reactions from backgrounds
reactions from contaminants in the target, or to determine the ratio between true and
background reactions.
6.5. Background from contaminants in the target

Figure 6.14: $T_A$ calculated from energy (A+C) and momentum (B+D) conservation using coincidence events with identified tritons (A+B) and deuterons (C+D). The three spectra are made with data from three different targets; blue: CD$_2$, red: C and green CH$_2$. 

\begin{figure}
\centering
\includegraphics[width=\textwidth]{figure6.14.png}
\caption{T$_A$ calculated from energy (A+C) and momentum (B+D) conservation using coincidence events with identified tritons (A+B) and deuterons (C+D). The three spectra are made with data from three different targets; blue: CD$_2$, red: C and green CH$_2$.}
\end{figure}
Identifying the various reaction channels

The reaction type for each event will be determined in this chapter. This includes determining the type of the outgoing particle and the excitation of the outgoing nucleus. The chapter contains a high particle energy part and a low particle energy part, the difference between the parts is the ability to distinguish protons from deuterons and tritons for the high energy particles.

7.1 How to identify a reaction

To calculate the differential cross section the reaction type has to be determined for each event. This includes both separating transfer reactions from scattering and determining the state of the final nucleus. If the energy of the light particles are high enough to go through the $\Delta E$-detectors and into the $E$-detectors the first can be done by making a $\Delta E - E$ plot. The state of the outgoing nucleus is determined from both the excitation energy calculated using the charged particle, and gamma gates, section 7.2. If the energy is to low transfer reactions and scattering events have to be analysed together complicating the analysis, section 7.3.

7.2 Excitation energy spectra from high energy particles

High energy particles is defined in this thesis as particles with energy enough to go through the $\Delta E$-detectors, enabling particle identification. The energy required to pass through the $\Delta E$-detector, hence enabling particle identification, is dependent on the particle type, the detector thickness and the angle between the detector and the particle. Fig. 7.1 shows the energy required for punch through as function of the laboratory angle for each of the three particle types (solid lines). The kinetic curves of protons, deuterons and tritons for reactions populating the ground states of $^{12,11,10}$Be is shown as well (dashed lines). The high energy particles occurs in the very forward
angles, hence only particles in the forward PSD’s and AD’s are analysed with particle identification.

Figure 7.1: The energy required for protons (black), deuterons (red) and tritons (blue) to punch through the $\Delta E$-detector (solid lines). The kinetic curves for the reactions populating the ground states of $^{12,11,10}$Be is shown as well.

7.2.1 Particle identification

If the light particles have enough energy to go through the $\Delta E$-detectors a separation of the light particle types can be made by looking at the energy deposited in the $\Delta E$- and the $E$-detector. There is a strong relation between the energy deposited in the two detectors and the particle type as mentioned in section 5.5. This is shown in a $\Delta E - E$ plot. Fig. 7.2 shows a $\Delta E - E$ plot for strip 5 in the forward PSD’s. The plot shows four curves corresponding to protons, deuterons, tritons and $\alpha$-particles. The shapes of the curves are very dependent on the angle between the detector and the particle, hence a $\Delta E - E$ plot has to be made for each strip in the PSD’s. To compensate for the different angles within the strip, a small correction is made, assuming a linear relation between the energy loss and the range in a material [7].

\[
\begin{align*}
\Delta E_{\text{cor}} &= \Delta E \cos \theta. \\
E_{\text{cor}} &= E + (1 - \cos \theta)\Delta E.
\end{align*}
\]
Here $\theta$ is the angle between the norm of the detector and the incoming particle. This is only valid for small angles, hence the requirement of individual plots for each strip. Corrected values are used in fig. 7.2.

![Figure 7.2: A plot of the $\Delta E$ vs. $E$ deposited in the strip 5 of the four forward PSD's. The gates made to identify p, d and t are shown as red lines. $^4$He particles are also seen, but omitted in the analysis.](image)

Gates have been made for each of the three $Z=1$ particles for each $\Delta E - E$ plot. The gates can be seen in fig. 7.2. All gates are of the form:

$$E_{\text{cor}} = \frac{N}{a\Delta E_{\text{cor}} + b},$$

(7.3)

with $N, a$ and $b$ being parameters determined individually. If an event lies within two neighboring lines, the event will be classified as being that given particle type, and analysed as such (Particles between the two lowest lines are protons).
7.2.2 Excitation energy spectra

Once the light particle is identified as either a proton, deuteron or triton the excitation of the nucleus can be determined from energy and momentum conservation:

\[ E^* = T_A - T_b - \frac{P_B^2}{2m_B} + Q. \]  
(7.4)

\[ \vec{P}_B = \vec{P}_A - \vec{P}_b. \]  
(7.5)

The excitation spectra for \( ^{10,11,12}\text{Be} \) can be seen in fig. 7.3. The ground states are clearly seen in all three spectra. Furthermore peaks at 2100 keV and 2700 keV in \( ^{12}\text{Be} \), and at 3300 keV and 6000 keV in \( ^{10}\text{Be} \) corresponding to the excited bound states in the two nuclei. The energy resolution from the light particles are 500 keV, which is too high to distinguish the excited states in \( ^{12}\text{Be} \) and \( ^{10}\text{Be} \) and to distinguish elastic from inelastic scattering. Gamma gates are used to improve the resolution, section 7.2.3.

![Figure 7.3: Excitation energy spectra made from high energy particles. A: \( ^{12}\text{Be} \). B: \( ^{11}\text{Be} \). C: \( ^{10}\text{Be} \).](image)
the deuterons and protons ($\approx 100 : 1$). The uncertainty in the proton background, described in section 6.5, will not affect the angular distributions for the excited states, as gamma gates are used for those, see next section, but the elastic scattering cross section will be affected. The effect is very small, almost negligible, due to the much larger cross section for (d,d) than for (p,d). With a 0.4 scaling (twice the upper limit) of the data from the proton target, the ratio between scattered deuterons and deuterons from (p,d) is less than 0.1 %.

Figure 7.4: Excitation energy spectra made from high energy particles including the background from reactions on carbon (red line) and protons (green line) in the target. The grey area is the total background.
7.2.3 Gamma gated spectra

The low resolution in the charged particle spectra makes gamma gates required to distinguish the population of the various states, fig. 2.2, 2.3 and 2.4. The gamma gates are done for each reaction ((d,p), (d,d) and (d,t)).

All gammas except one produced from the three reactions are emitted from a moving nuclei, which doppler shifts the emitted gammas. The only decay, that is not doppler shifted is the $0^+_2 \rightarrow 0^+_1$ decay in $^{12}$Be, section 7.2.3.2. The doppler shift can be corrected using eqn. 5.13 by isolating the gamma energy:

$$E = \gamma E' (1 - \beta \cos \theta).$$

(7.6)

7.2.3.1 $^{11}$Be

$^{11}$Be is the easiest case of the three nuclei with only two bound states. The elastic and the inelastic channels can be separated by gating on the 320 keV gamma line in fig. 7.5. The peak is placed on top of a large background, and in order to remove events from random coincidences with background gammas a background gate is made. The two gates can be seen table 7.1.

![Figure 7.5: The gamma energy spectrum made from gammas in coincidence with a deuteron. The gammas are Doppler corrected. A strong sharp peak is eminent at 320 keV.](image)

The background spectrum is subtracted from the peak spectrum, and the spectrum is plotted together with the total excitation energy spectrum for $^{11}$Be in fig. 7.6.
The gamma gated spectra is scaled to take into account for the MINIBALL efficiency given in table 5.2. It is clear, that the elastic scattering is dominating as expected. Still there is sufficient data to calculate a differential cross section for both the elastic and the inelastic scattering data, chapter ??.

![Figure 7.6](image-url)

**Figure 7.6**: The total excitation energy spectra for $^{11}$Be (blue) along with the excitation energy spectrum for the excited state in $^{11}$Be made from gamma gated deuterons (red).

### 7.2.3.2 $^{12}$Be

$^{12}$Be has three known bound excited states, which all primarily decays to the ground state, section 2.3. The life times of the $2^+_1$ and the $1^-_1$ states are very short, and the decay produces a 2107 keV and a 2700 keV gamma respectively, table 2.1. The peaks from the two gamma lines can easily be seen in fig. 7.7. The two peaks lies on top of a small but significant background. The background is especially important for the 2107 keV, which lies on the Compton edge of the 2700 keV line. Gates are made for the two peaks, along with a background gate for the 2107 keV. The gates are shown in table 7.1.
The $0^+_2$ state mainly decays to the ground state with an $e^+e^-$ pair creation corresponding to 511 keV signals in the germanium detectors, table 2.1. This leads to a long lifetime for the $0^+_2$ state. The lifetime is measured to be $331(12)$ ns [?]. The $^{12}$Be nucleus has to be stopped somewhere within the reaction chamber, otherwise it will have moved to the beam dump before decaying. The maximum angle of the outgoing $^{12}$Be in the ground state is shown in fig. 6.1. The angle is slightly smaller for the excited $^{12}$Be nuclei, making the maximum angle $10^\circ$, which is still large enough to hit the AD’s. If the nucleus is stopped in the AD’s the decays can be detected by the germanium detectors. This enables a possibility of making a gamma gated spectrum for the $0^+_2$ state. Two things have to be taken into account when the gamma gated events are used to determine the differential cross sections. The efficiency of the MINIBALL is determined from decays at the target and not at the AD’s, section ??, hence a lower efficiency should be expected. Furthermore the angular distribution of the detected outgoing particles could be altered due to the requirement on the angle of the $^{12}$Be nucleus.

The long lifetime of the $0^+_2$ state enables a second gate. Fig. 7.8 shows the time between the detected proton and the detected gamma ($\Delta t = t_\gamma - t_p$) vs. the gamma energy. The gamma energy is not doppler corrected, as the $^{12}$Be is stopped at the time of the decay. The majority of the events is placed close to $\Delta t = -250$ ns, but events with a larger time difference occur for a few gamma energies. Some are random coincidences from gamma background, like the 2100 keV line from $\beta$-decay of $^{11}$Be,
but two of the lines come from reaction events. The 677 keV line is an E0 transition in $^{74}$Ge emerging from an excitation of $^{74}$Ge in the MINIBALL through an inelastic scattering with neutrons [7]. This will be investigated in section 10.3.2. The 511 keV is partly from background electrons, but mainly from the E0 transition in $^{12}$Be. The time difference signal can be used to determine the lifetime of the $0^+_2$ state, which will be done in section 10.1, but it can also be used to lower the background for the $0^+_2$ spectrum by requiring a $\Delta t > 200$ ns, table 7.1.

\begin{figure}[h]
\centering
\includegraphics[width=\textwidth]{gamma_energy_vs_time.pdf}
\caption{The gamma energy vs. the time between the detection of the proton and the detection of the gamma.}
\end{figure}

The gamma gated excitation spectra are shown in fig. 7.9 with the backgrounds subtracted. The three peaks are centered at 2100 keV, 2200 keV and 2700 keV, which indicates true events and reliable gamma gates. The three gamma gated spectra are added together with the background spectra and compared to the total excitation energy spectrum. The overlay of the two spectra is almost perfect, but some events populating the excited states are still to be accounted for.
Figure 7.9: Top: Excitation energy spectra from gamma gated protons. Black: \(2^+_1\) (2107 keV), blue: \(0_2^+\) (511 keV) and red: \(1^-_1\) (2700 keV). All spectra are scaled to take the MINIBALL efficiency into account. Bottom: The total excitation energy spectrum (blue) and the excitation energy spectrum made from background runs and gamma gated spectra (red).

7.2.3.3 \(^{10}\text{Be}\)

\(^{10}\text{Be}\) is the most complicated of the three nuclei due to the four close lying states around \(E^* = 6\) MeV. It has been possible to identify five out of the six bound states in \(^{10}\text{Be}\). The decay lines of \(^{10}\text{Be}\) is shown in table 2.1. Four of the gamma lines are clearly seen, as shown in fig. 7.10. The figure shows the gamma energies versus the excitation energy calculated from the tritons. The projections onto the two axis are shown along with the total plot, showing the gamma spectrum for the vertical axis and the excitation energy spectra for the horizontal axis. The plot is made to show
the sequential decay. Horizontal lines situated around the diagonal \((E_\gamma = E^*)\) is the direct decay to the ground state. Both the 3367 keV decay of the first excited state and the 5959 keV decay of the \(1^+_1\)-state is seen. It is evident from the figure, that not all detected gammas around 3367 keV are from the \((d,t)\)-population of the first excited state. A large part of the gammas comes from a population of the \(2^+_1\)-state via a gamma decay from one of the higher lying states. Gating on the 3367 keV gamma line produces two peaks in an excitation energy spectrum, fig. 7.11. The peak at 3300 keV is the \(2^+_1\)-state. The other peak might contain components from all the higher lying states, making it harder to use. Instead the two gamma lines rising from the gamma decay too the \(2^+_1\)-state are used. The two sharp peaks at 2600 keV and 2900 keV can clearly be separated and the statistics from the two decays are sufficient to calculate differential cross sections, hence the 3367 keV gammas can be ignored for the high lying states, and is only used to calculate the differential cross section for the \(2^+_1\)-state.

The excitation energy spectra, made from gates on the 2600 keV and the 2900 keV gamma peaks, each have one peak, fig. 7.11. The mean value of the peaks are 5839 keV and 6151 keV corresponding to either the \(2^+_2\) or the \(1^+_1\)-state for the former and the \(0^+_2\) or the \(2^+_1\)-state for the latter. The amount of each state in the two peaks have to be determined. The \(2^+_2\) and the \(2^+_1\)-states only have one gamma decay channel (2590 keV and 2895 keV respectively) and no more details can be given for these states. The \(0^+_2\) and \(1^+_1\)-states, on the other hand, have two decay channels, which can be used to determine both the total amount of population of the two states, and the amount of each states in the two gamma gated spectra at 2600 keV and 2900 keV.

The \(1^+_1\)-state decays both to the ground state and the \(2^+_1\)-state with a ratio close to 2:1 (table 2.1) producing a 5959 keV and a 2590 keV respectively. The \(1^+_1\)-state is the only one of the four highly excited states, that decays to the ground state, and all the 6 MeV gammas seen fig. 7.10 can be seen as coming from the \(1^+_1\)-state. An excitation energy spectrum is made by gating on the 6 MeV peak including the Compton edge to gain enough statistics, table ???. The excitation energy spectrum is seen in fig. 7.11 as the green line. The spectra is scaled by the factors given in table ??, but the MINIBALL efficiency calculated in section 5.7 is done for the peak energies, and not for the peak+Compton edge. The efficiency will be larger if the Compton edge is taken into the gates, as is the case for the \(1^+_1\)-state. The new efficiency is estimated from a simple formula:

\[
\varepsilon_{\text{peak+compt}} = \frac{N_{\text{peak+compt}}}{N_{\text{peak}}} \varepsilon_{\text{peak}} = 9.5\%.
\]  

This is scaling factor used in fig. 7.11. The overlap between the 6 MeV spectrum and the 2600 keV is almost perfect. The mean value is the same as expected (the two states producing the 2600 keV gamma is only separated by 5 keV), but the integrals, after
Figure 7.10: A \( xy \)-plot of the experimental data from the \( d(^{11}\text{Be},t)^{10}\text{Be} \) reaction. The plot should be compared to the plots in fig. 6.2 and fig. 6.3.

scaling, are also similar \((N_{2600} = 2006 \text{ and } N_{6000} = 1951)\). Taking the branching ratios of the \( 1^- \)-decays and using the integral of the 6 MeV peak leads to a half of the events in the 2600 keV is from the \( 1^- \)-state, the rest is from the \( 2^+_2 \)-state.

The \( 0^+_2 \)-state decays to the \( 2^+_2 \)-state 34\% percent of the time, producing a 219.9 keV gamma, table 2.1. The 1:2 ratio between the 219.2 keV and the 2811 keV is fully compensated by the difference in detection efficiency of the MINIBALL (table 5.2), and the 219.2 keV should be the main peak of the two in the gamma spectrum. No indications of a gamma line at that energy is seen in fig. 7.10, and the population of the \( 0^+_2 \), in this experiment, is assumed to be negligible. Hence the 2900 keV peak is expected to be fully \( 2^+_1 \).

All four gamma gated spectra for \( ^{10}\text{Be} \) is now well understood, and the strong
populations of the $1^-_1$ and the $2^-_1$-states compared to the $2^+_2$ and the $0^-_2$-state are in excellent agreement with the requirement of two step processes for the latter two, section 2.4. Adding the four together, along with the background and comparing with the total excitation energy spectrum, fig. 7.11, a perfect overlay is seen.

<table>
<thead>
<tr>
<th>Nucleus</th>
<th>Decay</th>
<th>$E_{\gamma}$ [keV]</th>
<th>$E$-gate [keV]</th>
<th>$\Delta t$-gate [ns]</th>
</tr>
</thead>
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<tr>
<td>$^{11}$Be</td>
<td>$1/2^-_1 \rightarrow 1/2^+_1$</td>
<td>320</td>
<td>[300,340]</td>
<td>–</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>Background</td>
<td>[340,380]</td>
</tr>
<tr>
<td>$^{12}$Be</td>
<td>$2^+_1 \rightarrow 0^+_1$</td>
<td>2100</td>
<td>[2060,2160]</td>
<td>–</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>Background</td>
<td>[2160,2260]</td>
</tr>
<tr>
<td></td>
<td>$0^+_2 \rightarrow 0^+_1$</td>
<td>511</td>
<td>[490,520]</td>
<td>&gt; 200</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>Background</td>
<td>[475,490] or [520,535]</td>
</tr>
<tr>
<td></td>
<td>$1^-_1 \rightarrow 0^+_1$</td>
<td>2700</td>
<td>[2650,2800]</td>
<td>–</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>Background</td>
<td>[2800,3000]</td>
</tr>
<tr>
<td>$^{10}$Be</td>
<td>$2^+_2 \rightarrow 2^+_1$</td>
<td>2600</td>
<td>[2500,2750]</td>
<td>–</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>Background</td>
<td>[2800,2650]</td>
</tr>
<tr>
<td></td>
<td>$2^-_1 \rightarrow 2^+_1$</td>
<td>2900</td>
<td>[2800,3000]</td>
<td>–</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>Background</td>
<td>[3300,3500]</td>
</tr>
<tr>
<td></td>
<td>$2^+_1 \rightarrow 0^+_1$</td>
<td>3300</td>
<td>[3300,3500]</td>
<td>–</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>Background (LE)</td>
<td>[3500,3600]</td>
</tr>
<tr>
<td></td>
<td>$1^-_1 \rightarrow 0^+_1$</td>
<td>5900</td>
<td>[4600,6200]</td>
<td>–</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>Background (LE)</td>
<td>[6200,6600]</td>
</tr>
</tbody>
</table>

Table 7.1: The gamma energy and time gates used to separate the individual reactions from each other in the three nuclei. (LE) means the gate only has been used for low energy particles

### 7.3 Analysis of Low energy particles

A large fraction of the detected particles can not be identified through a $\Delta E - E$ identification, due to too low energy. Fig. 7.1 clearly shows, that at large laboratory angles the energy is too low to make particle identification. Large laboratory angles in inverse kinematics corresponds to small CM angles in direct kinematics, which holds
great information in the differential cross sections. Thus low energy particles are of a great importance, and the separation of the individual reactions is a significant part of the analysis.

Note that a lower limit of 2MeV is set due to noise in the setup, and no particles are detected with energy lower than 2MeV.

The analysis is done in two steps, one for the backward angles and one for the forward angles.

### 7.3.1 Backward laboratory angles

The only reaction producing particles, with high enough energy to be detected, in the backward angles is the d(\(^{11}\)Be,p)\(^{12}\)Be reaction. Due to the high lower limit and the low energy in the reaction, only the ground state can be identified in the backward angle. All particles are therefore analyzed as protons from a d(\(^{11}\)Be,p)\(^{12}\)Be reaction, and the excitation energy spectra for \(^{12}\)Be is calculated, fig. 7.12. Fig. 7.12A shows the total excitation spectra, and it is clearly dominated by background. By subtracting the background caused by contaminants in the target (section 6.5) a much cleaner spectrum can be made, fig. 7.12B. The ground state population is clearly seen, and can be used to calculate the differential cross section in the very forward CM angles. A significant background is still present, which can lead to an overestimate of the differential cross section.

![Figure 7.12: The excitation energy spectrum for low energy particles in the backward angles. The ground state of \(^{12}\)Be is clearly seen when the large background is subtracted.](image-url)
7.3.2 Forward laboratory angles

The analysis in the forward direction is more complicated due to the larger amount of reactions and populated states, but also due to a larger background from heavy fragments like scattered $^{11}$Be on carbon or $^{10}$Be as described in Chapter 6, but also $^8$Li and $^6$He, which have large outgoing angles, are detected.

The analysis can be divided into two steps identifying the reactions populating the excited states through gamma gates, and finally determining the reactions populating the ground states of $^{10,11,12}$Be. The last part requires a removal of the background.

7.3.2.1 Excited states

The gamma gates shown in table 7.1 should also be valid for the low energy particles, but it has to be checked. Fig. 7.14 shows the gamma energies after correction for doppler shift and the gamma energies versus the time difference between the gamma and the particle are shown in fig. 7.13. Fig. 7.13 clearly shows time delayed 511keV gammas, but ones again a background from other electrons could be significant. The gamma gated spectra for the $0^+_2 \rightarrow 0^+_1$ reaction shown in fig. 7.15A shows a larger background even after subtraction of the background determined from the background gate. The peak at 2200keV is clearly seen in the spectra, but the background could lead to an uncertainty in the final results.
Figure 7.13: The gamma energy vs. the time between the detection of a low energy particle and the detection of the gamma. To avoid saturation in the scatter plot only 0.4 of the events are plotted.
Figure 7.14: Gamma energy spectrum for gammas in coincidence with low energy particles. The spectrum is divided into three, due to the large difference (15000 to 100) in scales from low energy gammas to high energy gammas.

The large difference in background from 320keV to 6000keV makes it harder to identify all the gamma lines in a total gamma energy spectrum, instead the spectrum is divided into three energy intervals shown in fig. 7.14. The 320keV gamma line from the inelastic scattering is clearly seen, but the picture is much more complicated between $E_\gamma = 2\text{MeV}$ and $E_\gamma = 3.5\text{MeV}$ where five gamma lines are placed within 1.5MeV. The 2700keV gamma line from the $1^-_1 \rightarrow 0^-_1$ decay in $^{12}\text{Be}$ placed between the 2600keV and the 2900keV from decays in $^{10}\text{Be}$. Causing further uncertainty to the 2600keV line due to the Compton edge of the 2700keV line. Yet the resolution of the MINIBALL is small enough to distinguish all five states. The gating on the
Chapter 7. Identifying the various reaction channels

2100keV line from the $2^+_1 \rightarrow 0^+_1$ decay in $^{12}$Be is complicated by a large background peak at 2100keV caused by the $\beta$-decay of $^{11}$Be. $E_\gamma = 2100$keV is the strongest gamma line from the $\beta$-decay of $^{11}$Be, table 5.1. The gammas are not doppler corrected, as they come from stopped $^{11}$Be somewhere in the setup, and the peak is clearly seen in fig. 7.13. This peak will be divided into two peaks in the correction of the doppler shift, and can be seen in fig. 7.14 as the two outer peaks of the three at 2100keV. The one in the middle is the gammas from the decay in $^{12}$Be. To remove background from random coincidences with gamma from the $\beta$-decay a further gate is placed on gammas for the $2^+_1 \rightarrow 0^+_1$ decay:

$$E'_\gamma < 2100 \text{ or } 2140 < E'_\gamma.$$  \hspace{1cm} (7.8)

Here $E'_\gamma$ is the raw gamma energy without any correction for doppler shift.

The large gamma background becomes a problem for the 5959keV gamma line. No sharp peak is identified in the bottom plot in fig. 7.14, only a broad distribution starting at $E_\gamma = 4.2$MeV. A background has to be subtracted when making the excitation spectrum for the $1^-_1$-state in $^{10}$Be. The background is made at the high energy end (table 7.1) and has to be scaled with a factor given by:

$$\frac{N_{5959}}{N_{bg}} = \frac{\varepsilon_{4200} + \varepsilon_{6200}}{2\varepsilon_{4200}} \frac{L_{5959}}{L_{bg}} = 5.56.$$  \hspace{1cm} (7.9)

With $\varepsilon$ being the MINIBALL efficiency at the given energy, and $L$ is the length in energy interval for the two cuts. Normally the scaling factor is given only by the ratio in the length, but due to the large change in efficiency from 4200keV to 6200keV an extra factor has to be added.

Excitation energy spectra of $^{10,11,12}$Be are shown for the gamma gated events in fig. 7.15. The background is subtracted in all the spectra, and clear peaks with only limited background left are shown. The energy of the peaks correspond perfectly with the excitation energy of the states. The spectrum in the middle, corresponding to inelastic scattering, has a broad distribution ranging from 2 – 3.5 MeV. This background will be investigated in chapter 10.

Once again the amount of the $1^-_1$-state in the 2600keV gamma gated spectrum has to be determined. This is done similar to the high energy tritons and the result is 41 % of the peak is from the $1^-_1$-state, consistent with the high energy result.
7.3. Analysis of Low energy particles

Figure 7.15: The Excitation energy spectra from gamma gated low energy particles. The Particles are analysed as protons (top), deuterons (middle) and tritons (bottom). The colors correspond to the same as the in the excitation energy spectra from high energy particles.

7.3.2.2 Ground states

Gamma gates cannot be used to identify reactions populating ground states, hence another method has to be used. Like for the high energy particles, the ground states can be found by subtracting background and excited states from the total excitation
spectrum. Generally that would mean calculating three excitation energy spectra assuming all particles being protons, deuterons and tritons respectively, but of the three reactions populating the ground states of $^{12,11,10}$Be, only the elastic scattering produces low energy particles in the forward direction, fig. 7.1. Hence only one excitation spectrum has to be made, assuming all particles are scattered deuterons. The total excitation spectrum (top) and the total spectrum with excited states and background subtracted (bottom) are shown in fig. 7.16. A peak is clearly seen at 0MeV and no indication of further background is present in the bottom spectrum. The extra components at higher energies are a combination of inelastic scattered deuterons into the continuum of $^{11}$Be and other heavy fragments like $^8$Li and $^6$He, not investigated in this thesis.

Figure 7.16: Top: An excitation energy spectrum for $^{11}$Be assuming all low energy particles are scattered deuterons. Bottom: The same as the top one, but with background and reactions to excited states subtracted using gamma gates.
In summary, we have been able to identify reactions to all but one bound states in $^{12}\text{Be}$, $^{11}\text{Be}$ and $^{10}\text{Be}$ using gammas detected by the MINIBALL. The identification has been done in a large angular range both with and without particle identification through $\Delta E - E$. The high beam intensity has compensated for the low detection efficiency of the MINIBALL above 2MeV. Even at 6MeV with an efficiency of $\epsilon = 2\%$ we were able to get sufficient statistic to make a differential cross section for the $1^{-}$ state in $^{10}\text{Be}$. Comparisons of gamma gated and total excitation energy spectra have shown an almost complete identification of all high energy particles and 75\% of the low energy particles.

The calculation of the differential cross sections and the comparison with theoretical models will be described in the next chapter.
Figure 7.11: Top: The excitation energy spectra for the excited states in $^{10}$Be made from the gamma gates shown in table 7.1. Black: $E_{\gamma} = 3300$ keV, red: $E_{\gamma} = 2900$ keV, blue: $E_{\gamma} = 2600$ keV and green: $E_{\gamma} = 6000$ keV. Bottom: The total excitation energy spectrum (blue) and the excitation energy spectrum made from background runs and gamma gated spectra (red).
Differential cross sections and spectroscopic factors

The experimental differential cross sections for all the individual reactions will be determined in this chamber. The experimental cross sections will be compared to preliminary theoretical calculations. Preliminary spectroscopic factors will be determined and the effects of the deformation and the halo structure of $^{11}$Be will be investigated.

8.1 Differential cross sections

The aim of the experiment was to determine differential cross sections for the scattering and transfer reactions, and compare them to theoretical calculations. The comparisons are done separately for the three reactions ($(d,d)$, $(d,p)$ and $(d,t)$), and the theoretical calculations are done with two models. The difference between the models is the potential from $^{11}$Be felt by the deuteron. The complicated structure of $^{11}$Be described in chapter 2 has a large impact on the differential cross sections. This is well known and have been seen in various experiments [ ]. In this thesis, I will focus on the effect of the deformation of $^{11}$Be and the large coupling to the first excited state of $^{11}$Be in these reactions.

8.2 Calculating cross sections from the experimental data

The events were separated according to their reaction types in the previous chapter. With the reactions successfully identified the differential cross sections can be calculated using eqn. 3.6. Three of the parameters are already determined, $n_t = 7.47 \times 10^{-8}$ mb (section 3.2), $N_{A}^{\text{exp}} = 1.12 \times 10^{12}$ (section 6.4) and $N_{A}^{\text{sim}} = 1 \times 10^{6}$, leaving only the angular distribution ($\alpha$) to be determined.

The angular distributions are determined by splitting the excitation energy spectra, calculated in chapter 7, into small intervals of center of mass angles, fig. 8.1. Each spectrum covers an angular range of $3^\circ$ in center of mass. The same spectra are generated for the simulated data, and the scaling factor ($\alpha$) in a given center of mass angle
is determined by fitting the simulated data to the experimental data using eqn. 3.8. Fig. 8.1 shows ten of these excitation energy spectra for the $2^+_1$-state in $^{12}$Be along with the scaled spectra from the simulated data. The angular distributions of the excited states are determined from the gamma gated spectra. The fit is performed by adding the simulated spectra to the spectra from the background gates instead of subtracting the background gated spectra from the peak gated. This provides a more reliable uncertainty from the fit. The angular distributions for the ground states are calculated from the total excitation energy spectra. Again background spectra are made using background runs and gamma gated spectra, and the scaled simulation is added to these spectra in the fit.

Differential cross sections are calculated for all the bound states in $^{10,11,12}$Be except for the $0^+_2$-state in $^{10}$Be, which was not seen in the analysis. A differential cross section is determined from the total number of events in the 2600 keV gamma peak in $^{10}$Be. This cross section should be a combination of the differential cross sections from the $1^-_1$ and the $2^+_2$-states and the differential cross section of the $2^+_1$-state in $^{10}$Be is determined by subtracting the $1^-_1$ differential cross section from the 2600 keV one. The total ($2^+_2 + 1^-_1$) and the $2^+_2$ differential cross sections are shown as the last two plots in fig. 8.4. The differential cross sections will be shown along with the theoretic calculations in the next sections.
Figure 8.1: Excitation energy spectra made for small angular ranges, each one covering $3^\circ$ in center of mass. The simulated data (red) is scaled to best fit the experimental data (black). The scaling factor for each spectra provides the angular distribution.
8.3 Scattering data

An important part of DWBA calculations are the potential between the initial nucleus and particle ($V_{11\text{Be},d}$), section 8.4. This potential is normally determined from the elastic scattering data. The simplest approach is an optical model approach, which will be attempted first. The known deformation of $^{11}\text{Be}$ along with the small separation of the two bound states indicate a strong coupling of the two states. A deformed potential is tested as well. The deformed potential creates differential cross sections for both the ground and the first excited state.

8.3.1 Optical model calculation

The optical model method is described in section 3.3.1 and the calculations are performed with four different potentials shown in table 8.1. The potentials are taken from three papers, Satchler et al. [], Perey et al. [] and Kanungo et al.[]. The former two gives some generalised parameters for optical potentials derived from reactions of deuterons on stable nuclei. The latter provides two optical potentials used in a previous low energy $^{11}\text{Be}(d,p)^{12}\text{Be}$ experiment performed at TRIUMPH in 2005. The total optical potential is given by:

$$U(r) = -V_0 f(x_0) - i \left( W_c f(x_I) - W_d \frac{df(x_I)}{dx_I} \right) + V_{so} \frac{\hbar^2}{m_{cc} c^2} \frac{1}{r} \frac{df(x_{so})}{dx_{so}} (L \cdot s)$$

where $f(x)$ is the Wood-Saxon:

$$f(x) = \frac{1}{1 + \exp(x)}$$

$$x = \frac{r - r_i A_1}{a_i}.$$

Fig. 8.2 shows the experimental elastic scattering cross section (red dots) along with the four optical model calculations. None of the potentials can reproduce the experimental data and an attempt to adjust the potential depth of the real and the imaginary part to improve the overlap has been unsuccessful. The same was an attempt to use a large diffusiones for the imaginary part suggested by A. Bonnacorso et al.[]. The potential that reproduces the experimental data best is the Satchler potential and it will be used to make some first attempts to do DWBA calculations of the transfer reactions.
8.3. Scattering data

![Graph](https://example.com/graph.png)

**Figure 8.2:** The differential cross section for $^{11}\text{Be}(d,d)^{11}\text{Be}$. The experimental cross section (red dots) is plotted along with four different optical model calculations. The parameters are shown in table 8.1. Full line: Satchler et al. Dotted line: Perey-Perey et al. Dashed line: R. Kanungo et al. (set I). Dashed+dotted line: R. Kanungo et al. (set II).

The disagreement between theory and experiment could be caused by higher order reactions, which should be considered to improve the calculations. The first and only attempt made for these data is using a deformed potential, and will be described next. More complex reactions could also add to the differential cross section. Two contributions, that ought to be investigated is the compound and the exchange reactions. Compound reactions are known to be present in almost all direct reactions to some extent, and a significant contribution cannot be excluded for $^{11}\text{Be}(d,d)^{11}\text{Be}$. The exchange reaction is suggested because of the two loosely bound neutrons, the one in $^{11}\text{Be}$ and the one in the deuteron. Both nuclei can easily deliver a neutron to the other, and if they exchange a neutron it would look like a scattering reaction. No attempts to determine the contribution of these to reactions have been made.

### 8.3.2 Coupled channel calculation

A deformation of the optical potential is added in an attempt to improve the elastic cross section and to calculate a theoretic differential cross section for the inelastic...
### Table 8.1: Optical model parameters used to calculate the elastic scattering shown in fig. 8.2. All depths are in MeV and all distances in fm. The two reference in parentheses is the original references.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Satchler []</th>
<th>Perey []</th>
<th>Kanungo []</th>
<th>Kanungo []</th>
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<td>83.36</td>
<td>80.53</td>
<td>118.0</td>
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<td>0</td>
</tr>
<tr>
<td>$W_I$</td>
<td>4.38</td>
<td>15.744</td>
<td>4.71</td>
<td>5.80</td>
</tr>
<tr>
<td>$r_I$</td>
<td>2.452</td>
<td>1.34</td>
<td>1.56</td>
<td>1.57</td>
</tr>
<tr>
<td>$a_I$</td>
<td>0.264</td>
<td>0.68</td>
<td>0.8</td>
<td>0.78</td>
</tr>
<tr>
<td>$V_{so}$</td>
<td>6.0</td>
<td>0</td>
<td>3.54</td>
<td>5.80</td>
</tr>
<tr>
<td>$r_{so}$</td>
<td>0.9</td>
<td>0</td>
<td>1.23</td>
<td>0.87</td>
</tr>
<tr>
<td>$a_{so}$</td>
<td>0.9</td>
<td>0</td>
<td>0.81</td>
<td>0.91</td>
</tr>
</tbody>
</table>

scattering to the first excited state in $^{11}$Be. The deformation is set to:

$$\delta_2 = 0.84 \text{ fm} \quad \text{and} \quad \delta_4 = 1.27 \text{ fm}$$

taken from Hussein et al []. The deformation is added to the potential by Satchler et al. in a first attempt, full line in fig. 8.3. The elastic channel is not affected much and the inelastic channel is a factor 9 too low. The depth of the the reel and the imaginary potential is scaled to:

$$V = 120.18 \text{ MeV} \quad \text{and} \quad W = 19.535 \text{ MeV}$$

in order to improve the calculated cross section. While the shape of the elastic scattering differential cross section is improved at low angles, the agreement at large angles are worsened.
8.3. Scattering data

Figure 8.3: Differential cross sections for elastic (top) and inelastic scatterings (bottom). The experimental data (red dots) are compared to two deformed potentials, Satchler et al. (Full line) and a using modified depths for the Satchler potential (dashed line). The blue curves in the bottom are the theoretic curves scaled with a factor 9.

The inelastic scattering channel is still a factor of 9 off and the deformed potential has to be modified further. Like the Satchler potential in the optical model the
deformed potential with the modified depths will be used in the calculations of the transfer reactions.

### 8.4 Transfer reaction data

DWBA calculations are performed for both transfer reactions ((d,t) and (d,p)). The theory is described in section 3.3.3 and three potentials are needed to perform the integral in eqn. 3.35. The potentials involving deuterons are given in table 8.1 and in section 8.3.2. The potentials involving tritons and protons are listed in table 8.2 along with the papers they are taken from. The Be+p potential was derived from low energy scattering of protons on $^{12}$C. The potential was also used in the $^{11}$Be(d,p)$^{12}$Be experiment at TRIUMPH in combination with the last potential in table 8.1. The Be+t is derived for low energy scattering of $^3$He on $^9$Be. The shapes are given in eqn. 8.1-8.3.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Comfort [MeV]</th>
<th>Park [MeV]</th>
</tr>
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<tbody>
<tr>
<td>$V_0$</td>
<td>57.8</td>
<td>172.5</td>
</tr>
<tr>
<td>$r_0$</td>
<td>1.25</td>
<td>1.2</td>
</tr>
<tr>
<td>$a_0$</td>
<td>0.25</td>
<td>0.5</td>
</tr>
<tr>
<td>$W_d$</td>
<td>8.08</td>
<td>6.8</td>
</tr>
<tr>
<td>$r_I$</td>
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<tr>
<td>$a_I$</td>
<td>0.22</td>
<td>1.85</td>
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<tr>
<td>$V_{so}$</td>
<td>6.5</td>
<td>5.0</td>
</tr>
<tr>
<td>$r_{so}$</td>
<td>1.25</td>
<td>1.2</td>
</tr>
<tr>
<td>$a_{so}$</td>
<td>0.25</td>
<td>0.5</td>
</tr>
</tbody>
</table>

**Table 8.2:** Optical potentials used in the DWBA calculations for the (d,t) and (d,p) transfer reactions along with the optical potentials from table 8.1.

### 8.4.1 $^{11}$Be(d,t)$^{10}$Be

Two calculations are made for the (d,t) reaction. The core-core potential ($^{10}$Be+d) (Satchler et al) and the potential between the final nuclei ($^{10}$Be+t) (Park) will not be changed, but calculations using both the spherical (Satchler) and the deformed potential from section 8.3 for $^{11}$Be+d will be performed. The results are shown in fig. 8.4. The spectroscopic factors from the two calculations are given in table 8.3. None of the
models reproduce the experimental data very well, and only the ground state is reasonably well described. Especially the shape of the $2^+_2$-state cannot be reproduced. This may fall back on the experimental cross section. The inability to determine the $2^+_2$ cross section independently leads to a large uncertainty. The cross sections are too low for all the states in both calculations, leading to unrealistic high spectroscopic factors. Especially the deformed potential produces too low cross sections. Much more detailed calculations are required to provide more reliable results.

<table>
<thead>
<tr>
<th>State in $^{10}$Be</th>
<th>Spherical</th>
<th>Deformed</th>
</tr>
</thead>
<tbody>
<tr>
<td>$0^+_1$</td>
<td>0.45(5)</td>
<td>2</td>
</tr>
<tr>
<td>$2^+_1$</td>
<td>1.2(2)</td>
<td>4</td>
</tr>
<tr>
<td>$2^+_2$</td>
<td>1.7</td>
<td>6</td>
</tr>
<tr>
<td>$1^-_1$</td>
<td>2.5</td>
<td>4.5</td>
</tr>
<tr>
<td>$2^-_1$</td>
<td>1.4(5)</td>
<td>4.5</td>
</tr>
</tbody>
</table>

**Table 8.3:** Spectroscopic factors determined from $^{11}$Be(d,t)$^{10}$Be reactions. The results are only preliminary.
Figure 8.4: The experimental differential cross sections for (d,t) reactions (red dots) compared to two DWBA calculations, using a spherical (full line) potential and a deformed (dashed line). The $1^-_1 + 2^+_2$ cross section is the cross section determined from gamma gating on the 2600 keV line. The $2^+_2$ cross section is calculated by subtracting the $1^-_1$ from the $1^-_1 + 2^+_2$. 
8.4. Transfer reaction data

8.4.2 $^{11}\text{Be}(d,p)^{12}\text{Be}$

Three DWBA calculations are performed for the $^{11}\text{Be}(d,p)^{12}\text{Be}$ reactions. The spherical and the deformed potentials are tested. Furthermore calculations using potentials, taken from the previous $^{11}\text{Be}(d,p)^{12}\text{Be}$ [], are performed in order to test their calculations. Only angles in the very forward direction in CM were covered in the experiment at TRIUMPH and the data from this experiment can be used to validate the calculations at larger angles. The results are shown in fig. 8.5 and the spectroscopic factors are given in table 8.4 along with spectroscopic factors determined theoretically by Fortune et al [] and spectroscopic factors determined in the experiment at TRIUMPH.

Comparing the three DWBA calculations with the experimental data shows an almost perfect agreement for the ground state, especially the deformed potential reproduces the experimental data. The agreement worsens when going up in excitation energy. A very flat structure is seen in all the theoretic calculations and the experimental data for the $2^+_1$-state. The experimental data has a slightly steeper curve. Both the spherical and the deformed potential fails to reproduce the shape of the $0^+_2$ and the $1^-_1$, while the parameters taken from R. Kanungo et al almost perfectly reproduces the shape of $0^+_2$-state, though it still fails to describe the $1^-_1$-state. Once again higher order terms are expected to influence the population of the high lying states in $^{12}\text{Be}$.

<table>
<thead>
<tr>
<th>State in $^{12}\text{Be}$</th>
<th>Spherical</th>
<th>Deformed</th>
<th>Kanungo (R. Kanungo [])</th>
<th>TRIUMPH (H. Fortune [])</th>
<th>Theory</th>
</tr>
</thead>
<tbody>
<tr>
<td>$0^+_1$</td>
<td>0.15(2)</td>
<td>0.25(5)</td>
<td>0.15(3)</td>
<td>0.28$^{+0.03}_{-0.07}$</td>
<td>0.78</td>
</tr>
<tr>
<td>$2^+_1$</td>
<td>0.15(5)</td>
<td>0.30(1)</td>
<td>0.075(25)</td>
<td>0.1$^{+0.09}_{-0.07}$</td>
<td>0.52</td>
</tr>
<tr>
<td>$0^+_2$</td>
<td>0.25(10)</td>
<td>0.20(8)</td>
<td>0.25(8)</td>
<td>0.73$^{+0.27}_{-0.40}$</td>
<td>0.37</td>
</tr>
<tr>
<td>$1^-_1$</td>
<td>0.55(20)</td>
<td>0.50(20)</td>
<td>0.27(15)</td>
<td>$\approx 0.35$</td>
<td>–</td>
</tr>
</tbody>
</table>

Table 8.4: Spectroscopic factors determined from $^{11}\text{Be}(d,p)^{12}\text{Be}$ reactions using three different potential parameters. The results are only preliminary. The spectroscopic factors determined in the experiment at TRIUMPH are given along with theoretical calculated ones for comparison.

The spectroscopic factors determined using the same potentials as R. Kanungo et al are compared to the factors determined in the experiment at TRIUMPH. The factors are close to each other, except the $0^+_2$-state. The $0^+_2$ and the $2^+_1$-states were not separated in the old experiment, which lead to large uncertainties on the $0^+_2$-state. The value of 0.73 has later been questioned by Fortune et al []. The spectroscopic factors
give an indirect comparison of the differential cross sections determined in the two experiments, and the strong agreement indicates reliable experimental cross sections.
8.4. Transfer reaction data

Figure 8.5: The experimental differential cross sections for (d,p) reactions (red dots) compared to two DWBA calculations, using a spherical (full line) potential, a deformed potential (dashed line) and potentials taken from the old (d,p) experiment (dotted).
All three sets of spectroscopic factors determined in this experiments are in good agreement, except for the $2^+_2$-state, where the factor from the deformed potential is higher than the other two. The theoretic calculated spectroscopic factors on the other hand is far off, only the $0^+_2$-state is within a factor of 2 from the experimental data. A better agreement in the shape of the theoretic and experimental cross section is needed in order to fully disprove the theoretic spectroscopic factors though.

8.5 Summary

Experimental differential cross sections for all bound states, except one, in $^{10,11,12}$Be were determined. A comparison with the theoretic calculations was not successful, and only the ground state of $^{12}$Be could be successfully reproduced. Higher order reactions are expected to play a significant part in the reactions studied, and more detailed calculations are suggested.

The spectroscopic factors determined for the (d,t) reactions was unrealistic high and must be improved upon. The large disagreement between the two (d,t)-calculations further confirmed the unreliability of the calculations. The spectroscopic factors from the (d,p)-data, on the other hand, were internally in good agreement and close to previously experimental determined factors. The latest theoretic calculated spectroscopic factors did not agree with the experimental results and a further investigation is necessary.
While the primary reactions in the experiment were reactions of $^{11}\text{Be}$ on deuterons, the runs performed on regular polyethylene proved to have enough statistic for further direct reaction studies. The analysis of both (p,p) and (p,d) reactions seen in the runs of $^{11}\text{Be}$ on regular polyethylene are described in this chapter along with the differential cross sections and preliminary theoretic calculation.

### 9.1 Reactions on protons

During the experiment a few runs of $^{11}\text{Be}$ on a regular polyethylene target were performed, section ?. The runs were used to determine the background from reactions on protons in the deuterated polyethylene (chapter 7), but the data may also be used to study scattering and transfer reactions on protons. (p,d)-reactions have been used for years to study various nuclei including $^{10}\text{Be}$ []. $^{12}\text{Be}$ can not be reached using a proton target, but both $^{11}\text{Be}$ and $^{10}\text{Be}$ can be studied with these data.

### 9.2 The analysis of $^{11}\text{Be}+p$ data

The analysis of the reactions on protons is done exactly like the analysis of the reactions on deuterons described in the previous chapters, hence no detailed description will be given.

Both high energy deuterons and protons were identified in a $\Delta E - E$ plot and the excitation energies were calculated using eqn. 7.4. The excitation energy spectra are shown in fig. 9.1. Only the ground states of $^{11}\text{Be}$ and $^{10}\text{Be}$ are seen, hence only very limited inelastic scattering is possible. This is supported by the lack of a 320 keV peak in the gamma spectrum, fig. 9.2. None of the the excited states in $^{10}\text{Be}$ are populated and only the two ground state are studied.
Figure 9.1: Excitation energy spectra made from runs on regular polyethylene. Top: The excitation energy of $^{11}\text{Be}$ calculated from scattered proton. Bottom: The excitation energy of $^{10}\text{Be}$ from (p,d) reactions.
The analysis of $^{11}$Be+p data

The differential cross sections of $^{11}$Be(p,p)$^{11}$Be(gs) and $^{11}$Be(p,d)$^{10}$Be(gs) are calculated using the method described in section 8.2. An optical model calculation is done for the (p,p) scattering using a potential from J. R. Comfort et al. [1], seen in table 8.2. The optical model can once again not explain the experimental data. The disagreement between theory and experiment is even larger for (p,p) scattering than for (d,d). Higher order effects could play a significant part in the (p,p)-scattering. Especially compound reaction should be investigated further.
Figure 9.3: Experimentally determined differential cross section (red dots) for $^{11}\text{Be}(p,p)^{11}\text{Be}$ (top) and $^{11}\text{Be}(p,d)^{10}\text{Be}$ (bottom). The cross sections are compared to theoretical calculations using potentials from table 8.2 and 8.1.

Preliminary DWBA calculations of $^{11}\text{Be}(p,d)^{10}\text{Be}$ are performed using the spherical (Satchler) and deformed potentials, also used in chapter 8, for $^{10}\text{Be}+d$ and the potential from Comfort et al for Be+p. The shapes are much more consistent with the experimental data than the scattering events. Especially the deformed potential reproduces the experimental structure. The spectroscopic factors determined from
the DWBA calculations are:

\[
\text{Spherical: } S = 1.25(35) \quad (9.1) \\
\text{Deformed: } S = 1.15(15) \quad (9.2) \\
\text{Theory: } S = 0.87 \quad (9.3)
\]

The theoretic value is taken from ?? et al []. Again the spectroscopic factors are too high and the theoretic calculations of the cross sections have to be improved.
Further investigation of $^{12}\text{Be}$

The differential cross section and spectroscopical factors determined in the previous chapters were the main aim of the experiment, but a further study of the $(d,p)$ reaction led to interesting results. Results that will be presented in this chapter along with the analysis.

10.1 Lifetime measurement of the $0^+_2$ state in $^{12}\text{Be}$.

The lifetime of $0^+_2$ state has been determined using the time difference between the detection of the 511 keV gammas and the protons. The long lifetime of the state has already been seen in fig. 7.8. Fig. 10.1 shows the time difference spectrum gated on the energy interval in table 7.1. A decay is seen after the main peak and the data from $\Delta t = 200\,\text{ns}$ to $500\,\text{ns}$ are fitted to an exponential decay:

\[
N = N_0 \exp \left( -\frac{\Delta t - t_0}{\tau} \right).
\]

The fitted value for $\tau$ was:

\[
\tau = 357(22)\,\text{ns},
\]

close to and consistent with the previous measured value of $331(17)\,\text{ns}$ [?]. A better time resolution is needed to improve the lifetime measurement.
10.2 Investigation of a bound $0^{-}$ state in $^{12}$Be

A bound $0^{-}$-state was suggested in $^{12}$Be by C. Romero-Redondo et. al [?] from a three body calculation. The state has never been seen experimentally, but due to low resolutions in previous experiments it has not been disproved either. The strength and high resolution of the T-REX and MINIBALL have already been shown in chapter 7, where the $2^{+}_{1}$ and $0^{+}_{2}$ states were separated in the data even though the difference in excitation energy is 100keV. The setup should be able to detect and identify any bound $0^{-}$-state.

The excitation energy of the $0^{-}$ is only predicted with a large uncertainty in [?]. The state is above the $2^{+}_{1}$ ($E^{*} > 2100$ keV) and below the $1n$ threshold ($E^{*} = 3160$ keV). The decay channel and lifetime is dependent on the excitation energy of the $0^{-}_{1}$-state. If it is below the $1^{-}_{1}$-state it will decay to the $2^{+}_{1}$ through a M2 and it will be a long lived state ($\tau \approx 10^{-8}$ s). If it lies above the $1^{-}_{1}$ it will decay via a M1 transition to the $1^{-}_{1}$-state and it will have a short lifetime ($\tau \approx 10^{-11}$ s).

The population strength of a bound $0^{-}$ in a (d,p)-reaction should be comparable to the population strength of the $1^{-}_{1}$-state. The energies are similar, and both has
a $^{10}$Be+sp structure. The $1^-_1$ is strongly populated as shown in chapter 7, hence a significant amount of a $0^-$-state should be seen.

The investigation of a bound $0^-$-state is done in three steps. First by studying the interval $E^* \in [2800 \text{ keV}, 3160 \text{ keV}]$ looking for a gamma line between 100 keV and 500 keV in the gamma spectrum for $^{12}$Be, fig. 7.7. The short lifetime for the state above the $1^-_1$ would lead to a prompt doppler shifted gamma like the ones from the $2^+_1$ and the $1^-_1$ decay. No gamma peak is seen in this area even with the larger MINIBALL efficiency at those energies, ruling out a $0^-_1$-state above 2800 keV. Secondly the interval $E^* \in [2200 \text{ keV}, 2700 \text{ keV}]$ is studied. The identification of the long lived $0^+_2$ by looking at time delayed gammas shows the ability to identify long lived states in $^{12}$Be. A $0^-$-state between the $2^+_1$ and the $1^-_1$ states should produce a time delayed gamma line that can be seen in fig. 7.8 as a vertical line like the 511 keV. No other line below 600 keV is seen ruling out the second energy interval. This leaves the excitation energy of the $0^-_1$ to be within 100 keV above either the $2^+_1$ or the $1^-_1$ state. Gamma gates cannot be used to study these areas, due to the lower energy limit of the MINIBALL at 100 keV. Instead the excitation energy spectrum without gamma gates is used. The background and gamma gated spectra could not fully explain all detected protons corresponding to $E^* > 0 \text{ MeV}$, fig. 7.9. The gamma gated and background spectra are subtracted from the total to study this further, fig. 10.2. The ground state peak is still present as expected. A broad distribution ranging from 0.5 MeV to 2.8 MeV is also present. The distribution is too wide to be only one state and a part of it is expected to be background yet to be understood. The distribution is situated around 2.37 MeV and a bound $0^-_1$-state above the $1^-_1$ can be ruled out, leaving only the possible excitation energies of a bound $0^-_1$-state to be within the interval $E^* \in [2100 \text{ keV}, 2200 \text{ keV}]$. The mean value of the distribution contradicts this, but a better understanding of the background is needed to fully rule out a bound $0^-_1$-state in $^{12}$Be. The events in the broad distribution might also stem from protons populating the $0^+_2$-state in $^{12}$Be. The gamma gate for the $0^+_2$-state required a stopping of the $^{12}$Be nucleus within the reaction chamber and the extra time gate might also cut away true events. These effects were already mentioned in section 7.2.3.2 and a larger positive uncertainty should be assigned to the spectroscopic factors of the $0^+_2$-state in table 8.4.
Chapter 10. Further investigation of $^{12}\text{Be}$

Figure 10.2: The excitation energy spectrum left after subtraction of gamma gated and background spectra. The ground state is clearly seen, but also a significant distribution with excitation energy above 0 MeV.

A recent three body calculation has shown the possibility of a broad $0^-$-resonance between the 1n and the 2n threshold rather than a bound state \[?\]. This calculation is yet to be investigated, but might be by using the method described in the next section.

10.3 Investigation of low lying resonances

So far only bound states of $^{10,11,12}\text{Be}$ have been investigated, but also low lying resonances are populated in the experiment. Indications of the $5/2^-$-resonance in $^{11}\text{Be}$ were seen in fig. 7.16. The investigation of the resonances are complicated by the low energy and small angles of the outgoing particles. Fig. 10.3 shows the kinetic curves for the outgoing proton (red), deuteron (yellow) and triton (green) from reactions populating the lowest resonances in $^{12}\text{Be} (E^* = 4.5\text{MeV}), ^{11}\text{Be} (E^* = 1.8\text{MeV})$ and $^{10}\text{Be} (E^* = 7.3\text{MeV})$ respectively. The curves are plotted on top of a $\theta_{\text{lab}}$ vs. $E_{\text{par}}$ for low energy particles. Background and states identified from gamma-gates are subtracted from the plot. The elastic scattered events, already investigated, are clearly seen, but do not interfere with the resonances. The three curves are entangled, and furthermore $^{4,6}\text{He}, ^{7,8,9}\text{Li}$ and $^{10}\text{Be}$ will also be seen with similar energies and angles. A gate is needed to identify the three reaction types. Three attempts to make gates for the resonances is described in this section. All the attempts are based on the fact, that
the resonances decay through neutron emission. The first one has successfully been used on the lowest resonance in $^{12}\text{Be}$, and the analysis of this resonance will be fully described. Only some first attempts have been made for the last two, and a more detailed analysis is required to gain any information about resonances.

![Energy vs. angle for all low energy particles in the forward direction. The kinetic curves for (d,p) (red), (d,d) (yellow) and (d,t) (green) populating the lowest resonances in $^{12,11,10}\text{Be}$ are shown as well.](image)

**Figure 10.3:** The energy vs. angle for all low energy particles in the forward direction. The kinetic curves for (d,p) (red), (d,d) (yellow) and (d,t) (green) populating the lowest resonances in $^{12,11,10}\text{Be}$ are shown as well.

### 10.3.1 Gammas from decay products

The first method is to gate on gammas from the daughter nucleus. Hence the method requires neutron emission decay to an excited state in the daughter nucleus in order to get a subsequent gamma decay. Thus a study of the lowest resonances in $^{11}\text{Be}$ and $^{10}\text{Be}$ are impossible with this method, as they decay completely to the ground state of $^{10}\text{Be}$ and $^9\text{Be}$ respectively. The low lying $1/2^-$-state in $^{11}\text{Be}$ makes it energetically possible for the lowest resonance in $^{12}\text{Be}$ to decay to the excited state in $^{11}\text{Be}$, which will decay with a 320 keV gamma afterwards. The 320 keV is seen in coincidence with low energy particles, and has been used to identify the inelastic scattered deuterons, section 7.3.2.1. Fig. 10.4 shows the laboratory angle vs. the energy for the particles in coincidence with a 320 keV gamma. To curves are present in the plot. The strong curve around 60° is inelastic scattered deuterons, the wider and weaker curve at
angles between 10° and 40° is interpreted as protons from the d(\(^{11}\text{Be}, p\))\(^{12}\text{Be}\) reaction populating the lowest resonance in \(^{12}\text{Be}\). The red curve from fig. 10.3 fits perfectly with the experimental data.

Figure 10.4: The energy as a function of the laboratory angle for low energy particles in coincidence with a 320 keV gamma. The inelastic scattered events are clearly seen, but also a curve at smaller angles are present. The curve is expected to be events from a (d,p) reaction populating the lowest resonance in \(^{12}\text{Be}\).

The large separation between the deuterons and protons seen in the \(\theta_{\text{lab}}\) vs. \(E\) plot makes it possible to distinguished the deuterons from the protons in the analysis, as seen in fig. 7.15 (the large background in the spectrum, mentioned in section 7.3.2.1, are the protons). All the events are analysed as d(\(^{11}\text{Be}, p\))\(^{12}\text{Be}\) reactions and an excitation spectrum is shown in fig. 10.5. The low energy peak is the deuterons, which are ignored, but the peak centered at 4.5 MeV is the \(^{12}\text{Be}\) resonance. The width of the peak is determined by comparing the experimental data with simulations using various resonance widths. Three widths have been attempted: \(\Gamma = 0\) MeV (red), 200 MeV (green) and 500 MeV (yellow). The simulation with a delta function resonance is too narrow as expected. The offset in the mean value between the simulated and the experimental data, and the tail at 4 MeV, makes a precise measurement impossible. The best estimate is: \(\Gamma = 200(100)\) MeV, which is almost twice the accepted value of 107(17) MeV determined by Fortune et al [].
Figure 10.5: The excitation energy calculated assuming a (d,p) reaction for low energy particles in coincidence with a 320 keV. The low energy part is the inelastic scattered deuterons, and the peak at 4.5 MeV is the lowest known resonance in $^{12}$Be. The resonance is compared with three simulations. The different width of the resonance have been used in the simulations: red: $\Gamma = 0$ MeV, green: $\Gamma = 200$ MeV and yellow: $\Gamma = 500$ MeV.

The spin and parity of the resonance were initially predicted to be $2^+$ by Fortune et al. The strong population of the $1/2^-$ in $^{11}$Be through first a (d,p) population of the resonance and then a decay to the $1/2^-$-state, contradicts this prediction. The branching ratio of a decay from a $2^+$-resonance in $^{12}$Be to the $1/2^-$-state in $^{11}$Be has been estimated to less than 2% by E. Garrido et al. The branching ratio of the decay cannot be determined before a complete understanding of the last 25% of the low energy particles is established, but a lower limit can be given. The lower limit is determined by comparing the number of particles in the gamma gated spectrum (ignoring the scattered deuterons) with all unidentified particles within ±500 MeV of the red line in fig. 10.3. The lower limit is:

$$\text{BR}(\text{res}) \geq 10\%.$$  \hspace{1cm} (10.3)

This is a very conservative limit, but still higher than the 2% estimate for a $2^+$-resonance. A branching ratio of 10% would also lead to a differential cross section in the order of 50 mb/sr for the $^{11}$Be(d,p)$^{12}$Be(res) reaction. This is almost a factor of ten higher than all other (d,p)-cross sections and comparable to the elastic scattering cross section. A branching ratio above 50% seems more reliable.

New theoretical calculations predict the spin and parity of the resonance to be
either $1^{-}$ or $3^{-}$. None of these values can be confirmed or disproved with the present knowledge.

## 10.3.2 Neutron detection using Ge-detectors

The second method is to detect neutrons from the neutron emission decay. Detection of neutrons in studies of resonances is not a new technic, and has been used in various experiments. Normally dedicated neutron detectors, like plastic scintillators, are used. The setup for this experiment was not designed for studies of resonances, hence no neutron detectors were used.

Instead the MINIBALL Ge-detectors were used to indirectly detect the neutrons. The idea is to detect gammas from decays of excited germanium isotopes. The germanium isotopes are excited from inelastic scattering of neutrons ($\text{Ge}(n,n')\text{Ge}^*$). The method was already suggested by Chassman et al. in 1969. The three strongest gamma lines are shown in table 10.1, the fourth line in the table is from a long lived $0^+ \rightarrow 0^+$ transition in $^{72}\text{Ge}$, described in Jenkins et al. The last decay was already mentioned in chapter 7. The advantage of the last line is the long lifetime, which enables a time and energy gate, like the gate for the $0^+_{2}$-state in $^{12}\text{Be}$ made in section 7.2.3.2. The three first are much stronger, but unfortunately the energies are very close to gamma lines from fusion reactions of $^{11}\text{Be}$ on $^{12}\text{C}$, these lines are seen in table 10.1 as well. The gamma peaks is shown in fig. 10.6, all four peaks are visible, furthermore a very weak peak at 1210 keV ($\approx 2 \cdot 600$ keV) is seen indicating possibly detection of a two neutron emission. The long lifetime of the 690 keV gamma was shown in fig. 7.8.
10.3. Investigation of low lying resonances

<table>
<thead>
<tr>
<th>Reaction</th>
<th>$E_\gamma$ [keV]</th>
</tr>
</thead>
<tbody>
<tr>
<td>Ge(n,n')Ge</td>
<td>600</td>
</tr>
<tr>
<td></td>
<td>690</td>
</tr>
<tr>
<td></td>
<td>835</td>
</tr>
<tr>
<td></td>
<td>1035</td>
</tr>
<tr>
<td>$^{11}\text{Be} + ^{12}\text{C}$</td>
<td>637 ($^{22}\text{F}$)</td>
</tr>
<tr>
<td></td>
<td>821 ($^{21}\text{F}$)</td>
</tr>
<tr>
<td></td>
<td>980 ($^{6}\text{Li}$)</td>
</tr>
<tr>
<td>$^{22}\text{Ne} + d$</td>
<td>1017 ($^{23}\text{Ne}$)</td>
</tr>
</tbody>
</table>

Table 10.1: Table showing the four strongest gamma lines produced from inelastic neutron scattering on germanium isotopes. Some nearby gamma lines from $^{11}\text{Be} + ^{12}\text{C}$ and $^{22}\text{Ne} + d$ reactions are shown as well.

![Gamma energy spectrum](image_url)

Figure 10.6: The gamma energies detected in coincidence with a low energy particle. The energies are not corrected for doppler shift.

A gate has been made on all four peaks, both for reactions on deuterated polyethylene and on carbon. A $\theta_{lab}$ vs. $E$ plot is made with the carbon data subtracted, fig. 10.7. The statistic is low, and no clear curves are seen. An indication of an increased intensity following the yellow ($(d,d')$) line is seen. Population of the 1.8 MeV has already been indentified, but not studied, fig. 7.16. The large number of events still present at angles above 50 degrees indicates a significant background in the plot, which ruins
any clear identification of population of a resonance.

![Figure 10.7](image)

**Figure 10.7:** The energy vs. angle for low energy particles in the forward PSD’s in coincidence with an expected neutron. The background from reactions on carbon is subtracted. The kinetic curves for (d,p) (red), (d,d) (yellow) and (d,t) (green) populating the lowest resonances in $^{12}$Be, $^{11}$Be, $^{10}$Be are shown as well.

Furthermore, the neutron gate can not disentangle events from the three resonances, as they all decay by neutron emission. Gamma-gamma coincidences have been used in an attempt to disentangle the states. The lowest resonance of $^{12}$Be has already been proven to decay to the 1/2$^-$-state of $^{11}$Be, which produces a 320 keV gamma. First attempt is to make a gate requiring two gammas, a 320 keV and a neutron, fig. 10.8A. The second attempt is to look for two neutrons in coincidence coming from a two neutron decay of $^{12}$Be to the ground state of $^{10}$Be, fig. 10.8B. The resonances in $^{10}$Be and $^{11}$Be are both under the two neutron threshold, and only the $^{12}$Be resonance should decay with two-neutron emission. The statistic is extremely low for the particle-gamma-gamma coincidence events and a study of the 4.5 MeV-resonance in $^{12}$Be cannot be performed without improving the analysis.
10.3. Investigation of low lying resonances

Figure 10.8: The energy vs. angle for low energy particles in the forward PSD’s gated on gamma-gamma coincidence. Left: A coincidence of a neutron and a 320 keV gamma is required. Right: A coincidence of two neutrons is required. The red curve indicates the kinematics of the (d,p)-reaction populating the 4.5 MeV resonance in $^{12}$Be.

10.3.3 Coincidence events analysis.

Instead of detecting the neutron, which has been proven complicated, the heavy fragment can be detected. Coincidence events between tritons and $^{10}$Be have already been used in chapter 6 for beam diagnostic. This proves that coincidence events occurs in this experiment for reactions to the bound states of $^{10}$Be, and $^{11}$Be. The outgoing angle of the nuclei decreases with the excitation energy, and the resonances is at the limit of our angular range. The emission of a neutron should for some events lead to a larger outgoing angle of the heavy fragment, making it possible to have coincidence events for reactions populating resonances.

The reactions to the bound states and the reactions to resonances can be separated by looking at the missing energy in the latter. In a reaction to a bound state, like the ground state of $^{10}$Be, the two outgoing particles carries all the kinetic energy, while in a reaction populating a resonance some of the kinetic energy is taken by the undetected neutron. The two types of reactions can then be identified by calculating the excitation of the nucleus in two ways:

\[
E_1^* = T_A - T_b - T_B + Q \quad \text{(10.4)}
\]
\[
E_2^* = T_A - T_b - \frac{p_B^2}{2m_B} + Q \quad \text{(10.5)}
\]

The latter is the same as used in chapter 7 and provides the right excitation energy.
The former will give the excitation energy minus the kinetic energy of the neutron. The equations require knowledge of the reaction, hence the excitation energies are calculated assuming all three reactions ((d,t), (d,d) and (d,p)) individually. The two excitation energies are plotted against each other, fig. 10.9. If the two excitation energies are the same ($E_1 = E_2$) for an event this is a true event to a bound state. If $E_1 > E_2$, the particles stem from another reaction (mainly fusion reactions) Finally if $E_2 > E_1$ for an event, it is either another reaction or a population of a resonance. Unfortunately no events are found with $E_2 > E_1$, that cannot be explained by another reaction. Three clear peaks are seen in the plots. The lowest is yet to be understood. The middle is scattered deuterons and the top one is tritons from population of the 6 MeV states in $^{10}$Be.

This is only a quick first attempt to use this method and might be useful if improved upon.

![Figure 10.9](image.png)

**Figure 10.9:** The energy as a function of the laboratory angle for low energy particles in coincidence with a 320 keV gamma. The inelastic scattered events are clearly seen, but also a curve at smaller angles are present. The curve is expected to be events from a (d,p) reaction populating the lowest resonance in $^{12}$Be.

### 10.4 Summary

Studies of $^{12}$Be not related to the differential cross sections were described in this chapter. The experimental setup was not designed for these studies but the high statistic enabled it, though only results with large uncertainties could be provided.

The techniques used for the lifetime measurement and the study of the resonances should be applicable for other experiments and should provide as inspiration.
CHAPTER 11

Conclusion
## List of Figures

<table>
<thead>
<tr>
<th>Figure</th>
<th>Description</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.1</td>
<td>A picture of Sir Ernest Rutherford in his laboratory</td>
<td>1</td>
</tr>
<tr>
<td>1.2</td>
<td>A plot of the energy levels dependency on the deformation of the nuclei. The plot is derived using the model by S. G. Nilsson</td>
<td>4</td>
</tr>
<tr>
<td>1.3</td>
<td>The three shapes, prolate, spherical and oblate, of nuclei</td>
<td>5</td>
</tr>
<tr>
<td>2.1</td>
<td>A drawing of a $^{11}$Be nucleus indicating the halo structure, which leads to a deformed nucleus with a large matter radius</td>
<td>10</td>
</tr>
<tr>
<td>2.2</td>
<td>The bound states and the lowest resonances of $^{11}$Be. The neutron separation energy and the Q-value of the $\beta$-decay is indicated as well. The picture is taken from the TUNL datagroup</td>
<td>11</td>
</tr>
<tr>
<td>2.3</td>
<td>The bound states and the lowest resonances of $^{12}$Be. The one- and two-neutron separation energy and the Q-value of the $\beta$-decay is indicated as well. The picture is taken from the TUNL datagroup</td>
<td>12</td>
</tr>
<tr>
<td>2.4</td>
<td>The bound states and the lowest resonances of $^{10}$Be. The neutron separation energy and the Q-value of the $\beta$-decay is indicating as well. The picture is taken from the TUNL datagroup</td>
<td>13</td>
</tr>
<tr>
<td>3.1</td>
<td>Drawing of the three body model for the cluster model calculation, showing the four distances used</td>
<td>24</td>
</tr>
<tr>
<td>3.2</td>
<td>Drawing of the three body model for the transfer reaction. Left is the initial situation were the valence particle is connected to the blue core, and the right is the final situation were the valence particle is transfered to the red core</td>
<td>25</td>
</tr>
<tr>
<td>4.1</td>
<td>The ISOLDE facility</td>
<td>31</td>
</tr>
<tr>
<td>4.2</td>
<td>Diagram of the REX post accelerator. The low-energy beam from ISOLDE enters the penning trap, where it is bunched and sent to the EBIS. The EBIS strips the beam particles and send them through a $q/A$ separator to the LINAC. The LINAC consists of four different types of cavities, that can accelerate the beam up to 3 MeV/u</td>
<td>32</td>
</tr>
<tr>
<td>4.3</td>
<td>A picture of the MINIBALL setup. The eight germanium cluster is placed on an adjustable frame making it a versatile setup</td>
<td>34</td>
</tr>
<tr>
<td>4.4</td>
<td>A drawing of the T-REX setup including the laboratory frame described in section 5.2. The target is represented by a red dot. Half of the detectors on the top and left right side is omitted to give a view inside the detector. The beam is assumed to follow the $z$-direction</td>
<td>35</td>
</tr>
</tbody>
</table>
4.5 A MINIBALL cluster ........................................... 36

5.1 The histogram showing the address from an ADC. Each peak represents a strip in the top (400-700) or left AD (1000-1400). ............... 41

5.2 A linear fit to the four $\alpha$-energies for strip 0 in the top AD. .......... 42

5.3 Two plots showing the strong dependency between the position signal and the energy signal in the backward (A) and forward (B) PSD's. ... 43

5.4 (A) Two histograms of channel numbers made with two gates on the $\Delta E$-signal; Red: $\Delta E \approx 3$ MeV, Blue: $\Delta E \approx 2$ MeV. All three particles are seen at 3 MeV, only the two lightest are seen at 2 MeV. (B): A linear fit to all the identified peaks in one of the four forward PSD’s. ........... 44

5.5 Two gamma energy spectra made from the $d^{(22Ne,p)}^{23Ne}$ reaction. A peak close to 1016 keV, but slightly shifted, is seen in both spectra. The spectra are made from a detector placed before (red) and after (blue) the target. ................................................................. 45

5.6 Two gamma energy spectra made from the $d^{(22Ne,p)}^{23Ne}$ reaction. A peak close to 1016 keV, but slightly shifted, is seen in both spectra. The spectra are made from a detector placed before (red) and after (blue) the target. ................................................................. 47

5.7 The detection efficiency of the MINIBALL as a function of the gamma energy. The measured efficiencies are marked for the individual sources. The dotted line represents the best fit. ................................. 48

6.1 The outgoing angle of the heavy fragment ($\theta_B$) as a function of the outgoing angle of the light particle ($\theta_b$). The three lines in the plot represents: Blue: (d,t), Red: (d,d) and Black: (d,p). The grey area represent the area covered by the T-REX, the dotted line indicate the distinction between the AD’s and the PSD’s. ................................. 50

6.2 A plot of $x_A$ vs. $y_A$ for the four simulations in table 6.1. A-D refers to the simulation numbers. The shape of the peaks are caused by the setup, which does not cover $4\pi$. .................................................. 53

6.3 $xy$-plots for a uniform distribution (A+C) and a gaussian (B+D). A+B shows the 2D plot while C+D shows histograms of $x_A$. The difference in distribution is clearly seen. .................................................. 54

6.4 A plot of $\theta_A \cos \phi$ vs. $\theta_A \sin \phi$ for the four simulations in table 6.1. A-D refers to the simulation numbers. .................................................. 55

6.5 A plot of $x_A$ vs. $\theta_A \cos \phi$ for the four simulations in table 6.1. A-D refers to the simulation numbers. .................................................. 57

6.6 A plot of $x_A$ vs. $\theta_A \cos \phi$ for simulation C. (A) is from tritons hitting the left or right PSD. (B) is tritons hitting the top or bottom PSD. .... 58
6.7 Spectra of the incoming energy determined from energy conservation (left) and momentum conservation (right). The spectra are made from simulation B.

6.8 A $xy$-plot of the experimental data from the $d(^{11}\text{Be},t)^{10}\text{Be}$ reaction. The plot should be compared to the plots in fig. 6.2 and fig. 6.3.

6.9 A plot of $x_A$ vs. $\theta_A \cos \phi$ (A) and $y_A$ vs. $\theta_y$ (B) for the experimental data and for simulation using a beam with $w_A = 6$ mm, $s_A = -1.3$ mm, $\theta_A = 20$ mrad and $\phi_A = \pi/2$ rad (C) and (D). In order to avoid saturation in the scatter plot, only one third of the data is plotted.

6.10 The beam energy from the experimental data, determined from energy (left) and momentum (right) conservation.

6.11 The beam energy calculated from momentum conservation using the experimental data, and assuming a reaction in the beginning (green), in the middle (red) and in the end (blue) of the target.

6.12 The beam energy from the experimental data, determined from energy (left) and momentum (right) conservation.

6.13 Plots of the beam width at different times during the experiment. A: The beginning of the experiment with the intense beam. B: In the middle of the experiment at the time of large fluctuations in the intensity. C: In the end with a stable but low intensity beam.

6.14 $T_A$ calculated from energy (A+C) and momentum (B+D) conservation using coincidence events with identified tritons (A+B) and deuterons (C+D). The three spectra are made with data from three different targets; blue: $^{2}\text{CD}_2$, red: C and green $^{2}\text{CH}_2$.

7.1 The energy required for protons (black), deuterons (red) and tritons (blue) to punch through the $\Delta E$-detector (solid lines). The kinetic curves for the reactions populating the ground states of $^{12,11,10}\text{Be}$ is shown as well.

7.2 A plot of the $\Delta E$ vs. $E$ deposited in the strip 5 of the four forward PSD’s. The gates made to identify p, d and t are shown as red lines. $^4\text{He}$ particles are also seen, but omitted in the analysis.

7.3 Excitation energy spectra made from high energy particles. A: $^{12}\text{Be}$. B: $^{11}\text{Be}$. C: $^{10}\text{Be}$.

7.4 Excitation energy spectra made from high energy particles including the background from reactions on carbon (red line) and protons (green line) in the target. The grey area is the total background.
7.5 The gamma energy spectrum made from gammas in coincidence with a deuteron. The gammas are Doppler corrected. A strong sharp peak is eminent at 320 keV. .......................................................... 76

7.6 The total excitation energy spectra for $^{11}$Be (blue) along with the excitation energy spectrum for the excited state in $^{11}$Be made from gamma gated deuterons (red). .................................................. 77

7.7 The gamma energy spectrum made from gammas in coincidence with a proton. The gammas are Doppler corrected. A sharp peak at 2100 keV is seen, along with a small and broad peak at 2700 keV. ...................... 78

7.8 The gamma energy vs. the time between the detection of the proton and the detection of the gamma. .................................................. 79

7.9 Top: Excitation energy spectra from gamma gated protons. Black: $2^+_1$ (2107 keV), blue: $0^+_2$ (511 keV) and red: $1^-_1$ (2700 keV). All spectra are scaled to take the MINIBALL efficiency into account. Bottom: The total excitation energy spectrum (blue) and the excitation energy spectrum made from background runs and gamma gated spectra (red). .............. 80

7.10 A $xy$-plot of the experimental data from the d($^{11}$Be,t)$^{10}$Be reaction. The plot should be compared to the plots in fig. 6.2 and fig. 6.3 ............. 82

7.12 The excitation energy spectrum for low energy particles in the backward angles. The ground state of $^{12}$Be is clearly seen when the large background is subtracted. .................................................. 84

7.13 The gamma energy vs. the time between the detection of a low energy particle and the detection of the gamma. To avoid saturation in the scatter plot only 0.4 of the events are plotted. .............................. 86

7.14 Gamma energy spectrum for gammas in coincidence with low energy particles. The spectrum is divided into three, due to the large difference (15000 to 100) in scales from low energy gammas to high energy gammas. .................................................. 87

7.15 The Excitation energy spectra from gamma gated low energy particles. The particles are analysed as protons (top), deuterons (middle) and tritons (bottom). The colors correspond to the same as the in the excitation energy spectra from high energy particles. ...................... 89

7.16 Top: An excitation energy spectrum for $^{11}$Be assuming all low energy particles are scattered deuterons. Bottom: The same as the top one, but with background and reactions to excited states subtracted using gamma gates. .................................................. 90
7.11 Top: The excitation energy spectra for the excited states in $^{10}$Be made from the gamma gates shown in table 7.1. Black: $E_\gamma = 3300 \text{ keV}$, red: $E_\gamma = 2900 \text{ keV}$, blue: $E_\gamma = 2600 \text{ keV}$ and green: $E_\gamma = 6000 \text{ keV}$. Bottom: The total excitation energy spectrum (blue) and the excitation energy spectrum made from background runs and gamma gated spectra (red).

8.1 Excitation energy spectra made for small angular ranges, each one covering $3^\circ$ in center of mass. The simulated data (red) is scaled to best fit the experimental data (black). The scaling factor for each spectra provides the angular distribution.

8.2 The differential cross section for $^{11}$Be(d,d)$^{11}$Be. The experimental cross section (red dots) is plotted along with four different optical model calculations. The parameters are shown in table 8.1. Full line: Satchler et al. Dotted line: Perey-Perey et al. Dashed line: R. Kanungo et al. (set I). Dashed+dotted line: R. Kanungo et al. (set II).

8.3 Differential cross sections for elastic (top) and inelastic scatterings (bottom). The experimental data (red dots) are compared to two deformed potentials, Satchler et al. (Full line) and a using modified depths for the Satchler potential (dashed line). The blue curves in the bottom are the theoretic curves scaled with a factor 9.

8.4 The experimental differential cross sections for (d,t) reactions (red dots) compared to two DWBA calculations, using a spherical (full line) potential and a deformed (dashed line). The $1^- + 2^+_2$ cross section is the cross section determined from gamma gating on the 2600 keV line. The $2^+_2$ cross section is calculated by subtracting the $1^- + 2^+_2$.

8.5 The experimental differential cross sections for (d,p) reactions (red dots) compared to two DWBA calculations, using a spherical (full line) potential, a deformed potential (dashed line) and potentials taken from the old (d,p) experiment (dotted).

9.1 Excitation energy spectra made from runs on regular polyethylene. Top: The excitation energy of $^{11}$Be calculated from scattered proton. Bottom: The excitation energy of $^{10}$Be from (p,d) reactions.

9.2 Gamma energy spectrum from gammas in coincidence with protons.

9.3 Experimentally determined differential cross section (red dots) for $^{11}$Be(p,p)$^{11}$Be (top) and $^{11}$Be(p,d)$^{10}$Be (bottom). The cross sections are compared to theoretical calculations using potentials from table 8.2 and 8.1.
10.1 A lifetime measurement of the $0^+_2$ state in $^{12}$Be. A histogram of the time difference between the detected gamma and detected proton along with a fit to an exponential decay. ........................................ 114
10.2 The excitation energy spectrum left after subtraction of gamma gated and background spectra. The ground state is clearly seen, but also a significant distribution with excitation energy above 0 MeV. .............. 116
10.3 The energy vs. angle for all low energy particles in the forward direction. The kinetic curves for (d,p) (red), (d,d) (yellow) and (d,t) (green) populating the lowest resonances in $^{12,11,10}$Be are shown as well. .... 117
10.4 The energy as a function of the laboratory angle for low energy particles in coincidence with a 320 keV gamma. The inelastic scattered events are clearly seen, but also a curve at smaller angles are present. The curve is expected to be events from a (d,p) reaction populating the lowest resonance in $^{12}$Be. ................................................ 118
10.5 The excitation energy calculated assuming a (d,p) reaction for low energy particles in coincidence with a 320 keV. The low energy part is the inelastic scattered deuterons, and the peak at 4.5 MeV is the lowest known resonance in $^{12}$Be. The resonance is compared with three simulations. The Different width of the resonance have been used in the simulations: red: $\Gamma = 0$ MeV, green: $\Gamma = 200$ MeV and yellow: $\Gamma = 500$ MeV ................................................................. 119
10.6 The gamma energies detected in coincidence with a low energy particle. The energies are not corrected for doppler shift. ................. 121
10.7 The energy vs. angle for low energy particles in the forward PSD’s in coincidence with an expected neutron. The background from reactions on carbon is subtracted. The kinetic curves for (d,p) (red), (d,d) (yellow) and (d,t) (green) populating the lowest resonances in $^{12,11,10}$Be are shown as well. .................. 122
10.8 The energy vs. angle for low energy particles in the forward PSD’s gated on gamma-gamma coincidence. Left: A coincidence of a neutron and a 320 keV gamma is required. Right: A coincidence of two neutrons is required. The red curve indicates the kinematics of the (d,p)-reaction populating the 4.5 MeV resonance in $^{12}$Be. ............... 123
10.9 The energy as a function of the laboratory angle for low energy particles in coincidence with a 320 keV gamma. The inelastic scattered events are clearly seen, but also a curve at smaller angles are present. The curve is expected to be events from a (d,p) reaction populating the lowest resonance in $^{12}$Be. ..................... 124
List of Tables

2.1 Gamma lines ................................................................. 15
4.1 Targets used in the experiment ................................. 33
4.2 Detector types and thickness for the silicon detectors in T-REX .... 36
5.1 Gamma sources ............................................................. 46
5.2 MINIBALL efficiencies .................................................. 48
6.1 The beam structure for the four Geant4 simulation used to check the
   technique. All simulations are made with a uniform distribution on a
   disc in the xy-plane. w is the diameter, s is the offset, θ is the incoming
   angle and Δ the divergence of the beam. E∗ is the excitation of 10Be. .. 52
6.2 Table of the mean values from a gaussian fit to the four peaks shown
   in fig. 6.10 including the statistical uncertainty. The determined values
   are calculated into beam energies at the reaction point (T_{\text{reac}}) before
   the beam energy is determined by correcting for the energy loss in the
   target (T_{\text{in}}). Only uncertainties from the gaussian fit is given in the table. 65
6.3 Table of the sigma of a gaussian fit to the four peaks shown in fig. 6.10.
   The spread in the beam energies is determined from σ. ............... 67
6.4 Beam intensities ............................................................ 67
7.1 Gamma gates for 12Be ...................................................... 83
8.1 Optical model parameters used to calculate the elastic scattering shown
   in fig. 8.2. All depths are in MeV and all distances in fm. The two ref-
   erence in parentheses is the original references. ......................... 98
8.2 Optical potentials used in the DWBA calculations for the (d,t) and (d,p)
   transfer reactions along with the optical potentials from table 8.1. .. 100
8.3 Spectroscopic factors determined from 11Be(d,t)10Be reactions. The re-
   sults are only preliminary. .................................................. 101
8.4 Spectroscopic factors determined from 11Be(d,p)12Be reactions using
   three different potential parameters. The results are only preliminary.
   The spectroscopic factors determined in the experiment at TRIUMPH
   are given along with theoretical calculated ones for comparison. .... 103
10.1 Table showing the four strongest gamma lines produced from inelastic neutron scattering on germanium isotopes. Some nearby gamma lines from $^{11}\text{Be}+^{12}\text{C}$ and $^{22}\text{Ne}+\text{d}$ reactions are shown as well. . . . . . . . 121